Vector Analysis

1- Given points $M(-1, 2, 1)$, $N(3, -3, 0)$, and $P(-2, -3, -4)$, find:
(a) $\mathbf{R}_{MN}$; (*b*) $\mathbf{R}_{MN} + \mathbf{R}_{MP}$; (*c*) $|\mathbf{r}_M|$; (*d*) $\mathbf{a}_{MP}$; (*e*) $|2\mathbf{r}_P - 3\mathbf{r}_N|$.

2- The three vertices of a triangle are located at $A(6, -1, 2)$, $B(-2, 3, -4)$, and $C(-3, 1, 5)$. Find: (*a*) $\mathbf{R}_{AB}$; (*b*) $\mathbf{R}_{AC}$; (*c*) the angle θ_{BAC} at vertex A; (*d*) the (vector) projection of $\mathbf{R}_{AB}$ on $\mathbf{R}_{AC}$.

3− The three vertices of a triangle are located at $A(6, -1, 2)$, $B(-2, 3, -4)$, and $C(-3, 1, 5)$. Find: (*a*) $\mathbf{R}_{AB} \times \mathbf{R}_{AC}$; (*b*) the area of the triangle; (*c*) a unit vector perpendicular to the plane in which the triangle is located.

4- Transform to cylindrical coordinates: (*a*) $\mathbf{F} = 10\mathbf{a}_x - 8\mathbf{a}_y + 6\mathbf{a}_z$ at point $P(10, -8, 6)$; (*b*) $\mathbf{G} = (2x+y)\mathbf{a}x - (y-4x)\mathbf{a}y$ at point $Q(\rho, \phi, z)$. (*c*) Give the rectangular components of the vector $\mathbf{H} = 20\mathbf{a}_\rho - 10\mathbf{a}_\phi + 3\mathbf{a}_z$ at $P(x = 5, y = 2, z = -1)$.

5− Transform the following vectors to spherical coordinates at the points given: (*a*) $10\mathbf{a}_x$ at $P(x = -3, y = 2, z = 4)$; (*b*) $10\mathbf{a}_y$ at $Q(\rho = 5, \phi = 30^\circ, z = 4)$; (*c*) $10\mathbf{a}_z$ at $M(r = 4, \theta = 110^\circ, \phi = 120^\circ)$.

Coulomb's Law and Electric Field Intensity

6- A charge $Q_A = -20\,\mu C$ is located at $A(-6, 4, 7)$, and a charge $Q_B = 50\,\mu C$ is at $B(5, 8, -2)$ in free space. If distances are given in meters, find: **(a)** $\mathbf{R}_{AB}$; (*b*) R_{AB}. Determine the vector force exerted on Q_A by Q_B if $\epsilon_o =$ (*c*) $10^{-9}/(36\pi)$ F/m; (*d*) 8.854×10^{-12} F/m.

7- A charge of $-0.3\,\mu C$ is located at $A(25, -30, 15)$ (in cm), and a second charge of $0.5\,\mu C$ is at $B(-10, 8, 12)$ cm. Find $\mathbf{E}$ at: (*a*) the origin; (*b*) $P(15, 20, 50)$ cm.

8- Calculate the total charge within each of the indicated volumes: (a) $0.1 \le |x|, |y|, |z| \le 0.2$: $\rho_\nu = 1/(x^3 y^3 z^3)$; ($b$) $0 \le \rho \le 0.1$, $0 \le \phi \le \pi$, $2 \le z \le 4$; $\rho_\nu = \rho^2 z^2 \sin 0.6\phi$; ($c$) universe: $\rho_\nu = e^{-2r}/r^2$.

9- Infinite uniform line charges of 5 nC/m lie along the (positive and negative) x and y axes in free space. Find **E** at: (a) $P_A(0, 0, 4)$; (b) $P_B(0, 3, 4)$.

10− Three infinite uniform sheets of charge are located in free space as follows: 3 nC/m^2 at $z = -4$, 6 nC/m^2 at $z = 1$, and −8 nC/m2 at $z = 4$. Find **E** at the point: (a) $P_A(2, 5, -5)$; (b) $P_B(4, 2, -3)$; (c) $P_C(-1, -5, 2)$; (d) $P_D(-2, 4, 5)$.

Electric Flux Density, Gauss's Law, and Divergence

11− Given a 60-μC point charge located at the origin, find the total electric flux passing through: (a) that portion of the sphere $r = 26$ cm bounded by $0 < \theta < \pi/2$ and $0 < \phi < \pi/2$; (b) the closed surface defined by $\rho = 26$ cm and $z = \pm 26$ cm; (c) the plane $z = 26$ cm.

12- Calculate **D** in rectangular coordinates at point $P(2, -3, 6)$ produced by: (a) a point charge $Q_A = 55$ mC at $Q(-2, 3, -6)$; (b) a uniform line charge $\rho_{LB} = 20$ mC/m on the x axis; (c) a uniform surface charge density $\rho_{SC} = 120$ μC/m^2 on the plane $z = -5$ m.

13- Given the electric flux density, $\mathbf{D} = 0.3r^2\mathbf{a}_r$ nC/m^2 in free space:
(a) find **E** at point $P(r = 2, \theta = 25^\circ, \phi = 90^\circ)$; ($b$) find the total charge within the sphere $r = 3$; (c) find the total electric flux leaving the sphere $r = 4$.

14- A point charge of 0.25 μC is located at $r = 0$, and uniform surface charge densities are located as follows: 2 mC/m^2 at $r = 1$ cm, and −0.6 mC/m^2 at $r = 1.8$ cm. Calculate **D** at: (a) $r = 0.5$ cm; (b) $r = 1.5$ cm; (c) $r = 2.5$ cm. (d) What uniform surface charge density should be established at $r = 3$ cm to cause $\mathbf{D} = 0$ at $r = 3.5$ cm?

15- Determine an expression for the volume charge density associated with each **D** field: (*a*) $\mathbf{D} = 4xy/z\mathbf{a}_x + 2x^2/z\mathbf{a}_y - 2x^2 y/z^2 \mathbf{a}_z$; (*b*) $\mathbf{D} = z \sin\phi\ \mathbf{a}_\rho + z \cos\phi\ \mathbf{a}_\phi + \rho \sin\phi\ \mathbf{a}_z$; (*c*) $\mathbf{D} = \sin\theta \sin\phi\ \mathbf{a}_r + \cos\theta \sin\phi\ \mathbf{a}_\theta + \cos\phi\ \mathbf{a}_\phi$.

16− Given the field $\mathbf{D} = 6\rho \sin(½)\phi\ \mathbf{a}_\rho + 1.5\rho \cos(½)\phi\ \mathbf{a}_\phi$ C/m^2, evaluate both sides of the divergence theorem for the region bounded by $\rho = 2$, $\phi = 0$, $\phi = \pi$, $z = 0$, and $z = 5$.

Energy and Potential

17- Calculate the work done in moving a 4-C charge from $B(1, 0, 0)$ to $A(0, 2, 0)$ along the path $y = 2 - 2x$, $z = 0$ in the field $\mathbf{E} =$ (*a*) $5\mathbf{a}_x$V/m; (*b*) $5x\mathbf{a}_x$V/m; (*c*) $5x\mathbf{a}_x + 5y\mathbf{a}_y$V/m.

18- An electric field is expressed in rectangular coordinates by $\mathbf{E} = 6x^2\mathbf{a}_x + 6y\mathbf{a}_y + 4\mathbf{a}_z$V/m. Find: (*a*) V_{MN} if points M and N are specified by $M(2, 6, -1)$ and $N(-3, -3, 2)$; (*b*) V_M if $V = 0$ at $Q(4, -2, -35)$; (*c*) V_N if $V = 2$ at $P(1, 2, -4)$.

THE POTENTIAL FIELD OF A SYSTEM OF CHARGES: CONSERVATIVE PROPERTY

19- If we take the zero reference for potential at infinity, find the potential at $(0, 0, 2)$ caused by this charge configuration in free space (*a*) 12 nC/m on the line $\rho = 2.5$ m, $z = 0$; (*b*) point charge of 18 nC at $(1, 2, -1)$; (*c*) 12 nC/m on the line $y = 2.5$, $z = 0$, $-1.0 < x < 1.0$.

20− Given the potential field in cylindrical coordinates,
$V = 100/(z^2+1)\ \rho \cos\phi$ V, and point P at $\rho = 3$ m, $\phi = 60^\circ$, $z = 2$ m, find values at P for (*a*) V; (*b*) $\mathbf{E}$; (*c*) E; (*d*) dV/dN; (*e*) $\mathbf{a}_N$; (*f*) ρ_v in free space.

THE ELECTRIC DIPOLE

21- An electric dipole located at the origin in free space has a moment $\mathbf{p} = 3\mathbf{a}_x - 2\mathbf{a}_y + \mathbf{a}_z$ nC · m. (*a*) Find *V* at $P_A(2, 3, 4)$. (*b*) Find *V* at $r = 2.5$, $\theta = 30^\circ$, $\phi = 40^\circ$.

22− A dipole of moment $\mathbf{p} = 6\mathbf{a}_z$ nC · m is located at the origin in free space. (*a*) Find *V* at $P(r = 4, \theta = 20^\circ, \phi = 0^\circ)$. (*b*) Find **E** at *P*.

ENERGY DENSITY IN THE ELECTROSTATIC FIELD

23- Find the energy stored in free space for the region 2 mm $< r <$ 3mm, $0 < \theta < 90^\circ$, $0 < \phi < 90^\circ$, given the potential field *V* =: (*a*) $200/r$ V; (*b*) $300 \cos \theta / r^2$ V.

CURRENT AND CURRENT DENSITY

24- Given the vector current density $\mathbf{J} = 10\rho^2 z\mathbf{a}_\rho - 4\rho \cos^2\phi\, \mathbf{a}_\phi$ mA/m^2: (*a*) find the current density at $P(\rho = 3, \phi = 30^\circ, z = 2)$; (*b*) determine the total current flowing outward through the circular band $\rho = 3$, $0 < \phi < 2\pi$, $2 < z < 2.8$.

25− Current density is given in cylindrical coordinates as $\mathbf{J} = -10^6 z^{1.5}\mathbf{a}_z$ A/m^2 in the region $0 \leq \rho \leq 20\ \mu$m; for $\rho \geq 20\ \mu$m, $\mathbf{J} = 0$. (*a*) Find the total current crossing the surface $z = 0.1$ m in the $\mathbf{a}_z$ direction. (*b*) If the charge velocity is 2×10^6 m/s at $z = 0.1$ m, find ρ_ν there. (*c*) If the volume charge density at $z = 0.15$ m is -2000 C/m^3, find the charge velocity there.

METALLIC CONDUCTORS

26- Find the magnitude of the current density in a sample of silver for which $\sigma = 6.17 \times 10^7$ S/m and $\mu_e = 0.0056 \text{m}^2/\text{V} \cdot \text{s}$ if (*a*) the drift velocity is $1.5\ \mu$ m/s ; (*b*) the electric field intensity is 1 mV/m; (*c*) the sample is a cube 2.5 mm on a side having a voltage of 0.4 mV between opposite faces; (*d*) the sample is a cube 2.5 mm on a side carrying a total current of 0.5 A.

CONDUCTOR PROPERTIES AND BOUNDARY CONDITIONS

27- Given the potential field in free space, $V = 100 \sinh 5x \sin 5y$ *V*, and a point *P*(0.1, 0.2, 0.3), find at *P*: (*a*) *V*; (*b*) **E**; (*c*) $|\mathbf{E}|$; (*d*) $|\rho_S|$ if it is known that *P* lies on a conductor surface.

THE NATURE OF DIELECTRIC MATERIALS

28- A slab of dielectric material has a relative dielectric constant of 3.8 and contains a uniform electric flux density of 8 nC/m^2. If the material is lossless, find: (*a*) *E*; (*b*) *P*; (*c*) the average number of dipoles per cubic meter if the average dipole moment is 10^{-29} C · m.

BOUNDARY CONDITIONS FOR PERFECT DIELECTRIC MATERIALS

29- Let Region 1 ($z < 0$) be composed of a uniform dielectric material for which $\epsilon_r = 3.2$, while Region 2 ($z > 0$) is characterized by $\epsilon_r = 2$. Let $\mathbf{D}1 = -30\mathbf{a}_x + 50\mathbf{a}_y + 70\mathbf{a}_z$ nC/m^2 and find: (*a*) D_{N1}; (*b*) $\mathbf{D}_{t1}$; (*c*) D_{t1}; (*d*) D_1; (*e*) θ_1; (*f*) $\mathbf{P}_1$.

30− Continue Problem 29 by finding: (*a*) $\mathbf{D}_{N2}$; (*b*) $\mathbf{D}_{t2}$; (*c*) $\mathbf{D}_2$; (*d*) $\mathbf{P}_2$; (*e*) θ_2.

Capacitance: PARALLEL-PLATE CAPACITOR

31- Find the relative permittivity of the dielectric material present in a parallel-plate capacitor if: (*a*) $S = 0.12\text{m}^2$, $d = 80\,\mu\text{m}$, $V0 = 12$ V, and the capacitor contains 1 μJ of energy; (*b*) the stored energy density is 100 J/m^3, $V0 = 200$ V, and $d = 45\,\mu\text{m}$; (*c*) $E = 200$ kV/m and $\rho_S = 20\,\mu\text{C/m}^2$.

The Steady Magnetic Field

BIOT-SAVART LAW

32- Given the following values for P_1, P_2, and $I_1\Delta L_1$, calculate _$\mathbf{H}_2$: (*a*) $P_1(0, 0, 2)$, $P_2(4, 2, 0)$, $2\pi\mathbf{a}_z$ μA · m; (*b*) $P_1(0, 2, 0)$, $P_2(4, 2, 3)$, $2\pi\mathbf{a}_z$ μA · m; (*c*) $P_1(1, 2, 3)$, $P_2(-3, -1, 2)$, $2\pi(-\mathbf{a}x + \mathbf{a}_y + 2\mathbf{a}_z)$ μA · m.

33- A current filament carrying 15 A in the $\mathbf{a}_z$ direction lies along the entire z axis. Find **H** in rectangular coordinates at: (*a*) $P_A(\sqrt{20}, 0, 4)$; (*b*) $P_B(2, -4, 4)$.

AMPE` RE'S CIRCUITAL LAW

34- Use Ampere's law to obtain **H** due to an infinitely long, straight filament of current *I*.

35- Consider an infinitely long coaxial transmission line (cable) as shown in the figure. Its inner conductor is solid with radius a. The outer conductor is in the form of concentric cylinder whose inner radius is b and outer radius is c. This cable is placed along z-axis. The total current (I) is uniformly distributed in the inner conductor. While ($-I$) is uniformly distributed in the outer conductor. Use Ampere's law to find $\mathbf{H}$ in the regions: ***(a)*** inside inner conductor ($r < a$), ***(b)*** inside dielectric ($a < r < b$) ***(c)*** inside outer conductor ($b < r < c$), out of coaxial cable ($\boldsymbol{r} > \boldsymbol{c}$).