1. Show That

(a) $\Delta\left(\frac{u(x)}{v(x)}\right) = \frac{v(x)\Delta u(x) - u(x)\Delta v(x)}{v(x+h)v(x)}$.

(b) $\sum_{i=0}^{n-1} \Delta y_i = y_n - y_0$.

(c) $\sum_{i=0}^{n-1} u_i \Delta v_i = u_n v_n - u_0 v_0 - \sum_{i=0}^{n-1} v_{i+1} \Delta u_i$.

(d) $y_k = \sum_{i=0}^{k} \left(\begin{array}{c} k \\ i \end{array}\right) \Delta^i y_0$.

(e) $\nabla = \delta E^{-\frac{1}{2}} = 1 - E^{-1} = 1 - (1+D)^{-1}$.

(f) $\Delta^n f(x_0) = h^n f^{(n)}(x_0)$.

(g) $\Delta(\alpha u_i + \beta v_i) = \alpha \Delta u_i + \beta \Delta v_i$.

(h) $\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}}$.

(i) $\nabla = 1 - E^{-1}$.

(j) $E\Delta = \Delta E$

(k) $E\nabla = \nabla E = \Delta$.

2. Given the following pairs of values of x and y

x	1	2	4	8	10
y	0	1	5	21	27

Determine the value y at $x = 0.4$, use Divided difference interpolation polynomial.

3. Use mathematical induction to prove that $f[x_0, x_1, \cdots, x_n] = \frac{\Delta^n f(x_0)}{n! h^n}$.

4. Show that: If a function $g(x)$ interpolates the function $f(x)$ at $x_1, x_2, \ldots, x_{n-1}$, and $h(x)$ interpolates the function $f(x)$ at $x_2, x_3, \ldots, x_n$, then, $T(x) = g(x) + \frac{(x_1 - x)}{(x_n - x_1)}[g(x) - h(x)]$; Interpolate $f(x)$ at x_1, $x_2, \ldots$, x_{n-1}, x_n.

5. If the following values (a, s), $(a+h, t)$, $(a+2h, u)$ and $(a+3h, v)$ are obtained from polynomial of degree two. Prove that $f(a + 1.5h) = \frac{(t+u)}{2} + \frac{1}{24}[\frac{3}{2}(t + u - s - v)]$ by using Newton forward difference interpolation formula.

6. Use the definition of central difference operator to show that $\Delta^n y_k = \delta^n{}_{k+\frac{n}{2}}$.

7. Use the divided difference method to obtain a polynomial of least degree that fits the following values:

x	1	0	3	-1	5
y	1	-1	8	3	1

8. From the following values:

x	0	0.5	1	1.5	2
y	-1	-2	1	2	3

Find $f(0.6)$ by using Newton forward difference interpolation formula and Bessel interpolation formula.

9. If we interpolate the function $f(x) = e^{x-1}$ with a polynomial p of degree 12 using 13 nodes in $[-1, 1]$, what is a good upper bound for $|f(x) - p(x)|$ on $[-1, 1]$?

10. Compute a divided difference table for these function values: (3,1), (1,-3), (5,2) and (6,4) and also find $f(1.2)$ and $f(5.5)$ by using divided difference interpolation formula.

11. Suppose we know the values of $cos(x)$ at $x = -h$, $x = 0$, $x = h$, $(h > 0)$, write the interpolation polynomial $P(x)$ which interpolates $cos(x)$ at these points. Prove the error bound $|E_n(x)| = |\cos(x) - P(x)| \leq 0.065h^3$,for all $x \in [-h, h]$. Determine h such that the previous interpolation gives 4 exact decimals for any $x \in [-h, h]$.

12. Determine the maximum step size that can be used in tabular of $f(x) = e^x$ in $[0, 1]$, so that the error in the linear interpolation will be less than 5×10^{-4}. Find also the step size if quadratic interpolation is used.

13. Find the unique polynomial of degree 2 or less such that $P(1) = 1$, $P(3) = 27$, and $P(4) = 64$ by using each of the following methods: (i) Lagrange interpolating formula and (ii) Divided difference interpolating formula.

14. Calculate the n^{th} divided difference of $f(x) = \frac{1}{x}$.

15. If $f(x) = U(x)V(x)$, show that $f[x_0, x_1] = U[x_0]V[x_0, x_1] + U[x_0, x_1]V[x_1]$.

16. Use the Lagrange interpolation polynomial and Divided difference interpolation formula to estimate $f(3)$ from the following values:

x	0	1	2	3	5	6
$f(x)$	1	14	15	5	6	19

17. Find $\frac{dy}{dx}$ at $x = 0.6$ of the function $y = f(x)$ where

x	0.4	0.5	0.6	0.7	0.8
$f(x)$	1. 5836494	1. 7974426	2 .0442376	2.3275054	2. 6510818

18. Given the following pairs of values of x and y.

x	1	2	4	8	10
$f(x)$	0	1	5	21	27

Determine the first derivative at $x = 0.4$, use divided difference interpolation.

19. Find the first and second derivative at $x = 0.6$ for the following data: (0.4, 1.5836), (0.5,1.7974), (0.6,2.0442), (0.7, 2.3275) and (0.8, 2.6511).

20. A rod is moving in a plane, the following table given the angle θ in radian through which the rod has turned for various values of t seconds.

t	0	0.2	0.4	0.6	0.8	1	1.2
θ	0	0.12	0.49	1.12	2.02	3.2	4.67

Calculate the angular velocity=$\frac{d\theta}{dt}$ and angular accuracy=$\frac{d^2\theta}{dt^2}$ of the rod, when $x = 0.6$.

1. Find the linear lest-squares solution for the following table of values:

x	4	7	11	13	17
y	2	0	2	6	7

2. By using the method of least-squares, find the constant function that best fits the following data:

x	-1	2	3
y	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{5}{12}$

3. Find an equation of the form $y = ae^{x^2} + bx^3$ that best fits the points $(-1, 0)$, $(0, 1)$ and $(1, 2)$ in the least-squares sense.

4. Find the equation of a parabola of form $y = ax^2 + b$ that best represents the following data. Use the method of least squares.

x	-1	0	1
y	3.1	0.9	2.9

5. What straight line best fits the following data

x	1	2	3	4
y	0	1	1	2

in the least-squares sense?

6. What constant c makes the expression $\sum_{k=0}^{n}[f(x_k) - ce^{x_k}]^2$ as small as possible?

7. Find an equation of the form $y = (1 + b\,e^{ax})$, $y = a\,x + b\,x^2$, $y = a\,x + \frac{b}{\sqrt{x}}$, $y = a\,e^{-3x} + b\,e^{-2x}$, $y = \frac{b}{x+a}$, $y = \frac{b}{x(x-a)}$ and $y = a + bxy$ that best fits the points (1,4), (2,6), (3,8) and (4,9) in the least-squares sense.

8. Find the power fits $y = Ax^2$, $y = Bx^3$, $y = \frac{C}{x}$ and $y = \frac{D}{x^2}$ for the following data and use the least-squares error to determine which curve fits best.

x	2	2.3	2.6	2.9	3.2
y	3	3.4	3.8	5	5.2