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Runge-Kutta Methods

Research project

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Abstract

The Runge-Kutta method is a suitable and the most used method with less computational steps and accurate calculation. The Runge-Kutta method is the generalized form of the Euler method which is used for numerical solutions of ordinary differential equations. The numerical solutions of ordinary differential equations are solved by and Runge-Kutta methods and then their exact solutions are compared using tables.

Keywords: Ordinary Differential Equations, Numerical Solution of Equations, Exact Solution of Equations, Taylor’s Method, Runge-Kutta Method.

Introduction

First we talked about Runge Kutta method. We only talked about four types of range Kutta they were (RK1,RK2,RK3,RK4) first we found the exact price by first order linear differential equation then we talked about the orders by our clear (Taylor series). We have also used an example for orders. We have found the rules for order (RK1) and explained them. Then we have explained the rules for order (RK2) and explained the rules for orders (3,4) in the same way and Then we wrote code for all four orders and finally the final price of the last y of the (RK1,RK2,RK3,RK4) We compare it to the exact price and we find that y in (RK4) is closest to the exact price , Then we find the error. We find that the price of the order (RK4) has the least error. It is the best way for us to use it.

Chapter 1

Some important definitions

- **Taylor series:** is a representation of a function as an infinite sum of terms that are calculated from the values of the function's derivatives at a single point. It is named after the mathematician Brook Taylor.
- **Numerical methods:** - numerical methods, are techniques by Which Mathematical Problems are formulated So that they can be solved with arithmetic and logical operations Because digital computers excel at Performing such operation numerical method are sometimes referred to as computer mathematics.
- **numerical analysis:** - uses algorithms (asset of rules) to approximate solutions to mathematical Problems which cannot be solved analytically. or fit mathematical model to experimental data
- **error:** - is an action which means mistake in analytical chemistry error is a difference between standard value and observed.
- **exact solution:** - the exact solution of a system of equations can be found using algebraic method one such method is called substitution.

Exact

We have the differential equation $\frac{dy}{dx}$ = 3x + y , y(0)=0 with initial condition and want to solve it

$\frac{dy}{dx} = 3x + y$

$\frac{dy}{dx} - y = 3x$ **First Order l.DE**

$Y = e^{\int dx}\,[\int 3x\ \ e^{-\int dx}\,dx + 3]$

$= e^{x}\ [3\int x\, e^{-x}\,dx + c]$

$= e^{x}\,[3(-x\, e^{-x} + \int e^{-x}) + c]$

$= e^{x}\,[3(-xe^{-x} - e^{-x}) + c]$

$Y = -3x - 3 + ce^{x}$ g.solution

$0 = 0 - 3 + c \Rightarrow c = 3$

$Y = -3x - 3 + 3e^{x}$ p.solution

$Y(0.2) = -3(0.2) - 3 + 3e^{0.2}$

$= 0.064208274$

The Runge – Kutta method

The Runge-Kutta (R-K) technique is an efficient and commonly used approach for solving initial-value problems of differential equations.

It's used to generate high-order accurate numerical methods addresses Euler method challenge in selection a sufficiently short step size to provide satisfactory accuracy in problem resolution

Formula

Consider an ordinary differential equation $\frac{dy}{dx} = f(x, y)$with the initial condition $y(x_0) = y_0$. The formula for Runge-Kutta methods are define as follows

1st order R-K method

The formula is defined as follows.

Y1 = y0 + hf(X0,Y0) = U=y0 +h.y 0

This equation is equivalent to Euler's method.

The first order Runge-Kutta method (RK1)

Consider the initial value problem, the first order.

$$y'(x, y) = f(x, y); \quad y(x_0) = y_0 \tag{1}$$

where $\boldsymbol{y'}$ is the first order differential equation; $\boldsymbol{f(x, y)}$ is the function of $\boldsymbol{x}$ and $\boldsymbol{y}$; $\boldsymbol{y}$ is the solution to the differential equation in equation **(1)** at $\boldsymbol{x_0}$ given as $\boldsymbol{y_0}$; $\boldsymbol{y_0}$ is the value of y obtained at $\boldsymbol{x_0}$ and $\boldsymbol{x_0}$ is the point for which $\boldsymbol{y}$ is obtained as $\boldsymbol{y_0}$, we expand $\boldsymbol{y(x)}$ about $\boldsymbol{x_0}$

$$y(x) = y_0 + \frac{y'(x-x_0)}{1!} + \frac{y''(x-x_0)^2}{2!} + \frac{y'''(x-x_0)^3}{3!} + \cdots + \frac{y^n(x-x_0)^n}{n!} \tag{2}$$

Thus

$$y(x_1) = y(x_0 + h); \quad h = x_1 - x_0 \tag{3}$$

$$y(x_1) = y(x_0 + h) = y(x_0) + \frac{hy'(x_0)}{1!} + \frac{h^2y''(x_0)}{2!} + \frac{h^3y'''(x_0)}{3!} + \cdots + \frac{h^ny^n(x_0)}{n!} \tag{4}$$

Let $n = 1$

$k_1 = y'(x_i)$

$$y(x_1) = y(x_0) + hy'(x_0) \tag{5}$$

equation (5) is the same as

$$y_1 = y_0 + hf(x_0, y_0) \tag{6}$$

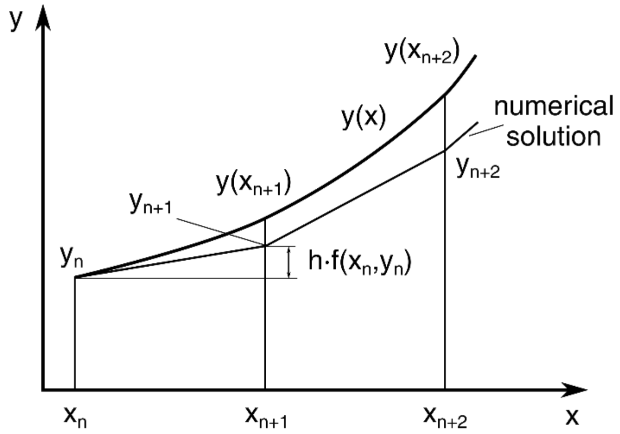
$$y_2 = y_1 + hf(x_1, y_1) \tag{7}$$

$$y_3 = y_2 + hf(x_2, y_2) \tag{8a}$$

$$y_{n+1} = y_n + hf(x_n, y_n) \tag{8b}$$

$$k_1 = y'(x_i) = f(x_i, y_i)$$

$$y_{i+1} = y_i + hk_1$$



find y(0.2) for *y′*=3*x*+*y*, *x*0=0,*y*0=0, with step length 0.04 using Euler method (1st order derivative)

Solution:

Given $y'=3x+y$, $y(0)=0$, $h=0.04$, $y(0.2)=?$

Euler Method

$y1=y0+hf(x0,y0)=0+(0.04)f(0,0)=0+(0.04)\cdot(0)=0+(0)=0$

$y2=y1+hf(x1,y1)=0+(0.04)f(0.04,0)=0+(0.04)\cdot(0.12)=0+(0.0048)=0.0048$

$y3=y2+hf(x2,y2)=0.0048+(0.04)f(0.08,0.0048)=0.0048+(0.04)\cdot(0.2448)=0.0048+(0.009792)=0.014592$

$y4=y3+hf(x3,y3)=0.014592+(0.04)f(0.12,0.014592)=0.014592+(0.04)\cdot(0.374592)=0.014592+(0.01498368)=0.02957568$

$y5=y4+hf(x4,y4)=0.02957568+(0.04)f(0.16,0.02957568)=0.02957568+(0.04)\cdot(0.50957568)=0.02957568+(0.020383027)=0.049958707$

$\therefore y(0.2)=0.049958707$

ODE

$y'(x) = f(x, y) = 3x + y(x)$

$y(x) = -3\,x - 3 + e^x\, c_1$

$y(0) = 0$

$y(x) = -3\,x - 3 + e^x$

$y(0.2) = 0.064208274$

```
syms x y Y(t)
format longg
f=3*x+y;
xx(1)=0;   yy(1)=0;   h=0.04;   xxn=0.2;     n=(xxn-xx(1))/h;
for i=1:n
    k1=subs(f,[x,y],[xx(i),yy(i)]);
    yy(i+1)=yy(i)+h*k1;
    xx(i+1)=xx(i)+h;
end
R_K_1=[xx' yy']
disp('-------ODE------------------')

ode= diff(Y,t) == 3*t+Y;
a=Y(xx(1))==yy(1);
S=subs(dsolve(ode,a),xxn);
fprintf('%1.10g\n',S)

disp('-------Err------------------')

en=abs(S-yy(n+1));
fprintf('%1.10g\n',en)
```

The second order Runge Kutta method (RK2)

The second _order Runge kutta method , also know as the midpoint ,is a numerical

Technique used for solving ordinary differential equations(ODE).it is given by the following formulas:

$$y(x) = y_0 + \frac{y'(x-x_0)}{1!} + \frac{y''(x-x_0)^2}{2!} + \frac{y'''(x-x_0)^3}{3!} + \cdots + \frac{y^n(x-x_0)^n}{n!}$$

$y_{(n+1)}=y(x_n)+(x_{n+1}+x_0)\frac{dy}{dx}+\frac{(x_{(n+1)}+x0)^2}{2!}\frac{d^2y}{dx^2}+0(h^3)$

$h = x_{n+1} - x_0, \frac{dy}{dx} = f(x,y)$

$y_n+hf(x_0,y_0)+\frac{h^2}{2}(\frac{\partial f}{\partial x}+ (\frac{\partial f}{\partial y}) f)+0(h^3)$

$\frac{d^2y}{dx^2} \approx \frac{\partial}{\partial x}\left(\frac{dy}{dx}\right) + \frac{\partial}{\partial y}\left(\frac{dy}{dx}\right).\frac{dy}{dx}$

$K2 = h\left(x_n + \frac{h}{2}, y_n + \frac{k1}{2}\right)$

$K1=h.f(x_n ,y_n)$
$K2=h.f(x_n+h/2,y_n+k1/2)$
$y_{n+1}=y_n +k2$
where
*k1 and k2 are intermediate slops
*y_{n+1} is the approximation of the solution at the next step
*$f(x_n,y_n)$ is the derivative of the function at the nth step
*h is the step size

Find y(0.2) for $y'=3x+y$, $x0=0,y0=0$, with step length 0.04 using Runge-Kutta 2 method

Solution:
Given $y'=3x+y$, $y(0)=0$, $h=0.04$, **$y(0.2)=?$**

Method-1 : Using formula $k2=hf(x0+h,y0+k1)$

Second order R-K method
$k1=hf(x0,y0)=(0.04)f(0,0)=(0.04)\cdot(0)=0$

$k2=hf(x0+h,y0+k1)=(0.04)f(0.04,0)=(0.04)\cdot(0.12)=0.0048$

$y1=y0+\frac{k_1+k_2}{2}=0+0.0024=0.0024$

$\therefore y(0.04)=0.0024$

Again taking $(x1,y1)$ in place of $(x0,y0)$ and repeat the process

$k1=hf(x1,y1)=(0.04)f(0.04,0.0024)=(0.04)\cdot(0.1224)=0.004896$

$k2=hf(x1+h,y1+k1)=(0.04)f(0.08,0.007296)=(0.04)\cdot(0.247296)=0.00989184$

$y2=y1+\frac{k1+k2}{2}=0.0024+0.00739392=0.00979392$

$\therefore y(0.08)=0.00979392$

Again taking $(x2,y2)$ in place of $(x1,y1)$ and repeat the process

$k1=hf(x2,y2)=(0.04)f(0.08,0.00979392)=(0.04)\cdot(0.24979392)=0.009991757$

$k2=hf(x2+h,y2+k1)=(0.04)f(0.12,0.019785677)=(0.04)\cdot(0.379785677)=0.015191427$

$y3=y2+\frac{k1+k2}{2}=0.00979392+0.012591592=0.022385512$
$\therefore y(0.12)=0.022385512$
Again taking $(x3,y3)$ in place of $(x2,y2)$ and repeat the process

$k1=hf(x3,y3)=(0.04)f(0.12,0.022385512)=(0.04)\cdot(0.382385512)=0.01529542$

$k2=hf(x3+h,y3+k1)=(0.04)f(0.16,0.037680932)=(0.04)\cdot(0.517680932)=0.020707237$

$y4=y3+\frac{k_1+k_2}{2}=0.022385512+0.018001329=0.040386841$

$\therefore y(0.16)=0.040386841$

Again taking ($x4$,$y4$) in place of ($x3$,$y3$) and repeat the process

$k1=hf(x4,y4)=(0.04)f(0.16,0.040386841)=(0.04)\cdot(0.520386841)=0.020815474$

$k2=hf(x4+h,y4+k1)=(0.04)f(0.2,0.061202314)=(0.04)\cdot(0.661202314)=0.026448093$

$y5=y4+\frac{k_1+k_2}{2}=0.040386841+0.023631783=0.064018624$

$\therefore y(0.2)=0.064018624$

Code

```
syms x y Y(t)
format longg
f=3*x+y;
xx(1)=0;   yy(1)=0;   h=0.04;   xxn=0.2;     n=(xxn-xx(1))/h;
for i=1:n
    k1=h*subs(f,[x,y],[xx(i),yy(i)]);
    k2=h*subs(f,[x,y],[xx(i)+h,yy(i)+k1]);
    yy(i+1)=yy(i)+(k1+k2)/2;
    xx(i+1)=xx(i)+h;
    fprintf('%1.10f   %1.10f\n',k1,k2)
end
R_K_1=[xx' yy']
disp('-------ODE------------------')

ode= diff(Y,t) == 3*t+Y;
a=Y(xx(1))==yy(1);
S=subs(dsolve(ode,a),xxn);
fprintf('%1.10g\n',S)

disp('-------Err------------------')

en=abs(S-yy(n+1));
fprintf('%1.10g\n',en)
```

Runge Kutta method order 3 (RK3)

The third -order Runge kutta method (RK3)is a numerical method used for solving ordinary differential equations (ODE)it is an extension of the second -order Runge -kutta method and provides higher accuracy.the RK3 method is given by following formulas :

$K1=h.f(x_0 , y_0)$

$K2=h.f(x_0 +\frac{h}{2} , y_0+\frac{k1}{2})$

$K3=h.f(x_0+h,y_0+2k2-k1)$

$y_{n+1}=y_n+\frac{1}{6}(k1+4k2+k3)$

*k1,k2 and k3 are intermediate slops
*y_{n+1} is the approximation of the solution at the next step
*$f(x_n,y_n)$ is the derivative of the function at the nth step
*h is the step size

Find y(0.2) for $y'=3x+y$, $x0=0,y0=0$, with step length 0.04 using Runge-Kutta 3 method
Solution:
Given $y'=3x+y$, $y(0)=0$, $h=0.04$, $y(0.2)=?$

Third order R-K method
$k1=hf(x0,y0)=(0.04)f(0,0)=(0.04)\cdot(0)=0$

$k2=hf(x0+h/2,y0+k1/2)=(0.04)f(0.02,0)=(0.04)\cdot(0.06)=0.0024$

$k3=hf(x0+h,y0+2k2-k1)=(0.04)f(0.04,0.0048)=(0.04)\cdot(0.1248)=0.004992$

$y1=y0+\frac{1}{6}(k1+4k2+k3)$

$y1=0+\frac{1}{6}[0+4(0.0024)+(0.004992)]$

$y1=0.002432$

$\therefore y(0.04)=0.002432$

Again taking $(x1,y1)$ in place of $(x0,y0)$ and repeat the process

$k1=hf(x1,y1)=(0.04)f(0.04,0.002432)=(0.04)\cdot(0.122432)=0.00489728$

$k2=hf(x1+h/2,y1+k1/2)=(0.04)f(0.06,0.00488064)=(0.04)\cdot(0.18488064)=0.007395226$

$k3=hf(x1+h,y1+2k2-k1)=(0.04)f(0.08,0.012325171)=(0.04)\cdot(0.252325171)=0.010093007$

$y2=y1+\frac{1}{6}(k1+4k2+k3)$

$y2=0.002432+\frac{1}{6}[0.00489728+4(0.007395226)+(0.010093007)]$

$y2=0.009860532$

$\therefore y(0.08)=0.009860532$

Again taking $(x2,y2)$ in place of $(x1,y1)$ and repeat the process

$k1=hf(x2,y2)=(0.04)f(0.08,0.009860532)=(0.04)\cdot(0.249860532)=0.009994421$

$k2=hf(x2+h/2,y2+k1/2)=(0.04)f(0.1,0.014857742)=(0.04)\cdot(0.314857742)=0.01259431$

$k3=hf(x2+h,y2+2k2-k1)=(0.04)f(0.12,0.02505473)=(0.04)\cdot(0.38505473)=0.015402189$

$y_3 = y_2 + \frac{1}{6}(k_1 + 4k_2 + k_3)$

$y_3 = 0.009860532 + \frac{1}{6}[0.009994421 + 4(0.01259431) + (0.015402189)]$

$y_3 = 0.022489506$

$\therefore y(0.12) = 0.022489506$

Again taking (x_3, y_3) in place of (x_2, y_2) and repeat the process

$k_1 = hf(x_3, y_3) = (0.04)f(0.12, 0.022489506) = (0.04) \cdot (0.382489506) = 0.01529958$

$k_2 = hf(x_3 + h/2, y_3 + k_1/2) = (0.04)f(0.14, 0.030139297) = (0.04) \cdot (0.450139297) = 0.018005572$

$k_3 = hf(x_3 + h, y_3 + 2k_2 - k_1) = (0.04)f(0.16, 0.04320107) = (0.04) \cdot (0.52320107) = 0.020928043$

$y_4 = y_3 + \frac{1}{6}(k_1 + 4k_2 + k_3)$

$y_4 = 0.022489506 + \frac{1}{6}[0.01529958 + 4(0.018005572) + (0.020928043)]$

$y_4 = 0.040531158$

$\therefore y(0.16) = 0.040531158$

Again taking (x_4, y_4) in place of (x_3, y_3) and repeat the process

$k_1 = hf(x_4, y_4) = (0.04)f(0.16, 0.040531158) = (0.04) \cdot (0.520531158) = 0.020821246$

$k_2 = hf(x_4 + h/2, y_4 + k_1/2) = (0.04)f(0.18, 0.050941781) = (0.04) \cdot (0.590941781) = 0.023637671$

$k_3 = hf(x_4 + h, y_4 + 2k_2 - k_1) = (0.04)f(0.2, 0.066985254) = (0.04) \cdot (0.666985254) = 0.02667941$

$y_5 = y_4 + \frac{1}{6}(k_1 + 4k_2 + k_3)$

$y_5 = 0.040531158 + \frac{1}{6}[0.020821246 + 4(0.023637671) + (0.02667941)]$

$y_5 = 0.064206382$

$\therefore y(0.2) = 0.064206382$

```
syms x y Y(t)
format longg
f=3*x+y;
xx(1)=0;   yy(1)=0;   h=0.04;   xxn=0.2;     n=(xxn-xx(1))/h;
for i=1:n
    k1=h*subs(f,[x,y],[xx(i),yy(i)]);
    k2=h*subs(f,[x,y],[xx(i)+h/2,yy(i)+k1/2]);
    k3=h*subs(f,[x,y],[xx(i)+h,yy(i)+2*k2-k1]);
    yy(i+1)=yy(i)+(k1+4*k2+k3)/6;
    xx(i+1)=xx(i)+h;
    fprintf('%1.10f   %1.10f   %1.10f\n',k1,k2,k3)
end
R_K_1=[xx' yy']
disp('-------ODE------------------')

ode= diff(Y,t) == 3*t+Y;
a=Y(xx(1))==yy(1);
S=subs(dsolve(ode,a),xxn);
fprintf('%1.10g\n',S)

disp('-------Err------------------')

en=abs(S-yy(n+1));
fprintf('%1.10g\n',en)
```

Runge kutta method order 4 (RK4)

RK4 stands for "Fourth-order Runge-Kutta method." It's a numerical method used for solving ordinary differential equations (ODEs). RK4 is a popular choice due to its simplicity and relatively high accuracy. It approximates the solution of an ODE

$K1=h.f(x_0 , y_0)$

$K2=h.f(x_0 +\frac{h}{2} , y_0+\frac{k1}{2})$

$K3=h.f(x_0 +\frac{h}{2} , y_0+\frac{k2}{2})$

$K4= h.f(x_0+h,y_0+k3)$

$y_{(n+1)}=y_n +\frac{1}{6} (k1+2k2+2k3+k4)$

*K1,K2,K3and k4 are intermediate slops
*y_{n+1} is the approximation of the solution at the next step
*$f(x_n,y_n)$ is the derivative of the function at the nth step
*h is the step size

Find y(0.2) for $y'=3x+y$, $x0=0, y0=0$, with step length 0.04 using Runge-Kutta 4 method (1st order derivative)

Solution:

Given $y'=3x+y$, $y(0)=0$, $h=0.04$, **$y(0.2)=?$**

Fourth order R-K method

$k1=hf(x0,y0)=(0.04)f(0,0)=(0.04)\cdot(0)=0$

$k2=hf(x0+h/2,y0+k_1/2)=(0.04)f(0.02,0)=(0.04)\cdot(0.06)=0.0024$

$k3=hf(x0+h/2,y0+k_2/2)=(0.04)f(0.02,0.0012)=(0.04)\cdot(0.0612)=0.002448$

$k4=hf(x0+h,y0+k3)=(0.04)f(0.04,0.002448)=(0.04)\cdot(0.122448)=0.00489792$

$y1=y0+\frac{1}{6}(k1+2k2+2k3+k4)$

$y1=0+\frac{1}{6}[0+2(0.0024)+2(0.002448)+(0.00489792)]$

$y1=0.00243232$

$\therefore y(0.04)=0.00243232$

Again taking $(x1,y1)$ in place of $(x0,y0)$ and repeat the process

$k1=hf(x1,y1)=(0.04)f(0.04,0.00243232)=(0.04)\cdot(0.12243232)=0.004897293$

$k2=hf(x1+h/2,y1+k1/2)=(0.04)f(0.06,0.004880966)=(0.04)\cdot(0.184880966)=0.007395239$

$k3=hf(x1+h/2,y1+k_2/2)=(0.04)f(0.06,0.006129939)=(0.04)\cdot(0.186129939)=0.007445198$

$k4=hf(x1+h,y1+k3)=(0.04)f(0.08,0.009877518)=(0.04)\cdot(0.249877518)=0.009995101$

$y2=y1+\frac{1}{6}(k1+2k2+2k3+k4)$

$y2=0.00243232+\frac{1}{6}[0.004897293+2(0.007395239)+2(0.007445198)+(0.009995101)]$

$y2=0.009861198$

$\therefore y(0.08)=0.009861198$

Again taking $(x2,y2)$ in place of $(x0,y0)$ and repeat the process

$k1=hf(x2,y2)=(0.04)f(0.08,0.009861198)=(0.04)\cdot(0.249861198)=0.009994448$

$k2=hf(x2+h/2,y2+k1/2)=(0.04)f(0.1,0.014858422)=(0.04)\cdot(0.314858422)=0.012594337$

$k3=hf(x2+h/2,y2+k2/2)=(0.04)f(0.1,0.016158366)=(0.04)\cdot(0.316158366)=0.012646335$

$k4=hf(x2+h,y2+k3)=(0.04)f(0.12,0.022507532)=(0.04)\cdot(0.382507532)=0.015300301$

$y3=y2+\frac{1}{6}(k1+2k2+2k3+k4)$

$y3=0.009861198+\frac{1}{6}[0.009994448+2(0.012594337)+2(0.012646335)+(0.015300301)]$

$y3=0.02249054$
$\therefore y(0.12)=0.0224905466$

Again taking $(x3,y3)$ in place of $(x0,y0)$ and repeat the process

$k1=hf(x3,y3)=(0.04)f(0.12,0.022490546)=(0.04)\cdot(0.382490546)=0.015299622$

$k2=hf(x3+h/2,y3+k1/2)=(0.04)f(0.14,0.030140357)=(0.04)\cdot(0.450140357)=0.018005614$

$k3=hf(x3+h/2,y3+k2/2)=(0.04)f(0.14,0.031493354)=(0.04)\cdot(0.451493354)=0.018059734$

$k4=hf(x3+h,y3+k3)=(0.04)f(0.16,0.040550281)=(0.04)\cdot(0.520550281)=0.020822011$

$y4=y3+\frac{1}{6}(k1+2k2+2k3+k4)$

$y4=0.022490546+\frac{1}{6}[0.015299622+2(0.018005614)+2(0.018059734)+(0.020822011)]$

$y4=0.040532601$

$\therefore y(0.16)=0.040532601$

Again taking $(x4,y4)$ in place of $(x0,y0)$ and repeat the process

$k1=hf(x4,y4)=(0.04)f(0.16,0.040532601)=(0.04)\cdot(0.520532601)=0.020821304$

$k2=hf(x4+h/2,y4+k1/2)=(0.04)f(0.18,0.050943253)=(0.04)\cdot(0.590943253)=0.02363773$

$k3=hf(x4+h/2,y4+k2/2)=(0.04)f(0.18,0.052351466)=(0.04)\cdot(0.592351466)=0.023694059$

$k4=hf(x4+h,y4+k3)=(0.04)f(0.2,0.06422666)=(0.04)\cdot(0.66422666)=0.026569066$

$y5=y4+\frac{1}{6}(k1+2k2+2k3+k4)$

$y5=0.040532601+\frac{1}{6}[0.020821304+2(0.02363773)+2(0.023694059)+(0.026569066)]$

$y5=0.064208259$

$\therefore y(0.2)=0.064208259$

```
syms x y Y(t)
format longg
f=3*x+y;
xx(1)=0;   yy(1)=0;   h=0.04;   xxn=0.2;     n=(xxn-xx(1))/h;
for i=1:n
    k1=h*subs(f,[x,y],[xx(i),yy(i)]);
    k2=h*subs(f,[x,y],[xx(i)+h/2,yy(i)+k1/2]);
    k3=h*subs(f,[x,y],[xx(i)+h/2,yy(i)+k2/2]);
    k4=h*subs(f,[x,y],[xx(i)+h,yy(i)+k3]);
    yy(i+1)=yy(i)+(k1+2*k2+2*k3+k4)/6;
    xx(i+1)=xx(i)+h;
    fprintf('%1.10f   %1.10f   %1.10f   %1.10f\n',k1,k2,k3,k4)
end
R_K_1=[xx' yy']
disp('-------ODE------------------')

ode= diff(Y,t) == 3*t+Y;
a=Y(xx(1))==yy(1);
S=subs(dsolve(ode,a),xxn);
fprintf('%1.10g\n',S)

disp('-------Err------------------')

en=abs(S-yy(n+1));
fprintf('%1.10g\n',en)
```

Conclusion

Runge kutta	Conclusion	Exact	Error
RK1	0.049958707	0.064208274	0.014249567
RK2	0.064018624	0.064208274	0.00018965
RK3	0.064206382	0.064208274	0.000001892
RK4	0.064208259	0.064208274	0.000000015

So we came to the conclusion that to solving O.D.E ،we need to use the Range Kuta order 4

Because we are getting exactly close to the exact price

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