



زانكۆی سه لاهه دین-هه ولێر

Salahaddin university-Erbil

Empirical Formula for (p, n) reaction Cross Sections of Thorium Isotopes at 1–20 MeV Neutrons

Research Project

Submitted to the department of (Physic) in partial fulfilment of the requirements for the degree of B.A or BSc. in Physic education Department

By:

Yaqub Zaid Younis

Matlub Othman Aziz

Tayfur Othman Aziz

Supervised by:

Hallo Mahmud kaka Abdulla

April-2024

Dedication

To all of our families who have toughs us the way of life ,brothers , sisters . and our supervisor Hallo Mahmud kaka Abdulla with all other teachers. And our friends that helped us with preparing this project we appreciate it .

Abstract:

This article describes how to calculate neutron-induced fission cross sections using an empirical formula and computer codes such as EMPIRE 3.2.3 and TALYS 1.95 nuclear codes, and calculations using this formula are shown for a number of nuclei up to the third fission plateau. In this study, the excitation functions of $^{228}\text{Th} (n, f)$, $^{229}\text{Th} (n, f)$, $^{230}\text{Th} (n, f)$, $^{231}\text{Th} (n, f)$, $^{232}\text{Th} (n, f)$ and $^{234}\text{Th} (n, f)$, nuclear reactions were calculated at neutron energies between 1 to 20 MeV. Furthermore, the cross sections were calculated with the empirical formula derived in study at 1 - 20MeV. The results were contrasted to measured values from the EXFOR library as well as evaluated data from ENDF such as (TENDL 2021, JENDL 3.3, JENDL 4, ENDF/B-VII.1, EAF 2010, ROSFOND 2010 and BROND 3.1) Overall, the calculated, experimental, and evaluated data fission cross-sections are in concordance.

Keywords: Cross-sections; (n, f) reactions; empirical formula; EMPIRE 3.2; TALYS 1.95

Contents

CHAPTER ONE: Introduction and Literature Review

1.1 Introduction.....	1
1.2 Literature Review.....	1

CHAPTER TWO: Theory and Models

2.1 Nuclear Reactions.....	2
2.1.1 Cross Section of Nuclear reactions.....	3
2.1.2 Energy Dependence of Nuclear Cross Sections.....	3
2.2 Theoretical Models of Nuclear Reactions	5
2.2.1 Optical model.....	5
2.2.2 Compound nuclear model.....	6
2.2.3 The Direct Reaction Model.....	7
2.2.4 Pre-equilibrium model.....	7
2.2.5 Nuclear level densities.....	7
2.2.6 The Statistical Multistep Direct (SMD) and The Statistical Multistep Compound (SMC).....	8
2.3 Neutron Activation Method.....	8

CHAPTER THREE: Methods

3.1 Empirical formulae for (n, f) Reaction Cross-Sections.....	9
3.2 Excitation function of for (n, f) Reaction Cross-Sections	9
3.3 Nuclear model Computer codes.....	10
3.3.1 The (n, f) Reaction Cross-Section Formula.....	10
3.3.2 Empire-3.2.3 code	11
3.3.3 Talys-1.95 Codes.....	13
3.4 Statistical Comparisons.....	14

CHAPTER FOUR: Results and Discussion

4.1 Excitation function of (n,f) reactions at 1 to 20 MeV.....	15
4.1.1 ²²⁸ Th (n, f) reaction.....	15
4.1.2 ²²⁹ Th (n, f) reaction.....	16
4.1.3 ²³⁰ Th (n, f) reaction.....	17
4.1.4 ²³¹ Th (n, f) reaction.....	18
4.1.5 ²³² Th (n, f) reaction.....	19
4.1.6 ²²³ Th (n, f) reaction.....	20
4.1.6 ²²⁴ Th (n, f) reaction.....	21
4-Conclusion	
Reference.....	22

CHAPTER ONE: Introduction and Literature Review

1.3 Introduction

Nuclear fission has been used to generate energy for more than 50 years and is increasingly seen as a crucial safe source of low-carbon energy for the world's future energy balance. The common of present-day nuclear power is based on uranium, thorium, protactinium, neptunium, plutonium fuels in light water reactors. However, they are often inefficient, particularly if the radioactive fuel is disposed of as garbage rather than reused [1].

Although it has been used widely as a standard in determining the cross sections of other fissionable materials, the absolute fission cross sections of ^{228}Th , ^{229}Th , ^{230}Th , ^{231}Th , ^{232}Th , ^{233}Th and ^{234}Th for fast neutrons are of special importance [2].

New theoretical calculating formula were carried out to obtain nuclear reactions besides the properties of excited states across a wide range of energies. Fast neutron collisions with heavy nuclei, in Weisskopf's view, result in the production of compound structures that are relatively stable as well as a more accurate statistical calculation of nuclear processes [3].

The use of analytical equations from the evaporation and pre-compound exciton models to derive empirical formulas is a development in enhancing the quality of Empirical Formula. Numerous authors using semi-empirical and empirical formulations has also been investigated systematic dependency of the, (n, p) , (n, n) , (n, α) , (n, f) and $(n, 2n)$ reaction cross sections in different neutron energies. To develop theoretical models to simulate nuclear reaction processes, further experimental data on (n, f) reaction cross section at the large energy range MeV is necessary [4]-[5]-[6].

This present study takes a step further by deriving novel empirical formulae for calculating the cross sections of (n, f) different target reactions of (^{228}Th , ^{229}Th , ^{230}Th , ^{231}Th , ^{232}Th , ^{233}Th and ^{234}Th) with three fitting parameters at neutron energy between 1 to 20 MeV. However, the (n, f) reaction cross sections were calculated using the EMPIRE 3.2.3 and TALYS 1.95 codes.

1.4 Literature Review

The following semi-empirical formula, provided by Anand et al. in 1976, is explained in order to determine neutron-induced fission cross-sections. This article presents calculations based on this formula for a number of trans-actinide nuclei up to the third fission plateau. The calculated fission cross-sections generally match experimental results to within 10%. Predicted cross-sections based on the current expression are also provided for the elements ^{233}Th and ^{233}Pa , for which there are no experimental results

CHAPTER TWO: Theory and Models

2.1 Nuclear Reactions

Nuclear reactions provide a significant portion of our information regarding the characteristics of nuclei. Three factors combine to determine how an incoming particle will behave when it scatters off a target nucleus: the internal structure of the nuclei involved, interaction between the projectile and the target and the reaction mechanism [7]. Nuclear reactions are divided into two main categories. The initial reaction X in the first category is a single atom or nucleus that undergoes spontaneous transformation by ejecting one or more particles, i.e.



this reaction is called radioactive decay. Most known nuclides are radioactive, according to the Chart of the Nuclides. The second large type of nuclear reactions includes binary reactions, in which two nuclear particles (n, p, α -ray and γ -ray) interact to form different nuclear particles.

Nuclear reactions generally generate two products for bombarding energies under 100 MeV, indicating by equation:



Where

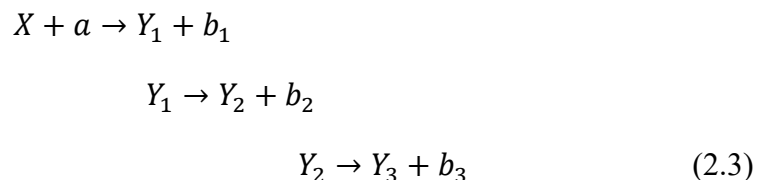
X = target (at rest in the lab. system), a = bombarding particle, Y = heavy reaction product

and b = light reaction product.

A reaction of the kind described above is abbreviated as follows:



Due to the binding energies of the participating nuclei, it is common for one reaction result to be light and the other heavy. In some instances, b and Y have comparable masses (fission or spallation reaction), or similar masses. In a capture reaction, where Y is the compound nucleus, we are assuming that b is a γ - ray. It is usually easy to depict the process as a rapid sequence of two-product reactions when more than two products are produced [7].



2.1.1 Cross Section of Nuclear reactions

The nuclear cross section is a quantitative measure of the probability that a nuclear reaction will occur. The principle of a nuclear cross section can be physically expressed in terms of "characteristic area," where a bigger area corresponds to a higher probability of interaction [8].

Assume a collimated particle beam that strikes target nuclei in an appropriate material specimen, interacts with them by scattering, absorption, and/or reaction, and attenuates as a result. The attenuation may be in energy or intensity, or both, in proportions that can be determined by measurements of the impact on the emerging radiation. The most simplistic way to understand the probability of interaction is to consider the incident beam as being composed of point particles that, if they directly impact a target nucleus or a portion of it, whereas if they miss the target nucleus, they will proceed unaffected. This simplistic conception, however, overlooks the finite interaction radius, which may be assumed to extend quite a distance beyond the immediate boundaries of the target nucleus, as well as the finite extension of the impinging particles [8].

Hence, rather of considering a nucleus' geometric cross-sectional area (πR^2), where R is the radius of nucleus as a measure of the probability of an interaction, As shown in Fig [2.1], it is meaningful to assign each nucleus an effective area σ that is perpendicular to the incident beam so that if a bombardment particle impinges on any portion of this imaginary disk, a reaction will take place, but otherwise there is no interaction. The magnitude of the cross-section a depends upon the reaction and the energy of the incident particles. The appropriate unit of measurement for its size is barns, ($1 \text{ barn} = 10^{-24} \text{ cm}^{-2}$).

2.1.2 Energy Dependence of Nuclear Cross Sections

The cross-sections of different nuclear reactions are extremely individually dependent on the bombarding energy. The excitation function for the specific reaction is frequently used to describe the intricate relationship between cross-section and bombarding energy [9].

There are two main categories for the theory describing the cross-section of nuclear processes in which a compound nucleus is created. The excited levels of the compound nucleus are discrete and can be widely separated at low bombarding energy. In this case, a resonance theory is used to explain the reaction cross-sections. The excited levels in the compound nucleus are closer together and partially overlapping at higher bombarding energy. In this energy range, the so-called continuum theory attempts to explain the broad variation in cross-section with bombarding energy [10].

When individual fluctuations and resonances are averaged, the energy dependence of nuclear cross-sections is described in terms of two inner nuclear structural characteristics. Consider the scenario associated with incident neutrons. The two parameters are the incident neutron's wave

number K and nuclear radius R after it has entered the compound nucleus. The neutron's internal wave number is K , and it is as follows:

$$K = k_0^2 + k^2 \quad (2.4)$$

where k_0 is the internal wave number if the bombarding energy is zero and k is the wave number of the incident neutron as it reaches the nucleus. k_0 is independent of the mass number A when the density of nuclei is assumed to remain constant. It should be made clear that R represents the distance at which the incident neutron's wave number transitions from its value k outside the nucleus to its value K within the nucleus. In reality, the change occurs over a finite distance on the order of $1/K$ [11].

The total, reaction and scattering cross-section are expressed in units of πR^2 .

$$\sigma_{tot,r,sc} = \pi R^2 F_{tot,r,sc}(x, X) \quad (2.5)$$

The three functions F_{tot} , F_r , F_{sc} depend only on the dimensionless variables $x = kR$, and $X_o = k_0 R$.

At large energies, where $\Gamma = 1/k \ll R$ (where Γ is the *de – Broglie wavelength*/ 2π of the incident wave) the scattering and reaction cross-sections for neutrons together approach the identical asymptotic value [12].

$$\sigma_r = \sigma_{sc} = \pi(R + \Gamma)^2 \quad (2.6)$$

As a result, the overall cross-section for neutrons with high energy is as follows:

$$\sigma_{tot} = \sigma_r + \sigma_{sc} = 2\pi(R + \Gamma)^2 \quad (2.7)$$

Moreover, at low energies, the σ_r shows a $1/v$ dependency where v is the neutron velocity (4). Equation (2.7) for the reaction cross-section of high energy neutrons becomes for low energies:

$$\sigma_r \approx \pi(R + \Gamma)^2 [4kK/(k + K)^2] \quad (2.9)$$

where the quantity included in square brackets is the common expression for the transparency of the potential step at the surface of the nucleus. For very small bombarding energies, $k \ll K$ and $R \ll \Gamma = 1/k$, so that the approximate reaction cross-section:

$$\sigma_r \approx 4\pi/kK \quad \text{for} \quad R \ll \Gamma \quad (2.10)$$

These simple relationships indicate a monotonic variation in the total cross-section between

$$\begin{aligned} \sigma_{tot} &\approx \pi(R + \Gamma)^2 && \text{for large energies} \\ \sigma_{tot} &\approx 4\pi/kK && \text{for very small energies} \end{aligned} \quad (2.11)$$

2.2 Theoretical Models of Nuclear Reactions

A nuclear reaction is a complex procedure that rearranges the atomic nucleus. It is practically not possible to discover a precise solution to the problem here, comparable to the case of nuclear structure. Several phenomena may manifest in a nuclear reaction when a projectile is thrown at the nucleus. A study of nuclear reactions includes an extensive range of knowledge on the complicated structure of an atom. nuclear reaction provides information about production possibilities of radionuclides. Reaction dynamics discusses about interaction involved in a reaction (e.g. direct interaction, compound formation, preequilibrium procedures). Excitation function of a nuclear reactions is significant for production and applications. Nuclear models are developed to study production cross-sections [11].

2.2.1 Optical model

The optical model treats the nucleus as a continuum that refracts and absorbs the de Broglie waves of the incident particles. According to quantum physics, the portion of the refractive index for a de Broglie waves is played by the Hamiltonian of interaction of the particle with the force field of the nucleus. Such a Hamiltonian is given the form when an imaginary part iW is added to it to represent absorption.

$$H_{in} = V(r) + iW(r) \quad (2.12)$$

Where this term $V(r)$ is the Hamiltonian. As a result, in the optical model, the interaction between the impinging nucleon and the nucleus is approximately represented by absorption and scattering of the nucleon by a force center. The optical model describes the cross sections of all inelastic processes, the integral elastic scattering and differential cross sections at different energies of scattering nucleons i.e. the absorption cross-section of the nucleons. In the incident energy ranges 10 - 20 MeV, where the contribution of direct processes is rather small, the absorption cross-section coincides with the compound nucleus formation cross-section by nuclei. The restrictive case of the optical model is the model of a blackbody in which the nucleus is assumed to absorb all particles that had hit it [13].

2.2.2 Compound nuclear model

Niels Bohr (1936) first proposed this model, in which the reaction occurs in two steps with the production of an intermediate nucleus, C known as the compound nucleus:



The compound nucleus principle is only valid when the life time of the compound nucleus' is long enough ($10^{-14}sec$), It is far longer than the normal nuclear period ($10^{-21}sec$) [11].

According to Niels Bohr (1936), nuclear reactions occur in two stages: firstly, an incoming projectile is absorbed by the nucleus resulting in an excited condition. A statistical equilibrium between the nucleons is achievable because of the equal distribution of energy. Second, the excited nucleus decay into residue nuclei through the emission of a single particle or groups of particles. Angular momentum, parity, and energy all affect the decay modes. There is no relationship between these stages [14]. These stages do not depend on each other as follow in fig. 2.1

Consequently, the cross-section of the compound nucleus reaction of the kind indicated by equation (2.11) can be displayed as the product of two multipliers: the cross-section σ_a of the formation of the compound nucleus by particle a and the probability Γ_b of the decay of the compound nucleus through the b channel.

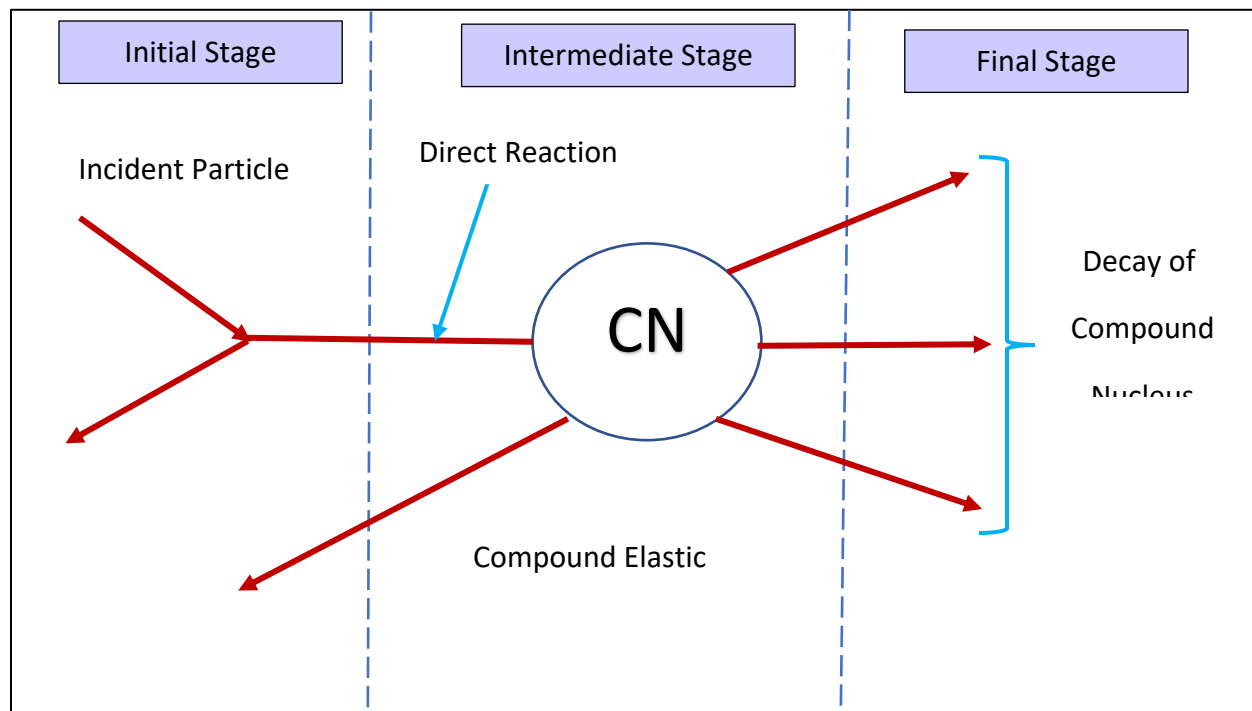


Fig. 2.1 Stages of nuclear reaction.

2-2-3 The Direct Reaction model

This is utilized if the time of reaction between the projectile particle and the target nucleus does not greater the characteristic nuclear time. In this event, the projectile effectively interacts one or two nucleons of the target nucleus without contacting the others or coming close to the surface of the target nucleus. Single nucleons, nucleon pairs, deuterons, ^3He nuclei, alpha particles, as well as more complicated nuclei like beryllium, lithium, etc., may escape from the target nucleus in direct processes. Direct reactions are divided into several types depending on the nature of the incident and emitted particles [5].

2.2.4 Pre-equilibrium model

Pre-equilibrium reactions have an important function in nuclear reactions with intermediate energy values. Pre-equilibrium reactions are a possibility in addition to direct nuclear reactions and decay modes. Many theoretical models lend support to this idea. Although the models are basic, they are often modified to study the dynamics of the reaction [137–138].

Both the statistical decay of the compound nucleus and the prompt emission after collision do not produce particles from the excited target nucleus in pre-equilibrium processes. In pre-equilibrium reactions, the semi-classical models provided by Griffin, the exciton model, and the hybrid model are utilized [139]. During the reaction, the exciton number may change as a result of collision. It is referred to as pre-equilibrium emission when a particle emits during the beginning of a reaction.

Pre-compound emission occurs 10^{-18}sec before direct interaction occurs, with a time scale of 10^{-21}sec . [140-141]

2.2.5 Nuclear level densities

Atomic structure as it relates to energy states is the basis for nuclear physics. "The number of levels per unit excitation energy" is the definition of nuclear level density. Level densities depend on the statistical characteristics of excited nuclear levels of the nuclei. Calculating the reaction cross-sections requires knowledge of level densities. It is described using phenomenological and microscopic models. These models' input parameters can be varied to achieve the best fit with the experimental data. The level densities are described using the Fermi gas model and models with constant temperature. Level densities can be accommodated via the Generalized Superfluid Model (GSM). It is essential that you acquire information of the level density from experimental data in order to utilize a statistical model. [144-147]

2.2.6 The Statistical Multistep Direct (SMD) and The Statistical Multistep Compound (SMC)

The (SMD) and (SMC) methods explain how the compound nucleus stabilizes through a succession of two-body collisions that could ultimately lead to the creation of a compound nucleus. The (SMC) mechanism is considered to be most applicable at low incident energies (10–20 MeV) and by definition only includes bound configurations embedded in the continuum, yielding angular distributions symmetric about 90 degrees center of mass [6].

2.3 Neutron Activation Method

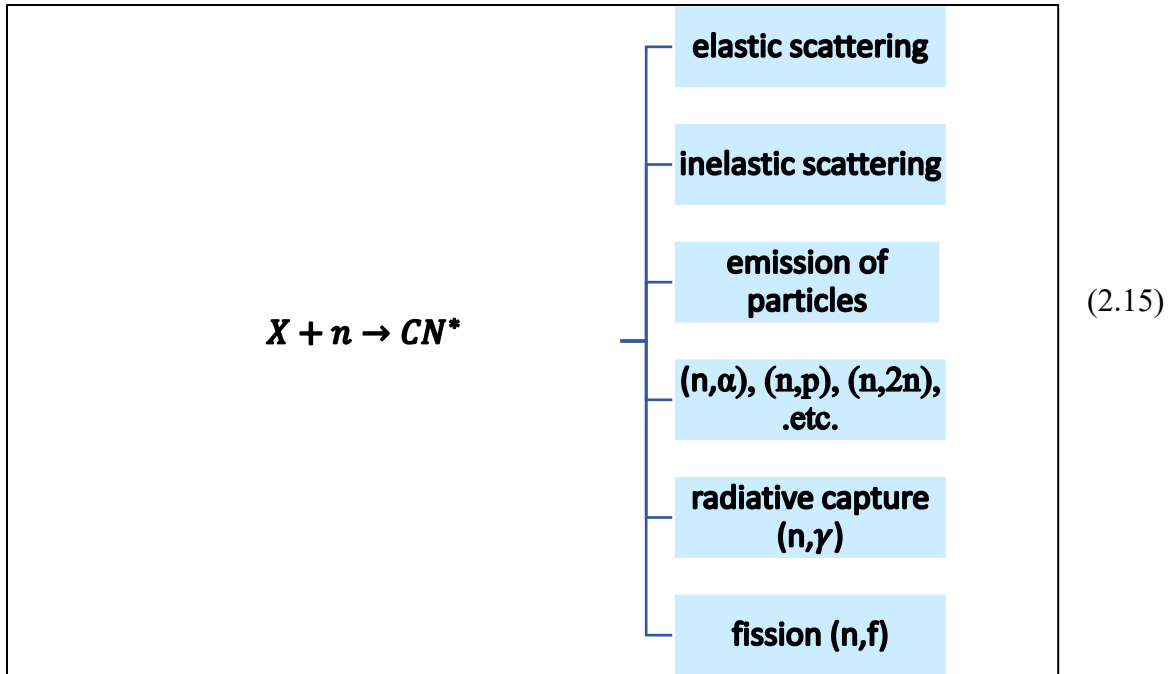
The neutron is neutral and mass 1.008665 amu. In the absence of other nuclear matter, a free neutron decays with a half-life of approximately 12.5 min into a proton, an electron and neutrino by the reaction:



Many of the features of neutrons depend on their kinetic energy. According to their energy, neutrons can be classified randomly as shown in:

- 1- Slow Neutrons: neutrons with energies up to 1 Kev. The most important subgroup of this class is the "thermal" neutrons with energies about 0.025 eV. Another crucial subgroup of slow neutrons is the Epithermal neutrons, which have energy between 1 eV and 1 KeV.
- 2- Intermediate Neutrons: These neutrons range in energy from around 1 to 500 KeV.
- 3- Fast neutrons are those with energies greater than 500 KeV.

Neutrons interact with target nuclei to generate compound nuclei. The amount of time the compound nucleus can stay in a highly excited state is finite-time. Independent of how the compound nucleus was created, de-excitation of the compound nucleus can occur in a variety of ways. The probability of each of these processes, as illustrated in equation (2.14), totally depends on the nuclear cross-section of each mode, which is connected to the excitation energy of the compound nucleus [8]. The following relationship illustrates the different ways that a compound nucleus can disintegrate when neutrons contact with target nucleus X.



The product nucleus that is produced can be radioactive or stable. The radioactive nuclide that is produced has a specific half-life, mode of decay, and energy emitted [8]. Since most radionuclides emit gamma rays with energy ranging from 40 to 1000 KeV and a relatively high penetrating power, measurements of gamma rays produced by radioactive nuclides have generally far wider uses in neutron activation technique. Therefore, gamma rays are subjected to slight losses by absorption in a sample throughout their measurements. The neutron activation technique for reaction cross-section measurements using gamma ray spectroscopy is an effective method as a result of this feature, as well as recent advancements in high-resolution and high-efficiency semiconductor detectors.

CHAPTER THREE: Methods

3.1 Empirical formulae for (n, p), (n, α) Reaction Cross-Sections

The activation method has been used to measure the cross-sections of the (n,p) and (n, α) reactions for a variety of elements and isotopes at neutron energies about 14.5 MeV [1-3]. Numerous studies have noted the impact of the $(N - Z)/A$ asymmetry parameter as well as the isotonic, isotopic, and even-odd characteristics of nuclei on the cross-sections [3-4]. For estimating the unknown data and choosing a suitable cross-section among the inconsistent experimental values, precise knowledge of the various systematics is important. Absolute normalization of the calculated and measured excitation functions was made possible by the accuracy of the mean neutron energy nearly equal 20 KeV and energy resolution approximately equal 100 KeV of D-T neutrons at 14.5 MeV. The major trends noticed in the cross-sections approximately 14.5 MeV neutron energy have been discussed in reference [3]. There was a significant $(N - Z)/A$ dependence in all of the reaction cross-sections. The empirical formula Levkovsky [4] suggested in the following form could sufficiently match the experimental data:

$$\sigma(n, x) = a_1 \sigma_{NE} \exp^{a_2(N-Z)/A} \quad (3.1)$$

where σ_{NE} is the term nonelastic cross-section, a_1 and a_2 are fitting parameters which are different for the $\sigma(n, p)$ and $\sigma(n, \alpha)$ data.

Although the cross-sections were determined using better methods and more accurate decay parameters, the data are still insufficient and frequently inconsistent at energies about 14.5 MeV. Recently, Cheng and Smith [5] provided a list of 83 activation reactions with insufficiently determined cross-sections. For reactions involving (n,p) and (n, α), the spread is considerable in both the old and new data [3,6]. In this work, studies were carried out to improve the data's accuracy and to analyze the behavior of the various $(N - Z)/A$ parameter trends. we substitute the nonelastic cross-section part in the empirical formulae with one that includes the neutron number, the average binding energy per nucleon and neutron number minus atomic number plus one.

3.2 Excitation function of some radionuclides used in nuclear medicine

The cross-sections for different nuclear reactions are highly individualistic and dependent on the bombarding energy of the incident particle. No two are the same. Experimental resemblances between reactions are typically limited to gross features and to overall trends. The detailed dependency of cross-sections on bombarding energy is frequently called the "excitation function" or the "transmutation function" for the specific reaction [1]. The precise understanding of excitation functions of fast neutron reactions is important from the perspectives of nuclear reaction theory (spin distribution parameters, applications of data in dosimetry, decay branching ratios), neutron flux standardization, elemental analysis, design of thermonuclear devices, etc.

3.3 Nuclear model Computer codes

The creation of computer code to carry out theoretical calculations uses theoretical models. Various codes include various models. Nuclear model help validate the results of experimental data. They support developing the reaction mechanisms needed to investigate reaction channels, projectiles, and ejectiles. The input parameters of the code can be varied within suitable limits. The most significant parameters are “optical model parameters, direct interactions, level density parameters, pre-equilibrium reactions, γ -ray transmission coefficients, equilibrated emissions and multiple emissions of particles” [61].

3.3.1 The (n, f) Reaction Cross-Section Formula

The Anand et al., formula was used to find empirical equations based on the (n, f) nuclear reaction cross sections' dependency on the penetration of the fission barrier in the neutron energy range of 1–20 MeV [15].

The fission cross section grows and saturates at the initial value of the fission plateau σ_1 , which is maintained up to 5–6 MeV, with increasing neutron energy E_n . Nuclei with neutron binding energies greater than the fission barrier do not show the rising component of the fission cross section. Due to the excitation energy being over the fission threshold, one neutron can evaporate without reducing the excitation energy of the other nuclei below it, which results in the second increase: In this case, a second opportunity for fission is offered to the system (n,nf). In a similar way to this, the (n, 2nf) reaction leads to an additional rise between 12 and 14 MeV after the second plateau, resulting in the third plateau. Since the processes (n,f), (n, nf), and (n, 2nf) are added together, the total neutron-induced fission cross-section up to the threshold of the fourth plateau on target A may be written as follows [6]:

$$\sigma_f(E_f) = P_1\sigma_1 + \left(1 - \frac{\sigma_1}{\sigma_R}\right) K \times \left\{P_2\sigma_2 + \left(1 - \frac{\sigma_2}{\sigma_R}\right) P_3\sigma_4\right\} \quad (3.2)$$

Where penetrability parameter for the i th chance fission of the Hill-Wheeler type is P_i ($i = 1, 2, 3$), as illustrated in [16]:

$$P_i = \left[1 + \exp\left(\frac{2\pi\{E_f - \epsilon_i\}}{\hbar\omega}\right)\right]^{-1} \quad (3.3)$$

According to the first, second, and third chance fissions, respectively, the excitation energies ϵ_i ($i = 1, 2, 3$) are as follows:

$$\epsilon_1 = E_n + S_n(A + 1), \epsilon_2 = E_n \text{ and } \epsilon_3 = E_n + S_n(A) \quad (3.4)$$

where $S_n(A)$ is the neutron separation energy for the mass number A [6]. For all the nuclei in e.g. (3), $\hbar\omega$ represents the barrier's curvature, has been assumed to be 1 MeV [17]. Additionally, the contribution from the fission of the initial residual nucleus A is represented by σ_2 in (1). It has been used as the target for the first plateau of the neutron-induced fission of the $(A - 1)$ nucleus.

Similar results may be obtained for the neutron plus ($A - 2$) target, which is characterized by the third fission plateau and is represented by σ_3 . The non-equilibrium neutron emission of the compound system is thought to be the cause of the parameter K , which is less than 1. From the examination of $(n, 2n)$ cross-sections, it is assumed that the factor K is the matching as that provided by [18], and is

$$K = 1 - \exp^{-10.605*(N-Z)/A} \quad (3.5)$$

where Z and N are the proton and neutron numbers of the target, respectively. The reaction cross section σ_R is given by [19]:

$$\sigma_R = \pi r^2 (A^{\frac{1}{3}} + 1)^2 \text{ (barn)} \quad (3.6)$$

In our work, we have modified e.g. (1-2) formula by taking into account the impact of the σ_R . The σ_i term is replaced by modified a free parameter a_i , and the range of investigated target nuclei is extended to $232 \leq U^A \leq 238$, at 1-20 MeV of the incident neutron energy:

$$\sigma_{(n,f)} = \sigma_R [a_1 P_1 + a_2 P_2 + a_3 P_3] \quad (3.7)$$

$$P_i = \left[1 + \exp \frac{2 \cdot \frac{1}{2} \pi \{E_f - \epsilon_i + \hbar \omega\}}{\hbar \omega} \right]^{-1} \quad (3.8)$$

where a_1, a_2 and a_3 are free fitting parameters. These are shown numerically in Table 1.

Table 3.1. The parameters that are used to calculate fission cross sections.

Target	free fitting parameters			Barrier heights $E_f(\text{MeV})[20]$			Neutron separation energies $S_n(\text{MeV})$	
	a_1	a_2	a_3	(A+1)	(A)	(A-1)	(A+1)	(A)
^{228}Th	0.15	0.15	0.15	6.06	6.52	6.98	5.25	7.10
^{229}Th	0.06	0.18	0.30	5.66	6.06	6.52	6.79	5.25
^{230}Th	0.11	0.18	0.05	5.55	5.66	6.06	5.11	6.79
^{231}Th	0.024	0.26	0.034	5.44	5.55	5.66	6.44	5.11
^{232}Th	0.056	0.08	0.05	5.47	5.44	5.55	4.78	4.78
^{233}Th	0.02	0.02	0.08	5.38	5.47	5.44	6.16	4.78
^{234}Th	0.027	0.05	0.025	5.8	5.38	5.47	4.66	6.19

3.3.2 EMPIRE-3.2.3 code

The computer software EMPIRE-3.2.3 is used to run nuclear reaction codes. It consists of various nuclear models and is capable of performing calculations with various incident particles and different energies. Moreover, it can be applied to the analysis of nuclear data. It has models such as the optical model, Multi-step Direct (ORION + TRISTAN), Coupled Channels and DWBA (ECIS06 and OPTMAN), exciton model (PCROSS), NVWY Multi-step Compound, the fully functional Hauser-Feshbach model, and hybrid Monte Carlo simulation (DDHMS)[154]. The EMPIRE 3.2.3 has various input parameters from RIPL-3 Library such as masses, incident energy, optical model parameter, etc [150]

EMPIRE 3.2.3, which deals with the statistical and dynamical treatment of the nuclear structure, incorporates the Multi-Step Direct (MSD) theory put forth by Tamurs, Lenske, and Udagawa. This theory allows for the study of projectiles with a wide range of energy. The Multi-Step Compound (MSC) Model from Nishioka et al. (NVWY) is likewise included in EMPIRE 3.2.3 [140]. It discusses the equilibrium of composite nuclei. The emission of the multi-step compound mechanism is explained by the model put forth by Hoering and Weidenmueller [142]. The Giant Dipole Resonance within MSC classes in this model is deexcited, which leads to the emission. Each nuclear state is considered to be the basis for GDR excitation on the Brink-Axel hypothesis since they all have identical characteristics [155–156].

In EMPIRE-3.2.3, the classical exciton model (PCROSS code) and the monte Carlo DDHMS (Blann's Hybrid Model) are both utilized as pre-equilibrium models. The classical model only calculates the pre-equilibrium input of the reaction. Mignerey and Blann suggest parameterization of the transition rate. It is approximately based on scattering data and the Pauli principle. Kalbach's method is used to calculate the nucleon emission rate [141]. The Hybrid Monte-Carlo Simulation (HMS) model for the preequilibrium emission of nucleons was developed by M. Blann.

This method avoids the multi-exciton level densities that Bisplingho proposed. The quantity of pre-equilibrium emissions is not physically limited. It contains linear momentum conservation further by M. Chadwick and P. Obllzinsky existing a comprehensive list of observables. It combines spin- and excitation-energy dependent populations as well as cross-section for light particle spectra. The statistical model employed in the EMPIRE is a powerful application of the Hauser-Feshbach theory. The binding energy in the HMS mode is accurate in terms of thermodynamics. It is capable of doing calculations up to 250 MeV.

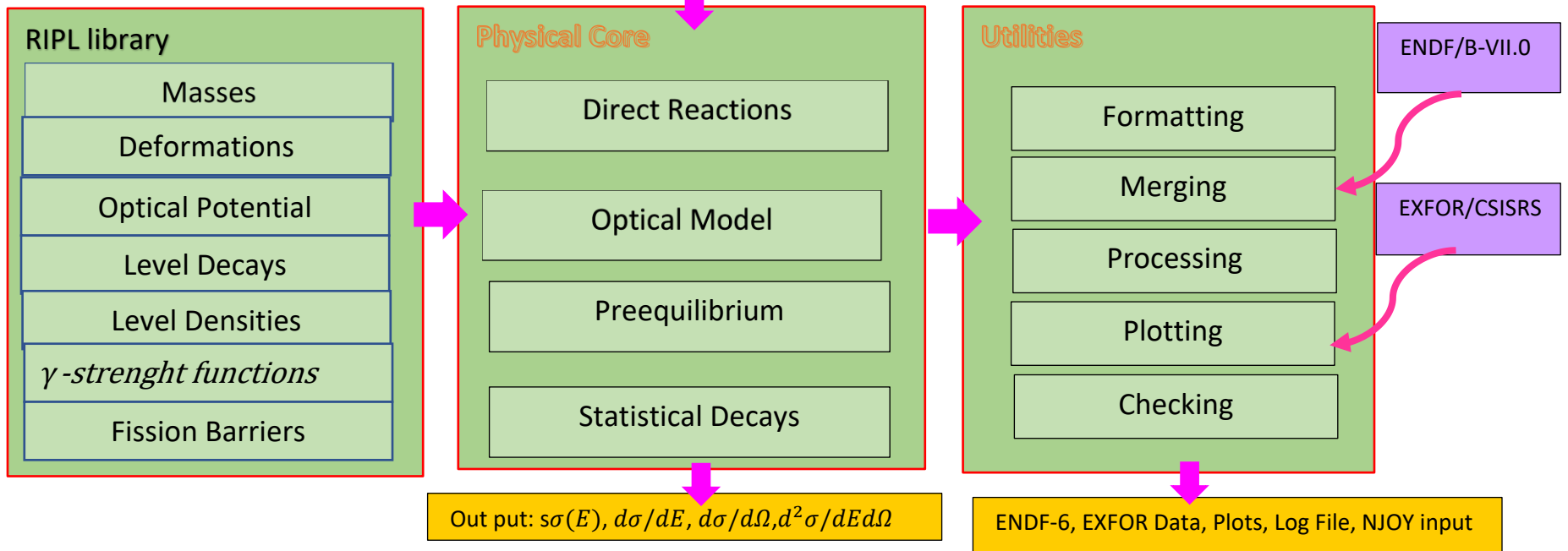
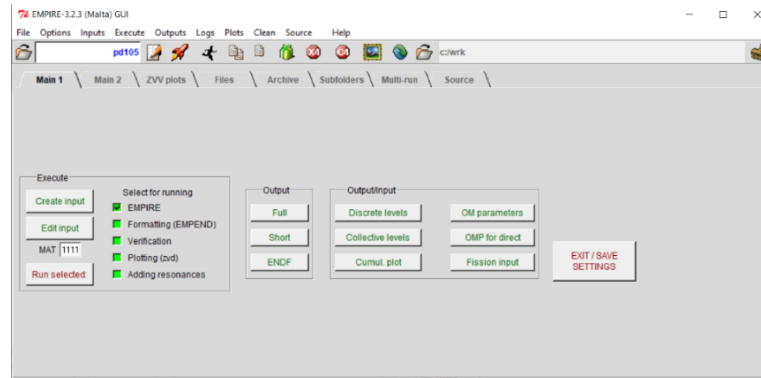


Fig. 3.1 Flow sheet of EMPIRE 3.2.3 [154].

3.3.3 TALYS-1.95 Codes

TALYS-1.95 is another code which is used for calculating of nuclear cross-sections and study of nuclear reactions. It was invented in 1998 at CEA Bruyres-le-Chtel, France, and NRG Petten, Netherlands. The first authorized version TALYS-1.95 was published on December 21, 2007 [148-149]. It has a wide energy range of calculating capabilities, from 1 keV to 200 MeV.

In TALYS-1.95, geometry can be modified with the "rvadjust" command to fit the data. The latest option, "ldmodel 6," is based on Hartree-Fock-Bogolyubov calculations that depend on temperature and apply the Gogny force. TALYS-1.95 applied Eric Bauge's MOM code as a subroutine to perform the so called Jeukenne-Lejeune-Mahaux (JLM) OMP calculations. For JLM calculations, the "Lvadjust" normalization factor is used to calculate the real central potential. Only spherical JLM OMP's are involved in TALYS 1.95. It is also possible to run TALYS-1.95 calculations using the semi-microscopic nucleon-nucleus spherical optical model potential in addition to the phenomenological OMP. The high-energy component of the continuum spectra is often explained by pre-equilibrium models [150-153]. It includes numerous phenomenological and microscopic level density models. These are a few key aspects listed below: -

1. Optical model and combined channels calculations have been fully integrated into the ECIS code.
2. Pre-equilibrium reactions have been modeled using both classical and quantum mechanical methods, and TALYS-1.95 combines both. In general, these models have a single-particle nature.
3. The most recent nuclear models for direct compound, preequilibrium, and fission reactions can all be implemented.
4. It automatically refers to the input parameters, such as masses, level densities, resonances, and density parameters from the IAEA (International Atomic Energy Agency) input parameter library.
5. It can be utilized to produce medicinal isotope yields depending on the energy and beam current.
6. Several samples are utilized in the calculations.

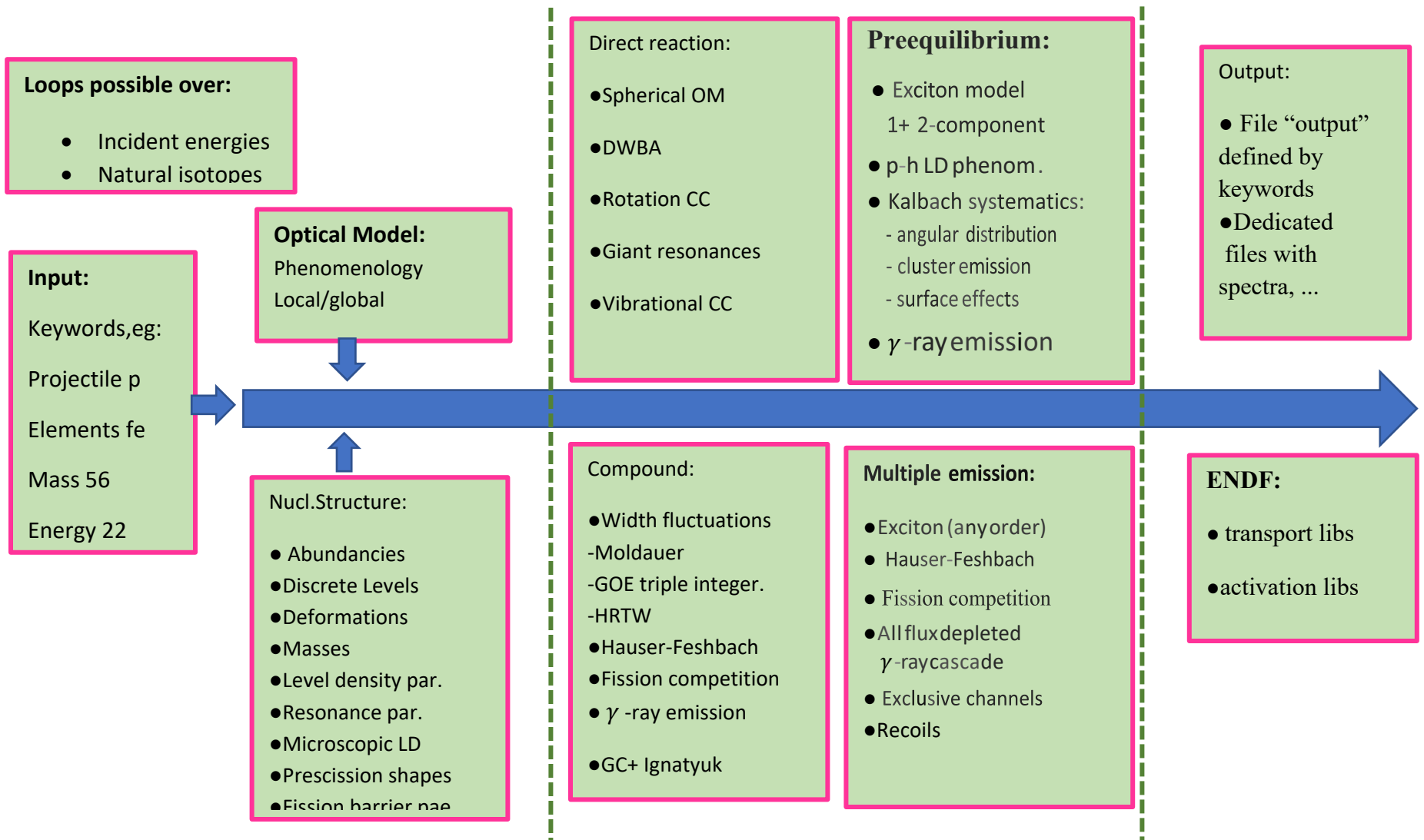


Fig. 3.2 Flow sheet of TALYS 1.95.

3.1 Excitation function of (n,f) reactions at 1 to 20 MeV

4.1.1 ^{228}Th (n, f) reaction

In addition to the newly calculated cross sections, the evaluated cross sections together with the results from previous experimental data are shown in Figs. 1-6. Neutron energies up to 20 MeV are now included in the calculation of $^{228}\text{Th}(n,f)$. Using the TALYS 1.95 and EMPIRE 3.2.3 codes and comparing the results with empirical formulae, experimental and evaluated data are given in Fig.1. The calculated values from Empirical formula, TALYS 1.95 and EMPIRE 3.2.3 codes are in excellent agreement with the evaluated data file (TENDL-2019, EAF-2010, JENDL 3.3, ENDF/B-VII.1 and ROSFOND-2010 and EXFOR data (G. D. James et al., 1984) and (P.E.Vorotnikov et al., 1973), except some shifting with Talys 1.95 at energy range 8-20 MeV.

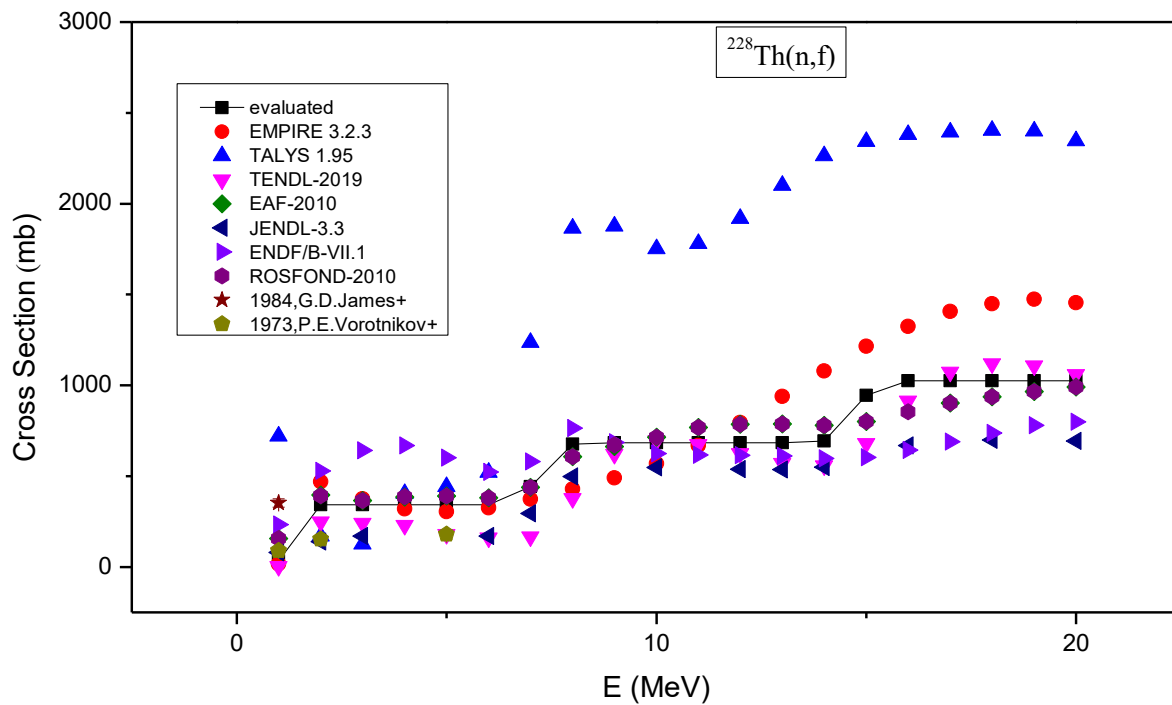


Fig.1. The ^{228}Th (n,f) reaction cross section by using empirical formulas, nuclear codes and EXFOR data.

4.1.2 ^{229}Th (n, f) reaction

In Fig. 2, the resulted of ^{229}U (n, f) reaction have been shown. The calculated data of Empirical formula agrees with the previous work (TENDL-2021, EAG-2010, JENDAL-4.0, JEFF-3.3, ROSFOND-2010) except some shifting with Empire 3.2.3 at energy range 8-20 MeV

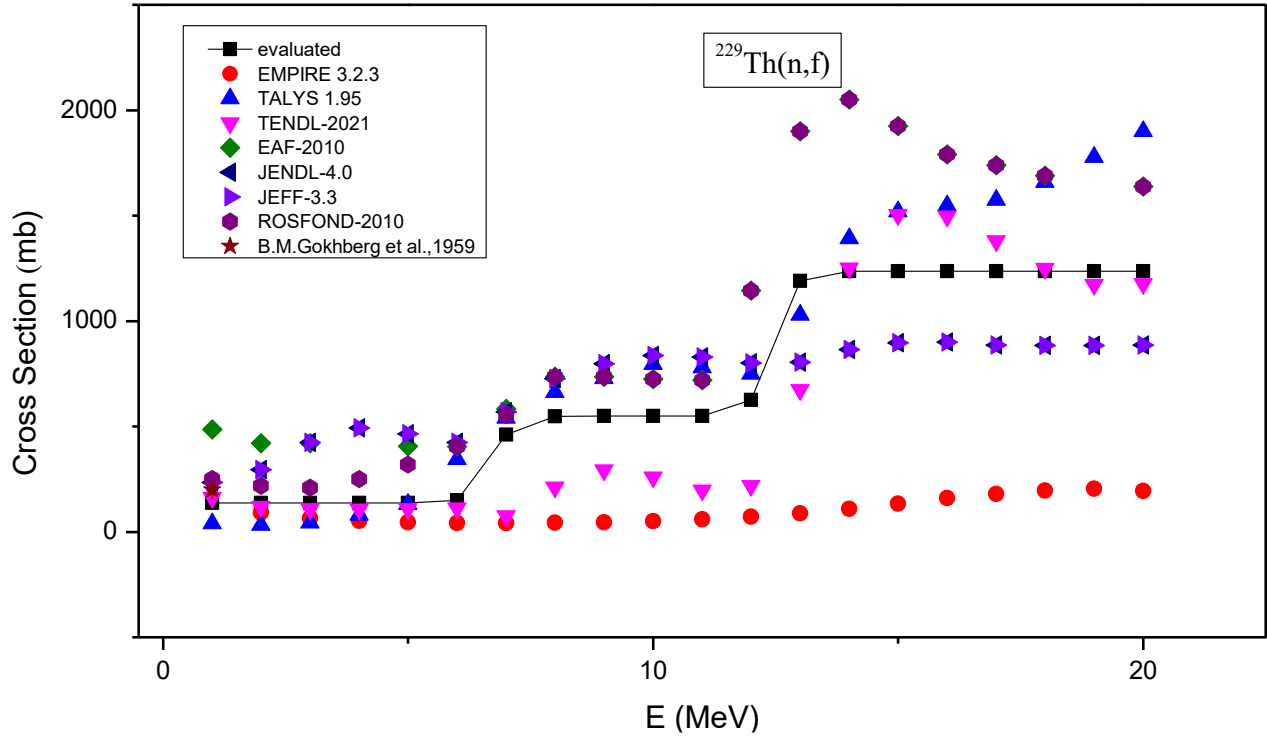


Fig.2. The ^{229}Th (n,f) reaction cross section by using empirical formulas, nuclear codes and EXFOR data.

4.1.3 ^{230}Th (n, f) reaction

Fig. 3 Shows the calculated and evaluated data for the ^{230}U (n, f) reaction cross sections. The cross-section data from Empirical formula and EMPIRE 3.2.3 codes are close to the evaluated data of (TENDL-2021, JENDL-3.3, JEFF-3.3, ROSFOND-2010, BROND-3.1), previous experimental data (D.W.Muir et al.,1971), (J.W.Meadows,1983), (J.Blons et al.,1980), (G.D.James et al.,1972), (D.W.Muir et al.,1971), (M.I.Kazarinova et al.,1960), (B.L.Goldblum et al.,2009) and (M.Petit et al.,2004).

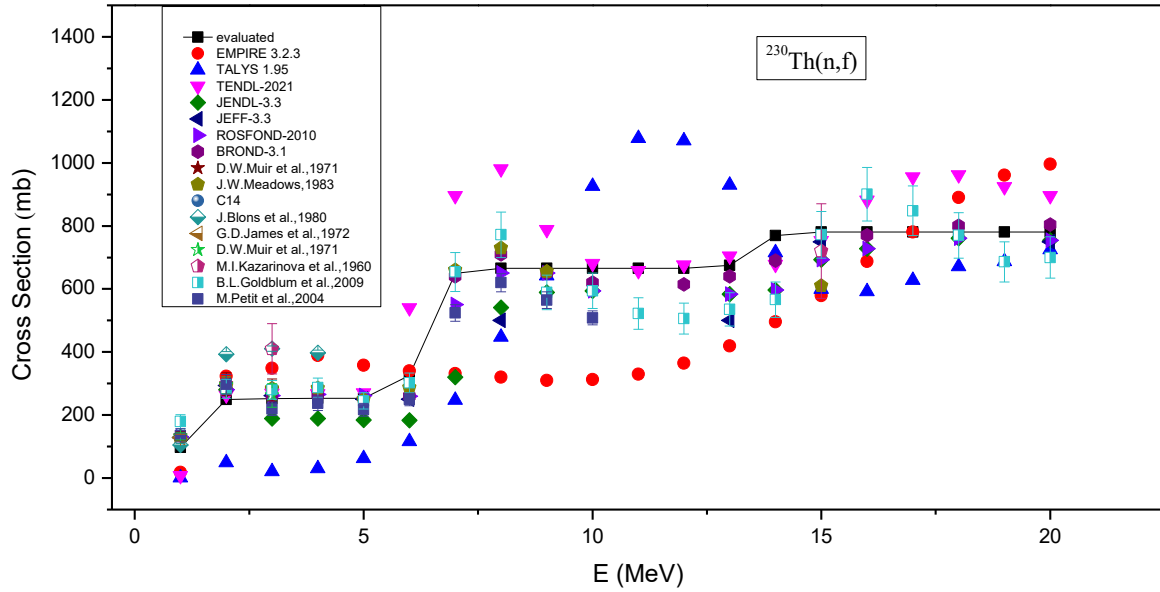


Fig.3. The ^{230}Th (n,f) reaction cross section by using empirical formulas, nuclear codes and EXFOR data.

4.1.4 ^{231}Th (n, f) reaction

Fig. 4 compares calculated data (EMPIRE 3.2.3, TALYS 1.95), all evaluated data (JENDL/AC-2008, EAF-2010, ROSFOND-2010, BROND-3.1), for the $^{231}\text{Th}(n,f)$ nuclear reactions agree Empirical formula, except TENDL-2021 has been shifted on line at energy range 8-20.

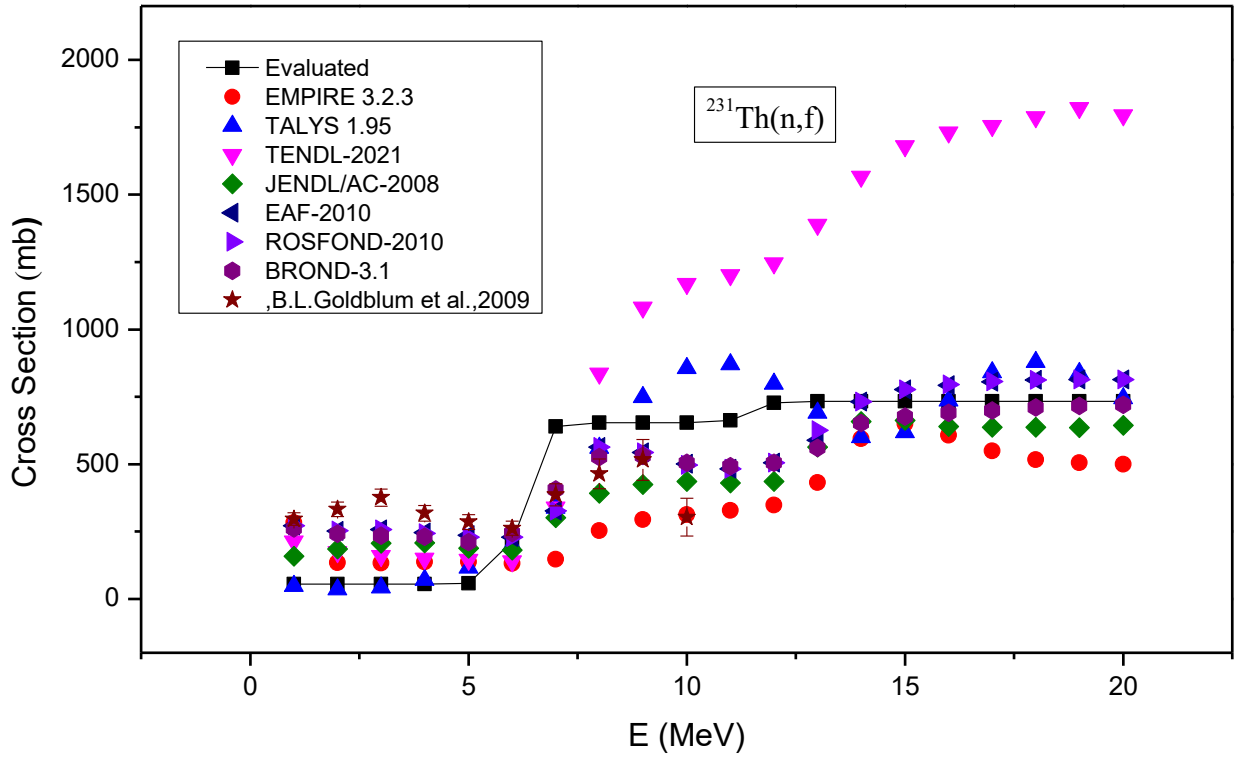


Fig.4. The ^{231}Th (n,f) reaction cross section by using empirical formulas, nuclear codes and EXFOR data.

4.1.5 ^{232}Th (n, f) reaction

As of ^{232}Th (n, f) nuclear reaction are displayed in Fig. 5. The Empirical formula are mostly in good agreement between 1 and 20 MeV with EMPIRE 3.2.3 code, TENDL-2019, and all experimental results such as (O.Shcherbakov et al.,2002) , (P.W.Lisowski et al.,1988), (J.Blons et al.,1980) except (J.W.Meadows,1983) and TALYS 1.95 formula show significant discrepancies.

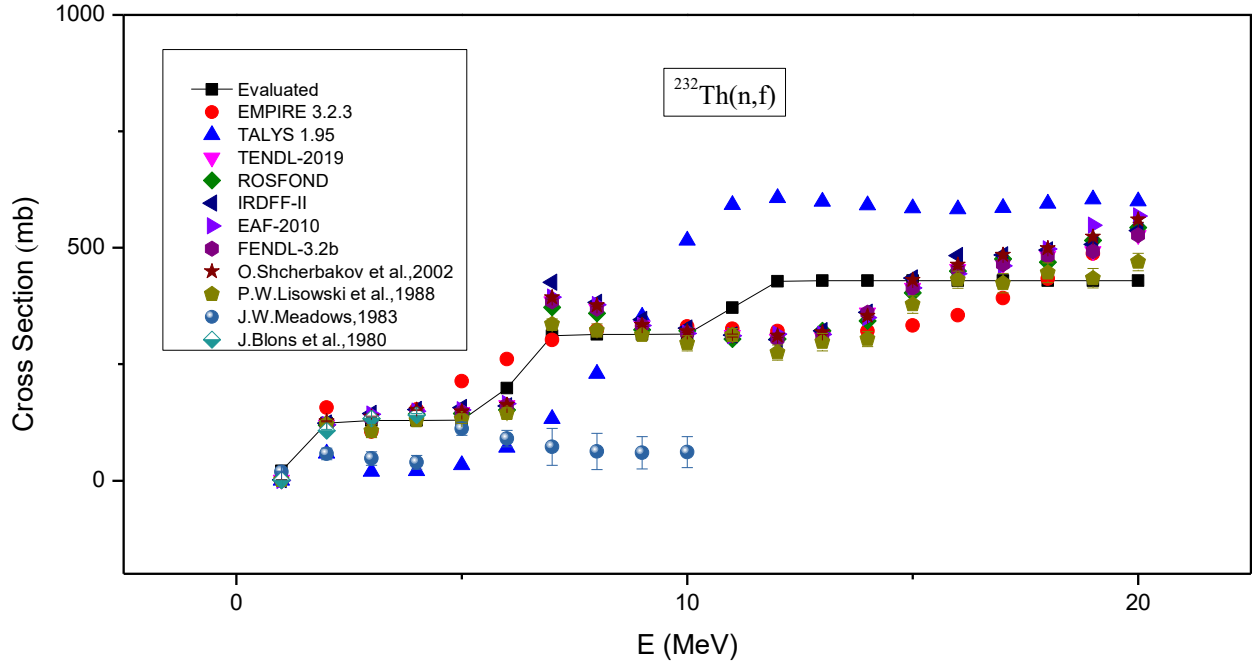


Fig.5. The ^{232}Th (n,f) reaction cross section by using empirical formulas, nuclear codes and EXFOR data.

4.1.6 ^{223}Th (n, f) reaction

Nuclear reactions as of ^{223}Th (n, f) are shown in Fig. 6. the calculated values from the empirical formula are in middle the evaluated data files (JEFF-3.3, JENDL 3.3). EMPIRE 3.2.3, TALYS 1.95 show significant discrepancies.

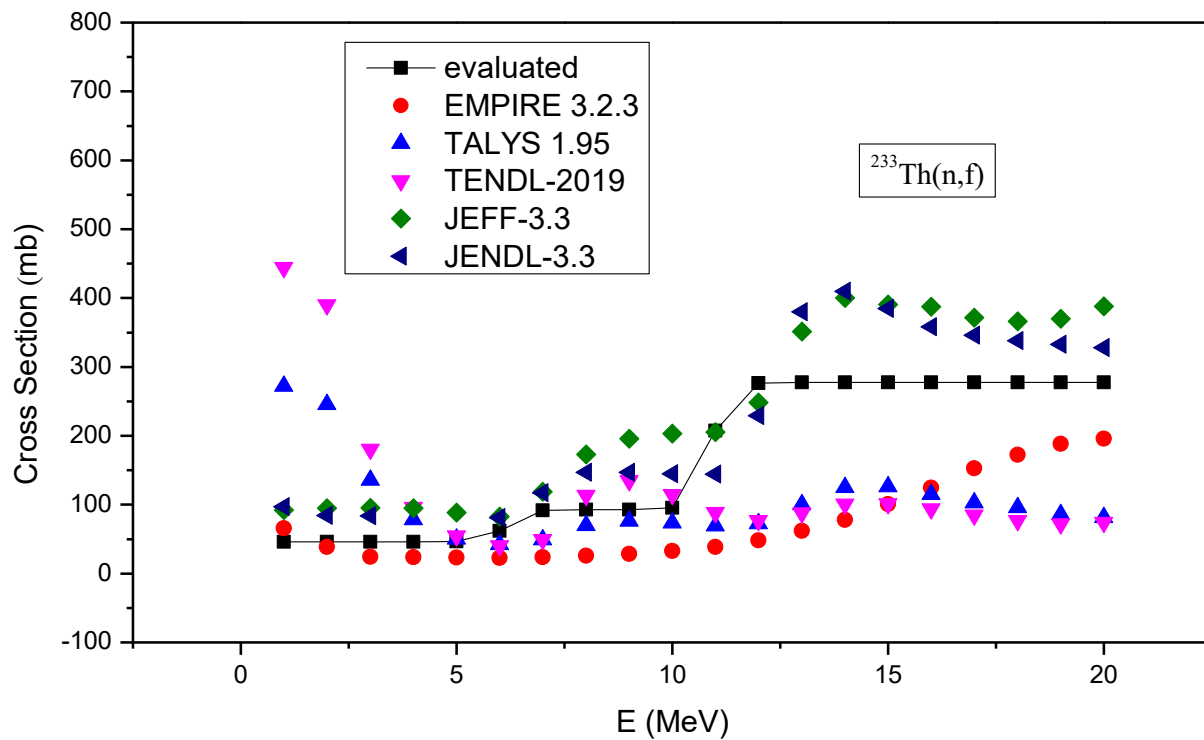


Fig.6. The ^{223}Th (n,f) reaction cross section by using empirical formulas, nuclear codes and EXFOR data

4.1.7 ^{224}Th (n, f) reaction

The resulted excitation function for ^{234}Th (n, f) reactions at the energies range 1 to 20 MeV are shown in Fig.7. The cross sections calculated with empirical formula, evaluated (TENDL-2019, EAF 2010, JENDL -5, ROSFOND-2010, JEEF-3.1 are often in good agreement

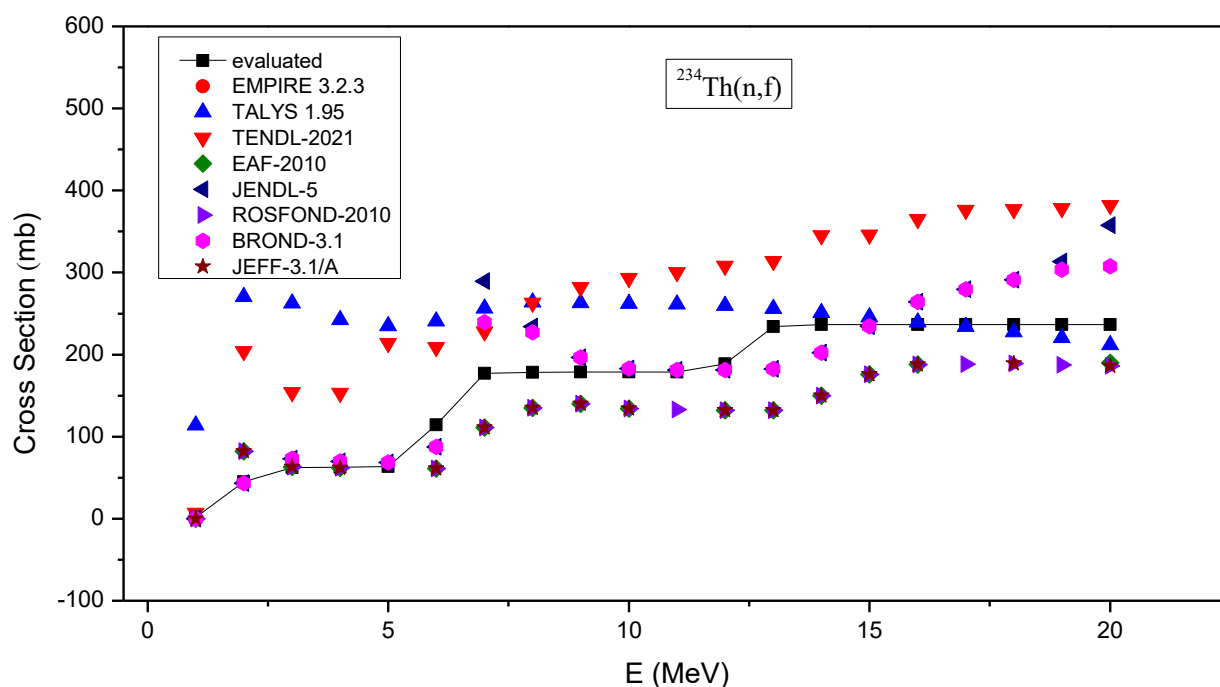


Fig.7. The ^{234}Th (n,f) reaction cross section by using empirical formulas, nuclear codes and EXFOR data

4-Conclusion

The modified empirical formula could be utilized to describe the fission cross-sections of many different thorium isotopes. Three parameters total have been used in our calculation. Besides σ_R and penetrability parameter dependencies of the cross-section formula. The outcomes of our reproduction formula indicate a significant development in the expected cross-sections in compared to the EMPIRE 3.2.3 code, TALYS 1.95 code, previous research in EXFOR library for describing the evaluated data. The existing empirical formula may be thought of as a very useful tool for immediate calculation of unknown or unmeasured (n, f) reaction cross-sections between 1 and 20 MeV neutron energy. The suggested semi-empirical expression's main benefit is its extreme simplicity, it may be used even with a desktop calculator.

References

- A. Ercan, M.N. Erduran, M. Suba\csi, E. Gueltekin, G. Tarcan, A. Baykal, M. Bostan, 14.6 MeV Neutron Induced Reaction Cross-Section Measurements, in: Nucl. Data Sci. Technol., 1992: pp. 376–377.
- B.C. Diven, Fission Cross Section of U 235 for Fast Neutrons, Phys. Rev. 105 (1957) 1350.
- E. Kondaiah, ($n, 2n$) cross sections and the statistical model predictions, J. Phys. A Math. Nucl. Gen. 7 (1974) 1457.
- E.T. Cheng, D.L. Smith, Nuclear data needs and status for fusion reactor technology, in: Nucl. Data Sci. Technol. Proc. an Int. Conf. Held Forschungszentrum Jülich, Fed. Rep. Ger. 13--17 May 1991, 1992: pp. 273–278.
- G.J. Csikai, CRC handbook of fast neutron generators, (1987).
- H.M. Abdullah, A.H. Ahmed, Empirical systematics for (n, p) reaction cross sections at 14--15 MeV neutrons, Indian J. Phys. (2022) 1–12.
- H.A. Mahmud, A.H. Ahmad, Empirical formulae for (n, p) reaction cross-sections at 14–15 MeV neutrons, Int. J. Mod. Phys. E. 27 (2022) 1850079.
- H.M. Abdullah, A.H. Ahmed, Semi-empirical formula for (n, α) reaction cross sections at 14--15 MeV neutrons, Appl. Radiat. Isot. (2022) 110396.
- J.R. Nix, The normal modes of oscillation of a uniformly charged drop about its saddle-point shape, Ann. Phys. (N. Y). 41 (1967) 52–107.
- M.L. Jhingan, R.P. Anand, S.K. Gupta, M.K. Mehta, Semi-empirical approach for predicting neutron-induced fission cross-sections in the energy range 1--18 MeV, Ann. Nucl. Energy. 6 (1979) 495–498.

- M. Herman, Overview of nuclear reaction models used in nuclear data evaluation, *Radiochim. Acta.* 89 (2001) 305–316.
- P. Möller, A.J. Sierk, T. Ichikawa, A. Iwamoto, M. Mumpower, Fission barriers at the end of the chart of the nuclides, *Phys. Rev. C.* 91 (2015) 24310.
- S.M. Qaim, others, *Handbook of spectroscopy*, CRC, Boca Rat. (1981).
- S. t Pearlstein, Analysis of (n, 2 n) Cross Sections for Medium and Heavy Mass Nuclei, *Nucl. Sci. Eng.* 23 (1965) 238–250.
- T.M. Letcher, *Future energy: improved, sustainable and clean options for our planet*, Elsevier, 2020.
- V.E. Viola Jr, B.D. Wilkins, Fission barriers and half-lives of the trans-radium elements, *Nucl. Phys.* 82 (1966) 65–90.
- V.N. Levkovskii, Empirical behavior of the (n, p) cross section for 14-15 MeV neutrons, *Zh. Eksperim i Teor. Fiz.* 45 (1963).
- V. Weisskopf, Statistics and nuclear reactions, *Phys. Rev.* 52 (1937) 295.
- W.E. Meyerhof, W.E. Meyerhof, *Elements of nuclear physics*, McGraw-Hill New York, 1967.