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(Newton Raphson's method for non-linear system)

Research project

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Abstract:

The Newton-Raphson method is the method of choice for solving nonlinear systems of equations. Many engineering software packages (especially finite element analysis software) that solve nonlinear systems of equations use the Newton-Raphson method

Introduction:

The goal of this research is to provide comprehensive understanding of newton Raphson method non-linear systems of Numerical methods are the study of methods in which we compute the numerical data. In this we find a general sequence of approximations with repeating the process again and again The research consists of two chapter in chapter one we have some basic definition of Numerical analysis and newton Raphson method non -linear system and Taylor series then in chapter two some of Program of Newton-Raphson Method for one variables and one equations and for two variables and two equations and for three variables and three equations.

Chapter 1

Numerical analysis: - the study of how to get approximate answers by using lots of simple calculations used when direct perfect answers are hard.

Algorithm: - is a mathematical process to solve a problem using a finite number of steps.

Numerical solution: - A numerical solution approximates the solution of a mathematical equation, often used where analytical solutions are hard or impossible to find.

Analytical solution: - involves framing the problem in a well – understood from and calculating the exact solution.

Taylor series: -The Taylor series of a real or complex-value function $f(x)$ that is infinitely differentiable at a real or complex number a is the power series.

$$f(a) + \frac{f'(a)}{1!}(x - a) + \frac{f''(a)}{2!}(x - a)^2 + \frac{f'''(a)}{3!}(x - a)^3 + \dots$$

where $n!$ denotes the **factorial** of n . In the more compact **sigma notation**, this can be written a

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x - a)^n$$

where $f^{(n)}(a)$ denotes the n th **derivative** of f evaluated at the point a . (The derivative of order zero of f is defined to be f itself and $(x - a)^0$ and When $a = 0$, the series is also called a **Maclaurin series**.

Examples:-

The Taylor series of any **polynomial** is the polynomial itself.

The Maclaurin series of $\frac{1}{1-x}$ is the **geometric series**.

$$1 + x + x^2 + x^3 + \dots$$

So, by substituting x for $1 - x$, the Taylor series of $\frac{1}{x}$ at $a = 1$ is.

$$1 - (x - 1) + (x - 1)^2 - (x - 1)^3 + \dots$$

By integrating the above Maclaurin series, we find the Maclaurin series of $\ln(1 - x)$, where \ln denotes the **natural logarithm**:

$$-x - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{4}x^4 - \dots$$

The corresponding Taylor series of $\ln x$ at $a = 1$ is

$$(x - 1) - \frac{1}{2}(x - 1)^2 + \frac{1}{3}(x - 1)^3 - \frac{1}{4}(x - 1)^4 + \dots$$

and more generally, the corresponding Taylor series of $\ln x$ at an arbitrary nonzero point a is:

$$\ln a + \frac{1}{a}(x - a) - \frac{1}{a^2} \frac{(x - a)^2}{2} + \dots$$

The Maclaurin series of the **exponential function** e^x is

$$\begin{aligned} \sum_{n=0}^{\infty} \frac{x^n}{n!} &= \frac{x^0}{0!} + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \dots \\ &= 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \dots \end{aligned}$$

The above expansion holds because the derivative of e^x with respect to x is also e^x , and e^0 equals 1. This leaves the terms $(x - 0)^n$ in the numerator and $n!$ in the denominator of each term at an infinite sum.

Error:- if (x) and (\tilde{x}) are true and computed values respectively , then error denoted by (E) is defined as

$$E = x - \tilde{x}$$

Linear function: - linear function is those whose graph is a straight line. A linear function has the following form. $y = f(x) = a + bx$. A liner function has one independent variable, and one dependent variable is (x) and the dependent variable is (y)

Nonlinear system: - in mathematics and science, a nonlinear system is a system in which the change of the output is not proportional to the change.

Newton Raphson method (for one variable)

In numerical analysis, Newton's method, also known as the Newton-Raphson method, named after Isaac Newton and Joseph Raphson. To adapt the Newton-Raphson method to simultaneous equations, we proceed as follows:

and let x_0 be an approximation to the solution λ of (1) assuming that (f) is sufficiently differentiable, $f(x)$ about x_0 using Taylor series for function of one variable:

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \dots$$

Assuming that x_0 sufficiently closed to λ so that higher order terms can be neglected, we equate the expansion through linear terms to zero. This gives us the following.

$$f'(x_0)(\lambda - x_0) \approx -f(x_0) \quad \dots \dots \dots (*)$$

Where it is understood that the function and its derivative in (*) will be evaluated at x_0 . We might then expect that the solution x_1 of (*) will be closer to the solution λ than x_0 . The solution of (*) yields.

$$\lambda \approx x_0 - \frac{f(x_0)}{f'(x_0)}$$

So, in general we get

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$i = 0, 1, \dots$$

Where $f'(x_i) \neq 0$ and any at x_i , we stop iteration if $|x_{i+1} - x_i| < \varepsilon$ for any (i) .

Example:- newton Raphson method find a root of the following let

$$f(x) = x^2 - 2 = 0 \quad x_0 = 0.5 \quad \text{and error} \leq 0.01.$$

Solution:- we follow these steps to apply the newton-raphson method :

1-define the function whose zero or root are be found, in this example, it is:

$$f(x) = x^2 - 2$$

$$2\text{-find the derivative of this function: } f'(x) = 2x$$

3-choose the starting point, in our example this value is given as follows:

$$x_0 = 0.5$$

$$4\text{-evaluate the function and its derivative at } x_0 : f(x_0) = (0.5)^2 - 2 = -1.75$$

$$f'(x_0) = (2 \times 0.5) = 1$$

5-Apply the newton Raphson iterative formula to find a first estimate

$$x_{i+1} = x_0 - \frac{f(x_0)}{f'(x_0)}$$

6-Repeat steps (4) and (5) until the estimate matches the desired number

Of decimal places:

$$f(x_1) = -1.75 \quad f'(x_1) = 1$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \quad x_2 = 1.5694$$

$$f(x_2) = 3.06245 \quad f'(x_2) = 4.5$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \quad x_3 = 1.4234$$

$$f(x_3) = 0.46301 \quad f'(x_3) = 3.1388$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} \quad x_4 = 1.4142$$

x_i	$f(x)$	$f'(x)$	$x_{i+1} = x_0 - \frac{f(x_0)}{f'(x_0)}$
0.5	-1.75	1	2.25
2.25	3.06245	4.5	1.5694
1.5694	0.46301	3.1388	1.4234
1.4234	0.0260	2.8468	1.4142

Newton-Raphson Method for two variables and two equations:

To adapt Newton-Raphson method to simultaneous equations, we proceed as follows: Let $\{x_0, y_0\}$ be an approximation to the solution $\{\lambda, \mu\}$ of (1) assuming that (f) and (g) are sufficiently differentiable, $f(x, y)$ about $\{x_0, y_0\}$ using Taylor series for function of two variables:

$$f(x, y) = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) + \dots$$

$$g(x, y) = g(x_0, y_0) + g_x(x_0, y_0)(x - x_0) + g_y(x_0, y_0)(y - y_0) + \dots$$

Assuming that $\{x_0, y_0\}$ sufficiently closed to $\{\lambda, \mu\}$ so that higher order terms can be neglected, we equate the expansion through linear terms to zero. This gives us the system.

$$f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) = f(x_0, y_0)$$

$$g_x(x_0, y_0)(x - x_0) + g_y(x_0, y_0)(y - y_0) = g(x_0, y_0)$$

Where it is understood that all function and derivatives in (*) are to be evaluated at $\{x_0, y_0\}$. We might then expect that the solution $\{x_1, y_1\}$ of (*) will be closer to the solution $\{\lambda, \mu\}$ than $\{x_0, y_0\}$. The solution of (*) by cramer's rule yields.

$$|x_1 - x_0| = \frac{\begin{vmatrix} -f & f_y \\ -g & g_y \end{vmatrix}}{\begin{vmatrix} f_x & f_y \\ g_x & g_y \end{vmatrix}} = \left[\frac{-fg_y + gf_y}{J(f,g)} \right]_{(x_0,y_0)}$$

$$|y_1 - y_0| = \frac{\begin{vmatrix} f_x & -f \\ g_x & -g \end{vmatrix}}{\begin{vmatrix} f_x & f_y \\ g_x & g_y \end{vmatrix}} = \left[\frac{-gf_x + fg_x}{J(f,g)} \right]_{(x_0,y_0)}$$

Provided that $J(f, g) = f_x g_y - g_x f_y \neq 0$ at $\{x_0, y_0\}$. The function $J(f, g)$ is called the Jacobin of the function f and g. the solution $\{x_1, y_1\}$ of this system now provides a new approximation to $\{\lambda, \mu\}$ repetition of this process leads to newton-raphson method for systems

$$x_{i+1} = x_i - \left[\frac{fg_y - gf_y}{J(f,g)} \right]_{(x_i,y_i)} ; y_{i+1} = y_i - \left[\frac{gf_x - fg_x}{J(f,g)} \right]_{(x_i,y_i)} \quad i=0,1$$

Where $J(f, g) = f_x g_y - g_x f_y$ and where all function involved are to be evaluated at $\{x_i, y_i\}$ stop iteration if $|x_{i+1} - x_i| < \varepsilon$ and $|y_{i+1} - y_i| < \varepsilon$ for any (i).

Example:- solve the system :

$$x^2 + y^2 = 1$$

$$x^2 - y^2 = -0.5$$

By using Newton-Raphson method at $\{x_0, y_0\} = \{0.1, 0.3\}$ with error $\leq 10^{-5}$.

Solution:

$$\text{Let } f(x, y) = x^2 + y^2 - 1 = 0$$

$$g(x, y) = x^2 - y^2 + 0.5 = 0$$

$$\therefore f'_x = 2x, f'_y = 2y, g'_x = 2x, g'_y = -2y$$

$$\text{At } \{x_0, y_0\} = \{0.1, 0.3\} \rightarrow f(0.1, 0.3) = -0.9, f_x = 0.2, f_y = 0.6, \\ g(0.1, 0.3) = 0.42, g_x = 0.2, g_y = -0.6$$

$$x_i = x_0 - \left[\frac{fg_y - gf_y}{J(f,g)} \right]_{(x_i, y_i)} = 0.1 - \left[\frac{(-0.9 \times -0.6) - (0.42 \times 0.6)}{(0.2 \times -0.6) - (0.6 \times 0.2)} \right] =$$

$$0.1 - \frac{0.288}{-0.24} = 1.3$$

$$y_i = y_0 - \left[\frac{gf_x - fg_x}{J(f,g)} \right]_{(x_i, y_i)} = 0.3 - \left[\frac{(0.42 \times 0.2) - (-0.9 \times 0.2)}{(0.2 \times -0.6) - (0.6 \times 0.2)} \right] =$$

$$0.3 - \frac{0.264}{-0.24} = 1.4$$

$$|1.3 - 0.1| > \varepsilon \text{ and } |1.4 - 0.3| > \varepsilon$$

i	x_i	y_i	max of error $_i$
1	0.808	0.8208	0.6208
2	0.93265766	0.93523598	0.12465766
3	0.97445167	0.97484219	0.041794002
4	0.98998734	0.99004899	0.015535669
5	0.99602719	0.996037	0.0060398584

Newton-Raphson Method for three variables and three equations:

Given a system of non-linear equations as:

$$\left. \begin{array}{l} f(x, y, z) = 0 \\ g(x, y, z) = 0 \\ w(x, y, z) = 0 \end{array} \right\} \dots \dots \dots \dots \dots \dots \dots \quad 1$$

To adapt Newton-Raphson method to simultaneous equations, we proceed as follows: Let $\{x_0, y_0, z_0\}$ be an approximation to the solution $\{\lambda, \mu, \theta\}$ of (1) assuming that (f) and (g) are sufficiently differentiable, $f(x, y, z)$ about $\{x_0, y_0, z_0\}$ using Taylor series for function of three variables:

$$f(x, y, z) = f(x_0, y_0, z_0) + f_x(x_0, y_0, z_0)(x - x_0) + f_y(x_0, y_0, z_0)(y - y_0) + f_z(x_0, y_0, z_0)(z - z_0) + \dots$$

$$g(x, y, z) = g(x_0, y_0, z_0) + g_x(x_0, y_0, z_0)(x - x_0) + g_y(x_0, y_0, z_0)(y - y_0) + g_z(x_0, y_0, z_0)(z - z_0) + \dots \quad (*)$$

$$w(x, y, z) = w(x_0, y_0, z_0) + w_x(x_0, y_0, z_0)(x - x_0) + w_y(x_0, y_0, z_0)(y - y_0) + w_z(x_0, y_0, z_0)(z - z_0) + \dots$$

Let $\{x_0, y_0, z_0\}$ sufficiently closed to $\{\lambda, \mu, \theta\}$ so that higher order terms can be neglected, we equate the expansion through linear terms to zero. This gives us the system.

$$f_x(x_0, y_0, z_0)(x - x_0) + f_y(x_0, y_0, z_0)(y - y_0) + f_z(x_0, y_0, z_0)(z - z_0) \\ = f(x_0, y_0, z_0)$$

$$g_x(x_0, y_0, z_0)(x - x_0) + g_y(x_0, y_0, z_0)(y - y_0) + g_z(x_0, y_0, z_0)(z - z_0) \\ = g(x_0, y_0, z_0)$$

$$w_x(x_0, y_0, z_0)(x - x_0) + w_y(x_0, y_0, z_0)(y - y_0) + w_z(x_0, y_0, z_0)(z - z_0) \\ = w(x_0, y_0, z_0)$$

Where it is understood that all function and derivatives in (*) are to be evaluated at $\{x_0, y_0, z_0\}$. We might then expect that the solution $\{x_1, y_1, z_1\}$ of (*) will be closer to the solution $\{\lambda, \mu, \theta\}$ than $\{x_0, y_0, z_0\}$. The solution of (*) by Cramer's rule yields.

$$\begin{aligned}
x_1 &= x_0 + \frac{\begin{vmatrix} -f & f_y & f_z \\ -g & g_y & g_z \\ -w & w_y & w_z \end{vmatrix}_{\{x_0, y_0, z_0\}}}{\begin{vmatrix} f_x & f_y & f_z \\ g_x & g_y & g_z \\ w_x & w_y & w_z \end{vmatrix}_{\{x_0, y_0, z_0\}}} \\
&= x_0 + \left[\frac{-fg_yw_z - wf_yg_z - gf_zw_y + wf_zg_y + gf_yw_z + fw_yg_z}{f_xg_yw_z + w_xf_yg_z + g_xf_zw_y - w_xf_zg_y - g_xf_yw_z - f_xw_yg_z} \right]_{\{x_0, y_0, z_0\}} \\
&= x_0 + \left[\frac{-fg_yw_z - wf_yg_z - gf_zw_y + wf_zg_y + gf_yw_z + fw_yg_z}{J(f, g, w)} \right]_{\{x_0, y_0, z_0\}} \\
y_1 &= y_0 + \frac{\begin{vmatrix} f_x & -f & f_z \\ g_x & -g & g_z \\ w_x & -w & w_z \end{vmatrix}_{\{x_0, y_0, z_0\}}}{\begin{vmatrix} f_x & f_y & f_z \\ g_x & g_y & g_z \\ w_x & w_y & w_z \end{vmatrix}_{\{x_0, y_0, z_0\}}} \\
&= y_0 + \left[\frac{-gf_xw_z - fw_xg_z - wg_xf_z + gw_xf_z + fg_xw_z + wf_xg_z}{f_xg_yw_z + w_xf_yg_z + g_xf_zw_y - w_xf_zg_y - g_xf_yw_z - f_xw_yg_z} \right]_{\{x_0, y_0, z_0\}} \\
&= y_0 + \left[\frac{-gf_xw_z - fw_xg_z - wg_xf_z + gw_xf_z + fg_xw_z + wf_xg_z}{J(f, g, w)} \right]_{\{x_0, y_0, z_0\}} \\
z_1 &+= z_0 + \frac{\begin{vmatrix} f_x & f_y & -f \\ g_x & g_y & -g \\ w_x & w_y & -w \end{vmatrix}_{\{x_0, y_0, z_0\}}}{\begin{vmatrix} f_x & f_y & f_z \\ g_x & g_y & g_z \\ w_x & w_y & w_z \end{vmatrix}_{\{x_0, y_0, z_0\}}}
\end{aligned}$$

$$\begin{aligned}
&= z_0 \\
&+ \left[\frac{-wf_xg_y - gw_xf_y - fg_xw_y + fw_xg_y + wg_xf_y + gf_xw_y}{f_xg_yw_z + w_xf_yg_z + g_xf_zw_y - w_xf_zg_y - g_xf_yw_z - f_xw_yg_z} \right]_{\{x_0, y_0, z_0\}} \\
&= z_0 + \left[\frac{-wf_xg_y - gw_xf_y - fg_xw_y + fw_xg_y + wg_xf_y + gf_xw_y}{J(f, g, w)} \right]_{\{x_0, y_0, z_0\}}
\end{aligned}$$

Provided that $J(f, g, w) \neq 0$ at $\{x_0, y_0, z_0\}$. The function $J(f, g, w)$ is called the Jacobin of the function f and g. the solution $\{x_1, y_1, z_1\}$ of this system now provides a new approximation to $\{\lambda, \mu, \theta\}$ repetition of this process leads to Newton-Raphson Method for systems

$$\begin{aligned}
&x_{i+1} = x_i \\
&- \left[\frac{fg_yw_z + wf_yg_z + gf_zw_y - wf_zg_y - gf_yw_z - fw_yg_z}{J(f, g, w)} \right]_{\{x_i, y_i, z_i\}}
\end{aligned}$$

$$\begin{aligned}
&y_{i+1} = y_i \\
&- \left[\frac{gf_xw_z + fw_xg_z + wg_xf_z - gw_xf_z - fg_xw_z - wf_xg_z}{J(f, g, w)} \right]_{\{x_i, y_i, z_i\}}
\end{aligned}$$

$$\begin{aligned}
&z_{i+1} = z_i \\
&+ \left[\frac{wf_xg_y + gw_xf_y + fg_xw_y - fw_xg_y - wg_xf_y - gf_xw_y}{J(f, g, w)} \right]_{\{x_i, y_i, z_i\}}
\end{aligned}$$

for $i = 0, 1, \dots$

We stop iteration if $|x_{i+1} - x_i| < \varepsilon$, $|y_{i+1} - y_i| < \varepsilon$ and $|z_{i+1} - z_i| < \varepsilon$ for any (i) .

Example 1 :- Solve the following nonlinear system

$$3x - \cos(yz) - \frac{1}{2} = 0$$

$$x^2 - 81(y + 0.1)^2 + \sin z + 1.06 = 0$$

$$e^{-xy} + 20z + \frac{10\pi - 3}{3} = 0$$

$$\{x_0, y_0, z_0\} = \{0.1, 0.1, -0.1\}$$

with *error* $\leq 10^{-8}$

Solution:- Define

$$f(x, y, z) = 3x - \cos(yz) - \frac{1}{2}$$

$$g(x, y, z) = x^2 - 81(y + 0.1)^2 + \sin(z) + 1.06$$

$$w(x, y, z) = e^{-xy} + 20z + \frac{10\pi - 3}{3}$$

The Jacobian matrix $J(f, g, w)$ for this system is

$$J(f, g, w) = \begin{bmatrix} 3 & x_3 \sin x_2 x_3 & x_2 \sin x_2 x_3 \\ 2x_1 & -162(x_2 + 0.1) & \cos x_3 \\ -x_2 e^{-x_1 x_2} & -x_1 e^{-x_1 x_2} & 20 \end{bmatrix}$$

$$J(f, g, w)_{\{x_0, y_0, z_0\}}$$

$$= \begin{bmatrix} 3 & 9.999833334 \times 10^{-4} & 9.999833334 \times 10^{-4} \\ 0.2 & -32.4 & 0.9950041653 \\ -0.09900498337 & -0.09900498337 & 20 \end{bmatrix}$$

The results using this iterative procedure are shown in the following table

i	x_i	y_i	z_i	max of error $_i$
1	0.4998696729	0.01946684854	-0.5215204719	0.6215
2	0.5000142402	0.00158859137	-0.5235569643	0.0179
3	0.5000001135	1.244478332e-05	-0.5235984501	0.0016
4	0.5	7.75785723e-10	-0.5235987756	1.2444e-05
5	0.5	-1.71571806e-17	-0.5235987756	7.7579e-10

Chapter 2

Program of Newton-Raphson Method for one variable and one equation:

```

clc
syms x
f=x^2-2;x0=0.5;e=0.01;k=0;
if subs(diff(f,x),x0)~=0
    x1=x0-(subs(f,x0)/subs(diff(f,x),x0));
    while abs(subs(f,x1))>=e
        k=k+1;
        x1=x0-(subs(f,x0)/subs(diff(f,x),x0));
        fprintf( '%5i          %g
',k,x1,subs(f,x1))
        x0=x1;
        if subs(diff(f,x),x0)==0,break,end
    end
end

```

Program of Newton-Raphson Method for two variables and two equations:

```
clc
clear
syms x y
F=0.1*x^2+0.1*y^2+0.8;
G=0.1*x+0.1*x*y^2+0.8;e=0.01;
x0=0.2;y0=0.2;i=1;
Fx=abs(subs(subs(diff(F,x),x0),y0));
Gx=abs(subs(subs(diff(G,x),x0),y0));
Fy=abs(subs(subs(diff(F,y),y0),x0));
Gy=abs(subs(subs(diff(G,y),y0),x0));
fprintf(' %1.8g      %1.8g      %1.8g
%1.8g\n',Fx,Gx,Fy,Gy)
if Fx+Gx>=1 || Fy+Gy>=1
    disp('Divergent'),else
        disp('Step      x          y')
        x1=subs(subs(F,x0),y0);
        y1=subs(subs(G,x0),y0);
        fprintf('%2i      %1.8g      %1.8g
%1.8g\n',i,x1,y1,max([abs(x1-x0),abs(y1-y0)]))
while abs(x1-x0)>=e || abs(y1-y0)>=e
    x0=x1;y0=y1;
    x1=subs(subs(F,x0),y0);
    y1=subs(subs(G,x0),y0);
    i=i+1;
    fprintf('%2i      %1.8g      %1.8g
%1.8g\n',i,x1,y1,max([abs(x1-x0),abs(y1-y0)]))
end
end
```

Program of Newton-Raphson Method for three variables and three equations:

```
clc
clear
syms x y z
F(x,y,z)=3*x-cos(y.*z)-1/2;
G(x,y,z)=x^2-81*(y+0.1)^2+sin(z)+1.06;
W(x,y,z)=exp(-x.*y)+20.*z+((10.*pi-3)/3);

e=10^(-8);X=[0.1;0.1;-0.1];
er=inf;
while abs(er)>=e
    FF=matlabFunction(F);
    GG=matlabFunction(G);
    WW=matlabFunction(W);
    Fx(x,y,z)=diff(F,x);Fxx=matlabFunction(Fx);
    Gx(x,y,z)=diff(G,x);Gxx=matlabFunction(Gx);
    Wx(x,y,z)=diff(W,x);Wxx=matlabFunction(Wx);
    Fy(x,y,z)=diff(F,y);Fyy=matlabFunction(Fy);
    Gy(x,y,z)=diff(G,y);Gyy=matlabFunction(Gy);
    Wy(x,y,z)=diff(W,y);Wyy=matlabFunction(Wy);
    Fz(x,y,z)=diff(F,z);Fzz=matlabFunction(Fz);
    Gz(x,y,z)=diff(G,z);Gzz=matlabFunction(Gz);
    Wz(x,y,z)=diff(W,z);Wzz=matlabFunction(Wz);

    Cr1=[FF(X(1),X(2),X(3)) Fyy(X(1),X(2),X(3))
        Fzz(X(1),X(2),X(3));
        GG(X(1),X(2),X(3)) Gyy(X(1),X(2),X(3))
        Gzz(X(1),X(2),X(3));
        WW(X(1),X(2),X(3)) Wyy(X(1),X(2),X(3))
        Wzz(X(1),X(2),X(3))];

    Cr2=[Fxx(X(1),X(2),X(3)) FF(X(1),X(2),X(3))
        Fzz(X(1),X(2),X(3));
        Gxx(X(1),X(2),X(3)) GG(X(1),X(2),X(3))
        Gzz(X(1),X(2),X(3));
```

```

Wxx(X(1),X(2),X(3)) WW(X(1),X(2),X(3))
Wzz(X(1),X(2),X(3))];

Cr3=[Fxx(X(1),X(2),X(3)) Fyy(X(1),X(2),X(3))
FF(X(1),X(2),X(3));
Gxx(X(1),X(2),X(3)) Gyy(X(1),X(2),X(3))
GG(X(1),X(2),X(3));
Wxx(X(1),X(2),X(3)) Wyy(X(1),X(2),X(3))
WW(X(1),X(2),X(3))];

J=[Fxx(X(1),X(2),X(3)) Fyy(X(1),X(2),X(3))
Fzz(X(1),X(2),X(3));
Gxx(X(1),X(2),X(3)) Gyy(X(1),X(2),X(3))
Gzz(X(1),X(2),X(3));
Wxx(X(1),X(2),X(3)) Wyy(X(1),X(2),X(3))
Wzz(X(1),X(2),X(3))];

XX(1)=X(1)-(det(Cr1)/det(J));
XX(2)=X(2)-(det(Cr2)/det(J));
XX(3)=X(3)-(det(Cr3)/det(J));
disp('X=')
for k=1:length(XX)
    fprintf('%1.10g\n',XX(k))
end
er=max(abs(XX-X))
X=XX;
end

```

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