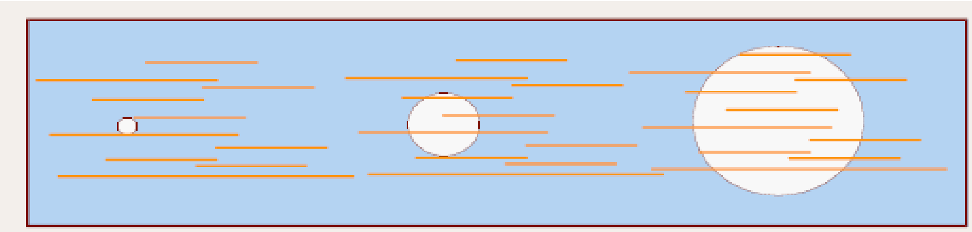
Chapter Six
CAVITY THEORY

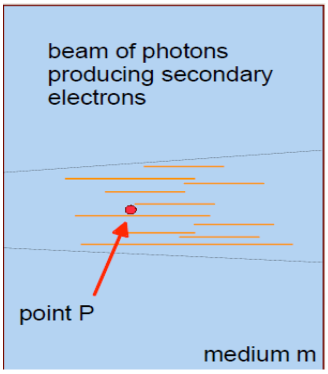
6.1 Introduction:

In order to measure the absorbed dose in a medium, it is necessary to introduce a radiation sensitive device (dosimeter) into the medium. Generally, the sensitive medium of the dosimeter will not be of the same material as the medium in which it is embedded. Cavity theory relates the absorbed dose in the dosimeter's sensitive medium (cavity) to the absorbed dose in the surrounding medium containing the cavity. Cavity sizes are referred to as small, intermediate or large in comparison with the ranges of secondary charged particles produced by photons in the cavity medium. If, for example, the range of charged particles (electrons) is much larger than the cavity dimensions, the cavity is regarded as small.

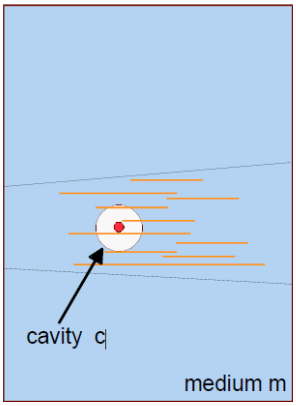


Consider a point P within a medium m within a beam of photon radiation (right).The absorbed dose at point P can be calculated by:

$$D_{med}(P) = \phi\left(\frac{\overline{S}}{\rho}\right)_{med} \qquad 6.1$$



In order to measure the absorbed dose at point P in the medium, it is necessary to introduce a radiation sensitive device (dosimeter) into the medium. The sensitive medium of the dosimeter is frequently called a cavity. Generally, the sensitive medium of the cavity will not be of the same material as the medium in which it is embedded.



The measured absorbed dose D_{cav} within the entire cavity can also be calculated by:

$$D_{cav} = \int_{V_{cav}} \int_{0}^{E_{max}} \Phi_{E,\vec{r}}(E,\vec{r}) \left(\frac{S_{cav}(E)}{\rho}\right)_{cav} dE d\vec{r} \qquad 6.2$$

If the material of the cavity differs in atomic number and density from that of the medium, the measured absorbed dose to the cavity will be different from the absorbed dose to the medium at point P.

$$D_{cav} \neq D_{med}(P) \qquad 6.3$$

Various cavity theories for photon beams have been developed, which depend on the size of the cavity; for example, the Bragg– Gray and Spencer–Attix theories for small cavities and the Burlin theory for cavities of intermediate sizes.

6.2. Bragg–Gray cavity theory

The Bragg–Gray cavity theory was the first cavity theory developed to provide a relation between the absorbed dose in a dosimeter and the absorbed dose in the medium containing the dosimeter.

The conditions for application of the Bragg–Gray cavity theory are:

(a) The cavity must be small when compared with the range of charged particles incident on it, so that its presence does not perturb the fluence of charged particles in the medium;

(b) The absorbed dose in the cavity is deposited solely by charged particles crossing it (i.e. photon interactions in the cavity are assumed negligible and thus ignored).

The result of condition (a) is that the electron fluences are the same and equal to the equilibrium fluence established in the surrounding medium. This condition can only be valid in regions of CPE or TCPE. In addition, the presence of a cavity always causes some degree of fluence perturbation that requires the introduction of a fluence perturbation correction factor.

Condition (b) implies that all electrons depositing the dose inside the cavity are produced outside the cavity and completely cross the cavity. No secondary electrons are therefore produced inside the cavity and no electrons stop within the cavity.

If one assumes that the energy of the crossers does not change within a small air cavity volume, the dose in the cavity is completely due to the crossers as:

$$D_{cav} = \int_0^{E_{\max}} \Phi_E(E) \left(\frac{S_{col}}{\rho} \right)_{cav} (E) dE \qquad 6.4$$

Where

E_k is the kinetic energy of crossers;

$E_{\max}$ is their highest energy equal to the initial energy of the secondary electrons produced by photons;

$\Phi_E(E)$ is the energy spectrum of all crossers.

Using the shorthand notation we have in the cavity:

$$D_{cav} = \Phi \left(\frac{\bar{S}}{\rho} \right)_{cav} \qquad 6.5$$

In the medium without the cavity:

$$D_{med}(P) = \Phi\left(\frac{\bar{S}}{\rho}\right)_{med} \quad 6.6$$

Under these two conditions, according to the Bragg–Gray cavity theory, the dose to the medium D_{med} is related to the dose in the cavity D_{cav} as follows, Since Φ is identical (not disturbed), it follows:

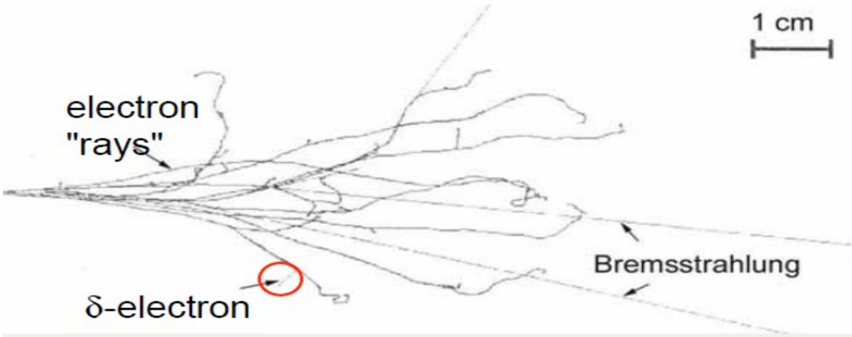
$$D_{med} = D_{cav}\left(\frac{\bar{S}}{\rho}\right)_{med,cav} \quad 6.7$$

Where

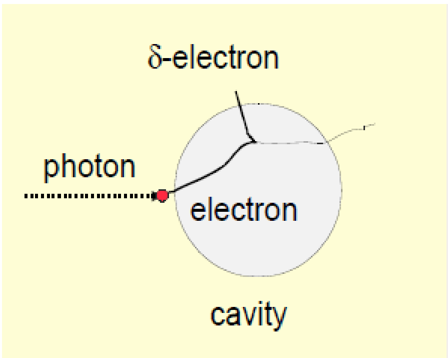
$\left(\frac{\bar{S}}{\rho}\right)_{med,cav}$ is the ratio of the average unrestricted mass collision stopping powers of the medium and the cavity. The use of unrestricted stopping powers rules out the production of secondary charged particles (or delta electrons) in the cavity and the medium. Although the cavity size is not explicitly taken into account in the Bragg–Gray cavity theory, the fulfilment of the two Bragg–Gray conditions will depend on the cavity size, which is based on the range of the electrons in the cavity medium, the cavity medium and the electron energy. A cavity that qualifies as a Bragg–Gray cavity for high energy photon beams, for example, may not behave as a Bragg–Gray cavity in a medium energy or low energy X-ray beam.

6.3. Spencer–Attix cavity theory

The Bragg–Gray cavity theory does not take into account the creation of secondary (delta) electrons generated as a result of hard collisions in the slowing down of the primary electrons in the sensitive volume of the dosimeter. The Spencer–Attix cavity theory is a more general formulation that accounts for the creation of these electrons that have sufficient energy to produce further ionization on their own account. The Spencer–Attix theory operates under the two Bragg–Gray conditions; however, these conditions now even apply to the secondary particle fluence in addition to the primary particle fluence.



Some of these electrons released in the gas cavity may have sufficient energy to **escape from the cavity** carrying some of their energy with them out of the volume.

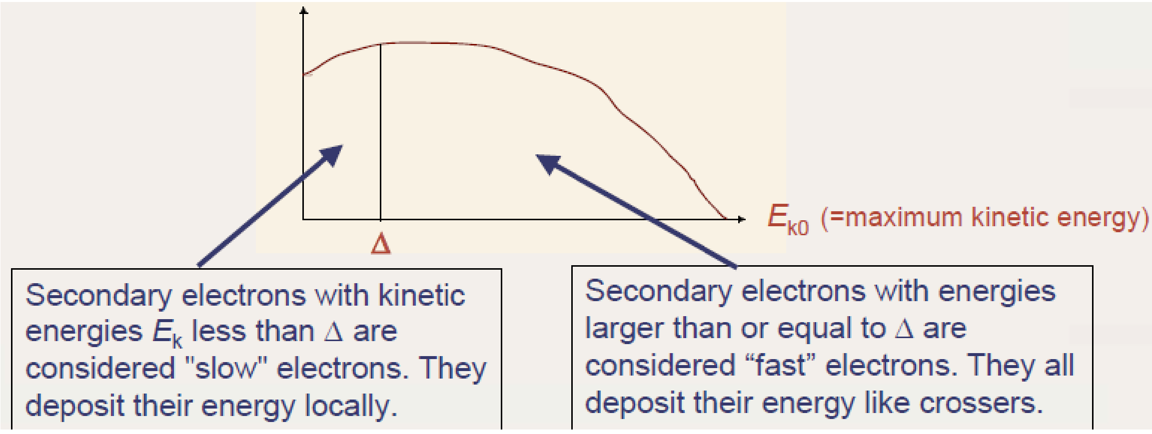


This reduces the energy absorbed in the cavity and requires a modification to the stopping power of the crossers in the gas.

- This is accomplished in the Spencer-Attix cavity theory by explicitly considering the δ electrons.
- Spencer-Attix cavity theory operates under the same two conditions as used in the Bragg-Gray cavity theory.
- However, these conditions are now applied also to the fluence of the δ electrons.

The concept of the Spencer-Attix cavity theory:

The secondary electron fluence in the Spencer–Attix theory is divided into two components based on a user defined energy threshold Δ. Secondary electrons with kinetic energies E_K less than Δ are considered slow electrons that deposit their energy locally; secondary electrons with energies larger than or equal to Δ are considered fast (slowing down) electrons and are part of the electron spectrum.



All secondary electrons with energies $E_k > \Delta$ are treated as crossers. It means that such δ electrons with $E_k > \Delta$ must be included in the entire electron spectrum.

$$D_{1,cav} = \int_0^{E_{ko}} \Phi_{E_k}^{\delta}(E_K)\left(\frac{S}{\rho}\right)_{cav}(E_K)dE_k \quad 6.8$$

Where

$\Phi_{E_k}^{\delta}(E_k)$ is now the energy spectrum of all electrons including the δ electrons with $E_k > \Delta$.

However, this equation is not correct because the energy of the δ electrons is now taken into account twice:

• as part of the spectrum of electrons

• in the unrestricted stopping power as the energy lost ranging

up to the maximum energy lost (including that larger than Δ)

Solution to this situation:

The calculation must refer to the restricted mass stopping power:

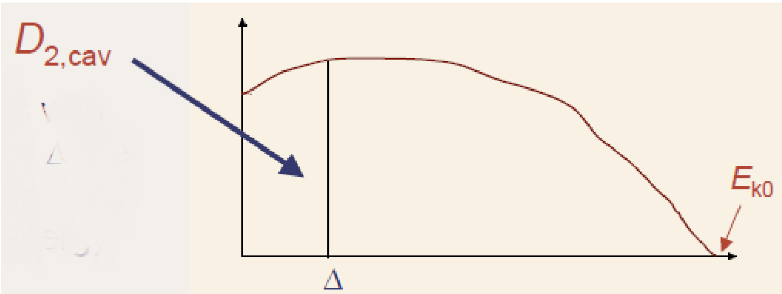
$$L_{\Delta} = \frac{dE_{\Delta}}{dl} \quad 6.9$$

$$D_{1,cav} = \int_0^{Eko} \Phi_{E_k}^{\delta}(E_K)\left(\frac{L_\Delta}{\rho}\right)_{cav}(E_K)dE_k \qquad 6.10$$

Secondary electrons withkinetic energies $K_E < \Delta$ are considered slow electrons. They deposit their energy "locally"

- "Locally" means that they can be treated as so-called "stoppers". $D_{2,cav}$ is sometimes called the "track-end term".
- Energy deposition of "stoppers" **cannot** be described by stopping power.
- Their energy lost is simply their (local) kinetic energy.

$$D_{2,cav} = \text{energy of stoppers per mass} \qquad 6.12$$



For practical calculations, the track-end term *TE* was approximated by A. Nahum as:

$$TE = \phi_{E_k}^{\delta}(\Delta)\frac{S(\Delta)}{\rho}\Delta \qquad 6.13$$

Finally we have:

$$D_{cav} = \int_0^{Eko} \Phi_{E_k}^{\delta}(E_K)\left(\frac{L_\Delta(E_k)}{\rho}\right)_{cav} dE_k + TE \qquad 6.14$$

In the Spencer-Attix cavity theory, the stopping power ratio is therefore obtained by:

$$\left(\frac{S}{\rho}\right)_{med,cav} = \frac{\int_0^{E_{ko}} \phi^{\delta}_{\mathrm{med,E_k}}(E_k)(L_{\Delta.med}(E_k)/\rho)dE_k + \phi^{\delta}_{\mathrm{med,E_k}}(\Delta)\frac{S_{med}(\Delta)}{\rho}\Delta}{\int_0^{E_{ko}} \phi^{\delta}_{\mathrm{med,E_k}}(E_k)(L_{\Delta,cav}(E_k)/\rho)dE_k + \phi^{\delta}_{\mathrm{cav,E_k}}(\Delta)\frac{S_{cav}(\Delta)}{\rho}\Delta} \qquad 6.15$$

6.4. Considerations in the application of cavity theory to ionization chamber calibration and dosimetry protocols

A dosimeter can be defined generally as any device that is capable of providing a reading that is a measure of the average absorbed dose deposited in its (the dosimeter's) sensitive volume by ionizing radiation. A dosimeter can generally be considered as consisting of a sensitive volume filled with a given medium, surrounded by a wall of another medium.

In the context of cavity theories, the sensitive volume of the dosimeter can be identified as the 'cavity', which may contain a gaseous, liquid or solid medium. Gas is often used as the sensitive medium, since it allows a relatively simple electrical means for collection of charges released in the sensitive medium by radiation.

The medium surrounding the cavity of an ionization chamber depends on the situation in which the device is used. In an older approach, the wall (often supplemented with a build up cap) serves as the buildup medium and the Bragg–Gray theory provides a relation between the dose in the gas and the dose in the wall. This is referred to as a thick walled ionization chamber and forms the basis of cavity chamber based air kerma in-air standards and of the C_{λ} based dosimetry protocols of the 1970s. If, however, the chamber is used in a phantom without a buildup material, since typical wall thicknesses are much thinner than the range of the secondary electrons, the proportion of the cavity dose due to electrons generated in the phantom greatly exceeds the dose contribution from the wall, and hence the phantom medium serves as the medium and the wall is treated as a perturbation to this concept.

In the case of a thick walled ionization chamber in a high energy photon beam, the wall thickness must be greater than the range of secondary electrons in the wall material to ensure that the electrons that cross the cavity arise in the wall and not in the medium. The Bragg–Gray cavity equation then relates the dose in the cavity to the dose in the wall of the chamber. The dose in the medium is related to the dose in the wall by means of a ratio of the mass– energy absorption coefficients of the medium and the wall $(\bar{\mu}_{en}/\rho)_{med,wall}$ by assuming that:

(a) The absorbed dose is the same as the collision kerma;

(b) The photon fluence is not perturbed by the presence of the chamber.

The dose to the cavity gas is related to the ionization produced in the cavity as follows:

$$D_{gas} = \frac{Q}{m}\left(\frac{\overline{W}_{gas}}{e}\right) \tag{6.16}$$

where Q is the charge (of either sign) produced in the cavity and m is the mass of the gas in the cavity, W_{gas} is the average energy expended in air per ion pair formed.

Spencer–Attix cavity theory can be used to calculate the dose in the medium as:

$$D_{med} = D_{wall}\left(\frac{\bar{\mu}_{en}}{\rho}\right)_{med,wall} = D_{gas}S_{wall,gass}\left(\frac{\bar{\mu}_{en}}{\rho}\right)_{med,wall}$$

$$= \frac{Q}{m}\left(\frac{\overline{W}_{gas}}{e}\right)S_{wall,gass}\left(\frac{\bar{\mu}_{en}}{\rho}\right)_{med,wall} \tag{6.17}$$

where $S_{wall,gas}$ is the ratio of restricted mass collision stopping powers for a cavity wall and gas with threshold Δ. In practice, there are additional correction factors associated with Eq. (6.16) to satisfy assumptions (a) and (b) made above.

A similar equation to Eq. (6.16) is used for air kerma in-air calibrations; however, here the quantity of interest is not the dose to the medium, but the air kerma in air. In this case, a substantial wall correction is introduced to ensure the presence of complete CPE in the wall to satisfy assumption (a) above.

In the case of a thin walled ionization chamber in a high energy photon or electron beam, the wall, cavity and central electrode are treated as a perturbation to the medium fluence, and the equation now involves the ratio of restricted collision stopping powers of the medium to that of the gas $S_{\text{med,gas}}$ as:

$$D_{med} = \frac{Q}{m}\left(\frac{\overline{W}_{gas}}{e}\right) S_{wall,gass}\, p_{fl}\, p_{dis}\, p_{wall}\, p_{cel} \tag{6.18}$$

Where:

p_{fl} is the electron fluence perturbation correction factor;
p_{dis} is the correction factor for displacement of the effective measurement point;
p_{wall} is the wall correction factor;
p_{cel} is the correction factor for the central electrode.

Values for these multiplicative correction factors are summarized for photon and electron beams in typical dosimetry protocols.

6.5. Large cavities in photon beams

A large cavity is a cavity with dimensions such that the dose contribution made by electrons inside the cavity originating from photon interactions outside the cavity can be ignored when compared with the contribution of electrons created by photon interactions within the cavity. For a large cavity the ratio of dose cavity to medium is calculated as the ratio of the collision kerma in the cavity to the medium and is therefore equal to the ratio of the average mass–energy absorption coefficients of the cavity gas to that of the medium $(\overline{\mu}_{en}/\rho)_{gas,med}$:

$$\frac{D_{gas}}{D_{med}} = (\overline{\mu}_{en}/\rho)_{gas,med} \tag{6.19}$$

where the mass–energy absorption coefficients have been averaged over the photon fluence spectra in the cavity gas (numerator) and in the medium (denominator).

6.6. Burlin cavity theory for photon beams

Burlin extended the Bragg–Gray and Spencer–Attix cavity theories to cavities of intermediate dimensions by introducing, on a purely phenomenological basis, a large cavity limit to the Spencer–Attix equation using a weighting technique. He provided a formalism to calculate the value of the weighting parameter.

The Burlin cavity theory can be written in its simplest form as follows:

$$\frac{D_{gas}}{D_{med}} = ds_{gas,med} + (1-d)\left(\bar{\mu}_{en}/\rho\right)_{gas,med} \qquad 6.20$$

Where

d is a parameter related to cavity size, approaching unity for small cavities and zero for large cavities;

$S_{\mathrm{gas,med}}$ is the mean ratio of the restricted mass stopping powers of the cavity and the medium;

D_{gas} is the absorbed dose in the cavity;

$\left(\bar{\mu}_{en}/\rho\right)_{gas,med}$ is the mean ratio of the mass–energy absorption coefficients for the cavity and the medium.

The Burlin theory effectively requires that:

● The surrounding medium and the cavity medium be homogeneous;

● A homogeneous photon field exist everywhere throughout the medium and the cavity;

● CPE exist at all points in the medium and the cavity that are further than the maximum electron range from the cavity boundary;

● The equilibrium spectra of secondary electrons generated in the medium and the cavity be the same.

Burlin provided a method for estimating the weighting parameter d in his theory. It is expressed as the average value of the electron fluence reduction in the medium. Consistent with experiments with β sources he proposed that the electron fluence in the medium Φ_{med}^{e-e} decays, on average, exponentially. The value of the weighting parameter d in conjunction with the stopping power ratio can be calculated as:

$$d = \frac{\int_0^L \Phi_{med}^{e-e} e^{-\beta l} dl}{\int_0^L \Phi_{med}^{e-e} dl} = \frac{1 - e^{-\beta L}}{\beta L} \qquad 6.21$$

where β is an effective electron fluence attenuation coefficient that quantifies the reduction in particle fluence from its initial medium fluence value through a cavity of average length L. For convex cavities and isotropic electron fluence distributions, L can be calculated as $4V/S$, where V is the cavity volume and S its surface area. Burlin described the buildup of the electron fluence Φ_{med}^{e-e} inside the cavity using a similar, complementary equation:

$$1 - d = \frac{\int_0^L \Phi_{med}^{e-e} (1 - e^{-\beta l}) dl}{\int_0^L \Phi_{med}^{e-e} dl} = \frac{\beta L - 1 + e^{-\beta L}}{\beta L} \qquad 6.22$$

Burlin's theory is consistent with the fundamental constraint of cavity theory: that the weighting factors of both terms add up to unity (i.e. d and $1 - d$). It had relative success in calculating ratios of absorbed dose for some types of intermediate cavities. More generally, however, Monte Carlo calculations show that, when studying ratios of directly calculated absorbed doses in the cavity to absorbed dose in the medium as a function of cavity size, the weighting method is too simplistic and additional terms are necessary to calculate dose ratios for intermediate cavity sizes. For these and other reasons, the Burlin cavity theory is no longer used in practice.