Electricity / 2 م.م حسن

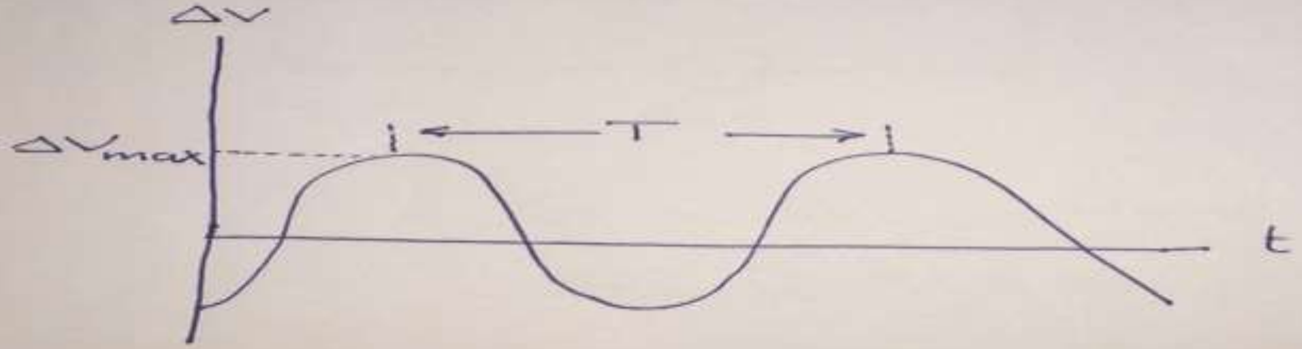
lec / 11

A.C Source :

An A.C circuit consists of circuit elements and power source the provides an alternating voltage ΔV. This time-varing voltage from the source is described by

$$\Delta V = \Delta V_{max} \sin \omega t$$

where ΔV_{max} : is the maximum output voltage of the source



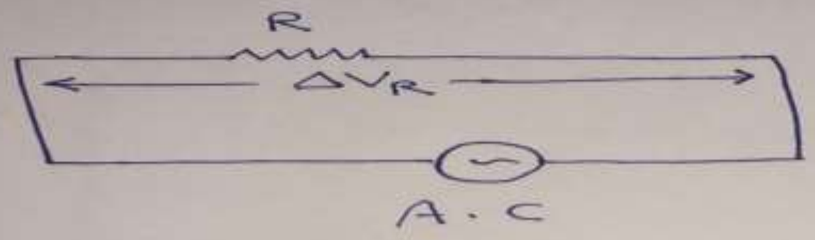
The voltage applied by an A.C source

$$f = \frac{1}{T} \Rightarrow T = \frac{1}{f}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

where f : is the frequency
T : is the period

① Resistors in an A.C circuit



$$\Delta V = \Delta V_{max} \sin \omega t$$

$$\Delta V - i_R R = 0$$

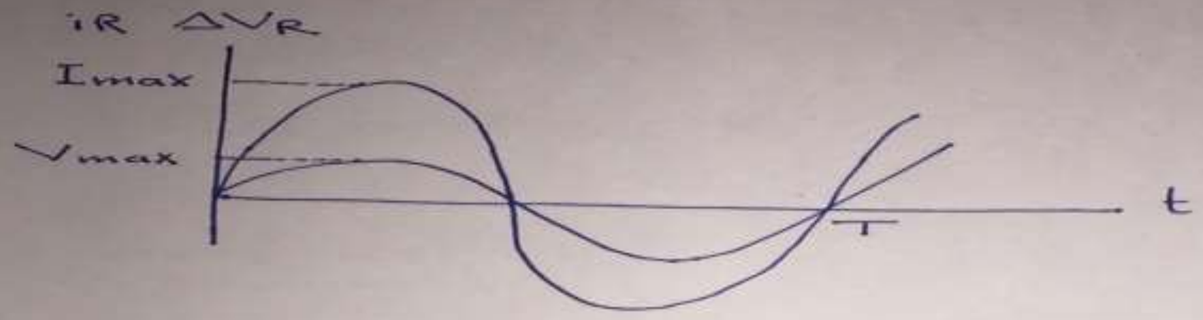
$$i_R = \frac{\Delta V}{R} = \frac{\Delta V_{max}}{R} \sin \omega t$$

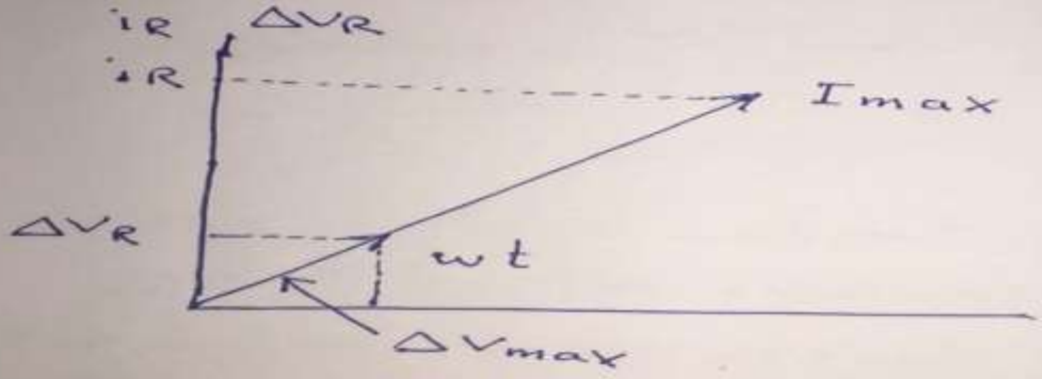
$$i_R = I_{max} \sin \omega t$$

where I_{max} is the maximum current

$$I_{max} = \frac{\Delta V_{max}}{R}$$

$$\Delta V_R = i_R R = I_{max} R \sin \omega t$$





rms current

$$I_{rms} = \sqrt{(i^2)_{ave}}$$

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = 0.707\, I_{max}$$

$$P_{ave} = I_{rms}^2 R$$

rms voltage

$$\Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}} = 0.707\, \Delta V_{max} .$$

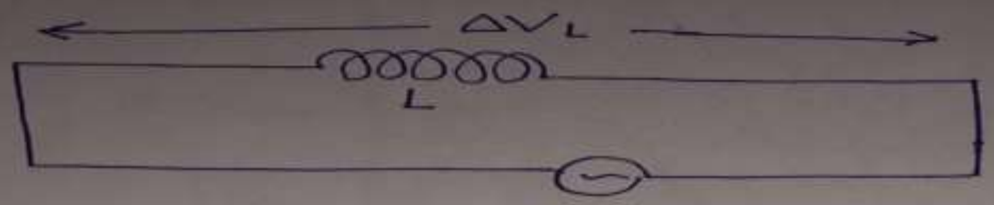
Exp: The voltage output of an AC source is given by the expression $\Delta V = (200V) \sin \omega t$. Find the rms current in the circuit when the source is connected to a 100 Ω resistor (R)

Solution:

$$\Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}} = \frac{200}{\sqrt{2}} = 141 \text{ volt}$$

$$I_{rms} = \frac{\Delta V_{rms}}{R} = \frac{141}{100} = 1.41 \text{ Amp}$$

(2) Inductors in A.C circuit



$$\Delta V = \Delta V_{max} \sin wt$$

$$\Delta V_L = V_L = -L\left(\frac{di_L}{dt}\right)$$

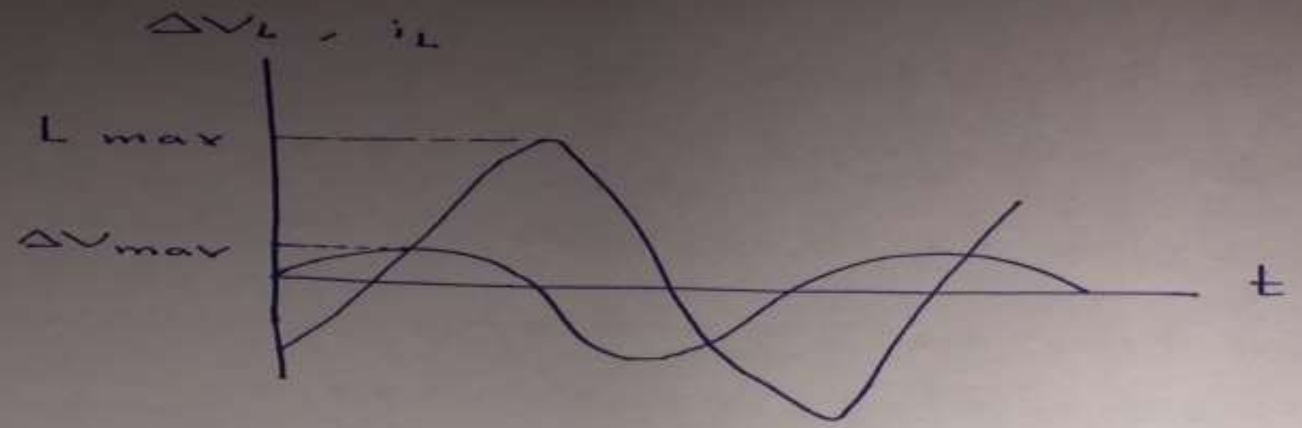
$$\Delta V + \Delta V_L = 0$$

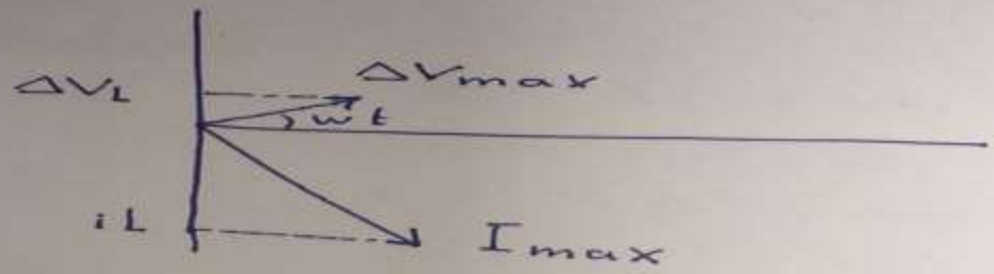
$$\Delta V - L\frac{di_L}{dt} = \Delta V_{max} \sin wt$$

$$di_L = \frac{\Delta V_{max}}{L} \sin wt \, dt$$

$$i_L = \frac{\Delta V_{max}}{L} \int \sin wt \, dt = \frac{\Delta V_{max}}{wt} \cos wt$$

$$i_L = \frac{\Delta V_{max}}{wL} \sin\left(wt - \frac{\pi}{2}\right)$$





when maximum value $\cos \omega t = \pm 1$

$$I_{max} = \frac{\Delta V_{max}}{\omega L}$$

Inductive reactance X_L

$$X_L = \omega L = 2\pi f L$$

$$I_{max} = \frac{\Delta V_{max}}{X_L}$$

Exp: In a purely induction A.C circuit $L = 25\, mH$ and rms voltage is 150 volts

Calculate: The inductive reactance (X_L) and rms current in the circuit if frequency is $f = 60\, Hz$.

Solution:

(a) $X_L = \omega L = 2\pi f L$

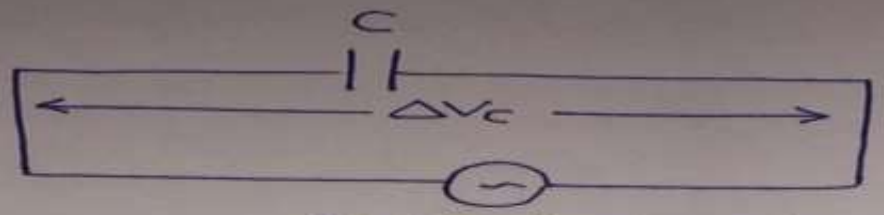
$$X_L = 2\pi(60 \times 25 \times 10^{-3})$$

$$X_L = 9.42\ \Omega.$$

(b) $$I_{rms} = \frac{\Delta V_{max}}{X_L} = \frac{150}{9.42}$$

$$I_{rms} = 15.9 \text{ Ampers.}$$

③ Capacitors in an A.C Circuit



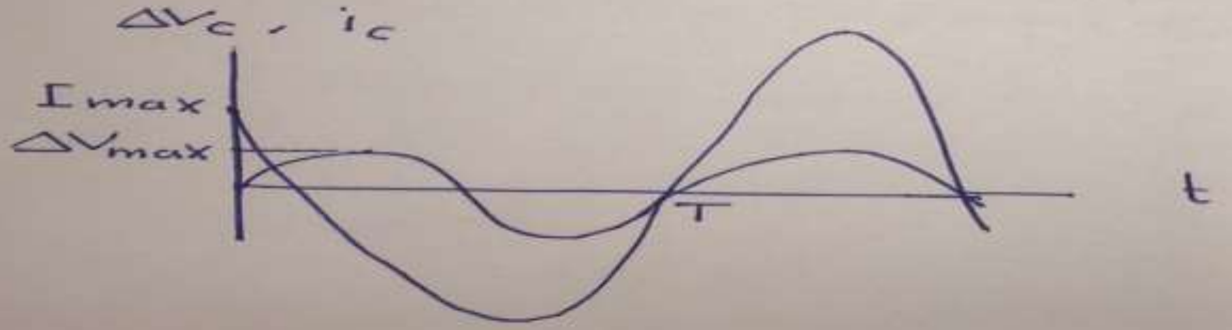
$$\Delta V = \Delta V_m \sin wt$$

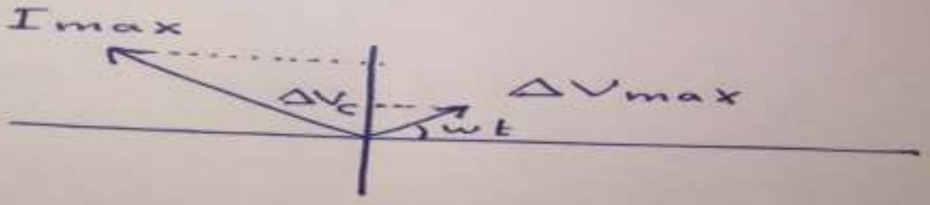
$$\Delta V - \Delta V_c = 0$$

$$\Delta V - \frac{q}{c} = 0$$

$$q = c\, \Delta V_{max} \sin wt$$

$$i_c = \frac{dq}{dt} = wc\, \Delta V_{max} \cos wt$$





$$\cos wt = \sin\left(wt + \frac{\pi}{2}\right)$$

$$i_c = wc\, \Delta V_{max} \sin\left(wt + \frac{\pi}{2}\right)$$

$$I_{max} = \omega C \Delta V_{max} = \frac{\Delta V_{max}}{\left(\frac{1}{\omega C}\right)}$$

capacitive reactance (X_C)

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$I_{max} = \frac{\Delta V_{max}}{X_C}$$

$$\Delta V_C = \Delta V_{max} \sin \omega t = I_{max} X_C \sin \omega t .$$

Exp : An a $8 \mu F$ capacitor is connected to the terminals of a 60 Hz . A.C source whose rms voltage is 150 volts.
Find : The capacitive reactance and the rms current in the circuit.

Solution :

(a) $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi (60 \times 8 \times 10^{-6})}$

$X_C = 332 \ \Omega$

(b) $I_{rms} = \frac{\Delta V_{rms}}{X_C} = \frac{150}{332} = 0.452$ Ampers