Salahaddin University-Erbil

College of Basic Education

[Block Matrix]

Research Project

Submitted to the department of (mathematics) in partial fulfillment of the requirements for the degree of Baccalaureus in Mathematics

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Signature

This research project has been written under my supervision and has been

Submitted for the award of the degree of Baccalaureus in mathematic.

Signature

Name: Mr Ghazi

Date: / /

I confirm that all requirments have been completed.

Signature:

Name:

Head of the Department of ……

Date: / /

Dedication

In the name of Allah, Most Merciful, Most Compassionate.

We dedicated this humble work

To our supervisor Mr.Ghazi

And to our beloved Department, Mathematic Department

In Salahaddin University, College of Basic Education

Acknowledgements

Find we would like to thank Allah for giving us strength and energy to complete this work.

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Abstract

This project presents block matrix and some operation as an addition and multiplication on it. We will use some technique or formula to find determinant and inverse of block matrix.

Keywords:

Block matrix; Inverse of matrix; Determinant

Introduction

This research is about block matrix and consist of two chapters. In chapter one we talk about basic information of matrix, operation on it, finding determinant, and finally investigate inverse of matrix.

Chapter two is about block matrix, then we will investigate some operation on it, finding determinant, and how we can find inverse of block matrix with some basic formula.

Chapter 1

1.1 Matrix

A matrix is a rectangular array of numbers, a matrix with m rows and n columns is called an $m \times n$ matrix .

Example:

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 3 \end{pmatrix}$$

1.2 Types of matrix

1.2.1 Square matrix:

If a matrix A has n rows and n columns, then we say it's a square matrix.

Example 1:

$$A=\begin{pmatrix} 1 & 2 & 4 \\ 8 & 3 & 2 \\ 7 & 9 & 5 \end{pmatrix}$$

1.2.2 The identity matrix:

A squrae matrix A whose elements $a_{ij} = 1$ for $i = j$, $a_{ij} = 0$ for $i > j$ is called upper triangular, A square matrix A whose elements $a_{ij} = 0$ for $i < j$ is called lower triangular, thus:

$I_n=\begin{pmatrix} 1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 1 \end{pmatrix}$ which is identity matrix with n rows and n colomns.

The matrix $I_2=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Chapter two

2.1 Block matrix: - A block matrix (partitioned matrix) is a matrix that is defined using smaller matrices, called blocks.

For **example**

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

Where A, B, C and D are themselves matrices, in the specific example

$$P=\begin{pmatrix} 0 & 2 & 3 & 3 & 3 \\ 2 & 0 & 3 & 3 & 3 \\ 4 & 4 & 5 & 0 & 5 \\ 4 & 4 & 0 & 5 & 0 \\ 4 & 4 & 5 & 0 & 5 \end{pmatrix}$$

$$A=\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} \quad ,B=\begin{pmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \end{pmatrix} \quad ,C=\begin{pmatrix} 4 & 4 \\ 4 & 4 \\ 4 & 4 \end{pmatrix} \quad ,D=\begin{pmatrix} 5 & 0 & 5 \\ 0 & 5 & 0 \\ 5 & 0 & 5 \end{pmatrix}$$

When two block matrices have to the same shape and their diagonal blocks are. Square matrices then. They multiply similarity to matrix multiplication for example

2.2 Property of addition

2.2.1 A+B=B+A

$$A = \begin{pmatrix} 1 & 3 & 5 & 4 & 2 \\ 0 & 2 & 3 & 1 & 2 \\ 4 & 3 & 2 & 0 & 5 \\ 0 & 2 & 4 & 1 & 2 \\ 1 & 3 & 2 & 0 & 3 \end{pmatrix} , B = \begin{pmatrix} 2 & -3 & -2 & 4 & 1 \\ -2 & 1 & 0 & 5 & 6 \\ 2 & 0 & 1 & -1 & -2 \\ 3 & -1 & 3 & 2 & 1 \\ 0 & 4 & 2 & 1 & 6 \end{pmatrix}$$

$$A=\begin{pmatrix} a1 & b1 \\ c1 & d1 \end{pmatrix} \quad ,B=\begin{pmatrix} a2 & b2 \\ c2 & d2 \end{pmatrix}$$

$a_1 = \begin{pmatrix} 1 & 3 & 5 \\ 0 & 2 & 3 \\ 4 & 3 & 2 \end{pmatrix}$ $b_1 = \begin{pmatrix} 4 & 2 \\ 1 & 2 \\ 0 & 5 \end{pmatrix}$ $c_1 = \begin{pmatrix} 0 & 2 & 4 \\ 1 & 3 & 2 \end{pmatrix}$ $d_1 = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$

$a_2 = \begin{pmatrix} 2 & -3 & -2 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$ $b_2 = \begin{pmatrix} 4 & 1 \\ 5 & 6 \\ -1 & -2 \end{pmatrix}$ $c_2 = \begin{pmatrix} 3 & -1 & 3 \\ 0 & 4 & 2 \end{pmatrix}$ $d_2 =$

$\begin{pmatrix} 2 & 1 \\ 1 & 6 \end{pmatrix}$

A +B = $\begin{pmatrix} a_1 + a_2 & b_1 + b_2 \\ c_1 + c_2 & d_1 + d_2 \end{pmatrix}$ => B+A = $\begin{pmatrix} a_2 + a_1 & b_2 + b_1 \\ c_2 + c_1 & d_2 + d_1 \end{pmatrix}$

$$\begin{pmatrix} 3 & 0 & 3 & 8 & 3 \\ -2 & 3 & 3 & 6 & 8 \\ 6 & 3 & 3 & -1 & 3 \\ 3 & 1 & 7 & 3 & 3 \\ 1 & 7 & 4 & 1 & 9 \end{pmatrix} = \begin{pmatrix} 3 & 0 & 3 & 8 & 3 \\ -2 & 3 & 3 & 6 & 8 \\ 6 & 3 & 3 & -1 & 3 \\ 3 & 1 & 7 & 3 & 3 \\ 1 & 7 & 4 & 1 & 9 \end{pmatrix}$$

2.2.2 A+0 = 0+A=A

$$A = \begin{pmatrix} 1 & 3 & 5 & 4 & 2 \\ 0 & 2 & 3 & 1 & 2 \\ 4 & 3 & 2 & 0 & 5 \\ 0 & 2 & 4 & 1 & 2 \\ 1 & 3 & 2 & 0 & 3 \end{pmatrix}$$

$$0 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ => A+0= $\begin{pmatrix} a+0 & b+0 \\ c+0 & d+0 \end{pmatrix}$ = 0+A=$\begin{pmatrix} 0+a & 0+b \\ 0+c & 0+d \end{pmatrix}$

$$=A = A = \begin{pmatrix} 1 & 3 & 5 & 4 & 2 \\ 0 & 2 & 3 & 1 & 2 \\ 4 & 3 & 2 & 0 & 5 \\ 0 & 2 & 4 & 1 & 2 \\ 1 & 3 & 2 & 0 & 3 \end{pmatrix}$$

A = $\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$, B = $\begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$

A*B = $\begin{pmatrix} a_2 + b_{1.}c_2 & a_{1.}b_2 + b_{1.}d_2 \\ c_{1.}\ a_2 + d_1.c_2 & c_{1.}b_{2+}\ d_{1.}d_2 \end{pmatrix}$

B*A = $\begin{pmatrix} a_{2.}a_1 + c_2b_1 & b_{2.}a_1 + d_{2.}b_1 \\ a_{2.}\ c_1 + c_2.d_1 & b_{2.}c_{1+}\ d_{2.}d_1 \end{pmatrix}$

A.B≠B.A

For **example**/ A = $\begin{pmatrix} 1 & 3 & 5 & 4 & 2 \\ 0 & 2 & 3 & 1 & 2 \\ 4 & 3 & 2 & 0 & 5 \\ 0 & 2 & 4 & 1 & 2 \\ 1 & 3 & 2 & 0 & 3 \end{pmatrix}$, B = $\begin{pmatrix} 2 & -3 & -2 & 4 & 1 \\ -2 & 1 & 0 & 5 & 6 \\ 2 & 0 & 1 & -1 & -2 \\ 3 & -1 & 3 & 2 & 1 \\ 0 & 4 & 2 & 1 & 6 \end{pmatrix}$

$a_1 = \begin{pmatrix} 1 & 3 & 5 \\ 0 & 2 & 3 \\ 4 & 3 & 2 \end{pmatrix}$ $b_1 = \begin{pmatrix} 4 & 2 \\ 1 & 2 \\ 0 & 5 \end{pmatrix}$ $c_1 = \begin{pmatrix} 0 & 2 & 4 \\ 1 & 3 & 2 \end{pmatrix}$ $d_1 = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$

$a_2 = \begin{pmatrix} 2 & -3 & -2 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$ $b_2 = \begin{pmatrix} 4 & 1 \\ 5 & 6 \\ -1 & -2 \end{pmatrix}$ $c_2 = \begin{pmatrix} 3 & -1 & 3 \\ 0 & 4 & 2 \end{pmatrix}$ $d_2 =$

$\begin{pmatrix} 2 & 1 \\ 1 & 6 \end{pmatrix}$

A*B = $\begin{pmatrix} a_{1.}a_2 + b_{1.}c_2 & a_{1.}b_2 + b_{1.}d_2 \\ c_{1.}\ a_2 + d_1.c_2 & c_{1.}b_{2+}\ d_{1.}d_2 \end{pmatrix}$ ≠ B*A

=$\begin{pmatrix} a_{2.}a_1 + c_2b_1 & b_{2.}a_1 + d_{2.}b_1 \\ a_{2.}\ c_1 + c_2.d_1 & b_{2.}c_{1+}\ d_{2.}d_1 \end{pmatrix} = \begin{pmatrix} 18 & 4 & 19 & 24 & 25 \\ 5 & 9 & 10 & 11 & 19 \\ 6 & 11 & 4 & 34 & 48 \\ 7 & 9 & 11 & 10 & 17 \\ 0 & 12 & 6 & 20 & 33 \end{pmatrix}$ ≠

$\begin{pmatrix} -5 & 5 & 15 & 9 & -1 \\ 4 & 24 & 25 & -2 & 26 \\ 4 & 1 & 4 & 7 & 1 \\ 16 & 23 & 28 & 13 & 26 \\ 14 & 34 & 32 & 5 & 38 \end{pmatrix}$

2.2 Transpose of block matrix:

To find transpose of block matrix M we use the following rule

$$M = \begin{pmatrix} A & B \\ C & \mathrm{D} \end{pmatrix}$$

$$M^T = \begin{pmatrix} A^T & C^T \\ B^T & D^T \end{pmatrix}$$

Example: 2.2

Let $M = \begin{pmatrix} 1 & 3 & 5 & 4 \\ 0 & 2 & 3 & 1 \\ 4 & 3 & 2 & 0 \\ 0 & 2 & 4 & 1 \end{pmatrix}$

$$M = \begin{pmatrix} A & B \\ C & \mathrm{D} \end{pmatrix}$$

A=$\begin{pmatrix} 1 & 3 \\ 0 & 2 \end{pmatrix}$, B=$\begin{pmatrix} 5 & 4 \\ 3 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 4 & 3 \\ 0 & 2 \end{pmatrix}$, $\mathrm{D} = \begin{pmatrix} 2 & 0 \\ 4 & 1 \end{pmatrix}$

A^T=$\begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}$, B^T=$\begin{pmatrix} 5 & 3 \\ 4 & 1 \end{pmatrix}$, $C^T = \begin{pmatrix} 4 & 0 \\ 3 & 2 \end{pmatrix}$, $\mathrm{D}^T = \begin{pmatrix} 2 & 4 \\ 0 & 1 \end{pmatrix}$

A=A^T , B=C^T , $C = B^T$, $\mathrm{D} = \mathrm{D}^T$

$$M^T = \begin{pmatrix} A^T & C^T \\ B^T & \mathrm{D}^T \end{pmatrix}$$

$$M^T = \begin{pmatrix} 1 & 0 & 5 & 3 \\ 3 & 2 & 4 & 1 \\ 4 & 0 & 2 & 4 \\ 3 & 2 & 0 & 1 \end{pmatrix}$$

2.1 Lemma:

Let M be any matrix, then $(M^T)^T = M$

Example: 2.2.1

Let $M = \begin{pmatrix} 1 & 3 & 5 & 4 \\ 0 & 2 & 3 & 1 \\ 4 & 3 & 2 & 0 \\ 0 & 2 & 4 & 1 \end{pmatrix} \Rightarrow M^T = \begin{pmatrix} 1 & 0 & 5 & 3 \\ 3 & 2 & 4 & 1 \\ 4 & 0 & 2 & 4 \\ 3 & 2 & 0 & 1 \end{pmatrix}$

$A^T = \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}$, $B^T = \begin{pmatrix} 5 & 3 \\ 4 & 1 \end{pmatrix}$, $C^T = \begin{pmatrix} 4 & 0 \\ 3 & 2 \end{pmatrix}$, $\mathrm{D}^T = \begin{pmatrix} 2 & 4 \\ 0 & 1 \end{pmatrix}$

$(A^T)^T = \begin{pmatrix} 1 & 3 \\ 0 & 2 \end{pmatrix}$, $(B^T)^T = \begin{pmatrix} 5 & 4 \\ 3 & 1 \end{pmatrix}$, $(C^T)^T = \begin{pmatrix} 4 & 3 \\ 0 & 2 \end{pmatrix}$, $(D^T)^T = \begin{pmatrix} 2 & 0 \\ 4 & 1 \end{pmatrix}$

$(M^T)^T = \begin{pmatrix} 1 & 3 & 5 & 4 \\ 0 & 2 & 3 & 1 \\ 4 & 3 & 2 & 0 \\ 0 & 2 & 4 & 1 \end{pmatrix} = \mathrm{M}$

2.3 The determinant

The determinant can be expressed by the use of

C1=A-B*D^{-1}*C

C2=D-C*A^{-1}B

AS

Det $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$=det (D)*det($c_1$) = det(A)*det ($c_2$)

Example/

$$M = \begin{pmatrix} 1 & 3 & 5 & 4 & 2 \\ 0 & 2 & 3 & 1 & 2 \\ 4 & 3 & 2 & 0 & 5 \\ 0 & 2 & 4 & 1 & 2 \\ 1 & 3 & 2 & 0 & 3 \end{pmatrix} \quad A = \begin{pmatrix} 1 & 3 & 5 \\ 0 & 2 & 3 \\ 4 & 3 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 4 & 2 \\ 1 & 2 \\ 0 & 5 \end{pmatrix} \quad C = \begin{pmatrix} 0 & 2 & 4 \\ 1 & 3 & 2 \end{pmatrix}$$

$$D = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix} \quad D^{-1} = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & \frac{-2}{3} \\ 0 & \frac{1}{3} \end{pmatrix}$$

$$C1 = A - BD^{-1}C \rightarrow C1 = \begin{pmatrix} 3 & 1 & -7 \\ 0 & 0 & -1 \\ \frac{7}{3} & -2 & -\frac{4}{3} \end{pmatrix}$$

$$\text{Det} \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \det(D) * \det(c_1) = \det(A) * \det(c_2)$$

$$C2 = D - C * A^{-1} B \rightarrow C2 = \begin{pmatrix} -\frac{23}{9} & \frac{4}{3} \\ \frac{11}{3} & -3 \end{pmatrix}$$

$$\text{Det} \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \det(D) * \det(c_1) = \det(A) * \det(c_2)$$

$$= 3 * \frac{-25}{3} = -9 * \frac{25}{9} \Rightarrow -25 = -25$$

2.3.1 Property of determinate

1) $|A^T| = |A|$

Example /

$$M = \begin{pmatrix} 1 & 3 & 5 & 4 & 2 \\ 0 & 2 & 3 & 1 & 2 \\ 4 & 3 & 2 & 0 & 5 \\ 0 & 2 & 4 & 1 & 2 \\ 1 & 3 & 2 & 0 & 3 \end{pmatrix}$$

$C1 = A - B\, D^{-1} C$

C2=D-C*A^{-1}B

Det $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$=det (D)*det($c_1$) = det(A)*det ($c_2$)

A = $\begin{pmatrix} 1 & 3 & 5 \\ 0 & 2 & 3 \\ 4 & 3 & 2 \end{pmatrix}$ B = $\begin{pmatrix} 4 & 2 \\ 1 & 2 \\ 0 & 5 \end{pmatrix}$ C = $\begin{pmatrix} 0 & 2 & 4 \\ 1 & 3 & 2 \end{pmatrix}$ D = $\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$

$D^{-1} = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & \frac{-2}{3} \\ 0 & \frac{1}{3} \end{pmatrix}$,$A^{-1} = \begin{pmatrix} 1 & \frac{-7}{4} & \frac{1}{4} \\ 0 & \frac{-1}{4} & \frac{3}{4} \\ 0 & \frac{1}{2} & \frac{-1}{2} \end{pmatrix}$, C1 =

$\begin{pmatrix} 3 & 1 & -7 \\ 0 & 0 & -1 \\ \frac{7}{3} & -2 & -\frac{4}{3} \end{pmatrix}$

Det (M) = -25

$M^T = \begin{pmatrix} 1 & 0 & 4 & 0 & 1 \\ 3 & 2 & 3 & 2 & 3 \\ 5 & 3 & 2 & 4 & 2 \\ 4 & 1 & 0 & 1 & 0 \\ 2 & 2 & 5 & 2 & 3 \end{pmatrix}$

$A^T = \begin{pmatrix} 1 & 0 & 4 \\ 3 & 2 & 3 \\ 5 & 3 & 2 \end{pmatrix}$ $B^T = \begin{pmatrix} 0 & 1 \\ 2 & 3 \\ 4 & 2 \end{pmatrix}$ $C^T = \begin{pmatrix} 4 & 1 & 0 \\ 2 & -2 & 5 \end{pmatrix}$ $D^T = \begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}$

$(D^T)^{-1} = \begin{pmatrix} 1 & 0 \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix}$, $(A^T)^{-1} = \begin{pmatrix} \frac{5}{9} & -\frac{4}{3} & \frac{8}{9} \\ -1 & 2 & -1 \\ \frac{1}{9} & \frac{1}{3} & -\frac{2}{9} \end{pmatrix}$, C1= $\begin{pmatrix} 3 & \frac{4}{3} & \frac{7}{3} \\ 1 & 4 & -2 \\ -7 & \frac{5}{3} & -\frac{4}{3} \end{pmatrix}$

det(M^T) = -25 = det (M) =-25

2) $|A\ B| = |A||B|$

$$A = \begin{pmatrix} 1 & 3 & 5 & 4 & 2 \\ 0 & 2 & 3 & 1 & 2 \\ 4 & 3 & 2 & 0 & 5 \\ 0 & 2 & 4 & 1 & 2 \\ 1 & 3 & 2 & 0 & 3 \end{pmatrix}$$

C1 = A -B D^{-1}C

C2=D-C*A^{-1}B

Det $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$=det (D)*det ($c_1$) = det (A)*det ($c_2$)

$$e=\begin{pmatrix} 1 & 3 & 5 \\ 0 & 2 & 3 \\ 4 & 3 & 2 \end{pmatrix} \quad f = \begin{pmatrix} 4 & 2 \\ 1 & 2 \\ 0 & 5 \end{pmatrix} \quad j = \begin{pmatrix} 0 & 2 & 4 \\ 1 & 3 & 2 \end{pmatrix} \quad w = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$$

$$w^{-1} = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & \frac{-2}{3} \\ 0 & \frac{1}{3} \end{pmatrix}, \text{ c1}=\begin{pmatrix} 3 & 1 & -7 \\ 0 & 0 & -1 \\ \frac{7}{3} & -2 & -\frac{4}{3} \end{pmatrix}$$

Det(A) = -25

$$B = \begin{pmatrix} 2 & -3 & -2 & 4 & 1 \\ -2 & 1 & 0 & 5 & 6 \\ 2 & 0 & 1 & -1 & -2 \\ 3 & -1 & 3 & 2 & 1 \\ 0 & 4 & 2 & 1 & 6 \end{pmatrix}$$

$$x=\begin{pmatrix} 2 & -3 & -2 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix} \quad y = \begin{pmatrix} 4 & 1 \\ 5 & 6 \\ -1 & -2 \end{pmatrix} \quad z=\begin{pmatrix} 3 & -1 & 3 \\ 0 & 4 & 2 \end{pmatrix} \quad u = \begin{pmatrix} 2 & 1 \\ 1 & 6 \end{pmatrix}$$

$$u^{-1}=\begin{pmatrix} 2 & 1 \\ 1 & 6 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{6}{11} & \frac{-1}{11} \\ \frac{-1}{11} & \frac{2}{11} \end{pmatrix}, \quad c_1=\begin{pmatrix} \frac{-47}{11} & -\frac{2}{11} & -\frac{87}{11} \\ -\frac{94}{11} & \frac{7}{11} & \frac{-86}{11} \\ \frac{34}{11} & \frac{8}{11} & \frac{29}{11} \end{pmatrix}$$

Det (B) = 369

Det(A) *Det(B)= -25* 369 =-9225

$$A*B = \begin{pmatrix} 18 & 4 & 19 & 24 & 25 \\ 5 & 9 & 10 & 11 & 19 \\ 6 & 11 & 4 & 34 & 48 \\ 7 & 9 & 11 & 10 & 17 \\ 0 & 12 & 6 & 20 & 33 \end{pmatrix}$$

$$P=\begin{pmatrix} 18 & 4 & 19 \\ 5 & 9 & 10 \\ 6 & 11 & 4 \end{pmatrix} \quad Q = \begin{pmatrix} 24 & 25 \\ 11 & 19 \\ 34 & 48 \end{pmatrix} \quad R = \begin{pmatrix} 7 & 9 & 11 \\ 0 & 12 & 6 \end{pmatrix} \quad S = \begin{pmatrix} 10 & 17 \\ 20 & 33 \end{pmatrix}$$

$$S^{-1}=\begin{pmatrix} \frac{-33}{10} & \frac{17}{10} \\ 2 & -1 \end{pmatrix}, \quad C1 = \begin{pmatrix} \frac{1112}{5} & \frac{386}{5} & \frac{1227}{5} \\ \frac{-69}{10} & \frac{-27}{10} & \frac{-69}{10} \\ \frac{597}{5} & \frac{196}{5} & \frac{617}{5} \end{pmatrix}$$

Det (A*B) = -9225 = det (A) = -25 * det (B) = 369

3)$|A|$ = 0 if it has two equal lines

Let $M = \begin{pmatrix} 1 & 3 & 5 & 4 & 2 \\ 1 & 3 & 5 & 4 & 2 \\ 4 & 3 & 2 & 0 & 5 \\ 0 & 2 & 3 & 1 & 2 \\ 1 & 3 & 2 & 0 & 3 \end{pmatrix}$ So, to find determinant of A we begin with this rule det $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$= det (A)*det ($c_2$) where $C_2 = D - C * A^{-1}B$

$$A=\begin{pmatrix} 1 & 3 & 5 \\ 1 & 3 & 5 \\ 4 & 3 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 4 & 2 \\ 4 & 2 \\ 0 & 5 \end{pmatrix} \quad C = \begin{pmatrix} 0 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} \quad D = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$$

$$C2 = \begin{pmatrix} -\frac{23}{9} & \frac{4}{3} \\ \frac{11}{3} & -3 \end{pmatrix}$$

det(A)=0 , det(C2) =- $\frac{113}{9}$

So, det $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$= det(A)*det ($c_2$)=0

1) Triangular determent is the product of the diagonal elements

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 4 & 3 & 2 & 0 & 0 \\ 1 & 2 & 4 & 1 & 0 \\ 1 & 3 & 2 & 0 & 3 \end{pmatrix}$$

A= $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 4 & 3 & 2 \end{pmatrix}$ B = $\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$ C = $\begin{pmatrix} 1 & 2 & 4 \\ 1 & 3 & 2 \end{pmatrix}$ D = $\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$

D^{-1} = $\begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{pmatrix}$, C1 = A -B D^{-1}C , C1= $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 4 & 3 & 2 \end{pmatrix}$

Det $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$=det (D)*det($C1$)

Det (d) = 3 * det (c1) = 4 = 12

Matrix inversion in block form:

Let a $(m + n \times m + n)$ matrix M be partitioned into a block from

M=$\begin{pmatrix} A & B \\ C & D \end{pmatrix}$

Where a $m \times m$ matrix A and a $n \times n$ matrix D are invertible then we have

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} X & Y \\ Z & U \end{pmatrix}=\begin{pmatrix} I_m & 0 \\ 0 & I_n \end{pmatrix} \rightarrow$$

$$\begin{pmatrix} X & Y \\ Z & U \end{pmatrix}=\begin{pmatrix} (A-B*D^{-1}*C)^{-1} & -A^{-1}*B(D-C*A^{-1}*B)^{-1} \\ -D^{-1}*C(A-B*D^{-1}*C)^{-1} & (D-C*A^{-1}*B)^{-1} \end{pmatrix}$$

$$\begin{pmatrix} X & Y \\ Z & U \end{pmatrix}\begin{pmatrix} A & B \\ C & D \end{pmatrix}=\begin{pmatrix} I_m & 0 \\ 0 & I_n \end{pmatrix} \rightarrow \begin{pmatrix} X & Y \\ Z & U \end{pmatrix}$$

$$=\begin{pmatrix} (A-B*D^{-1}*C)^{-1} & -(A-B*D^{-1}*C)^{-1}*B*D^{-1} \\ -(D-C*A^{-1}*B)^{-1}*C*A^{-1} & (D-C*A^{-1}*B)^{-1} \end{pmatrix}$$

Example:let

$$M=\begin{pmatrix} 1 & 3 & 5 & 4 & 2 \\ 0 & 2 & 3 & 1 & 2 \\ 4 & 3 & 2 & 0 & 5 \\ 0 & 2 & 4 & 1 & 2 \\ 1 & 3 & 2 & 0 & 3 \end{pmatrix}$$

$$A=\begin{pmatrix} 1 & 3 & 5 \\ 0 & 2 & 3 \\ 4 & 3 & 2 \end{pmatrix} \quad B=\begin{pmatrix} 4 & 2 \\ 1 & 2 \\ 0 & 5 \end{pmatrix} \quad C=\begin{pmatrix} 0 & 2 & 4 \\ 1 & 3 & 2 \end{pmatrix} \quad D=\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$$

$$A^{-1}=\begin{pmatrix} 1 & \frac{-7}{4} & \frac{1}{4} \\ 0 & \frac{-1}{4} & \frac{3}{4} \\ 0 & \frac{1}{2} & \frac{-1}{2} \end{pmatrix}$$

$$D^{-1} = \begin{pmatrix} 1 & \frac{-2}{3} \\ 0 & \frac{1}{3} \end{pmatrix}$$

$$X=(A - B * D^{-1} * C)^{-1}$$

$$\left(\begin{pmatrix} 1 & 3 & 5 \\ 0 & 2 & 3 \\ 4 & 3 & 2 \end{pmatrix} - \begin{pmatrix} 4 & 2 \\ 1 & 2 \\ 0 & 5 \end{pmatrix} * \begin{pmatrix} 1 & \frac{-2}{3} \\ 0 & \frac{1}{3} \end{pmatrix} * \begin{pmatrix} 0 & 2 & 4 \\ 1 & 3 & 2 \end{pmatrix} \right)^{-1}$$

$$B * D^{-1} = \begin{pmatrix} 4 & 2 \\ 1 & 2 \\ 0 & 5 \end{pmatrix} * \begin{pmatrix} 1 & \frac{-2}{3} \\ 0 & \frac{1}{3} \end{pmatrix}^{-1} = \begin{pmatrix} 4 & -2 \\ 1 & 0 \\ 0 & \frac{5}{3} \end{pmatrix}$$

$$F=\begin{pmatrix} 4 & -2 \\ 1 & 0 \\ 0 & \frac{5}{3} \end{pmatrix}$$

$$F*C=\begin{pmatrix} 4 & -2 \\ 1 & 0 \\ 0 & \frac{5}{3} \end{pmatrix} \begin{pmatrix} 0 & 2 & 4 \\ 1 & 3 & 2 \end{pmatrix} = \begin{pmatrix} -2 & 2 & 12 \\ 0 & 2 & 4 \\ \frac{5}{3} & 5 & \frac{10}{3} \end{pmatrix}$$

$$\left(\begin{pmatrix} 1 & 3 & 5 \\ 0 & 2 & 3 \\ 4 & 3 & 2 \end{pmatrix} - \begin{pmatrix} -2 & 2 & 12 \\ 0 & 2 & 4 \\ \frac{5}{3} & 5 & \frac{10}{3} \end{pmatrix} \right)^{-1} = \begin{pmatrix} 3 & 1 & -7 \\ 0 & 0 & -1 \\ \frac{7}{3} & -2 & \frac{-4}{3} \end{pmatrix}^{-1} = \begin{pmatrix} \frac{6}{25} & \frac{-46}{25} & \frac{3}{25} \\ \frac{7}{25} & \frac{-37}{25} & \frac{-9}{25} \\ 0 & -1 & 0 \end{pmatrix}$$

Y=$-A^{-1} * B(D - C * A^{-1} * B)^{-1}$

F=A^{-1}*B

$$F=\begin{pmatrix} \frac{-11}{9} & \frac{1}{3} \\ \frac{10}{3} & -3 \\ \frac{-23}{9} & \frac{4}{3} \end{pmatrix}$$

W=$C * A^{-1} * B$

$$W=\begin{pmatrix} \frac{-32}{9} & \frac{2}{3} \\ \frac{-11}{9} & 6 \end{pmatrix}$$

$$Y=\begin{pmatrix} \frac{-11}{9} & \frac{1}{3} \\ \frac{10}{3} & -3 \\ \frac{-23}{9} & \frac{4}{3} \end{pmatrix}\left(\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix} - \begin{pmatrix} \frac{-32}{9} & \frac{2}{3} \\ \frac{-11}{9} & 6 \end{pmatrix}\right)^{-1} = \begin{pmatrix} \frac{22}{25} & \frac{7}{25} \\ \frac{9}{25} & \frac{29}{25} \\ 1 & 0 \end{pmatrix}$$

Z=$-D^{-1} * C(A - B * D^{-1} * C)^{-1}$

$$N==-D^{-1} * C=\begin{pmatrix} \frac{2}{3} & 0 & \frac{-8}{3} \\ \frac{-1}{3} & -1 & \frac{-2}{3} \end{pmatrix}$$

$$K= B * D^{-1} = \begin{pmatrix} 4 & -2 \\ 1 & 0 \\ 0 & \frac{5}{3} \end{pmatrix}$$

$$H=\begin{pmatrix} 4 & -2 \\ 1 & 0 \\ 0 & \frac{5}{3} \end{pmatrix} * \begin{pmatrix} 0 & 2 & 4 \\ 1 & 3 & 2 \end{pmatrix} = \begin{pmatrix} -2 & 2 & 12 \\ 0 & 2 & 4 \\ \frac{5}{3} & 5 & \frac{10}{3} \end{pmatrix} \left(\begin{pmatrix} 3 & 1 & -7 \\ 0 & 0 & -1 \\ \frac{7}{3} & -2 & \frac{-4}{3} \end{pmatrix}\right)^{-1}$$

$$R=\begin{pmatrix} 1 & 3 & 5 \\ 0 & 2 & 3 \\ 4 & 3 & 2 \end{pmatrix} - \begin{pmatrix} -2 & 2 & 12 \\ 0 & 2 & 4 \\ \frac{5}{3} & 5 & \frac{10}{3} \end{pmatrix} = \begin{pmatrix} 3 & 1 & -7 \\ 0 & 0 & -1 \\ \frac{7}{3} & -2 & \frac{-4}{3} \end{pmatrix}$$

$$R=\left(\begin{pmatrix} 3 & 1 & -7 \\ 0 & 0 & -1 \\ \frac{7}{3} & -2 & \frac{-4}{3} \end{pmatrix}\right)^{-1} = \begin{pmatrix} \frac{6}{25} & \frac{-46}{25} & \frac{3}{25} \\ \frac{7}{25} & \frac{-37}{25} & \frac{-9}{25} \\ 0 & -1 & 0 \end{pmatrix}$$

$$Z=\begin{pmatrix} \frac{2}{3} & 0 & \frac{-8}{3} \\ \frac{-1}{3} & -1 & \frac{-2}{3} \end{pmatrix} * \begin{pmatrix} \frac{6}{25} & \frac{-46}{25} & \frac{3}{25} \\ \frac{7}{25} & \frac{-37}{25} & \frac{-9}{25} \\ 0 & -1 & 0 \end{pmatrix} = \begin{pmatrix} \frac{4}{25} & \frac{36}{25} & \frac{2}{25} \\ \frac{-9}{25} & \frac{69}{25} & \frac{8}{25} \end{pmatrix}$$

U=$(D - C * A^{-1} * B)^{-1}$

N=$C * A^{-1}$

$$N= \begin{pmatrix} 0 & 2 & 4 \\ 1 & 3 & 2 \end{pmatrix} * \begin{pmatrix} 1 & \frac{-7}{4} & \frac{1}{4} \\ 0 & \frac{-1}{4} & \frac{3}{4} \\ 0 & \frac{1}{2} & \frac{-1}{2} \end{pmatrix} = \begin{pmatrix} \frac{8}{9} & 0 & \frac{-2}{9} \\ \frac{-5}{3} & 3 & \frac{2}{3} \end{pmatrix}$$

$$W=N*B=\begin{pmatrix} \frac{8}{9} & 0 & \frac{-2}{9} \\ \frac{-5}{3} & 3 & \frac{2}{3} \end{pmatrix} * \begin{pmatrix} 4 & 2 \\ 1 & 2 \\ 0 & 5 \end{pmatrix} = \begin{pmatrix} \frac{32}{9} & \frac{2}{3} \\ \frac{-11}{9} & 6 \end{pmatrix}$$

$$U=(D - W)^{-1}=\left(\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix} - \begin{pmatrix} \frac{32}{9} & \frac{2}{3} \\ \frac{-11}{9} & 6 \end{pmatrix}\right)^{-1} = \begin{pmatrix} \frac{-27}{25} & \frac{-12}{25} \\ \frac{-33}{25} & \frac{-23}{25} \end{pmatrix}$$

$$\begin{pmatrix} \frac{6}{25} & \frac{-46}{25} & \frac{3}{25} \\ \frac{7}{25} & \frac{-37}{25} & \frac{-9}{25} \\ 0 & -1 & 0 \end{pmatrix}$$

$$M^{-1} = \begin{pmatrix} \frac{6}{25} & \frac{-46}{25} & \frac{3}{25} & \frac{22}{25} & \frac{7}{25} \\ \frac{7}{25} & \frac{-37}{25} & \frac{-9}{25} & \frac{9}{25} & \frac{29}{25} \\ 0 & -1 & 0 & 1 & 0 \\ \frac{4}{25} & \frac{36}{25} & \frac{2}{25} & \frac{-27}{25} & \frac{-12}{25} \\ \frac{-9}{25} & \frac{69}{25} & \frac{8}{25} & \frac{-33}{25} & \frac{-23}{25} \end{pmatrix}$$

M*M^{-1}=$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

$M * M^{-1} = I$

For the other side

$M^{-1}M = I$

$$M^{-1} = \begin{pmatrix} \frac{6}{25} & \frac{-46}{25} & \frac{3}{25} & \frac{22}{25} & \frac{7}{25} \\ \frac{7}{25} & \frac{-37}{25} & \frac{-9}{25} & \frac{9}{25} & \frac{29}{25} \\ 0 & -1 & 0 & 1 & 0 \\ \frac{4}{25} & \frac{36}{25} & \frac{2}{25} & \frac{-27}{25} & \frac{-12}{25} \\ \frac{-9}{25} & \frac{69}{25} & \frac{8}{25} & \frac{-33}{25} & \frac{-23}{25} \end{pmatrix}$$

A=$\begin{pmatrix} \frac{6}{25} & \frac{-46}{25} & \frac{3}{25} \\ \frac{7}{25} & \frac{-37}{25} & \frac{-9}{25} \\ 0 & -1 & 0 \end{pmatrix}$ B= $\begin{pmatrix} \frac{22}{25} & \frac{7}{25} \\ \frac{9}{25} & \frac{29}{25} \\ 1 & 0 \end{pmatrix}$ C= $\begin{pmatrix} \frac{4}{25} & \frac{36}{25} & \frac{2}{25} \\ \frac{-9}{25} & \frac{69}{25} & \frac{8}{25} \end{pmatrix}$ D= $\begin{pmatrix} \frac{-27}{25} & \frac{-12}{25} \\ \frac{-33}{25} & \frac{-23}{25} \end{pmatrix}$

A^{-1}=$\begin{pmatrix} \frac{6}{25} & \frac{-46}{25} & \frac{3}{25} \\ \frac{7}{25} & \frac{-37}{25} & \frac{-9}{25} \\ 0 & -1 & 0 \end{pmatrix}$ $= \begin{pmatrix} 3 & 1 & -7 \\ 0 & 0 & -1 \\ \frac{7}{3} & -2 & \frac{-4}{3} \end{pmatrix}$

D^{-1}=$\begin{pmatrix} \frac{-27}{25} & \frac{-12}{25} \\ \frac{-33}{25} & \frac{-23}{25} \end{pmatrix}$ $= \begin{pmatrix} \frac{-23}{9} & \frac{4}{3} \\ \frac{11}{3} & -3 \end{pmatrix}$

X=$(A - B * D^{-1} * C)^{-1}$

F=B*D^{-1}=$\begin{pmatrix} \frac{22}{25} & \frac{7}{25} \\ \frac{9}{25} & \frac{29}{25} \\ 1 & 0 \end{pmatrix}$ $*\begin{pmatrix} \frac{-23}{9} & \frac{4}{3} \\ \frac{11}{3} & -3 \end{pmatrix}$=$\begin{pmatrix} \frac{-11}{9} & \frac{1}{3} \\ \frac{10}{3} & -3 \\ \frac{-23}{9} & \frac{4}{3} \end{pmatrix}$

H=$F * C = \begin{pmatrix} \frac{-23}{9} & \frac{4}{3} \\ \frac{11}{3} & -3 \end{pmatrix} * \begin{pmatrix} \frac{4}{25} & \frac{36}{25} & \frac{2}{25} \\ \frac{-9}{25} & \frac{69}{25} & \frac{8}{25} \end{pmatrix}$=$\begin{pmatrix} \frac{71}{225} & \frac{-21}{25} & \frac{2}{225} \\ \frac{121}{75} & \frac{-87}{25} & \frac{-52}{75} \\ \frac{-8}{9} & 0 & \frac{2}{9} \end{pmatrix}$

$$X=(A-H)^{-1}=\left(\begin{pmatrix}\frac{6}{25} & \frac{-46}{25} & \frac{3}{25}\\ \frac{7}{25} & \frac{-37}{25} & \frac{-9}{25}\\ 0 & -1 & 0\end{pmatrix}-\begin{pmatrix}\frac{71}{225} & \frac{-21}{25} & \frac{2}{225}\\ \frac{121}{75} & \frac{-87}{25} & \frac{-52}{75}\\ \frac{-8}{9} & 0 & \frac{2}{9}\end{pmatrix}\right)^{-1}=$$

$$\begin{pmatrix}\frac{5}{9} & -1 & \frac{1}{9}\\ \frac{-4}{3} & 2 & \frac{1}{3}\\ \frac{8}{9} & 0 & \frac{-2}{9}\end{pmatrix}^{-1}=\begin{pmatrix}1 & 3 & 5\\ 0 & 2 & 3\\ 4 & 3 & 2\end{pmatrix}$$

$$Y=-(A-B*D^{-1}*C)^{-1}*B*D^{-1}$$

$$J=B*D^{-1}=\begin{pmatrix}\frac{22}{25} & \frac{7}{25}\\ \frac{9}{25} & \frac{29}{25}\\ 1 & 0\end{pmatrix}*\begin{pmatrix}\frac{-23}{9} & \frac{4}{3}\\ \frac{11}{3} & -3\end{pmatrix}=\begin{pmatrix}\frac{-11}{9} & \frac{1}{3}\\ \frac{9}{25} & -3\\ \frac{-23}{9} & \frac{4}{3}\end{pmatrix}$$

$$F=J*C=\begin{pmatrix}\frac{-11}{9} & \frac{1}{3}\\ \frac{9}{25} & -3\\ \frac{-23}{9} & \frac{4}{3}\end{pmatrix}*\begin{pmatrix}\frac{4}{25} & \frac{36}{25} & \frac{2}{25}\\ \frac{-9}{25} & \frac{69}{25} & \frac{8}{25}\end{pmatrix}=\begin{pmatrix}\frac{-71}{225} & \frac{-21}{25} & \frac{2}{225}\\ \frac{121}{75} & \frac{-87}{25} & \frac{-52}{75}\\ \frac{-8}{9} & 0 & \frac{2}{9}\end{pmatrix}$$

$$N=-(A-F)^{-1}=-\left(\begin{pmatrix}\frac{6}{25} & \frac{-46}{25} & \frac{3}{25}\\ \frac{7}{25} & \frac{-37}{25} & \frac{-9}{25}\\ 0 & -1 & 0\end{pmatrix} - \begin{pmatrix}\frac{-71}{225} & \frac{-21}{25} & \frac{2}{225}\\ \frac{121}{75} & \frac{-87}{25} & \frac{-52}{75}\\ \frac{-8}{9} & 0 & \frac{2}{9}\end{pmatrix}\right)^{-1}=\begin{pmatrix}-1 & -3 & -5\\ 0 & -2 & -3\\ -4 & -3 & -2\end{pmatrix}$$

$$H=B*D^{-1}=\begin{pmatrix}\frac{22}{25} & \frac{7}{25}\\ \frac{9}{25} & \frac{29}{25}\\ 1 & 0\end{pmatrix}*\begin{pmatrix}\frac{-23}{9} & \frac{4}{3}\\ \frac{11}{3} & -3\end{pmatrix}=\begin{pmatrix}\frac{-11}{9} & \frac{1}{3}\\ \frac{9}{25} & -3\\ \frac{-23}{9} & \frac{4}{3}\end{pmatrix}$$

$$Y=N*H=\begin{pmatrix}-1 & -3 & -5\\ 0 & -2 & -3\\ -4 & -3 & -2\end{pmatrix}*\begin{pmatrix}\frac{-11}{9} & \frac{1}{3}\\ \frac{9}{25} & -3\\ \frac{-23}{9} & \frac{4}{3}\end{pmatrix}=\begin{pmatrix}4 & 2\\ 1 & 2\\ 0 & 5\end{pmatrix}$$

$$Z=-(D-C*A^{-1}*B)^{-1}*C*A^{-1}$$

$$I=C*A^{-1}=\begin{pmatrix}\frac{4}{25} & \frac{36}{25} & \frac{2}{25}\\ \frac{-9}{25} & \frac{69}{25} & \frac{8}{25}\end{pmatrix}*\begin{pmatrix}3 & 1 & -7\\ 0 & 0 & -1\\ \frac{7}{3} & -2 & \frac{-4}{3}\end{pmatrix}=\begin{pmatrix}\frac{2}{3} & 0 & \frac{-8}{3}\\ \frac{-1}{3} & -1 & \frac{-2}{3}\end{pmatrix}$$

$$G=C*A^{-1}=\begin{pmatrix}\frac{4}{25} & \frac{36}{25} & \frac{2}{25}\\ \frac{-9}{25} & \frac{69}{25} & \frac{8}{25}\end{pmatrix}*\begin{pmatrix}3 & 1 & -7\\ 0 & 0 & -1\\ \frac{7}{3} & -2 & \frac{-4}{3}\end{pmatrix}=\begin{pmatrix}\frac{2}{3} & 0 & \frac{-8}{3}\\ \frac{-1}{3} & -1 & \frac{-2}{3}\end{pmatrix}$$

$$W=G*B=\begin{pmatrix} \frac{2}{3} & 0 & \frac{-8}{3} \\ \frac{-1}{3} & -1 & \frac{-2}{3} \end{pmatrix} * \begin{pmatrix} \frac{22}{25} & \frac{7}{25} \\ \frac{9}{25} & \frac{29}{25} \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \frac{-52}{25} & \frac{14}{75} \\ \frac{-33}{25} & \frac{-94}{75} \end{pmatrix}$$

$$P=-(D-W)^{-1}=-\left(\begin{pmatrix} \frac{-27}{25} & \frac{-12}{25} \\ \frac{-33}{25} & \frac{-23}{25} \end{pmatrix} - \begin{pmatrix} \frac{-52}{25} & \frac{14}{75} \\ \frac{-33}{25} & \frac{-94}{75} \end{pmatrix}\right)^{-1} =$$

$$\begin{pmatrix} \frac{-52}{25} & \frac{14}{75} \\ \frac{-33}{25} & \frac{-94}{75} \end{pmatrix}^{-1} = \begin{pmatrix} -1 & -2 \\ 0 & -3 \end{pmatrix}$$

$$Z=P*I=\begin{pmatrix} -1 & -2 \\ 0 & -3 \end{pmatrix} * \begin{pmatrix} \frac{2}{3} & 0 & \frac{-8}{3} \\ \frac{-1}{3} & -1 & \frac{-2}{3} \end{pmatrix} = \begin{pmatrix} 0 & 2 & 4 \\ 1 & 3 & 2 \end{pmatrix}$$

$$U=(D-C*A^{-1}*B)^{-1}$$

$$G=C*A^{-1}=\begin{pmatrix} \frac{4}{25} & \frac{36}{25} & \frac{2}{25} \\ \frac{-9}{25} & \frac{69}{25} & \frac{8}{25} \end{pmatrix} * \begin{pmatrix} 3 & 1 & -7 \\ 0 & 0 & -1 \\ \frac{7}{3} & -2 & \frac{-4}{3} \end{pmatrix} = \begin{pmatrix} \frac{2}{3} & 0 & \frac{-8}{3} \\ \frac{-1}{3} & -1 & \frac{-2}{3} \end{pmatrix}$$

$$W=G*B=\begin{pmatrix} \frac{2}{3} & 0 & \frac{-8}{3} \\ \frac{-1}{3} & -1 & \frac{-2}{3} \end{pmatrix} * \begin{pmatrix} \frac{22}{25} & \frac{7}{25} \\ \frac{9}{25} & \frac{29}{25} \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \frac{-52}{25} & \frac{14}{75} \\ \frac{-33}{25} & \frac{-94}{75} \end{pmatrix}$$

$$P=-(D-W)^{-1}=\left(\begin{pmatrix} \frac{-27}{25} & \frac{-12}{25} \\ \frac{-33}{25} & \frac{-23}{25} \end{pmatrix} - \begin{pmatrix} \frac{-52}{25} & \frac{14}{75} \\ \frac{-33}{25} & \frac{-94}{75} \end{pmatrix}\right)^{-1} =$$

$$\begin{pmatrix} \frac{-52}{25} & \frac{14}{75} \\ \frac{-33}{25} & \frac{-94}{75} \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$$

$$M^{-1} * M = \begin{pmatrix} \frac{6}{25} & \frac{-46}{25} & \frac{3}{25} & \frac{22}{25} & \frac{7}{25} \\ \frac{7}{25} & \frac{-37}{25} & \frac{-9}{25} & \frac{9}{25} & \frac{29}{25} \\ 0 & -1 & 0 & 1 & 0 \\ \frac{4}{25} & \frac{36}{25} & \frac{2}{25} & \frac{-27}{25} & \frac{-12}{25} \\ \frac{-9}{25} & \frac{69}{25} & \frac{8}{25} & \frac{-33}{25} & \frac{-23}{25} \end{pmatrix} *$$

$$\begin{pmatrix} 1 & 3 & 5 & 4 & 2 \\ 0 & 2 & 3 & 1 & 2 \\ 4 & 3 & 2 & 0 & 5 \\ 0 & 2 & 4 & 1 & 2 \\ 1 & 3 & 2 & 0 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

2.3.1 Property of inversion:

1.$(NM)^{-1} = M^{-1}N^{-1}$

M=$\begin{pmatrix} 1 & 3 & 5 & 4 & 2 \\ 0 & 2 & 3 & 1 & 2 \\ 4 & 0 & 1 & 0 & 5 \\ 0 & 2 & 4 & 1 & 2 \\ 1 & 3 & 2 & 0 & 3 \end{pmatrix}$ N=$\begin{pmatrix} 1 & 4 & 5 & 4 & 2 \\ 0 & 1 & 3 & 1 & 2 \\ 4 & 3 & 2 & 0 & 5 \\ 0 & 2 & 4 & 1 & 2 \\ 1 & 3 & 2 & 0 & 3 \end{pmatrix}$

N*M = $\begin{pmatrix} A & B \\ C & D \end{pmatrix} * \begin{pmatrix} A1 & B1 \\ C1 & D1 \end{pmatrix} = \begin{pmatrix} A * A1 + B * C1 & A * B1 + B * D1 \\ C * A1 + D * C1 & C * B1 + D * D1 \end{pmatrix}$

A=$\begin{pmatrix} 1 & 4 & 5 \\ 0 & 1 & 3 \\ 4 & 3 & 2 \end{pmatrix}$ B=$\begin{pmatrix} 4 & 2 \\ 1 & 2 \\ 0 & 5 \end{pmatrix}$ C=$\begin{pmatrix} 0 & 2 & 4 \\ 1 & 3 & 2 \end{pmatrix}$ D=$\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$ $A^{-1} =$

$$\begin{pmatrix} 1 & \frac{-7}{4} & \frac{1}{4} \\ 0 & \frac{-1}{4} & \frac{3}{4} \\ 0 & \frac{1}{2} & \frac{-1}{2} \end{pmatrix}^{-1}$$

$$D^{-1} = \begin{pmatrix} 1 & \frac{-2}{3} \\ 0 & \frac{1}{3} \end{pmatrix}^{-1}$$

X=$(A - B * D^{-1} * C)^{-1}$

$$X=\left(\begin{pmatrix}1 & 3 & 5\\ 0 & 2 & 3\\ 4 & 3 & 2\end{pmatrix} - \begin{pmatrix}4 & 2\\ 1 & 2\\ 0 & 5\end{pmatrix} * \begin{pmatrix}1 & \frac{-2}{3}\\ 0 & \frac{1}{3}\end{pmatrix}^{-1} * \begin{pmatrix}0 & 2 & 4\\ 1 & 3 & 2\end{pmatrix}\right)^{-1} = \begin{pmatrix}\frac{2}{69} & \frac{-50}{69} & \frac{9}{23}\\ \frac{7}{69} & \frac{-37}{69} & \frac{-3}{23}\\ \frac{7}{69} & \frac{-32}{69} & \frac{-3}{23}\end{pmatrix}$$

$$Y=-(A - B * D^{-1} * C)^{-1} * B * D^{-1}$$

$$Y=-\begin{pmatrix}4 & 2\\ 1 & 2\\ 0 & 5\end{pmatrix} * \begin{pmatrix}1 & \frac{-7}{4} & \frac{1}{4}\\ 0 & \frac{-1}{4} & \frac{3}{4}\\ 0 & \frac{1}{2} & \frac{-1}{2}\end{pmatrix}\left(\begin{pmatrix}1 & 3 & 5\\ 0 & 2 & 3\\ 4 & 3 & 2\end{pmatrix} - \begin{pmatrix}4 & 2\\ 1 & 2\\ 0 & 5\end{pmatrix} * \begin{pmatrix}1 & \frac{-7}{4} & \frac{1}{4}\\ 0 & \frac{-1}{4} & \frac{3}{4}\\ 0 & \frac{1}{2} & \frac{-1}{2}\end{pmatrix} * \begin{pmatrix}0 & 2 & 4\\ 1 & 3 & 2\end{pmatrix}\right)^{-1} = \begin{pmatrix}\frac{14}{23} & \frac{-14}{69}\\ \frac{3}{23} & \frac{29}{69}\\ \frac{20}{23} & \frac{-29}{69}\end{pmatrix}$$

$$Z=-(D - C * A^{-1} * B)^{-1} * C * A^{-1}$$

$$Z=-\begin{pmatrix}1 & \frac{-2}{3}\\ 0 & \frac{1}{3}\end{pmatrix}^{-1} * \begin{pmatrix}0 & 2 & 4\\ 1 & 3 & 2\end{pmatrix}\left(D = \begin{pmatrix}1 & 2\\ 0 & 3\end{pmatrix} - \begin{pmatrix}4 & 2\\ 1 & 2\\ 0 & 5\end{pmatrix} * \begin{pmatrix}1 & \frac{-2}{3}\\ 0 & \frac{1}{3}\end{pmatrix}^{-1} * \begin{pmatrix}0 & 2 & 4\\ 1 & 3 & 2\end{pmatrix}\right)^{-1} = \begin{pmatrix}\frac{20}{69} & \frac{52}{69} & \frac{-2}{23}\\ \frac{-1}{23} & \frac{25}{23} & \frac{-2}{23}\end{pmatrix}$$

$$U=(D - C * A^{-1} * B)^{-1}$$

$$U=\left(D=\begin{pmatrix}1 & 2\\ 0 & 3\end{pmatrix} - \begin{pmatrix}4 & 2\\ 1 & 2\\ 0 & 5\end{pmatrix} * \begin{pmatrix}1 & \frac{-2}{3}\\ 0 & \frac{1}{3}\end{pmatrix}^{-1} * \begin{pmatrix}0 & 2 & 4\\ 1 & 3 & 2\end{pmatrix}\right)=\begin{pmatrix}\frac{-21}{23} & \frac{4}{69}\\ \frac{-21}{23} & \frac{9}{23}\end{pmatrix}$$

$$N^{-1}=\begin{pmatrix}\frac{2}{69} & \frac{-50}{69} & \frac{9}{23} & \frac{14}{23} & \frac{-14}{23}\\ \frac{7}{69} & \frac{-37}{69} & \frac{-3}{23} & \frac{3}{23} & \frac{29}{69}\\ \frac{-1}{69} & \frac{-32}{69} & \frac{3}{23} & \frac{2}{23} & \frac{-29}{69}\\ \frac{20}{69} & \frac{52}{69} & \frac{-2}{23} & \frac{-21}{23} & \frac{4}{69}\\ \frac{-1}{23} & \frac{25}{23} & \frac{-2}{23} & \frac{-21}{23} & \frac{9}{23}\end{pmatrix}$$

$$M=\begin{pmatrix}1 & 3 & 5 & 4 & 2\\ 0 & 2 & 3 & 1 & 2\\ 4 & 0 & 1 & 0 & 5\\ 0 & 2 & 4 & 1 & 2\\ 1 & 3 & 2 & 0 & 3\end{pmatrix}$$

$$A_1=\begin{pmatrix}1 & 3 & 5\\ 0 & 2 & 3\\ 4 & 0 & 1\end{pmatrix} \quad B_1=\begin{pmatrix}4 & 2\\ 1 & 2\\ 0 & 5\end{pmatrix} \quad C_1=\begin{pmatrix}0 & 2 & 4\\ 1 & 3 & 2\end{pmatrix} \quad D_1=\begin{pmatrix}1 & 2\\ 0 & 3\end{pmatrix}$$

$$A_1{}^{-1}=\begin{pmatrix}-1 & \frac{3}{2} & \frac{1}{2}\\ -6 & \frac{19}{2} & \frac{3}{2}\\ 4 & -6 & -1\end{pmatrix}$$

$$D_1{}^{-1}=\begin{pmatrix}1 & \frac{-2}{3}\\ 0 & \frac{1}{3}\end{pmatrix}$$

$$X=\left(A_1 - B_1 * D_1{}^{-1} * C_1\right)^{-1}$$

$$X=\left(\begin{pmatrix}1 & 3 & 5\\ 0 & 2 & 3\\ 4 & 0 & 1\end{pmatrix} - \begin{pmatrix}4 & 2\\ 1 & 2\\ 0 & 5\end{pmatrix} * \begin{pmatrix}1 & \frac{-2}{3}\\ 0 & \frac{1}{3}\end{pmatrix}^{-1} * \begin{pmatrix}0 & 2 & 4\\ 1 & 3 & 2\end{pmatrix}\right)^{-1} = \begin{pmatrix}\frac{15}{52} & \frac{-28}{13} & \frac{3}{52}\\ \frac{7}{52} & \frac{-7}{13} & \frac{-9}{52}\\ 0 & -1 & 0\end{pmatrix}$$

$$Y=-\left(A_1 - B_1 * {D_1}^{-1} * C_1\right)^{-1} * B_1 * {D_1}^{-1}$$

$$Y=-\begin{pmatrix}-1 & \frac{3}{2} & \frac{1}{2}\\ -6 & \frac{19}{2} & \frac{3}{2}\\ 4 & -6 & -1\end{pmatrix} * \begin{pmatrix}4 & 2\\ 1 & 2\\ 0 & 5\end{pmatrix}\left(\begin{pmatrix}1 & 3 & 5\\ 0 & 2 & 3\\ 4 & 3 & 2\end{pmatrix} - \begin{pmatrix}4 & 2\\ 1 & 2\\ 0 & 5\end{pmatrix} * \begin{pmatrix}-1 & \frac{3}{2} & \frac{1}{2}\\ -6 & \frac{19}{2} & \frac{3}{2}\\ 4 & -6 & -1\end{pmatrix} * \begin{pmatrix}0 & 2 & 4\\ 1 & 3 & 2\end{pmatrix}\right)^{-1} = \begin{pmatrix}1 & \frac{25}{52}\\ 0 & \frac{29}{52}\\ 1 & 0\end{pmatrix}$$

$$Z=-\left(D_1 - C_1 * {A_1}^{-1} * B_1\right)^{-1} * C_1 * {A_1}^{-1}$$

$$Z=-\begin{pmatrix}1 & \frac{-2}{3}\\ 0 & \frac{1}{3}\end{pmatrix}^{-1} * \begin{pmatrix}0 & 2 & 4\\ 1 & 3 & 2\end{pmatrix}\left(D = \begin{pmatrix}1 & 2\\ 0 & 3\end{pmatrix} - \begin{pmatrix}4 & 2\\ 1 & 2\\ 0 & 5\end{pmatrix} * \begin{pmatrix}1 & \frac{-2}{3}\\ 0 & \frac{1}{3}\end{pmatrix}^{-1} * \begin{pmatrix}0 & 2 & 4\\ 1 & 3 & 2\end{pmatrix}\right)^{-1} = \begin{pmatrix}\frac{5}{26} & \frac{16}{13} & \frac{1}{26}\\ \frac{-3}{13} & \frac{25}{13} & \frac{2}{13}\end{pmatrix}$$

$$U=\left(D_1 - C_1 * {A_1}^{-1} * B_1\right)^{-1}$$

$$U=\left(D = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix} - \begin{pmatrix} 4 & 2 \\ 1 & 2 \\ 0 & 5 \end{pmatrix} * \begin{pmatrix} 1 & \frac{-2}{3} \\ 0 & \frac{1}{3} \end{pmatrix}^{-1} * \begin{pmatrix} 0 & 2 & 4 \\ 1 & 3 & 2 \end{pmatrix} \right) = \begin{pmatrix} -1 & \frac{-9}{26} \\ -1 & \frac{-5}{13} \end{pmatrix}$$

$$M^{-1} = \begin{pmatrix} \frac{15}{52} & \frac{-28}{13} & \frac{3}{25} & 1 & \frac{25}{52} \\ \frac{7}{52} & \frac{-37}{13} & \frac{-9}{52} & 0 & \frac{29}{52} \\ 0 & -1 & 0 & 1 & 0 \\ \frac{5}{26} & \frac{16}{13} & \frac{1}{26} & -1 & \frac{-9}{26} \\ \frac{3}{13} & \frac{25}{13} & \frac{2}{13} & -1 & \frac{-5}{13} \end{pmatrix}$$

$$M^{-1} * N^{-1} = \begin{pmatrix} \frac{15}{52} & \frac{-28}{13} & \frac{3}{25} & 1 & \frac{25}{52} \\ \frac{7}{52} & \frac{-37}{13} & \frac{-9}{52} & 0 & \frac{29}{52} \\ 0 & -1 & 0 & 1 & 0 \\ \frac{5}{26} & \frac{16}{13} & \frac{1}{26} & -1 & \frac{-9}{26} \\ \frac{3}{13} & \frac{25}{13} & \frac{2}{13} & -1 & \frac{-5}{13} \end{pmatrix} * \begin{pmatrix} \frac{6}{25} & \frac{-46}{25} & \frac{3}{25} & \frac{22}{25} & \frac{7}{25} \\ \frac{7}{25} & \frac{-37}{25} & \frac{-9}{25} & \frac{9}{25} & \frac{29}{25} \\ 0 & -1 & 0 & 1 & 0 \\ \frac{4}{25} & \frac{36}{25} & \frac{2}{25} & \frac{-27}{25} & \frac{-12}{25} \\ \frac{-9}{25} & \frac{69}{25} & \frac{8}{25} & \frac{-33}{25} & \frac{-23}{25} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{95}{1794} & \frac{854}{389} & \frac{163}{598} & \frac{-1683}{1196} & \frac{-312}{365} \\ \frac{-103}{1794} & \frac{373}{425} & \frac{31}{598} & \frac{-775}{1196} & \frac{-55}{3588} \\ \frac{13}{69} & \frac{89}{69} & \frac{1}{23} & \frac{-24}{23} & \frac{-25}{69} \\ \frac{-133}{897} & \frac{-956}{491} & \frac{11}{299} & \frac{921}{598} & \frac{347}{1794} \\ \frac{-30}{299} & \frac{-630}{299} & \frac{-60}{299} & \frac{451}{299} & \frac{201}{299} \end{pmatrix}$$

$$N * \mathrm{M} = \begin{pmatrix} 1 & 3 & 5 & 4 & 2 \\ 0 & 1 & 3 & 1 & 2 \\ 4 & 3 & 2 & 0 & 5 \\ 0 & 2 & 4 & 1 & 2 \\ 1 & 3 & 2 & 0 & 3 \end{pmatrix} * \begin{pmatrix} 1 & 4 & 3 & 5 & 2 \\ 0 & 1 & 3 & 1 & 2 \\ 4 & 3 & 2 & 0 & 5 \\ 0 & 2 & 4 & 1 & 2 \\ 1 & 3 & 2 & 0 & 3 \end{pmatrix} = \begin{pmatrix} 23 & 38 & 44 & 11 & 47 \\ 0 & 21 & 20 & 3 & 27 \\ 13 & 30 & 32 & 16 & 28 \\ 18 & 24 & 22 & 3 & 32 \\ 12 & 42 & 24 & 7 & 27 \end{pmatrix}$$

$$\mathrm{S} = \begin{pmatrix} 23 & 38 & 44 \\ 0 & 21 & 20 \\ 13 & 30 & 32 \end{pmatrix} \quad \mathrm{P} = \begin{pmatrix} 11 & 47 \\ 3 & 27 \\ 16 & 28 \end{pmatrix} \quad \mathrm{Q} = \begin{pmatrix} 18 & 24 & 22 \\ 12 & 42 & 24 \end{pmatrix} \quad \mathrm{R} = \begin{pmatrix} 3 & 32 \\ 7 & 27 \end{pmatrix}$$

$$\mathrm{X} = (S - P * R^{-1} * Q)^{-1}$$

$$S^{-1} = \begin{pmatrix} \frac{18}{245} & \frac{26}{245} & \frac{-41}{245} \\ \frac{-47}{245} & \frac{41}{245} & \frac{39}{245} \\ \frac{3}{20} & \frac{-1}{5} & \frac{-1}{20} \end{pmatrix}$$

$$R^{-1} = \begin{pmatrix} -\frac{27}{143} & \frac{32}{143} \\ \frac{7}{143} & \frac{-3}{143} \end{pmatrix}$$

$$\mathrm{X} = \left(\begin{pmatrix} 23 & 38 & 44 \\ 0 & 21 & 20 \\ 13 & 30 & 32 \end{pmatrix} - \begin{pmatrix} 11 & 47 \\ 3 & 27 \\ 16 & 28 \end{pmatrix} * \begin{pmatrix} -\frac{27}{143} & \frac{32}{143} \\ \frac{7}{143} & \frac{-3}{143} \end{pmatrix} * \begin{pmatrix} 18 & 24 & 22 \\ 12 & 42 & 24 \end{pmatrix} \right)^{-1} =$$

$$\begin{pmatrix} \frac{15}{52} & \frac{-28}{245} & \frac{3}{25} \\ \frac{7}{52} & \frac{-37}{13} & \frac{-9}{52} \\ 0 & -1 & 0 \end{pmatrix}$$

$$\mathrm{Y} = -(S - P * R^{-1} * Q)^{-1} * P * R^{-1}$$

$$Y = -\left(\begin{pmatrix} 23 & 38 & 44 \\ 0 & 21 & 20 \\ 13 & 30 & 32 \end{pmatrix} - \begin{pmatrix} 11 & 47 \\ 3 & 27 \\ 16 & 28 \end{pmatrix} * \begin{pmatrix} -\frac{27}{143} & \frac{32}{143} \\ \frac{7}{143} & \frac{-3}{143} \end{pmatrix} * \begin{pmatrix} 18 & 24 & 22 \\ 12 & 42 & 24 \end{pmatrix}\right)^{-1} * \begin{pmatrix} 11 & 47 \\ 3 & 27 \\ 16 & 28 \end{pmatrix} * \begin{pmatrix} -\frac{27}{143} & \frac{32}{143} \\ \frac{7}{143} & \frac{-3}{143} \end{pmatrix} = \begin{pmatrix} 1 & \frac{25}{52} \\ 0 & \frac{29}{52} \\ 1 & 0 \end{pmatrix}$$

$$Z = -(R - Q * S^{-1} * P)^{-1} * Q * S^{-1}$$

$$-\begin{pmatrix} -\frac{27}{143} & \frac{32}{143} \\ \frac{7}{143} & \frac{-3}{143} \end{pmatrix} * \begin{pmatrix} 18 & 24 & 22 \\ 12 & 42 & 24 \end{pmatrix} \left(\begin{pmatrix} 3 & 32 \\ 7 & 27 \end{pmatrix} - \begin{pmatrix} 11 & 47 \\ 3 & 27 \\ 16 & 28 \end{pmatrix} * \begin{pmatrix} -\frac{27}{143} & \frac{32}{143} \\ \frac{7}{143} & \frac{-3}{143} \end{pmatrix} * \begin{pmatrix} 18 & 24 & 22 \\ 12 & 42 & 24 \end{pmatrix}\right)^{-1} = \begin{pmatrix} \frac{5}{26} & \frac{16}{13} & \frac{1}{26} \\ \frac{3}{13} & \frac{25}{13} & \frac{2}{13} \end{pmatrix}$$

$$U = (R - Q * S^{-1} * P)^{-1}$$

$$U = \left(\begin{pmatrix} 3 & 32 \\ 7 & 27 \end{pmatrix} - \begin{pmatrix} 18 & 24 & 22 \\ 12 & 42 & 24 \end{pmatrix} * \begin{pmatrix} \frac{18}{245} & \frac{26}{245} & \frac{-41}{245} \\ \frac{-47}{245} & \frac{41}{245} & \frac{39}{245} \\ \frac{3}{20} & \frac{-1}{5} & \frac{-1}{20} \end{pmatrix} * \begin{pmatrix} 11 & 47 \\ 3 & 27 \\ 16 & 28 \end{pmatrix}\right)^{-1} = \begin{pmatrix} -1 & \frac{-9}{26} \\ -1 & \frac{-5}{13} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{95}{1794} & \frac{854}{389} & \frac{163}{598} & \frac{-1683}{1196} & \frac{-312}{365} \\ \frac{-103}{1794} & \frac{373}{425} & \frac{31}{598} & \frac{-775}{1196} & \frac{-55}{3588} \\ \frac{13}{69} & \frac{89}{69} & \frac{1}{23} & \frac{-24}{23} & \frac{-25}{69} \\ \frac{-133}{897} & \frac{-956}{491} & \frac{11}{299} & \frac{921}{598} & \frac{347}{1794} \\ \frac{-30}{299} & \frac{-630}{299} & \frac{-60}{299} & \frac{451}{299} & \frac{201}{299} \end{pmatrix}$$

2. $(M^T)^{-1}$=$(M^{-1})^T$

$$M = \begin{pmatrix} 1 & 3 & 5 & 4 & 2 \\ 0 & 2 & 3 & 1 & 2 \\ 4 & 3 & 2 & 0 & 5 \\ 0 & 2 & 4 & 1 & 2 \\ 1 & 3 & 2 & 0 & 3 \end{pmatrix}$$

A=$\begin{pmatrix} 1 & 3 & 5 \\ 0 & 2 & 3 \\ 4 & 3 & 2 \end{pmatrix}$ B=$\begin{pmatrix} 4 & 2 \\ 1 & 2 \\ 0 & 5 \end{pmatrix}$ C=$\begin{pmatrix} 0 & 2 & 4 \\ 1 & 3 & 2 \end{pmatrix}$ D=$\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$

A^T=$\begin{pmatrix} 1 & 0 & 4 \\ 3 & 2 & 3 \\ 5 & 3 & 2 \end{pmatrix}$, B^T=$\begin{pmatrix} 4 & 1 & 0 \\ 2 & 2 & 5 \end{pmatrix}$, C^T=$\begin{pmatrix} 0 & 1 \\ 2 & 3 \\ 4 & 2 \end{pmatrix}$, D^T=$\begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}$

A = A^T , B = C^T , C = B^T, D = D^T

$$M^T = \begin{pmatrix} 1 & 0 & 4 & 0 & 1 \\ 3 & 2 & 3 & 2 & 3 \\ 5 & 3 & 2 & 4 & 2 \\ 4 & 1 & 0 & 1 & 0 \\ 2 & 2 & 5 & 2 & 3 \end{pmatrix}$$

A_1=$\begin{pmatrix} 1 & 0 & 4 \\ 3 & 2 & 3 \\ 5 & 3 & 2 \end{pmatrix}$, B_1=$\begin{pmatrix} 4 & 1 & 0 \\ 2 & 2 & 5 \end{pmatrix}$, C_1=$\begin{pmatrix} 0 & 1 \\ 2 & 3 \\ 4 & 2 \end{pmatrix}$, D_1=$\begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}$

$$A_1{}^{-1}=\begin{pmatrix} \frac{5}{9} & \frac{-4}{3} & \frac{8}{9} \\ -1 & 2 & -1 \\ \frac{1}{9} & \frac{1}{3} & \frac{-2}{9} \end{pmatrix}$$

$$D_1{}^{-1}=\begin{pmatrix} 1 & 0 \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix}$$

$$\text{X}=\left(A_1 - B_1 * D_1{}^{-1} * C_1\right)^{-1}$$

$$\text{X}=\left(\begin{pmatrix} 1 & 0 & 4 \\ 3 & 2 & 3 \\ 5 & 3 & 2 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 2 & 3 \\ 4 & 2 \end{pmatrix} * \begin{pmatrix} 1 & 0 \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} * \begin{pmatrix} 4 & 1 & 0 \\ 2 & 2 & 5 \end{pmatrix}\right)^{-1} =$$

$$\begin{pmatrix} \frac{6}{25} & \frac{7}{25} & 0 \\ \frac{-46}{25} & \frac{-37}{25} & -1 \\ \frac{3}{25} & \frac{-9}{25} & 0 \end{pmatrix}$$

$$\text{Y}=-A_1{}^{-1} * B\left(D_1 - C_1 * A_1{}^{-1} * B_1\right)^{-1}$$

$$\text{Y}=-\begin{pmatrix} 0 & 1 \\ 2 & 3 \\ 4 & 2 \end{pmatrix} * \begin{pmatrix} \frac{5}{9} & \frac{-4}{3} & \frac{8}{9} \\ -1 & 2 & -1 \\ \frac{1}{9} & \frac{1}{3} & \frac{-2}{9} \end{pmatrix} \left(\begin{pmatrix} 1 & 0 & 4 \\ 3 & 2 & 3 \\ 5 & 3 & 2 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 2 & 3 \\ 4 & 2 \end{pmatrix} * \right.$$

$$\left.\begin{pmatrix} \frac{5}{9} & \frac{-4}{3} & \frac{8}{9} \\ -1 & 2 & -1 \\ \frac{1}{9} & \frac{1}{3} & \frac{-2}{9} \end{pmatrix} * \begin{pmatrix} 4 & 1 & 0 \\ 2 & 2 & 5 \end{pmatrix}\right)^{-1} = \begin{pmatrix} \frac{22}{25} & \frac{9}{25} & 1 \\ \frac{7}{25} & \frac{29}{25} & 0 \end{pmatrix}$$

$$\text{Z}=-D_1{}^{-1} * C_1\left(A_1 - B_1 * D_1{}^{-1} * C_1\right)^{-1}$$

$$Z= -\begin{pmatrix} 1 & 0 \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} * \begin{pmatrix} 4 & 1 & 0 \\ 2 & 2 & 5 \end{pmatrix} * \left(\left(\begin{pmatrix} 1 & 0 & 4 \\ 3 & 2 & 3 \\ 5 & 3 & 2 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 2 & 3 \\ 4 & 2 \end{pmatrix} * \begin{pmatrix} 1 & 0 \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} * \begin{pmatrix} 4 & 1 & 0 \\ 2 & 2 & 5 \end{pmatrix} \right)^{-1} = \begin{pmatrix} \frac{4}{25} & \frac{-9}{25} \\ \frac{36}{25} & \frac{69}{25} \\ \frac{2}{25} & \frac{8}{25} \end{pmatrix}$$

$$U=\left(D_1 - C_1 * {A_1}^{-1} * B_1\right)^{-1}$$

$$U=\left(\begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix} - \begin{pmatrix} 4 & 1 & 0 \\ 2 & 2 & 5 \end{pmatrix} * \begin{pmatrix} 1 & 0 \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} * \begin{pmatrix} 0 & 1 \\ 2 & 3 \\ 4 & 2 \end{pmatrix} \right)^{-1} = \begin{pmatrix} \frac{-27}{25} & \frac{-33}{25} \\ \frac{-12}{25} & \frac{-23}{25} \end{pmatrix}$$

$$M^{T^{-1}} = \begin{pmatrix} \frac{6}{25} & \frac{7}{25} & 0 & \frac{4}{25} & \frac{7}{25} \\ \frac{-46}{25} & \frac{-37}{25} & -1 & \frac{36}{25} & \frac{69}{25} \\ \frac{3}{25} & \frac{-9}{25} & \frac{2}{25} & 0 & \frac{8}{25} \\ \frac{22}{25} & \frac{9}{25} & 1 & \frac{-27}{25} & \frac{-33}{25} \\ \frac{7}{25} & \frac{29}{25} & 0 & \frac{-12}{25} & \frac{-23}{25} \end{pmatrix}$$

$$M=\begin{pmatrix} 1 & 4 & 3 & 5 & 2 \\ 0 & 1 & 3 & 1 & 2 \\ 4 & 3 & 2 & 0 & 5 \\ 0 & 2 & 4 & 1 & 2 \\ 1 & 3 & 2 & 0 & 3 \end{pmatrix}$$

$$A=\begin{pmatrix} 1 & 3 & 5 \\ 0 & 2 & 3 \\ 4 & 3 & 2 \end{pmatrix} \quad B=\begin{pmatrix} 4 & 2 \\ 1 & 2 \\ 0 & 5 \end{pmatrix} \quad C=\begin{pmatrix} 0 & 2 & 4 \\ 1 & 3 & 2 \end{pmatrix} \quad D=\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 1 & \frac{-7}{4} & \frac{1}{4} \\ 0 & \frac{-1}{4} & \frac{3}{4} \\ 0 & \frac{1}{2} & \frac{-1}{2} \end{pmatrix}$$

$$D^{-1} = \begin{pmatrix} 1 & \frac{-2}{3} \\ 0 & \frac{1}{3} \end{pmatrix}$$

X=$(A - B * D^{-1} * C)^{-1}$

$$\text{X}=\left(\begin{pmatrix} 1 & 3 & 5 \\ 0 & 2 & 3 \\ 4 & 3 & 2 \end{pmatrix} - \begin{pmatrix} 4 & 2 \\ 1 & 2 \\ 0 & 5 \end{pmatrix} * \begin{pmatrix} 1 & \frac{-2}{3} \\ 0 & \frac{1}{3} \end{pmatrix} * \begin{pmatrix} 0 & 2 & 4 \\ 1 & 3 & 2 \end{pmatrix} \right)^{-1} = \begin{pmatrix} \frac{6}{25} & \frac{-46}{25} & \frac{3}{25} \\ \frac{7}{25} & \frac{-37}{25} & \frac{-9}{25} \\ 0 & -1 & 0 \end{pmatrix}$$

Y=$-(A - B * D^{-1} * C)^{-1} * B * D^{-1}$

$$\text{Y}=-\begin{pmatrix} 4 & 2 \\ 1 & 2 \\ 0 & 5 \end{pmatrix} * \begin{pmatrix} 1 & \frac{-7}{4} & \frac{1}{4} \\ 0 & \frac{-1}{4} & \frac{3}{4} \\ 0 & \frac{1}{2} & \frac{-1}{2} \end{pmatrix} \left(\begin{pmatrix} 1 & 3 & 5 \\ 0 & 2 & 3 \\ 4 & 3 & 2 \end{pmatrix} - \begin{pmatrix} 4 & 2 \\ 1 & 2 \\ 0 & 5 \end{pmatrix} * \begin{pmatrix} 1 & \frac{-7}{4} & \frac{1}{4} \\ 0 & \frac{-1}{4} & \frac{3}{4} \\ 0 & \frac{1}{2} & \frac{-1}{2} \end{pmatrix} * \begin{pmatrix} 0 & 2 & 4 \\ 1 & 3 & 2 \end{pmatrix} \right)^{-1} = \begin{pmatrix} \frac{22}{25} & \frac{7}{25} \\ \frac{9}{25} & \frac{29}{25} \\ 1 & 0 \end{pmatrix}$$

Z=$-D^{-1} * C(D - B * D^{-1} * C)^{-1}$

$$Z=-\begin{pmatrix} 1 & \frac{-2}{3} \\ 0 & \frac{1}{3} \end{pmatrix} * \begin{pmatrix} 0 & 2 & 4 \\ 1 & 3 & 2 \end{pmatrix} \left(D = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix} - \begin{pmatrix} 4 & 2 \\ 1 & 2 \\ 0 & 5 \end{pmatrix} * \begin{pmatrix} 1 & \frac{-2}{3} \\ 0 & \frac{1}{3} \end{pmatrix} * \begin{pmatrix} 0 & 2 & 4 \\ 1 & 3 & 2 \end{pmatrix} \right)^{-1} = \begin{pmatrix} \frac{4}{25} & \frac{36}{25} & \frac{2}{25} \\ \frac{-9}{25} & \frac{69}{25} & \frac{8}{25} \end{pmatrix}$$

$$U=(D - C * A^{-1} * B)^{-1}$$

$$U=\left(D = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix} - \begin{pmatrix} 4 & 2 \\ 1 & 2 \\ 0 & 5 \end{pmatrix} * \begin{pmatrix} 1 & \frac{-2}{3} \\ 0 & \frac{1}{3} \end{pmatrix} * \begin{pmatrix} 0 & 2 & 4 \\ 1 & 3 & 2 \end{pmatrix} \right) = \begin{pmatrix} \frac{-27}{25} & \frac{-12}{25} \\ \frac{-33}{25} & \frac{-23}{25} \end{pmatrix}$$

$$M^{-1} = \begin{pmatrix} \frac{6}{25} & \frac{-46}{25} & \frac{3}{25} & \frac{22}{25} & \frac{7}{25} \\ \frac{7}{25} & \frac{-37}{25} & \frac{-9}{25} & \frac{9}{25} & \frac{29}{25} \\ 0 & -1 & 0 & 1 & 0 \\ \frac{4}{25} & \frac{36}{25} & \frac{2}{25} & \frac{-27}{25} & \frac{-12}{25} \\ \frac{-9}{25} & \frac{69}{25} & \frac{8}{25} & \frac{-33}{25} & \frac{-23}{25} \end{pmatrix}$$

$$A^{-1}=\begin{pmatrix} \frac{6}{25} & \frac{7}{25} & 0 \\ \frac{-46}{25} & \frac{-37}{25} & -1 \\ \frac{3}{25} & \frac{-9}{25} & 0 \end{pmatrix} \qquad B^{-1}=\begin{pmatrix} \frac{4}{25} & \frac{-9}{25} \\ \frac{36}{25} & \frac{36}{25} \\ \frac{2}{25} & \frac{8}{25} \end{pmatrix} \qquad C^{-1}=\begin{pmatrix} \frac{22}{25} & \frac{9}{25} & 1 \\ \frac{7}{25} & \frac{29}{25} & 0 \end{pmatrix}$$

$$D^{-1}=\begin{pmatrix} \frac{-27}{25} & \frac{-33}{25} \\ \frac{-27}{25} & \frac{-23}{25} \end{pmatrix}$$

$$A^{-1} = (A^{-1})^T \ , B^{-1} = (C^{-1})^T \ , C^{-1} = (B^{-1})^T \ , \ D^{-1} = (D^{-1})^T$$

$$M^{-1^T} = \begin{pmatrix} \frac{6}{25} & \frac{7}{25} & 0 & \frac{4}{25} & \frac{7}{25} \\ \frac{-46}{25} & \frac{-37}{25} & -1 & \frac{36}{25} & \frac{69}{25} \\ \frac{3}{25} & \frac{-9}{25} & \frac{2}{25} & 0 & \frac{8}{25} \\ \frac{22}{25} & \frac{9}{25} & 1 & \frac{-27}{25} & \frac{-33}{25} \\ \frac{7}{25} & \frac{29}{25} & 0 & \frac{-12}{25} & \frac{-23}{25} \end{pmatrix}$$

3. $(M^{-1})^{-1}$=M

For example:

$$M=\begin{pmatrix} 1 & 4 & 3 & 5 & 2 \\ 0 & 1 & 3 & 1 & 2 \\ 4 & 3 & 2 & 0 & 5 \\ 0 & 2 & 4 & 1 & 2 \\ 1 & 3 & 2 & 0 & 3 \end{pmatrix}$$

$$A=\begin{pmatrix} 1 & 3 & 5 \\ 0 & 2 & 3 \\ 4 & 3 & 2 \end{pmatrix} \quad B=\begin{pmatrix} 4 & 2 \\ 1 & 2 \\ 0 & 5 \end{pmatrix} \quad C=\begin{pmatrix} 0 & 2 & 4 \\ 1 & 3 & 2 \end{pmatrix} \quad D=\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 1 & \frac{-7}{4} & \frac{1}{4} \\ 0 & \frac{-1}{4} & \frac{3}{4} \\ 0 & \frac{1}{2} & \frac{-1}{2} \end{pmatrix}$$

$$D^{-1} = \begin{pmatrix} 1 & \frac{-2}{3} \\ 0 & \frac{1}{3} \end{pmatrix}$$

X=$(A - B * D^{-1} * C)^{-1}$

$$X=\left(\begin{pmatrix} 1 & 3 & 5 \\ 0 & 2 & 3 \\ 4 & 3 & 2 \end{pmatrix} - \begin{pmatrix} 4 & 2 \\ 1 & 2 \\ 0 & 5 \end{pmatrix} * \begin{pmatrix} 1 & \frac{-2}{3} \\ 0 & \frac{1}{3} \end{pmatrix} * \begin{pmatrix} 0 & 2 & 4 \\ 1 & 3 & 2 \end{pmatrix}\right)^{-1} = \begin{pmatrix} \frac{6}{25} & \frac{-46}{25} & \frac{3}{25} \\ \frac{7}{25} & \frac{-37}{25} & \frac{-9}{25} \\ 0 & -1 & 0 \end{pmatrix}$$

Y=$-(A - B * D^{-1} * C)^{-1} * B * D^{-1}$

$$Y=-\begin{pmatrix}4 & 2\\1 & 2\\0 & 5\end{pmatrix}*\begin{pmatrix}1 & \frac{-7}{4} & \frac{1}{4}\\0 & \frac{-1}{4} & \frac{3}{4}\\0 & \frac{1}{2} & \frac{-1}{2}\end{pmatrix}\left(\begin{pmatrix}1 & 3 & 5\\0 & 2 & 3\\4 & 3 & 2\end{pmatrix}-\begin{pmatrix}4 & 2\\1 & 2\\0 & 5\end{pmatrix}*\right.$$

$$\left.\begin{pmatrix}1 & \frac{-7}{4} & \frac{1}{4}\\0 & \frac{-1}{4} & \frac{3}{4}\\0 & \frac{1}{2} & \frac{-1}{2}\end{pmatrix}*\begin{pmatrix}0 & 2 & 4\\1 & 3 & 2\end{pmatrix}\right)^{-1}=\begin{pmatrix}\frac{22}{25} & \frac{7}{25}\\\frac{9}{25} & \frac{29}{25}\\1 & 0\end{pmatrix}$$

$$Z=-D^{-1}*C(D-B*D^{-1}*C)^{-1}$$

$$Z=-\begin{pmatrix}1 & \frac{-2}{3}\\0 & \frac{1}{3}\end{pmatrix}^{-1}*\begin{pmatrix}0 & 2 & 4\\1 & 3 & 2\end{pmatrix}\left(D=\begin{pmatrix}1 & 2\\0 & 3\end{pmatrix}-\begin{pmatrix}4 & 2\\1 & 2\\0 & 5\end{pmatrix}*\begin{pmatrix}1 & \frac{-2}{3}\\0 & \frac{1}{3}\end{pmatrix}^{-1}*\right.$$

$$\left.\begin{pmatrix}0 & 2 & 4\\1 & 3 & 2\end{pmatrix}\right)^{-1}=\begin{pmatrix}\frac{4}{25} & \frac{36}{25} & \frac{2}{25}\\\frac{-9}{25} & \frac{69}{25} & \frac{8}{25}\end{pmatrix}$$

$$U=(D-C*A^{-1}*B)^{-1}$$

$$U=\left(D=\begin{pmatrix}1 & 2\\0 & 3\end{pmatrix}-\begin{pmatrix}4 & 2\\1 & 2\\0 & 5\end{pmatrix}*\begin{pmatrix}1 & \frac{-2}{3}\\0 & \frac{1}{3}\end{pmatrix}^{-1}*\begin{pmatrix}0 & 2 & 4\\1 & 3 & 2\end{pmatrix}\right)=\begin{pmatrix}\frac{-27}{25} & \frac{-12}{25}\\\frac{-33}{25} & \frac{-23}{25}\end{pmatrix}$$

$$M^{-1} = \begin{pmatrix} \frac{6}{25} & \frac{-46}{25} & \frac{3}{25} & \frac{22}{25} & \frac{7}{25} \\ \frac{7}{25} & \frac{-37}{25} & \frac{-9}{25} & \frac{9}{25} & \frac{29}{25} \\ 0 & -1 & 0 & 1 & 0 \\ \frac{4}{25} & \frac{36}{25} & \frac{2}{25} & \frac{-27}{25} & \frac{-12}{25} \\ \frac{-9}{25} & \frac{69}{25} & \frac{8}{25} & \frac{-33}{25} & \frac{-23}{25} \end{pmatrix}$$

X=$\begin{pmatrix} \frac{6}{25} & \frac{-46}{25} & \frac{3}{25} \\ \frac{7}{25} & \frac{-37}{25} & \frac{-9}{25} \\ 0 & -1 & 0 \end{pmatrix}$ Y= $\begin{pmatrix} \frac{22}{25} & \frac{7}{25} \\ \frac{9}{25} & \frac{29}{25} \\ 1 & 0 \end{pmatrix}$ Z= $\begin{pmatrix} \frac{4}{25} & \frac{36}{25} & \frac{2}{25} \\ \frac{-9}{25} & \frac{69}{25} & \frac{8}{25} \end{pmatrix}$ U= $\begin{pmatrix} \frac{-27}{25} & \frac{-12}{25} \\ \frac{-33}{25} & \frac{-23}{25} \end{pmatrix}$

$$X^{-1}=\begin{pmatrix} \frac{6}{25} & \frac{-46}{25} & \frac{3}{25} \\ \frac{7}{25} & \frac{-37}{25} & \frac{-9}{25} \\ 0 & -1 & 0 \end{pmatrix}^{-1} = \begin{pmatrix} 3 & 1 & -7 \\ 0 & 0 & -1 \\ \frac{7}{3} & -2 & \frac{-4}{3} \end{pmatrix}$$

$$U^{-1}=\begin{pmatrix} \frac{-27}{25} & \frac{-12}{25} \\ \frac{-33}{25} & \frac{-23}{25} \end{pmatrix}^{-1} = \begin{pmatrix} \frac{-23}{9} & \frac{4}{3} \\ \frac{11}{3} & -3 \end{pmatrix}$$

A=$(X - Y * U^{-1} * Z)^{-1}$

$$A=\left(\begin{pmatrix}\frac{6}{25} & \frac{-46}{25} & \frac{3}{25}\\ \frac{7}{25} & \frac{-37}{25} & \frac{-9}{25}\\ 0 & -1 & 0\end{pmatrix}-\begin{pmatrix}\frac{22}{25} & \frac{7}{25}\\ \frac{9}{25} & \frac{29}{25}\\ 1 & 0\end{pmatrix} * \begin{pmatrix}\frac{-23}{9} & \frac{4}{3}\\ \frac{11}{3} & -3\end{pmatrix} * \begin{pmatrix}\frac{4}{25} & \frac{36}{25} & \frac{2}{25}\\ \frac{-9}{25} & \frac{69}{25} & \frac{8}{25}\end{pmatrix}\right)^{-1} = \begin{pmatrix}1 & 3 & 5\\ 0 & 2 & 3\\ 4 & 3 & 2\end{pmatrix}$$

$$B=-(U-Y*U^{-1}*Z)^{-1}*Y*U^{-1}$$

$$B=\left(\begin{pmatrix}\frac{-27}{25} & \frac{-12}{25}\\ \frac{-33}{25} & \frac{-23}{25}\end{pmatrix}-\begin{pmatrix}\frac{22}{25} & \frac{7}{25}\\ \frac{9}{25} & \frac{29}{25}\\ 1 & 0\end{pmatrix} * \begin{pmatrix}\frac{-23}{9} & \frac{4}{3}\\ \frac{11}{3} & -3\end{pmatrix} * \begin{pmatrix}\frac{4}{25} & \frac{36}{25} & \frac{2}{25}\\ \frac{-9}{25} & \frac{69}{25} & \frac{8}{25}\end{pmatrix}\right)^{-1} * \begin{pmatrix}\frac{22}{25} & \frac{7}{25}\\ \frac{9}{25} & \frac{29}{25}\\ 1 & 0\end{pmatrix} * \begin{pmatrix}\frac{-23}{9} & \frac{4}{3}\\ \frac{11}{3} & -3\end{pmatrix} = \begin{pmatrix}4 & 2\\ 1 & 2\\ 0 & 5\end{pmatrix}$$

$$C=-(\boldsymbol{U}-\boldsymbol{Z}*\boldsymbol{X}^{-1}*\boldsymbol{Y})^{-1}*\boldsymbol{Z}*\boldsymbol{X}^{-1}$$

$$C=-\left(\begin{pmatrix}\frac{-27}{25} & \frac{-12}{25}\\ \frac{-33}{25} & \frac{-23}{25}\end{pmatrix}-\begin{pmatrix}\frac{4}{25} & \frac{36}{25} & \frac{2}{25}\\ \frac{-9}{25} & \frac{69}{25} & \frac{8}{25}\end{pmatrix} * \begin{pmatrix}3 & 1 & -7\\ 0 & 0 & -1\\ \frac{7}{3} & -2 & \frac{-4}{3}\end{pmatrix} * \begin{pmatrix}\frac{22}{25} & \frac{7}{25}\\ \frac{9}{25} & \frac{29}{25}\\ 1 & 0\end{pmatrix}\right)^{-1} \begin{pmatrix}\frac{4}{25} & \frac{36}{25} & \frac{2}{25}\\ \frac{-9}{25} & \frac{69}{25} & \frac{8}{25}\end{pmatrix} * \begin{pmatrix}3 & 1 & -7\\ 0 & 0 & -1\\ \frac{7}{3} & -2 & \frac{-4}{3}\end{pmatrix} = \begin{pmatrix}0 & 2 & 4\\ 1 & 3 & 2\end{pmatrix}$$

$$D=(\boldsymbol{U}-\boldsymbol{Z}*\boldsymbol{X}^{-1}*\boldsymbol{Y})^{-1}$$

$$\boldsymbol{D} = \left(\begin{pmatrix} \frac{\mathbf{-27}}{\mathbf{25}} & \frac{\mathbf{-12}}{\mathbf{25}} \\ \frac{\mathbf{-33}}{25} & \frac{-23}{25} \end{pmatrix} - \begin{pmatrix} \frac{4}{25} & \frac{36}{25} & \frac{2}{25} \\ \frac{-9}{25} & \frac{69}{25} & \frac{8}{25} \end{pmatrix} * \begin{pmatrix} 3 & 1 & -7 \\ 0 & 0 & -1 \\ \frac{7}{3} & -2 & \frac{-4}{3} \end{pmatrix} * \begin{pmatrix} \frac{22}{25} & \frac{7}{25} \\ \frac{9}{25} & \frac{29}{25} \\ 1 & 0 \end{pmatrix} \right)^{-1} = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3 & 5 & 4 & 2 \\ 0 & 2 & 3 & 1 & 2 \\ 4 & 3 & 2 & 0 & 5 \\ 0 & 2 & 4 & 1 & 2 \\ 1 & 3 & 2 & 0 & 3 \end{pmatrix} = M$$

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