5.1. Given the current density $\mathbf{J} = -10^4[\sin(2x)e^{-2y}\mathbf{a}_x + \cos(2x)e^{-2y}\mathbf{a}_y]\,\text{kA/m}^2$:

a) Find the total current crossing the plane $y = 1$ in the $\mathbf{a}_y$ direction in the region $0 < x < 1$, $0 < z < 2$: This is found through

$$I = \int\int_S \mathbf{J}\cdot\mathbf{n}\Big|_S da = \int_0^2\int_0^1 \mathbf{J}\cdot\mathbf{a}_y\Big|_{y=1} dx\,dz = \int_0^2\int_0^1 -10^4\cos(2x)e^{-2}\,dx\,dz$$
$$= -10^4(2)\frac{1}{2}\sin(2x)\Big|_0^1 e^{-2} = \underline{-1.23\,\text{MA}}$$

b) Find the total current leaving the region $0 < x, x < 1$, $2 < z < 3$ by integrating $\mathbf{J}\cdot\mathbf{dS}$ over the surface of the cube: Note first that current through the top and bottom surfaces will not exist, since $\mathbf{J}$ has no z component. Also note that there will be no current through the $x = 0$ plane, since $J_x = 0$ there. Current will pass through the three remaining surfaces, and will be found through

$$I = \int_2^3\int_0^1 \mathbf{J}\cdot(-\mathbf{a}_y)\Big|_{y=0} dx\,dz + \int_2^3\int_0^1 \mathbf{J}\cdot(\mathbf{a}_y)\Big|_{y=1} dx\,dz + \int_2^3\int_0^1 \mathbf{J}\cdot(\mathbf{a}_x)\Big|_{x=1} dy\,dz$$
$$= 10^4\int_2^3\int_0^1 \left[\cos(2x)e^{-0} - \cos(2x)e^{-2}\right] dx\,dz - 10^4\int_2^3\int_0^1 \sin(2)e^{-2y}\,dy\,dz$$
$$= 10^4\left(\frac{1}{2}\right)\sin(2x)\Big|_0^1(3-2)\left[1-e^{-2}\right] + 10^4\left(\frac{1}{2}\right)\sin(2)e^{-2y}\Big|_0^1(3-2) = \underline{0}$$

c) Repeat part b, but use the divergence theorem: We find the net outward current through the surface of the cube by integrating the divergence of $\mathbf{J}$ over the cube volume. We have

$$\nabla\cdot\mathbf{J} = \frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} = -10^{-4}\left[2\cos(2x)e^{-2y} - 2\cos(2x)e^{-2y}\right] = \underline{0}\ \text{ as expected}$$

5.2. Let the current density be $\mathbf{J} = 2\phi\cos^2\phi\,\mathbf{a}_\rho - \rho\sin 2\phi\,\mathbf{a}_\phi$ A/m^2 within the region $2.1 < \rho < 2.5$, $0 < \phi < 0.1$ rad, $6 < z < 6.1$. Find the total current I crossing the surface:

a) $\rho = 2.2$, $0 < \phi < 0.1$, $6 < z < 6.1$ in the $\mathbf{a}_\rho$ direction: This is a surface of constant ρ, so only the radial component of $\mathbf{J}$ will contribute: At $\rho = 2.2$ we write:

$$I = \int \mathbf{J}\cdot d\mathbf{S} = \int_6^{6.1}\int_0^{0.1} 2(2)\cos^2\phi\,\mathbf{a}_\rho\cdot\mathbf{a}_\rho\,2d\phi dz = 2(2.2)^2(0.1)\int_0^{0.1}\frac{1}{2}(1+\cos 2\phi)\,d\phi$$
$$= 0.2(2.2)^2\left[\frac{1}{2}(0.1) + \frac{1}{4}\sin 2\phi\Big|_0^{0.1}\right] = \underline{97\,\text{mA}}$$

b) $\phi = 0.05$, $2.2 < \rho < 2.5$, $6 < z < 6.1$ in the $\mathbf{a}_\phi$ direction: In this case only the ϕ component of $\mathbf{J}$ will contribute:

$$I = \int \mathbf{J}\cdot d\mathbf{S} = \int_6^{6.1}\int_{2.2}^{2.5} -\rho\sin 2\phi\big|_{\phi=0.05}\,\mathbf{a}_\phi\cdot\mathbf{a}_\phi\,d\rho\,dz = -(0.1)^2\frac{\rho^2}{2}\Big|_{2.2}^{2.5} = \underline{-7\,\text{mA}}$$

5.18. Let us assume a field $\mathbf{E} = 3y^2z^3\,\mathbf{a}_x + 6xyz^3\,\mathbf{a}_y + 9xy^2z^2\,\mathbf{a}_z$ V/m in free space, and also assume that point $P(2, 1, 0)$ lies on a conducting surface.

a) Find ρ_v just adjacent to the surface at P:

$$\rho_v = \nabla \cdot \mathbf{D} = \epsilon_0 \nabla \cdot \mathbf{E} = 6xz^3 + 18xy^2z \text{ C/m}^3$$

Then at P, $\rho_v = \underline{0}$, since $z = 0$.

b) Find ρ_s at P:

$$\rho_s = \mathbf{D} \cdot \mathbf{n}\Big|_P = \epsilon_0 \mathbf{E}\dot{\mathbf{n}}\Big|_P$$

Note however, that this computation involves evaluating $\mathbf{E}$ at the surface, yielding a value of 0. Therefore the surface charge density at P is $\underline{0}$.

c) Show that $V = -3xy^2z^3$ V: The simplest way to show this is just to take $-\nabla V$, which yields the given field: A more general method involves deriving the potential from the given field: We write

$$E_x = -\frac{\partial V}{\partial x} = 3y^2z^3 \;\Rightarrow\; V = -3xy^2z^3 + f(y, z)$$

$$E_y = -\frac{\partial V}{\partial y} = 6xyz^3 \;\Rightarrow\; V = -3xy^2z^3 + f(x, z)$$

$$E_z = -\frac{\partial V}{\partial z} = 9xy^2z^2 \;\Rightarrow\; V = -3xy^2z^3 + f(x, y)$$

where the integration "constants" are functions of all variables other than the integration variable. The general procedure is to adjust the functions, f, such that the result for V is the same in all three integrations. In this case we see that $f(x, y) = f(x, z) = f(y, z) = 0$ accomplishes this, and the potential function is $V = -3xy^2z^3$ as given.

d) Determine V_{PQ}, given $Q(1, 1, 1)$: Using the potential function of part c, we have

$$V_{PQ} = V_P - V_Q = 0 - (-3) = \underline{3 \text{ V}}$$

5.19. Let $V = 20x^2yz - 10z^2$ V in free space.

a) Determine the equations of the equipotential surfaces on which $V = 0$ and 60 V: Setting the given potential function equal to 0 and 60 and simplifying results in:

$$\text{At 0 V}: \;\; 2x^2y - z = 0$$

$$\text{At 60 V}: \;\; 2x^2y - z = \frac{6}{z}$$

b) Assume these are conducting surfaces and find the surface charge density at that point on the $V = 60$ V surface where $x = 2$ and $z = 1$. It is known that $0 \le V \le 60$ V is the field-containing region: First, on the 60 V surface, we have

$$2x^2y - z - \frac{6}{z} = 0 \;\Rightarrow\; 2(2)^2y(1) - 1 - 6 = 0 \;\Rightarrow\; y = \frac{7}{8}$$

7.7. Let $V = (\cos 2\phi)/\rho$ in free space.

a) Find the volume charge density at point $A(0.5, 60^\circ, 1)$: Use Poisson's equation:

$$\rho_v = -\epsilon_0 \nabla^2 V = -\epsilon_0 \left(\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} \right)$$
$$= -\epsilon_0 \left(\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\frac{-\cos 2\phi}{\rho}\right) - \frac{4}{\rho^2}\frac{\cos 2\phi}{\rho} \right) = \frac{3\epsilon_0 \cos 2\phi}{\rho^3}$$

So at A we find:

$$\rho_{vA} = \frac{3\epsilon_0 \cos(120^\circ)}{0.5^3} = -12\epsilon_0 = \underline{-106\,\text{pC/m}^3}$$

b) Find the surface charge density on a conductor surface passing through $B(2, 30^\circ, 1)$: First, we find $\mathbf{E}$:

$$\mathbf{E} = -\nabla V = -\frac{\partial V}{\partial \rho}\,\mathbf{a}_\rho - \frac{1}{\rho}\frac{\partial V}{\partial \phi}\,\mathbf{a}_\phi$$
$$= \frac{\cos 2\phi}{\rho^2}\,\mathbf{a}_\rho + \frac{2\sin 2\phi}{\rho^2}\,\mathbf{a}_\phi$$

At point B the field becomes

$$\mathbf{E}_B = \frac{\cos 60^\circ}{4}\,\mathbf{a}_\rho + \frac{2\sin 60^\circ}{4}\,\mathbf{a}_\phi = 0.125\,\mathbf{a}_\rho + 0.433\,\mathbf{a}_\phi$$

The surface charge density will now be

$$\rho_{sB} = \pm|\mathbf{D}_B| = \pm\epsilon_0|\mathbf{E}_B| = \pm 0.451\epsilon_0 = \underline{\pm 0.399\,\text{pC/m}^2}$$

The charge is positive or negative depending on which side of the surface we are considering. The problem did not provide information necessary to determine this.

7.21. In free space, let $\rho_v = 200\epsilon_0/r^{2.4}$.

a) Use Poisson's equation to find $V(r)$ if it is assumed that $r^2 E_r \to 0$ when $r \to 0$, and also that $V \to 0$ as $r \to \infty$: With r variation only, we have

$$\nabla^2 V = \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dV}{dr}\right) = -\frac{\rho_v}{\epsilon} = -200r^{-2.4}$$

or

$$\frac{d}{dr}\left(r^2 \frac{dV}{dr}\right) = -200r^{-.4}$$

Integrate once:

$$\left(r^2 \frac{dV}{dr}\right) = -\frac{200}{.6}r^{.6} + C_1 = -333.3r^{.6} + C_1$$

or

$$\frac{dV}{dr} = -333.3r^{-1.4} + \frac{C_1}{r^2} = \nabla V \text{ (in this case)} = -E_r$$

Our first boundary condition states that $r^2 E_r \to 0$ when $r \to 0$ Therefore $C_1 = 0$. Integrate again to find:

$$V(r) = \frac{333.3}{.4}r^{-.4} + C_2$$

From our second boundary condition, $V \to 0$ as $r \to \infty$, we see that $C_2 = 0$. Finally,

$$V(r) = \underline{833.3r^{-.4} \text{ V}}$$

b) Now find $V(r)$ by using Gauss' Law and a line integral: Gauss' law applied to a spherical surface of radius r gives:

$$4\pi r^2 D_r = 4\pi \int_0^r \frac{200\epsilon_0}{(r')^{2.4}}(r')^2 dr = 800\pi\epsilon_0 \frac{r^{.6}}{.6}$$

Thus

$$E_r = \frac{D_r}{\epsilon_0} = \frac{800\pi\epsilon_0 r^{.6}}{.6(4\pi)\epsilon_0 r^2} = 333.3r^{-1.4} \text{ V/m}$$

Now

$$V(r) = -\int_\infty^r 333.3(r')^{-1.4} dr' = \underline{833.3r^{-.4} \text{ V}}$$

8.7. Given points $C(5,-2,3)$ and $P(4,-1,2)$; a current element $I\,d\mathbf{L} = 10^{-4}(4,-3,1)\ \mathrm{A}\cdot\mathrm{m}$ at C produces a field $d\mathbf{H}$ at P.

a) Specify the direction of $d\mathbf{H}$ by a unit vector $\mathbf{a}_H$: Using the Biot-Savart law, we find

$$d\mathbf{H} = \frac{I\,d\mathbf{L} \times \mathbf{a}_{CP}}{4\pi R_{CP}^2} = \frac{10^{-4}[4\mathbf{a}_x - 3\mathbf{a}_y + \mathbf{a}_z] \times [-\mathbf{a}_x + \mathbf{a}_y - \mathbf{a}_z]}{4\pi 3^{3/2}} = \frac{[2\mathbf{a}_x + 3\mathbf{a}_y + \mathbf{a}_z] \times 10^{-4}}{65.3}$$

from which

$$\mathbf{a}_H = \frac{2\mathbf{a}_x + 3\mathbf{a}_y + \mathbf{a}_z}{\sqrt{14}} = \underline{0.53\mathbf{a}_x + 0.80\mathbf{a}_y + 0.27\mathbf{a}_z}$$

b) Find $|d\mathbf{H}|$.

$$|d\mathbf{H}| = \frac{\sqrt{14} \times 10^{-4}}{65.3} = 5.73 \times 10^{-6}\ \mathrm{A/m} \ = \underline{5.73\ \mu\mathrm{A/m}}$$

c) What direction $\mathbf{a}_l$ should $I\,d\mathbf{L}$ have at C so that $d\mathbf{H} = 0$? $I\,d\mathbf{L}$ should be collinear with $\mathbf{a}_{CP}$, thus rendering the cross product in the Biot-Savart law equal to zero. Thus the answer is $\mathbf{a}_l = \underline{\pm(-\mathbf{a}_x + \mathbf{a}_y - \mathbf{a}_z)/\sqrt{3}}$

8.27. The magnetic field intensity is given in a certain region of space as

$$\mathbf{H} = \frac{x+2y}{z^2}\,\mathbf{a}_y + \frac{2}{z}\,\mathbf{a}_z \text{ A/m}$$

a) Find $\nabla \times \mathbf{H}$: For this field, the general curl expression in rectangular coordinates simplifies to

$$\nabla \times \mathbf{H} = -\frac{\partial H_y}{\partial z}\,\mathbf{a}_x + \frac{\partial H_y}{\partial x}\,\mathbf{a}_z = \underline{\frac{2(x+2y)}{z^3}\,\mathbf{a}_x + \frac{1}{z^2}\mathbf{a}_z \text{ A/m}}$$

b) Find $\mathbf{J}$: This will be the answer of part a, since $\nabla \times \mathbf{H} = \mathbf{J}$.

c) Use $\mathbf{J}$ to find the total current passing through the surface $z = 4$, $1 < x < 2$, $3 < y < 5$, in the $\mathbf{a}_z$ direction: This will be

$$I = \int\int \mathbf{J}\big|_{z=4} \cdot \mathbf{a}_z\,dx\,dy = \int_3^5 \int_1^2 \frac{1}{4^2}dx\,dy = \underline{1/8 \text{ A}}$$

d) Show that the same result is obtained using the other side of Stokes' theorem: We take $\oint \mathbf{H} \cdot d\mathbf{L}$ over the square path at $z = 4$ as defined in part c. This involves two integrals of the y component of $\mathbf{H}$ over the range $3 < y < 5$. Integrals over x, to complete the loop, do not exist since there is no x component of $\mathbf{H}$. We have

$$I = \oint \mathbf{H}\big|_{z=4} \cdot d\mathbf{L} = \int_3^5 \frac{2+2y}{16}\,dy + \int_5^3 \frac{1+2y}{16}\,dy = \frac{1}{8}(2) - \frac{1}{16}(2) = \underline{1/8 \text{ A}}$$

8.28. Given $\mathbf{H} = (3r^2/\sin\theta)\mathbf{a}_\theta + 54r\cos\theta\mathbf{a}_\phi$ A/m in free space:

a) find the total current in the $\mathbf{a}_\theta$ direction through the conical surface $\theta = 20^\circ$, $0 \le \phi \le 2\pi$, $0 \le r \le 5$, by whatever side of Stokes' theorem you like best. I chose the line integral side, where the integration path is the circular path in ϕ around the top edge of the cone, at $r = 5$. The path direction is chosen to be *clockwise* looking down on the xy plane. This, by convention, leads to the normal from the cone surface that points in the positive $\mathbf{a}_\theta$ direction (right hand rule). We find

$$\oint \mathbf{H} \cdot d\mathbf{L} = \int_0^{2\pi} \left[(3r^2/\sin\theta)\mathbf{a}_\theta + 54r\cos\theta\mathbf{a}_\phi\right]_{r=5,\theta=20} \cdot 5\sin(20^\circ)\,d\phi\,(-\mathbf{a}_\phi)$$
$$= -2\pi(54)(25)\cos(20^\circ)\sin(20^\circ) = \underline{-2.73 \times 10^3 \text{ A}}$$

This result means that there is a component of current that enters the cone surface in the $-\mathbf{a}_\theta$ direction, to which is associated a component of $\mathbf{H}$ in the positive $\mathbf{a}_\phi$ direction.

b) Check the result by using the other side of Stokes' theorem: We first find the current density through the curl of the magnetic field, where three of the six terms in the spherical coordinate formula survive:

$$\nabla \times \mathbf{H} = \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(54r\cos\theta\sin\theta\right)\,\mathbf{a}_r - \frac{1}{r}\frac{\partial}{\partial r}\left(54r^2\cos\theta\right)\,\mathbf{a}_\theta + \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{3r^3}{\sin\theta}\right)\,\mathbf{a}_\phi = \mathbf{J}$$

Thus

$$\mathbf{J} = 54\cot\theta\,\mathbf{a}_r - 108\cos\theta\,\mathbf{a}_\theta + \frac{9r}{\sin\theta}\,\mathbf{a}_\phi$$

8.28b. (continued)

The calculation of the other side of Stokes' theorem now involves integrating **J** over the surface of the cone, where the outward normal is positive $\mathbf{a}_\theta$, as defined in part *a*:

$$\int_S (\nabla \times \mathbf{H}) \cdot d\mathbf{S} = \int_0^{2\pi} \int_0^5 \left[54 \cot\theta \, \mathbf{a}_r - 108 \cos\theta \, \mathbf{a}_\theta + \frac{9r}{\sin\theta} \, \mathbf{a}_\phi \right]_{\theta=20^\circ} \cdot \mathbf{a}_\theta \, r \sin(20^\circ) \, dr \, d\phi$$
$$= -\int_0^{2\pi} \int_0^5 108 \cos(20^\circ) \sin(20^\circ) r dr d\phi = -2\pi(54)(25) \cos(20^\circ) \sin(20^\circ)$$
$$= \underline{-2.73 \times 10^3 \text{ A}}$$

8.29. A long straight non-magnetic conductor of 0.2 mm radius carries a uniformly-distributed current of 2 A dc.

a) Find **J** within the conductor: Assuming the current is $+z$ directed,

$$\mathbf{J} = \frac{2}{\pi(0.2 \times 10^{-3})^2} \mathbf{a}_z = \underline{1.59 \times 10^7 \, \mathbf{a}_z \text{ A/m}^2}$$

b) Use Ampere's circuital law to find **H** and **B** within the conductor: Inside, at radius ρ, we have

$$2\pi\rho H_\phi = \pi\rho^2 J \quad \Rightarrow \quad \mathbf{H} = \frac{\rho J}{2} \mathbf{a}_\phi = \underline{7.96 \times 10^6 \rho \, \mathbf{a}_\phi \text{ A/m}}$$

Then $\mathbf{B} = \mu_0 \mathbf{H} = (4\pi \times 10^{-7})(7.96 \times 10^6)\rho \mathbf{a}_\phi = \underline{10\rho \, \mathbf{a}_\phi \text{ Wb/m}^2}$.

c) Show that $\nabla \times \mathbf{H} = \mathbf{J}$ within the conductor: Using the result of part *b*, we find,

$$\nabla \times \mathbf{H} = \frac{1}{\rho} \frac{d}{d\rho} (\rho H_\phi) \, \mathbf{a}_z = \frac{1}{\rho} \frac{d}{d\rho} \left(\frac{1.59 \times 10^7 \rho^2}{2} \right) \mathbf{a}_z = \underline{1.59 \times 10^7 \, \mathbf{a}_z \text{ A/m}^2} = \mathbf{J}$$

d) Find **H** and **B** *outside* the conductor (note typo in book): Outside, the entire current is enclosed by a closed path at radius ρ, and so

$$\mathbf{H} = \frac{I}{2\pi\rho} \mathbf{a}_\phi = \underline{\frac{1}{\pi\rho} \, \mathbf{a}_\phi \text{ A/m}}$$

Now $\mathbf{B} = \mu_0 \mathbf{H} = \underline{\mu_0/(\pi\rho) \, \mathbf{a}_\phi \text{ Wb/m}^2}$.

e) Show that $\nabla \times \mathbf{H} = \mathbf{J}$ outside the conductor: Here we use **H** outside the conductor and write:

$$\nabla \times \mathbf{H} = \frac{1}{\rho} \frac{d}{d\rho} (\rho H_\phi) \, \mathbf{a}_z = \frac{1}{\rho} \frac{d}{d\rho} \left(\rho \frac{1}{\pi\rho} \right) \mathbf{a}_z = \underline{0} \text{ (as expected)}$$

8.41. Assume that $\mathbf{A} = 50\rho^2\mathbf{a}_z$ Wb/m in a certain region of free space.

a) Find **H** and **B**: Use

$$\mathbf{B} = \nabla \times \mathbf{A} = -\frac{\partial A_z}{\partial \rho}\mathbf{a}_\phi = \underline{-100\rho\,\mathbf{a}_\phi \text{ Wb/m}^2}$$

Then $\mathbf{H} = \mathbf{B}/\mu_0 = \underline{-100\rho/\mu_0\,\mathbf{a}_\phi \text{ A/m}}$.

b) Find **J**: Use

$$\mathbf{J} = \nabla \times \mathbf{H} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho H_\phi)\mathbf{a}_z = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\frac{-100\rho^2}{\mu_0}\right)\mathbf{a}_z = \underline{-\frac{200}{\mu_0}\mathbf{a}_z \text{ A/m}^2}$$

c) Use **J** to find the total current crossing the surface $0 \le \rho \le 1$, $0 \le \phi < 2\pi$, $z = 0$: The current is

$$I = \int\int \mathbf{J}\cdot d\mathbf{S} = \int_0^{2\pi}\int_0^1 \frac{-200}{\mu_0}\mathbf{a}_z \cdot \mathbf{a}_z\,\rho\,d\rho\,d\phi = \frac{-200\pi}{\mu_0} \text{ A} = \underline{-500 \text{ kA}}$$

d) Use the value of H_ϕ at $\rho = 1$ to calculate $\oint \mathbf{H}\cdot d\mathbf{L}$ for $\rho = 1$, $z = 0$: Have

$$\oint \mathbf{H}\cdot d\mathbf{L} = I = \int_0^{2\pi} \frac{-100}{\mu_0}\mathbf{a}_\phi \cdot \mathbf{a}_\phi\,(1)d\phi = \frac{-200\pi}{\mu_0} \text{ A} = \underline{-500 \text{ kA}}$$

10.11. Let the internal dimension of a coaxial capacitor be $a = 1.2$ cm, $b = 4$ cm, and $l = 40$ cm. The homogeneous material inside the capacitor has the parameters $\epsilon = 10^{-11}$ F/m, $\mu = 10^{-5}$ H/m, and $\sigma = 10^{-5}$ S/m. If the electric field intensity is $\mathbf{E} = (10^6/\rho)\cos(10^5 t)\mathbf{a}_\rho$ V/m (note missing t in the argument of the cosine in the book), find:

a) $\mathbf{J}$: Use

$$\mathbf{J} = \sigma\mathbf{E} = \underline{\left(\frac{10}{\rho}\right)\cos(10^5 t)\mathbf{a}_\rho \ \text{A/m}^2}$$

b) the total conduction current, I_c, through the capacitor: Have

$$I_c = \int\int \mathbf{J}\cdot d\mathbf{S} = 2\pi\rho l J = 20\pi l\cos(10^5 t) = \underline{8\pi\cos(10^5 t)\ \text{A}}$$

c) the total displacement current, I_d, through the capacitor: First find

$$\mathbf{J}_d = \frac{\partial\mathbf{D}}{\partial t} = \frac{\partial}{\partial t}(\epsilon\mathbf{E}) = -\frac{(10^5)(10^{-11})(10^6)}{\rho}\sin(10^5 t)\mathbf{a}_\rho = -\frac{1}{\rho}\sin(10^5 t)\ \text{A/m}$$

Now

$$I_d = 2\pi\rho l J_d = -2\pi l\sin(10^5 t) = \underline{-0.8\pi\sin(10^5 t)\ \text{A}}$$

d) the ratio of the amplitude of I_d to that of I_c, the quality factor of the capacitor: This will be

$$\frac{|I_d|}{|I_c|} = \frac{0.8}{8} = \underline{0.1}$$

10.15. Let $\mu = 3 \times 10^{-5}$ H/m, $\epsilon = 1.2 \times 10^{-10}$ F/m, and $\sigma = 0$ everywhere. If $\mathbf{H} = 2\cos(10^{10}t - \beta x)\mathbf{a}_z$ A/m, use Maxwell's equations to obtain expressions for $\mathbf{B}$, $\mathbf{D}$, $\mathbf{E}$, and β: First, $\mathbf{B} = \mu\mathbf{H} = \underline{6 \times 10^{-5}\cos(10^{10}t - \beta x)\mathbf{a}_z \text{ T}}$. Next we use

$$\nabla \times \mathbf{H} = -\frac{\partial \mathbf{H}}{\partial x}\mathbf{a}_y = 2\beta \sin(10^{10}t - \beta x)\mathbf{a}_y = \frac{\partial \mathbf{D}}{\partial t}$$

from which

$$\mathbf{D} = \int 2\beta \sin(10^{10}t - \beta x)\, dt + C = \underline{-\frac{2\beta}{10^{10}}\cos(10^{10}t - \beta x)\mathbf{a}_y \text{ C/m}^2}$$

where the integration constant is set to zero, since no dc fields are presumed to exist. Next,

$$\mathbf{E} = \frac{\mathbf{D}}{\epsilon} = -\frac{2\beta}{(1.2 \times 10^{-10})(10^{10})}\cos(10^{10}t - \beta x)\mathbf{a}_y = \underline{-1.67\beta\cos(10^{10}t - \beta x)\mathbf{a}_y \text{ V/m}}$$

Now

$$\nabla \times \mathbf{E} = \frac{\partial E_y}{\partial x}\mathbf{a}_z = 1.67\beta^2 \sin(10^{10}t - \beta x)\mathbf{a}_z = -\frac{\partial \mathbf{B}}{\partial t}$$

So

$$\mathbf{B} = -\int 1.67\beta^2 \sin(10^{10}t - \beta x)\mathbf{a}_z dt = (1.67 \times 10^{-10})\beta^2 \cos(10^{10}t - \beta x)\mathbf{a}_z$$

We require this result to be consistent with the expression for $\mathbf{B}$ originally found. So

$$(1.67 \times 10^{-10})\beta^2 = 6 \times 10^{-5} \quad \Rightarrow \quad \beta = \underline{\pm 600 \text{ rad/m}}$$