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Laser Principles and Applications

Chapter (1): Introduction to the Laser

1.1- Historical Review of the Laser

1.2- Properties of Laser Light.

Chapter (2): Operation of Laser Principle

2.1- Emission and Absorption of Light   2.2- population Inversion

2.3- Energy pumping (3-Level system, 4-Level system)

Chapter (3): Optical Cavities

3.1- Optical Resonators (Cavity Configuration)

3.2- Laser Modes (Axial Modes, Transverse Electromagnetic Modes (TEM))

Chapter (4): Spectroscope of the Laser Light

4.1- Line Width properties   4.2- Homogeneous Broadening

4.3- Non Homogeneous Broadening   4.4- Longitudinal Mode Selection Methods

Chapter (5): High power Techniquies

5.1- Q-Switching (Mechanical, chemical, electro-optic)

5.2- Mode Locking (Active and Passive) Methods

Chapter (6): Types of Lasers

6.1- Solid State Lasers   6.2- Gas Lasers   6.3- Liquid Dye Lasers

6.4- Semiconductor Lasers

Chapter (7): Applications of Lasers

7.1- Industrial Applications   7.2- Medical Applications   7.3- Holography

References: 1- Lasers, Principles and Applications; J. Wilson and J. Hawkes
2- Principles of Lasers; O. Svelto and D. Hanna.

Chapter (1): Introduction to the Laser

The laser is one of the most outstanding achievements of science and engineering in the 20th century. Since the invention of the first ruby laser in 1960, lasers and laser systems brought about a revolution in various fields of science and technology. Today, wide applications of lasers in life, industry, medicine and communications are employed, due to the specific properties of the laser radiation, high coherence, monochromaticity, and directivity.

1.1- Historical Review of the Laser

The proposal that particles of light (Photons) with energy of a particular frequency could stimulate atomic electrons to emit radiant energy as light of the same frequency was made by Albert Einstein in 1917. This phenomena is the key to the operation of the laser. The term "Laser" in fact, is an acronym for Light amplification by stimulated emission of radiation.

In 1958, C. H. Townes and A.L. Shawlow published a proposal that the principles employed in microwave amplification by stimulated emission, to produce the maser, could be extended to the amplification of light. A few years later, the first laser was invented by T. H. Maiman. It consisted of a ruby rod (crystal), with mirrored ends, that was surrounded by a helical flash lamp. Shortly after this, a helium – neon (He-Ne) gas laser was developed, by Javan, Bennett, and Herriott. The first carbon dioxide (CO_2) laser belongs to C. Patel, who did his work at Bell laboratories. In 1962-63 the first semiconductor lasers were demonstrated in the United States and in the former Soviet Union. During the 1970s scientists began to discover and developed ~~more~~ various types of lasers.

1-2 Properties of Laser Light

Laser radiation is characterized by an extremely high degree of properties such as, monochromaticity, coherence, directionality, and brightness.

Monochromaticity;

The laser radiation consists of a narrow range of frequencies. The laser linewidth (stimulated emission) is often much narrower by as much as six order of magnitude than the usual linewidth of usual sources (spontaneous emission), Fig.(1.1)

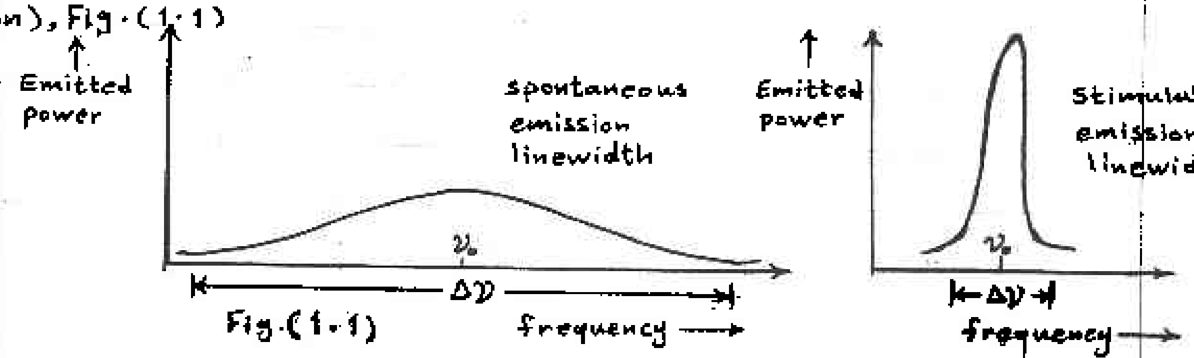


Fig.(1.1)

The property of monochromaticity is occured due to the following two circumstances:

(1) Only an electromagnetic wave of frequency ν_0 of the active laser transition can be amplified.

(2) Oscillation can occur only at the resonant frequencies of resonant cavity (forms from two mirror arrangement).

Coherence;

The laser radiation is spatially and temporally coherent. The electromagnetic field at all points in the radiation oscillates in a steady, predicatable phase from one point in time to any other point in time.

Spatial coherence; Consider two points P_1 and P_2, that, at time $t=0$, lie on the same wavefront of some given electromagnetic wave and let $E_1(t)$ and $E_2(t)$ be the corresponding electric fields at these points. By definition, the difference between the phases of the two fields at time $t=0$ is zero. Now if this difference remains zero at any time $t>0$, we will say that there is a perfect coherence between two points. If this occurs for any two points of the electromagnetic wave front, we will say that the wave have perfect spatial coherence, Fig (1.2).

Fig. (1.2)

Temporal coherence; consider the electric field of the electromagnetic wave at a given point p, at times t and $t+\tau$. If, for a given time delay τ, the phase difference between the two field values remains the same for any time t, we will say that there is temporal coherence over time τ. If this occurs for any value of τ, the electromagnetic wave will be said to have perfect temporal coherence, Fig (1.2).

Directionality;

The laser radiation is limited to small angle of divergence, from a few tenths of a milliradian to a few milliradians. Directionality occurs due to the optical resonant cavity that, confines the path of the laser beam. When laser light trapped between two mirrors, and propagating along the cavity direction, low divergence of output laser beam can be achived, Fig. (1.3)

The laser beam divergence is given by

$$\theta = \sin^{-1}\left(\frac{\lambda}{\pi W_o}\right) \quad \ldots\ldots (1.1)$$

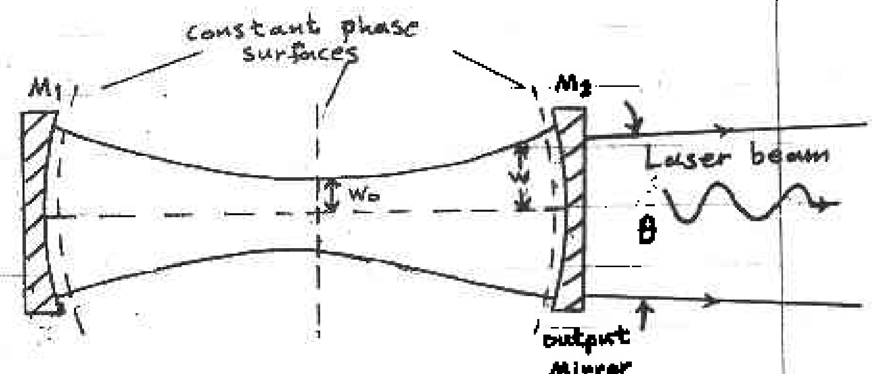


Fig. (1.3)

Brightness;

Brightness is defined as the power emitted per unit area per unit solid angle (the SI units of brightness are thus $W\, m^{-2}\, sr^{-1}$), Fig.(1.4)

$$\text{Brightness}\,(B) = \frac{\text{Power emitted}\,(P)}{\text{Aarea}\,(A) \times \text{Solid angle}\,(\Omega)} \quad \ldots\ldots (1.2)$$

The laser radiation has ~~a~~ brightness which is about one to ten orders of magnitude greater than ~~the~~ brightness of the conventional sources. This is mainly due to the high directionality of the laser beam.

High brightness is essential for the delivery of high power per unit area to a target.

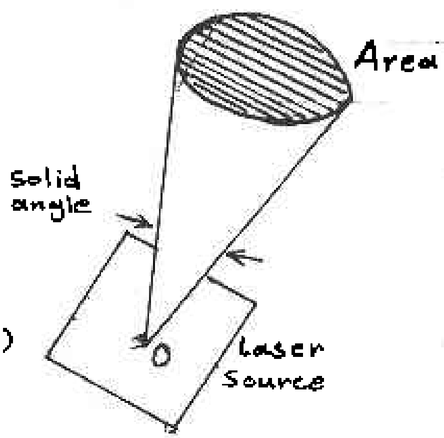


Fig.(1-4)

Problems Chapter(1): (Properties of Laser Light)

1.1 The part of the e.m. spectrum that is of interest in the laser field starts from the submillimeter wave region and goes down in wavelength to the x-ray region. This covers the following regions in succession: (1) far infrared; (2) near infrared; (3) visible; (4) ultraviolet (uv); (5) vacuum ultraviolet (VUV); (6) soft x-ray; (7) x-ray. From standard textbooks find the wavelength intervals of the above regions.
(Memorize or record these intervals since they are frequently used in this study)

Solution: The electromagnetic spectrum. (regions)

Type of radiation	wavelength (m)	Frequency (Hz)	Quantum energy (eV)
	100 km	3×10^{3}	1.2×10^{-11}
Radio waves			
	300 mm	10^{9}	4×10^{-6}
Microwaves			
	0.3 mm	10^{12}	4×10^{-3}
Infrared			
	0.7 μm	4.3×10^{14}	1.8
Visible			
	0.4 μm	7.5×10^{14}	3.1
Ultraviolet			
	0.03 μm	10^{16}	40
X rays			
	0.1 nm	3×10^{18}	1.2×10^{4}
γ rays			
	1.0 pm	3×10^{20}	1.2×10^{6}

Laser spectrum (mm - nm)

1.2 Argon ion laser light of wavelength $\lambda = 488$ nm falls onto a metal surface which has a work function ϕ of 2.2 eV. Calculate the maximum kinetic energy of the photoelectrons emitted.

Solution:

Einstein in 1905 explained the photoelectric effect (radiation incident on metal surfaces releases electrons) very simply in terms of the incident light consisting of small bundles of energy or particles which he called photons. He said that the energy of a photon was proportional to its frequency, that is, $E = h\nu$, where h is the Planck constant.
An incident photon can then impart its energy to a single electron giving it a sufficient amount of energy, ϕ, the work function, to overcome the forces holding it to the surface of the metal and to impart to it a certain amount of kinetic energy. That is,

$$E = h\nu = \phi + \frac{1}{2}mv^2,$$

where m is the mass of the electron and v its velocity

The energy of the incident photons is

$$\frac{hc}{\lambda} = \frac{6.626 \times 10^{-34}\ \text{Js} \times 2.998 \times 10^{8}\ \text{ms}^{-1}}{488 \times 10^{-9}\ \text{m}}$$

$$= 4.07 \times 10^{-19}\ \text{J}$$

The work function is

$$\phi = 2.2\ \text{eV}$$

$$= 2.2 \times 1.6 \times 10^{-19} = 3.52 \times 10^{-19}\ \text{J}$$

Therefore, maximum kinetic energy of the photoelectrons is

$$(4.07 - 3.52) \times 10^{-19} = 0.55 \times 10^{-19}\ \text{J} = 0.34\ \text{eV}.$$

1.3 Calculate the maximum value of the work function of a photocathode which could be used in a photomultiplier to detect the green line of the HeNe laser of wavelength 543.5 nm.

Solution;

The maximum value of the work function of such a photocathode, is. when the incident photon impart its energy to a single electron giving it only amount of energy, ϕ, the work function. i.e.,

$$E = h\nu = \phi + \frac{1}{2}mv^2 = \phi_m$$

The maximum work function is

$$\phi_m = \frac{hc}{\lambda}$$

$$= \frac{6.626 \times 10^{-34}\ \text{Js} \times 2.998 \times 10^{8}\ \text{ms}^{-1}}{543.5 \times 10^{-9}\ \text{m}}$$

$$= 3.66 \times 10^{-19}\ \text{J} \quad \text{or} \quad 2.29\ \text{eV}.$$

1.4 In Young's experiment for interference, the separation of neighboring bright fringes is approximately $\lambda D/d$, where λ is the wavelength of the light used, D the distance from light sources to screen and d is the separation of the sources. Given that the sources are 2mm apart and the screen is 0.85 m apart from the sources, find the fringe spacing for the red line of a HeNe laser ($\lambda = 632.8$ nm).

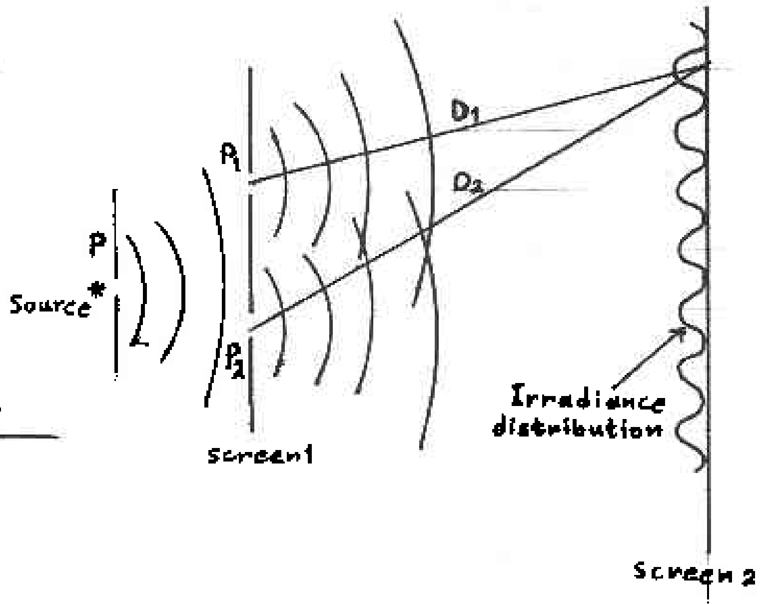
Solution:

The separation of neighboring bright fringes is

$$L = \frac{\lambda D}{d}$$

$$= \frac{632.8 \times 10^{-9}\ \text{m} \times 0.85\ \text{m}}{2 \times 10^{-3}\ \text{m}}$$

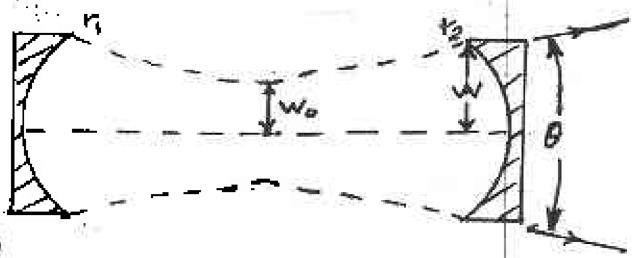
$$= 0.27\ \text{mm}$$



1.5 Calculate the Gaussion beam divergence of HeNe laser ($\lambda = 633$ nm) which has a confocal cavity with a minimum beam radius (waist) $W_0 = 0.22$ mm.

Solution:

The laser beam divergence is given by



$$\theta_{confocal} = \sin^{-1}\left(\frac{\lambda}{\pi W_0}\right)$$

$$= \sin^{-1}\left(\frac{633 \times 10^{-9}\ m}{\pi \times 2.2 \times 10^{-4}\ m}\right)$$

$$= 0.916 \text{ mrad}$$

$$\left(\text{rad} = \frac{180}{\pi}\text{ degree}\right)$$

$$= 0.052^\circ$$

1.6 Calculate the beam divergences in a semiconductor (GaAs) laser ($\lambda = 900$ nm) whose active region has cross-sectional dimensions of 3 μm X 10 μm, perpendicular to the plane of the junction.

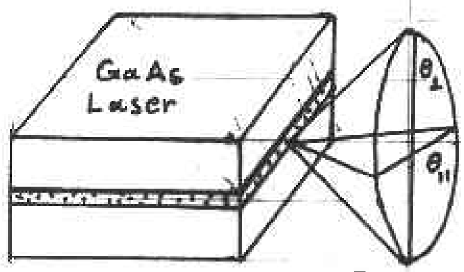
Solution:

An aperture of dimension d gives rise to an angular divergence given by

$$\theta = \sin^{-1}(k\lambda/d),$$

wher k is a number of order unity (k=1).

therefore,



$$\theta_\perp = \sin^{-1}\left(\frac{900 \times 10^{-9}\ m}{3 \times 10^{-6}\ m}\right) = 0.3 \text{ rad} = 17^\circ$$

$$\theta_\parallel = \sin^{-1}\left(\frac{900 \times 10^{-9}\ m}{10 \times 10^{-6}\ m}\right) = 0.09 \text{ rad} = 5.2^\circ$$

1.7 Compare the coherence lengths of conventional and laser radiation sources. If the radiation emitted from a low-pressure Sodium lamp with a typical linewidth of Sodium D lines (both lines taken together) at $\lambda = 589\ \mu m$ is 5.1×10^{11} Hz, and if the laser radiation at $\lambda = 633$ nm emitted from a HeNe laser operating in a single mode with linewidth of 1 MHz.

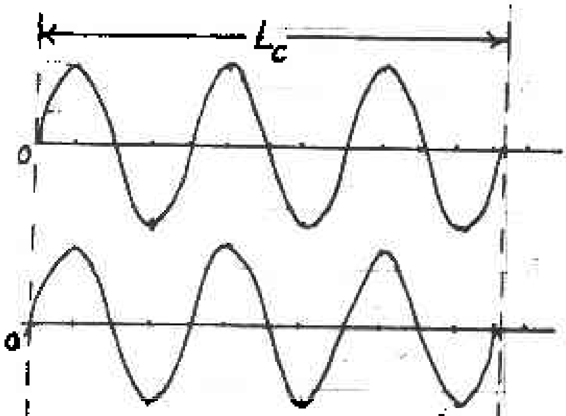
Solution:

The coherence time ($t_c = L_c/c$) is the time taken for a source to emit a wavelength of length L_c - where c is the velocity of light.
The coherence time is related to the linewidth of the emission ($\Delta\nu$) via the equation

$$t_c = \frac{1}{\Delta\nu}$$

$$t_c = \frac{1}{5.1 \times 10^{11}\ Hz} = 2 \times 10^{-12}\ s$$

then, $L_c = C t_c = 3 \times 10^{8}\ ms^{-1} \times 2 \times 10^{-12}\ s$
$= 6 \times 10^{-4}\ m = 0.6\ mm$



Two identical wavetrains of Length L_c

We may contrast these values with those applicable to the HeNe laser. If the laser is operating in a single mode stabilized to 1 MHz, then, the coherence time is,

$$t_c = \frac{1}{1 \times 10^{6}\ Hz} = 1 \times 10^{-6}\ s$$

then, $L_c = 3 \times 10^{8}\ ms^{-1} \times 10^{-6}\ s = 300 m$

Now if the HeNe laser is operating in many modes, with linewidth is about 1500 MHz, giving a coherence length of

$$L_c = 3 \times 10^{8}\ ms^{-1} \times \frac{1}{1500 \times 10^{6}\ s^{-1}} = 0.2\ m$$

then, the coherence length is some 1500 times less.

1.8 Calculate the brightness of a laser beam from a HeNe laser with an output of 5 mW, and a beam divergence of 5.2×10^{-5} rad. If the radius of the laser spot is 0.3 mm.

Solution:

Brightness is defined as the power emitted per unit area per unit solid angle.

The beam divergence 5.2×10^{-5} rad corresponds to a solid angle of $(\pi \times \theta^2)$.

$$\Omega = \pi\theta^2 = \pi(5.2 \times 10^{-5})^2 = 8.5 \times 10^{-9}\ \text{Sr}.$$

The laser spot are is $(\pi \times r^2)$,

$$A = \pi r^2 = \pi (0.3 \times 10^{-3}\ m)^2 = 2.8 \times 10^{-7}\ m.$$

The brightness of the laser beam is,

$$B = \frac{P}{A \times \Omega} = \frac{5 \times 10^{-3}\ W}{(2.8 \times 10^{-7} m^2)(8.5 \times 10^{-9} Sr)} = 2 \times 10^{12}\ W m^{-2} sr^{-1}.$$

For comparison purposes the brightness of the sun is a mere $1.3 \times 10^6\ W m^{-2} sr^{-1}$.

Note: Brightness values as high as $10^{21}\ W m^{-2} sr^{-1}$ have been achieved using an Nd: glass laser followed by optical amplifiers.

1.9 Consider a 1 mW HeNe laser ($\lambda = 633$ nm) with a focusing lens of F number equal 1. Determine the power per unit area of the focusing laser beam.

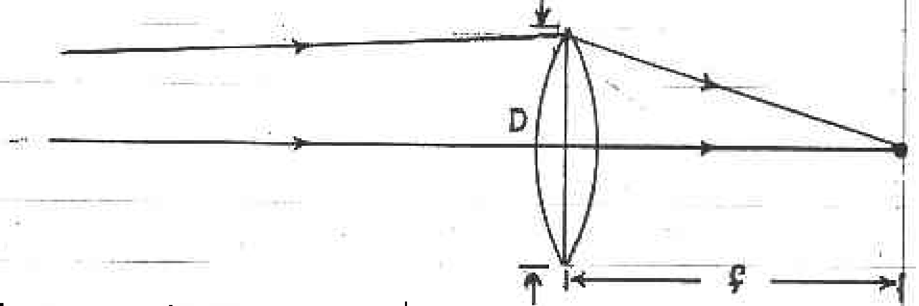
Solution:

All laser beams posses a small degree of divergence. The Gaussian laser beam will diverge (spread) at comparatively large distances from the laser output. A sutiable focusing (converging) lens inserted into the diverging beam reverses the situation, that is to produce a converging beam that is focused at distance nearly equal to f, (the focal length of the lens).

The focused beam radius is given by

$$r_s = \frac{2}{\pi}\lambda F,$$

where F is known as the F number of the lens and is given by $F = f/D$, where f and D are the focal length and diameter of the lens.



The power per unit area (circle) is equal to

$$\frac{P}{A} = \frac{P}{\pi r_s^2} = \frac{P}{\pi\left(\frac{2}{\pi}\lambda F\right)^2} = \frac{\pi P}{4\lambda^2 F^2}$$

$$= \frac{3.14 \times (1 \times 10^{-3}\,W)}{4 \times (633 \times 10^{-9}\,m)^2 \times (1)^2} = 2 \times 10^{9}\ W m^{-2}$$

Chapter (2): Operation of Laser

2.1 Emission and Absorption of Light

Consider an atom having only two energy levels, an upper level E_2 and lower level E_1, as shown in Fig.(2.1) Under normal conditions the atom will be in the lower level as physical systems tend to the lowest possible energy state. Suppose that the atom in the lower level is exposed to radiation of frequency ν_{21}, where $\nu_{21} = (E_2 - E_1)/h$. .... (2.1)

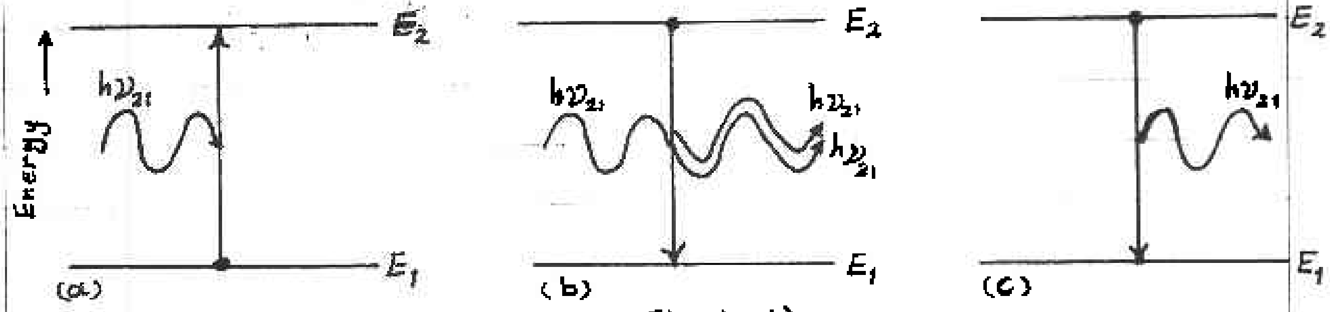


Fig. (2.1)

The atom will absorb a photon and be excited to the upper level E_2. This process is referred to as stimulated absorption. Fig. 2.1(a). After a few nanoseconds, the excited atom (atom in the higher energy level) will emit a photon of energy $h\nu_{21} = E_2 - E_1$ and return to the lower level. In 1917 Einstein showed that the emission process can occur in two quite distinct ways, which are stimulated emission, Fig. 2.1(b) or spontaneous emission, Fig. 2.1(c).

There are two very important properties of the photon produced by stimulated emission. Firstly, the stimulated photon has the same energy as the stimulating photon and hence the associated photon (wave) must have the same frequency. Secondly the light waves associated with the two photons are in phase and have the same state of polarization. This means that the stimulated wave adds to the incident wave on a constructive basis, thereby increasing its amplitude, which give rise for the possibility of 'light amplification by stimulated emission of radiation'. Stimulated radiation is coherent, that is, all of the waves making up a beam of such radiation are identical in phase and energy (wavelength).

Under normal conditions of thermal equilibrium spontaneous emission is much more probable than stimulated emission (by a factor of about $10^{37} : 1$) and therefore the radiation from most radiation source is incoherent.

The laser action is possible only if the atomic system is in non thermal equilibrium condition.

The Einstein relations:

Einstein showed that the three processes of stimulated absorption, stimulated emission and spontaneous-emission are related mathematically, that the rate of upward transitions (from E_1 to E_2) must equal the rate of downward transitions (from E_2 to E_1) for atomic system in thermal equilibrium.

If there are N_1 atoms in the energy level E_1, then

$$\text{stimulated absorption rate} = N_1 \rho_\nu B_{12} \qquad \cdots (2.2)$$

where ρ_ν is the energy density of the absorbed photon $\rho_\nu = nh\nu$, (n is the number of photons per unit volume.

B_{12} is a constant coefficient for a given pair of energy level.

Similarly if there are N_2 atoms per unit volume in the energy level E_2, then

$$\text{Stimulated emission rate} = N_2 \rho_\nu B_{21} \qquad \cdots (2.3)$$

where again B_{21} is a constant for the pair of energy levels involved.

The spontaneous emission rate depends on the average lifetime, τ_{21}, of the atoms in the excited state (level) E_2.

$$\text{Spontaneous emission rate} = N_2 A_{21} \qquad \cdots (2.4)$$

The constants A_{21}, B_{12} and B_{21} are called the Einestein coefficients, and the relationships between them for the atomic system in thermal equilibrium must be,

$$N_1 \rho_\nu B_{12} = N_2 \rho_\nu B_{21} + N_2 A_{21} \qquad \cdots (2.5)$$

From this equation

$$\rho_\nu = \frac{A_{21} / B_{21}}{(B_{12} N_1 / B_{21} N_2) - 1} \qquad \cdots (2.6)$$

The number N_j of atoms in the jth level (or populations) of the various energy levels E_i of a system in thermal equilibrium is given by Boltzmann distribution (statistics),

$$N_j = N_o \frac{\exp(-E_j / kT)}{\sum_i \exp(-E_i / kT)} \qquad \cdots (2.7)$$

where N_o is the total number of atoms and E_j is the energy of the jth level.

From this equation, the ratio of the populations N_1 and N_2 in the levels E_1 and E_2 is

$$\frac{N_1}{N_2} = \exp[(E_2 - E_1)/KT] \qquad \ldots (2.8)$$

and therefore substituting Eq. (2.8) into Eq. (2.6) and using Eq. (2.1), get

$$\rho_\nu = \frac{A_{21}/B_{21}}{B_{12}/B_{21}\exp(h\nu/KT) - 1} \qquad \ldots (2.9)$$

Also, because the atomic system is in equilibrium, the radiation of atoms must be identical to blackbody radiation which can be described by this equation:

$$\rho_\nu = \frac{8\pi h \nu^3}{c^3}\left(\frac{1}{\exp(h\nu/KT) - 1}\right) \qquad \ldots (2.10)$$

Comparing Eqs. (2.) and (2.) for ρ_ν, get

$$B_{12} = B_{21} \qquad \ldots (2.11)$$

and

$$A_{21} = B_{21}\frac{8\pi h \nu^3}{c^3} \qquad \ldots (2.12)$$

These equations are known as the Einstein relations.

From the last equation (2.12), the ratio of the rates of spontaneous and stimulated emission can be evaluated for a given pair of energy levels in thermal equilibrium with the radiation.

So that this ratio, R, is

$$R = \frac{N_2 A_{21}}{N_2 B_{21} \rho_\nu} = \frac{8\pi h \nu^3}{\rho_\nu c^3} \qquad \ldots (2.13)$$

Substituting for ρ_ν from Eq. (2.8) then gives

$$R = \exp\left(\frac{h\nu}{KT} - 1\right) \qquad \ldots (2.14)$$

Under conditions of thermal equilibrium, the rate of spontaneous emission is much more probable than the rate of stimulated emission. Hence to obtain laser action, a nonthermal equilibrium conditions must be satsfied, i.e. the population density of the higher energy levels must be increased at the expanse of lower energy levels. This is known as population inversion.

2.2 Population Inversion

The Boltzmann distribution corresponding to thermal equilibrium of atomic system of two energy levels is given by

$$N_2 = N_1 \exp\left[-(E_2 - E_1)/KT\right] \qquad \cdots (2.15)$$

From this Eq. (2.15), N_2 can approach, but never exceed N_1, if the atomic system is in thermal equilibrium, Fig 2.2(a). Hence for N_2 to exceed N_1, i.e. to create a population inversion in the atomic system, a nonthermal equilibrium conditions must be achieved. Population inversion can be created, when the atoms within the laser medium is excited or pumped into a nonequilibrium distribution, Fig 2.2(b), through the application of a large amount of energy to the medium from an external source.

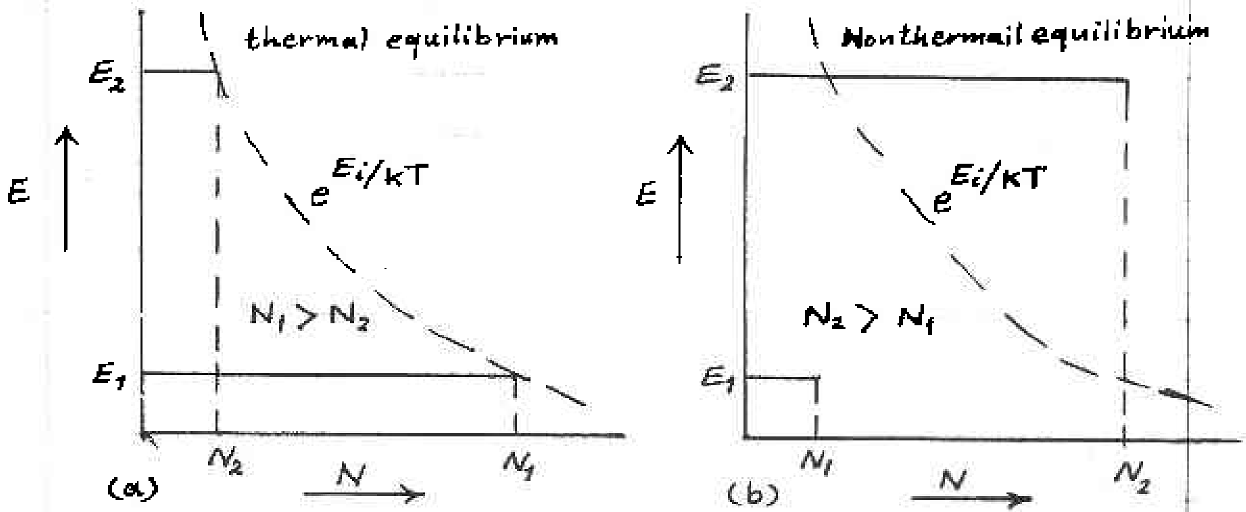


Fig. (2.2)

In nonthermal equilibrium or nonequilibrium distribution, population inversion is increased and a large number of transition can then be stimulated from the higher level to the lower level with the emission of coherent radiation.

For an atomic system of three Energy levels, Laser action with the emission of radiation then occurs due to stimulated transitions from level two to the ground level. Ruby laser is one of the three energy levels system which emit red laser at 694 nm wavelength.

2-3 Energy Pumping

There are several methods of energy pumping a collection of atom of laser active material, which are optical radiation, electrical discharge, passage of a current, free electron bombardment, release a chemical energy, and others. Optical and electrical discharge pumpings are the most common methods applied in lasers with different energy level schemes.

If we consider the problem of producing a population inversion to a simple two-energy level system by optical pumping. Many atoms will absorb the radiation and be excited from level E_1 to E_2. At best, because the constant coefficients $B_{12} = B_{21}$, even with very intense irradiation the condition of the populations of the upper and lower energy levels can only be made equal ($N_2 = N_1$), then, the absorption and stimulated processes will compensate one another. This situtation is often referred to as two-level saturation. We can see, with just that two-level system, it is impossible therefore to produce a population inversion. Hence optical pumping, and indeed most other puming methods, requires either a three- or a four-energy level systems.

Three Energy levels System

In three energy levels system Fig.(2-3), the laser active medium is illuminated by intense radiation from a flash tube and a large number of atoms are pumped into the upper energy level E_2 from the ground energy level E_o by absorption of radiation of frequency ν_{02}.

From level E_2 the atoms decay into level E_1 and a population inversion occurs between the levels E_1 and E_o, when N_1 exceeds N_o. The minimum pumping for population inversion required rapid transition of atoms from E_2 to E_1, (Less than the lifetime of atom in the excited levels of the order of 10^{-8} to 10^{-9} seconds).

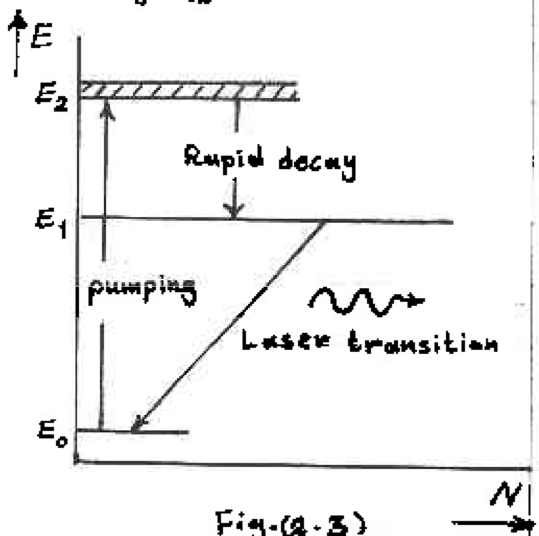


Fig.(2-3)

If these conditions are satisfied atoms can be pumped quickly from E_0 via E_2 into E_1, where they accumalate, so that the population N_1 builds up. Since the population inversion is achieved, amplification of radiation of frequency $\nu_{10} = (E_1 - E_0)/h$ can be obtained by stimulated emission from E_1 to E_0.

The three-levels system requires very high pumping power, because the energy required to pump half of the total number of atoms in the system into E_1 via E_2 is wasted. Therefore, the three-energy levels systems is inefficient.

Four Energy Levels System

In four energy levels system Fig.(2.4), the population of the energy levels E_1, E_2, and E_3 are all effectively zero before pumping commences. Pumping excites atoms from the ground energy level E_0 into level E_3, whence they decay rapidly into the metastable energy level E_2, so that N_2 increases rapidly to give population inversion between E_2 and E_1, and then laser transition of frequency,

$$\nu_{21} = (E_2 - E_1)/h$$

can be obtained by stimulated emission from E_2 to E_1.

If the lifetime of the transition (decay) from level E_1 to level E_0 is short, then the population inversion can be maintained easily with modest pumping.

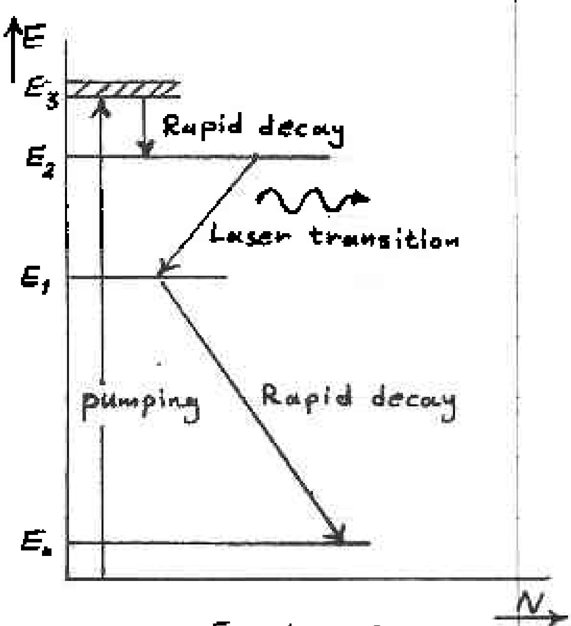


Fig.(2.4)

Problems Chapter (2): (Operation of Laser)

2.1 Calculate the ratio of the rates of spontaneous and stimulated emission (R) for the light emitted by an electric discharge in a gas such as Neon in the Helium-Neon (HeNe) laser. If the discharge temperature is 370 k for the red line produced by this laser, which has a frequency of $\nu = 4.74 \times 10^{14}$ Hz.

Solution:

The ratio of the rates of spontaneous and stimulated emission is given by the Eq;

$$R = \frac{N_2 A_{21}}{N_2 B_{21} \rho_\nu}$$

Where, A_{21}, B_{21} are constant called Einstein coefficients and the relationship betwen them is,

$$A_{21} = B_{21} \frac{8\pi h \nu^3}{c^3},$$

N_2, is the atoms per unit volume in energy level E_2, and

ρ_ν, is the energy density of such photons and can be described by the Eq;

$$\rho_\nu = \frac{8\pi h \nu^3}{c^3} \left(\frac{1}{\exp(h\nu/KT) - 1} \right)$$

Substituting for, A_{21} and ρ_ν Eqs, in the ratio Eq., then gives

$$R = \exp\left(\frac{h\nu}{KT}\right) - 1$$

$$R = \exp\left(\frac{6.63 \times 10^{-34}\ \text{J.s} \times 4.74 \times 10^{14}\ \text{s}^{-1}}{1.38 \times 10^{-23}\ \text{J.K}^{-1} \times 370\ \text{K}}\right)$$

$$R \simeq e^{61.5}$$

$$R \simeq 5 \times 10^{26}$$

2.2 In a material at 300 k two energy levels have a wavelength separation of 1 μm. Determine

(a) the ratio of upper to lower level population densities when the material is in thermal equilibrium.

(b) the effective temperature when the levels are equally populated;

(c) the effective temperature when the upper level is twice as densely populated as the lower.

Solution:

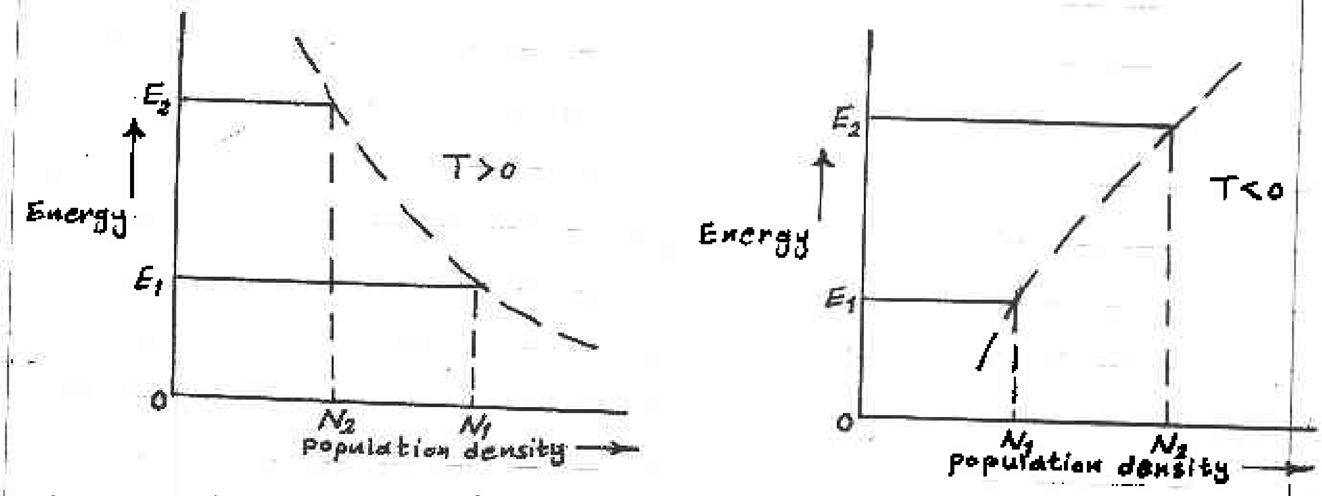
At normal room temperature, the lower energy level N_1 in a material is more densely populated than a higher energy level N_2 according to the Boltzmann equation as

$$\frac{N_1}{N_2} = \frac{e^{-E_1/kT}}{e^{-E_2/kT}} = e^{(E_2 - E_1)/kT}$$

or

$$T = \frac{(E_2 - E_1)}{k \ln (N_1 / N_2)}$$

and since $E_2 > E_1$ and $N_1 > N_2$, the temperature is positive as illustrated in Fig.



However, it is possible under certain conditions to make $N_2 > N_1$ with $E_2 > E_1$. This is known as population inversion and from the expression for T above, it will be seen that T has a negative value and the effect is therefore equivalent to a negative temperature. This is illustrated in Fig.

(a)
$$\frac{N_2}{N_1} = \frac{e^{-E_2/kT}}{e^{-E_1/kT}}$$

and for transitions between energy levels $\lambda = hc/(E_2 - E_1)$. Hence

$$\frac{(E_2 - E_1)}{kT} = \frac{hc}{\lambda kT} = \frac{6.626 \times 10^{-34}\ \text{J s} \times 3 \times 10^{8}\ \text{m s}^{-1}}{10^{-6}\ \text{m} \times 1.38 \times 10^{-23}\ \text{J K}^{-1} \times 300\ \text{K}} = 48.01$$

with $\quad N_2/N_1 = e^{-48.01} \simeq 10^{-21}$

(b) $\quad N_2/N_1 = e^{-(E_2 - E_1)/kT} = 1$

or $\quad \dfrac{-(E_2 - E_1)}{kT} = \ln 1 = 0$

and $\quad T \longrightarrow \pm \infty$

(c) $\quad N_2/N_1 = e^{-(E_2 - E_1)/kT} = 2$

or $\quad \dfrac{-(E_2 - E_1)}{kT} = 0.693$

with $\quad T = \dfrac{-6.626 \times 10^{-34}\ \text{J s} \times 3 \times 10^{8}\ \text{m s}^{-1}}{10^{-6}\ \text{m} \times 1.38 \times 10^{-23}\ \text{J K}^{-1} \times 0.693} \simeq -21\,000\ \text{K}$

2.3 For a system in thermal equilibrium calculate the temperature at which the rates of spontaneous and stimulated emission are equal for a wavelength of 10 μm, and the wavelength at which these rates are equal at temperature of 4000 K.

Solution:

At $\lambda = 10\ \mu m$ and $R = 1$

$R = \exp\left(\frac{hc}{\lambda kT}\right) = 1$, then solve for T, to get $T = 2018$ K

For $T = 4000$ K and $R = 1$

$R \simeq \exp\left(\frac{hc}{\lambda kT}\right) = 1$, then solve for λ, to get $\lambda = 5.2\ \mu m$

Chapter (3): Optical Cavities (Resonators)

3-1 Optical Resonant Cavity Configurations

The optical resonant cavity, plays a most significant role in the operation of laser. In the majority of cases the gain of a pumped active medium is quite small, so that the amplification of an optical beam passing once through the medium is minimal. Amplification is increased by plasing highly reflecting mirrors (reflectance approaching 100%) at each end of the medium. The optical beam then bounces to end and fro through the the active medium about one hundred times, thereby increasing the effective length of the active medium. The mirrors form an optical resonant Cavity (often called as Fabry-perot resonator), and together with the active medium constitutes an optical oscillator rather than an amplifier. Fig.(3.1) shows laser resonant cavity.

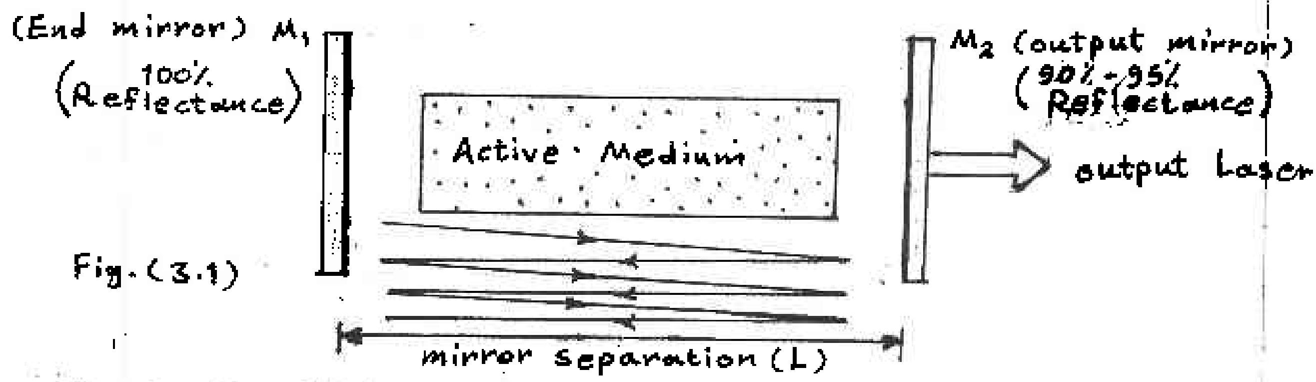


Fig. (3.1)

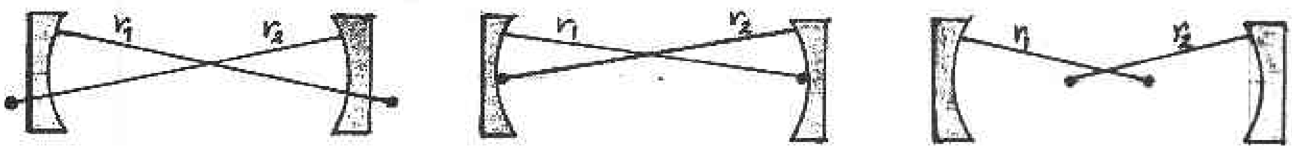
The mirrors introduce optical feedback to the amplifing medium in an entirely analogous way to positive feedback in an electronic amplifier. At the end mirror most of the energy is returned into the cavity and passes through the medium, being amplified on route, to the other mirror where the process is repeated. The amplitude of the disturbance at the transition frequency grows until a steady-state level of oscillation is reached. At this stage, growth of the wave amplitude within the cavity ceases and any additional energy produced by stimulated emission appears as the laser output.

There are several additional resonant cavity configurations that are more practical than is the original plane - parallel (Fabry - Perot) setup (Fig. 3.1). For example, if the planar - mirrors are replaced by identical concave spherical mirrors separated by a distance very nearly equal to their radius of curvature, we have the confocal resonator, the focal points are almost coincident on the axis midway between mirrors. If one of the spherical mirrors is made planer, the cavity is termed a hemispherical or hemiconcentric, resonator. Both these configurations are considerably easier to align than is the plane - parallel form. Laser cavities are said to be either stable or unstable to the degree that the beam tends to retrace itself and so remain relatively close to the optical axis (Fig. 3.2).

(a) Nearly planar (convex) $-r_1, -r_2 >> L$ unstable

(b) planar $r_1 = r_2 = \infty$ marginally stable

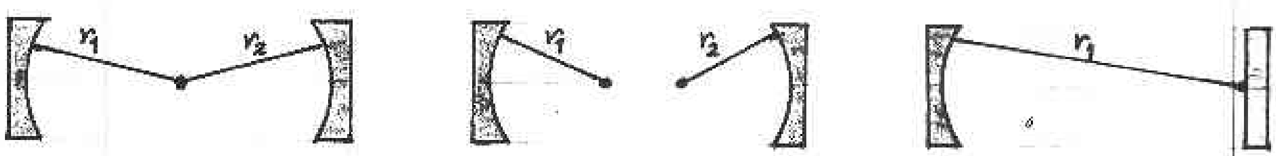
(c) Nearly planar (concave) $r_1, r_2 >> L$ stable



(d) Nearly confocal $r_1, r_2 \gtrsim L$ stable

(e) confocal $r_1 = r_2 = L$ marginally stable

(f) Nearly Concentric $r_1 \gtrsim L/2$; $r_2 \gtrsim L/2$ stable



(g) Concentric $r_1 = r_2 = L/2$ marginally stable

(h) Nearly concentric $r_1 \lesssim L/2$; $r_2 \lesssim L/2$ unstable

(i) Hemi - concentric $r_1 = L$; $r_2 = \infty$ marginally stable

Fig. (3.2)

A beam in an unstable cavity will walk out, going farther from the axis on each reflection until it quickly leaves the cavity altogether. By contrast, in a stable configuration (with mirrors that are, say, 100% and 98% reflective) the beam might traverse the resonator 50 times or more. Unstable resonators are commonly used in high-power lasers, where the fact that the beam traces across a wide region of the active medium enhances the amplification and allows for more energy to be extracted.
This approach will be especially useful for media (like carbon dioxide, CO_2 or Argon, Ar) wherein the beam gains more energy on each trip in the cavity.

The selection of a resonator configuration is actually governed by the specific requirements of the laser system – there is no universally best resonator arrangement.

Cavity Stability Criteria

The stability condition for a laser resonator cavity composed of two mirrors, separated by a distance L on the axis of the resonator is given by the very simple expression.

$$0 < \left(1 - \frac{L}{r_1}\right)\left(1 - \frac{L}{r_2}\right) < 1$$

or

$$0 < g_1 g_2 < 1$$

This condition can be expressed in the form of a stability diagram, as shown in Fig. (3.3)

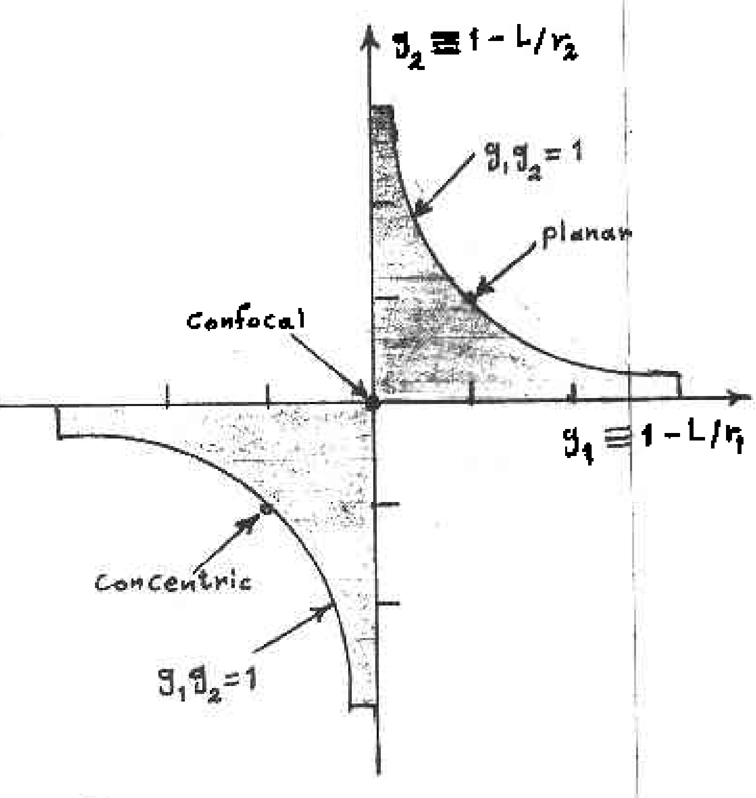


Fig. (3-3)

Threshold Gain Coefficient

To sustain laser oscillations the gain coefficient must be at least large enough to overcome the losses in the laser system. The sources of loss include the following:

(1) Transmission, absorption and scattering by the mirrors.

(2) Diffraction around the boundary of the mirrors.

(3) Absorption and scattering in the laser active medium due to transitions other than the desired one and optical inhomogenities.

The minimum or threshold gain coefficient k_{th} required from the condition that the round trip gain, G, in the irradiance of the beam must be at least unity.

If G < unity then the oscillations would die out, and

G > unity then the oscillations would grow.

In traviling from M_1 to M_2 in the laser resonator cavity, Fig. (3.4), the beam irradiances increases from I_o to I_1, where,

$$I_1 = I_o e^{(k-\gamma)L}$$

and,

γ is the effective volume loss coefficient which reduce the effective gain coefficient to $(k-\gamma)$.

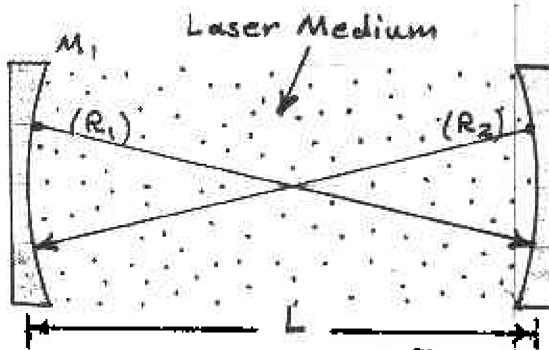


Fig. (3.4)

R_1: Reflectance at M_1

R_2: Reflectance at M_2

After reflection at M_2 the beam irradiance will be $I_o R_2 e^{(k-\gamma)L}$ and after a complete round trip the irradiance will be $I_o R_1 R_2 e^{2(k-\gamma)L}$, so that the round trip gain is

$$G = \frac{\text{final irradiance}}{\text{initial irradiance}} = \frac{I_o R_1 R_2 e^{2(k-\gamma)L}}{I_o} = R_1 R_2 e^{2(k-\gamma)L}$$

The threshold condition for laser oscillations is

$$G = R_1 R_2 e^{2(k_{th}-\gamma)L} = 1 \qquad \ldots \quad (3.1)$$

where k_{th}, the threshold gain coefficient, is given by

$$k_{th} = \gamma + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right) . \qquad \ldots \quad (3.2)$$

The first term in Eq. (3.2) represents the volume losses while the second is the loss in the form of the useful output; thus the condition for steady-state laser operation is that the gain equals the sum of the losses.
In lasers designed for continuous output (CW) the gain becomes constant at the threshold value. This is because only when round trip gain $G = 1$, the cavity energy (and hence the laser output) settles down to a steady-state value. This phenomenon is referred to as gain saturation.
The actual value of the gain depends on the population inversion and on the physical properties of the medium. If k is high then it is relatively easy to achieve laser action and mirror alignment and cleanliness are not too critical. With low gain media, the mirrors must be accurately aligned, have high reflectances and clean.

Circulating Power

For a continuous output laser, the loss of laser power from the output (transmitted) mirror will be compensated by the active (gain) medium inside the resonator. The output laser power and the circulating laser power inside the resonator are in a steady state or saturated value, Fig. (3.5), and is given by the expression,

$$P_{out} = T\,P_{cir} \qquad \cdots (3.3)$$

where,
T is the transmittance of the output mirror.

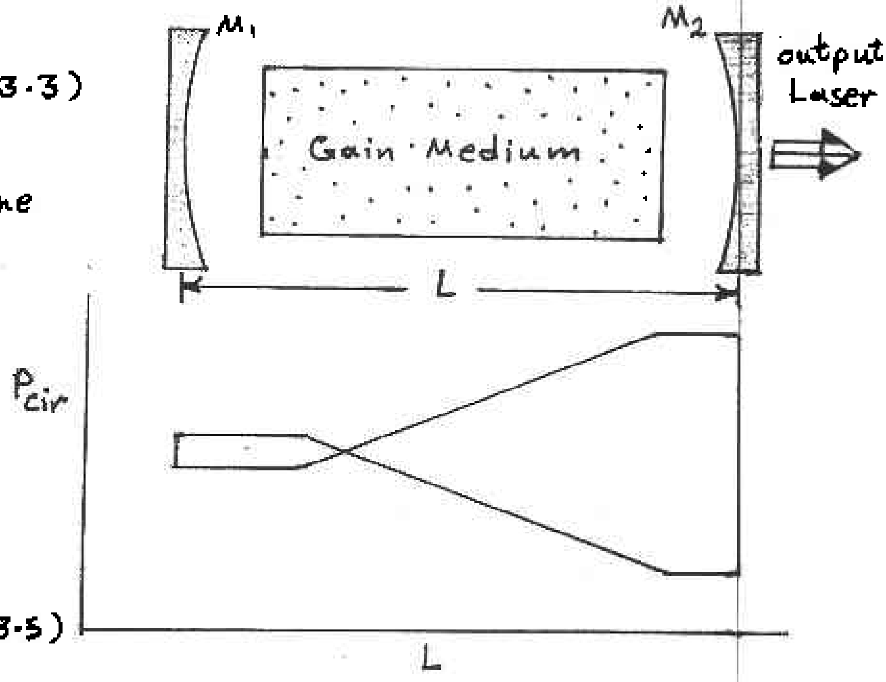


Fig.(3-5)

3.2 Laser Modes

The output laser consists of a number of very closely spaced, discrate frequency components (very narrow spectral lines) covering a moderately broad spectral range. The discrete components are called laser modes and the spectral range they occupy is approximately the fluorescent linewidth of atomic transition giving rise to the laser output.

Longitudinal (Axial) Modes

Laser oscillations occur, when the wave within the cavity replicate itself after two reflections so that the electric fields add in phase. In other words, the mirrors form a resonant cavity and standing-wave patterns are set up. The cavity resonates when there is an integer number (m) of half wavelengths spanning the region between the mirrors, as shown in Fig. (3.6). That there must be a node at each mirror, and this can only happen when the seperation of the mirrors (L) equals a whole number multiple of $\lambda/2$.

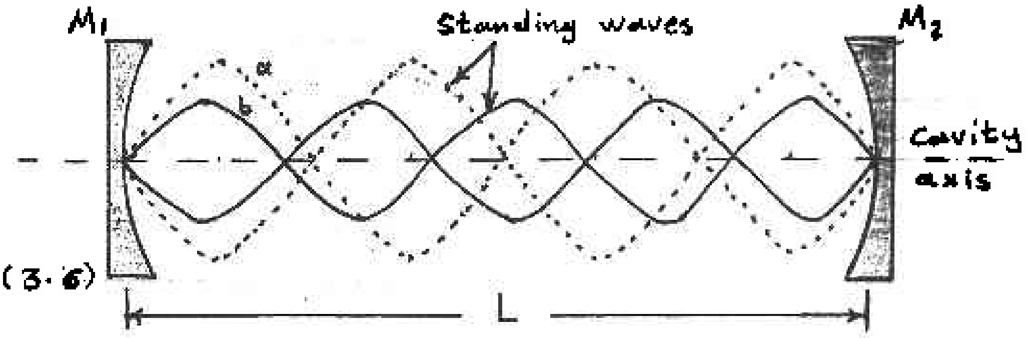


Fig. (3.6)

Thus,

$$m\frac{\lambda}{2} = L \quad , \text{ or}$$

$$m = \frac{L}{\lambda/2} , \qquad \cdots (3.4)$$

and if the refractive index of the active medium is unity, the frequency is given as,

$$\nu_m = \frac{c}{\lambda} , \text{ or}$$

$$\nu_m = \frac{mc}{2L} \qquad \cdots (3.5)$$

Therefore, there are an infinite number of possible oscillatory longitudinal cavity modes, each with a distinctive frequency ν_m, the frequency separation $\Delta\nu$ between adjacent modes $\Delta m = 1$ is given by

$$\nu_{m+1} - \nu_m = \Delta\nu = \frac{c}{2L} \qquad \cdots (3.6)$$

The longitudinal modes of the laser cavity thus consist of a large number of frequencies given by Eq. (3.5) and separated by $c/2L$, these modes are shown in Fig. (3.7).

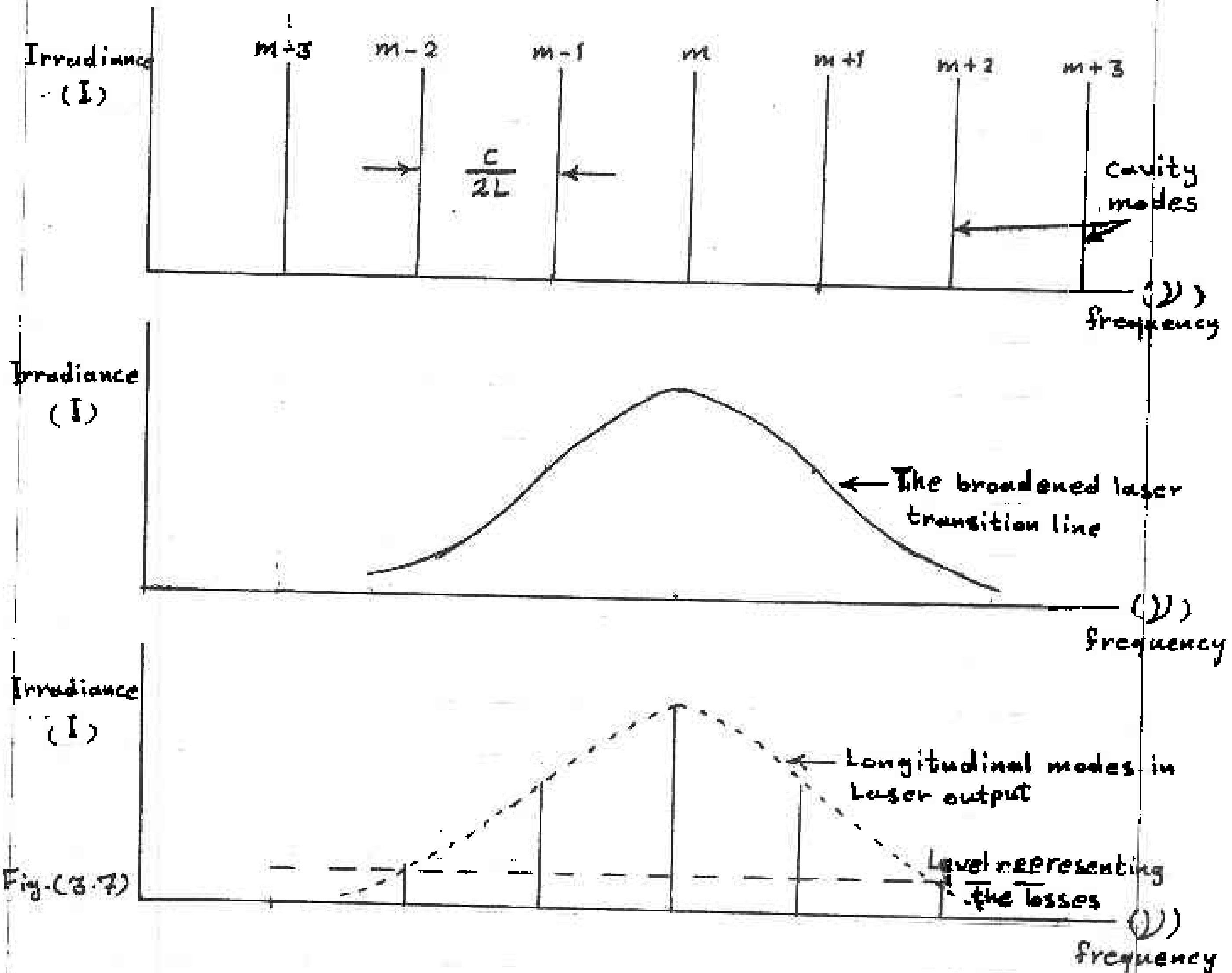


Fig.(3.7)

The given mode can only oscillate if there is gain at that frequency, in other words, if it lies within the frequency range of the fluoresent line and if the gain exceeds the losses.

Transverse Modes

The longitudinal modes all contribute to a single spot of light in the laser output, whereas in general if the laser beam is shone onto a screen we observe a patern of spots. These are due to the transverse modes of the cavity.

In most cases, however, waves which are traveling just off-axis and are able to replicate themselves after covering a more complex closed path in the resonator are referred to as transverse electromagnetic modes TEM_{mn}. They are characterized by two integers m and n, so that as Fig. (3-8) shows, we have TEM_{00}, TEM_{01}, TEM_{11} etc. modes (m gives the number of minima (or phase reversals) as the beam is scanned horizontally and n the number of minima as it scanned vertically).

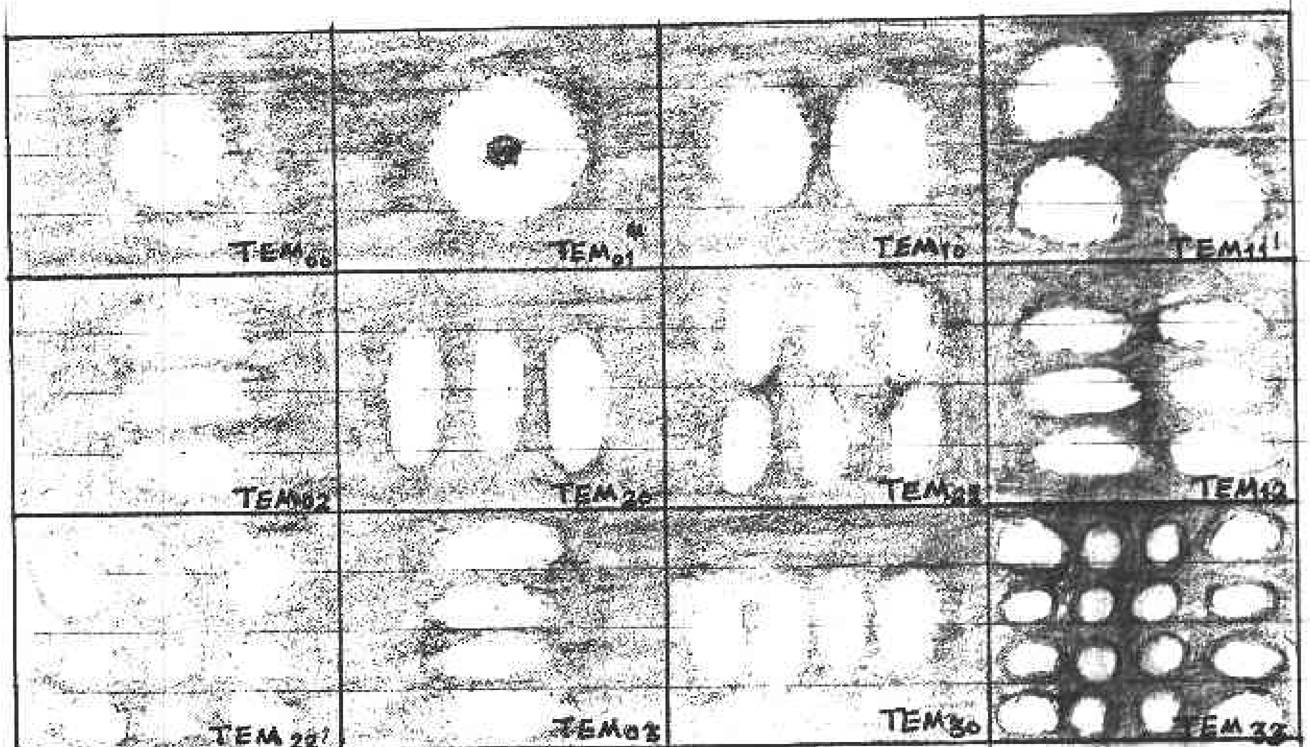


Fig. (3.8)

The TEM_{00} transverse mode is the most widly used, and this for several resons: the flux density is ideally Gaussian over the beam cross section; there are no phase shifts in the electric field across the beam, as there are in other modes, and so it is completely spatially coherent; the beam's angular divergence is the smallest; and it can be focused down to the smallest-sized spot.

Problems Chapter (3): Optical Cavities

3.1 Determine the stability of the following cavity resonators:

(i) $L = 1.5\,m$, $r_1 = 3\,m$, $r_2 = 2\,m$

(ii) $L = 1\,m$, $r_1 = 0.5\,m$, $r_2 = 2\,m$

(iii) $L = 1\,m$, $r_1 = 3\,m$, $r_2 = -2\,m$

Solution;

The stability condition for a laser resonator cavity is given by the expression,

$$0 < g_1 g_2 < 1 \quad , \quad g_1 \equiv 1 - L/r_1 \ \text{and}\ g_2 \equiv 1 - L/r_2$$

(i) $$\left(1-\frac{1.5}{3}\right)\left(1-\frac{1.5}{2}\right) = (1-0.5)(1-0.75) = (0.5)(0.25) = 0.125$$

since,

$$0 < 0.125 < 1$$

the cavity resonator is stable.

(ii) $$\left(1-\frac{1}{0.5}\right)\left(1-\frac{1}{2}\right) = (1-2)(0.5) = (-1)(0.5) = -0.5$$

Since,

$$g_1 g_2 = -0.5 < 0$$

the cavity resonator is unstable.

(iii) $$\left(1-\frac{1}{3}\right)\left(1-\frac{1}{-2}\right) = \left(\frac{2}{3}\right)\left(\frac{3}{2}\right) = 1$$

Since,

$$g_1 g_2 = 1$$

the cavity resonator is marginally stable.

3.2 Verify that the confocal, hemispherical, and plane paralle cavities are stable according to stability criteria.

3.3 In a ruby laser ($\lambda = 694.3$ nm), the ruby crystal is 0.1 m long and the mirror reflactances are 95% and 90%. Given that the losses are 10% per round trip. Calculate the threshold gain coefficient

Solution;

The threshold gain coefficient for a ruby laser, is given by the equation

$$k_{th} = \gamma + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right)$$

$$k_{th} = 0.1 + \frac{1}{2 \times 0.1} \ln\left(\frac{1}{(0.95) \times (0.90)}\right)$$

$$k_{th} = 0.88 \ m^{-1}$$

3.4 A HeNe laser has a confocal cavity with mirror separation 0.5 m. Given that the width of the gain curve is 1.5 GHz, Calculate the maximum number of Longitudinal mode frequencies which can oscillate in the cavity.

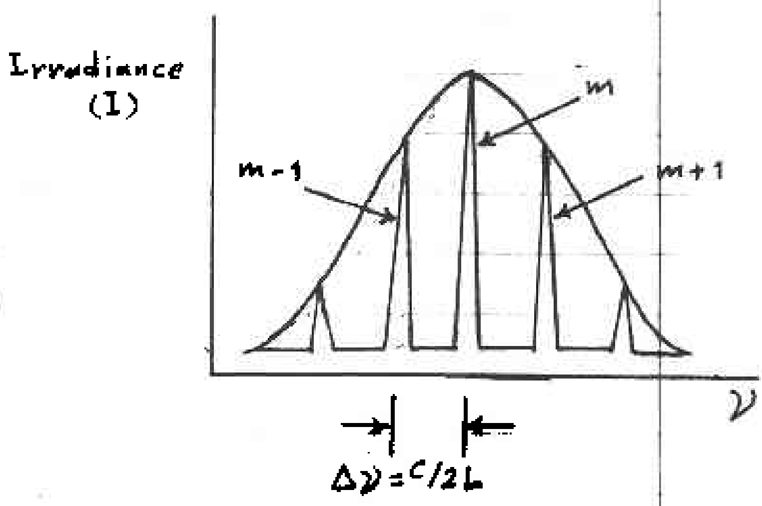
Solution;

The maximum number of longitudinal modes for a HeNe laser with spectral width of the 632.8 nm laser transition 1.5 GHz is given by the equation

$$m = \frac{2 \nu_m L}{c}$$

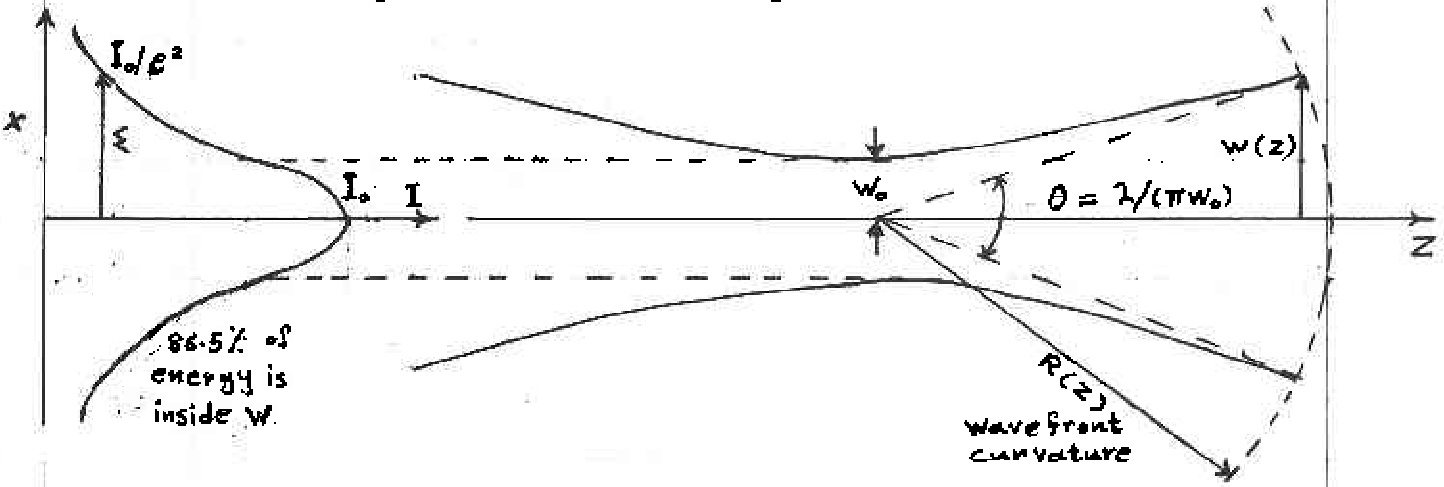
$$m = \frac{2 \times 1.5 \times 10^{9} s^{-1} \times 0.5 \, m}{3 \times 10^{8} \, m s^{-1}}$$

$m = 5$ (longitudinal modes)



3.5 Discuss the properties and propagation of a Gaussian laser beam.

In a gain medium located within an optical resonator, the TEM_{00} Gaussian mode that develops when the single-pass gain exceeds the cavity losses have a Gaussian profile at the mirrors in the direction transverse to the direction of propagation of the beam, Fig. (3.10)



A Gaussian laser beam has the following properties:

1. The beam have a Gaussian transverse profile at all locations. such a Gaussian beam can be characterized completely at any spatial location by defining both its "beam waist" and its "wavefront curvature" at a specific location of the beam.

2. A Gaussian beam always has a minimum beam waist (w_o) at one location in space.

3. The transverse distributions of the intensity of a simple Gaussian beam is of the form $I = I_o e^{-2x^2/w^2}$
where I_o is the maximum intensity and w is the beam radius inside of which 86.5% of the energy is concentrated, as shown in Fig (3.10).

The Gaussian beam minimum waist w_o for a typical laser resonator mode occurs in the region between the two mirrors of an optical resonator. For example, the minimum beam waist w_o in a confocal optical resonator ($r_1 = r_2 = L$) occurs halfway between the two mirrors. As the Gaussian beam propagate it expands and diverges from that location, such that the beam waist at a distance of $\pm z$ from the minimum beam waist W_o can be described as

$$W_z = W_o \left[1 + \left(\frac{\lambda z}{\pi W_o^2} \right)^2 \right]^{1/2}$$

The beam wavefront curvature of a Gaussian beam at a location z, in term of the minimum beam waist W_o and the wavelength λ, is given by

$$R_z = z \left[1 + \left(\frac{\pi W_o^2}{\lambda z} \right)^2 \right]$$

The angular spread of a Gaussian beam for a value of z is given by

$$\theta_z = \frac{\lambda}{\pi W_o},$$

as shown in Fig (3.10).

The θ_z term is the full angle, at a given location z, over which the beam reduces to half of its maximum intensity at the center of the beam.

Example: An HeNe laser operating in a single TEM_{oo} mode at 632.8 nm has a mirror separation of 0.5 m with mirrors $r_1 = r_2 = 1\,m$, Calculate the radius and wavefront curvature of the Gaussian laser beam at a distance of 10 m away from the minimum beam radius (W_o) of 0.3 mm.

Solution:

The beam radius of the HeNe Gaussian laser beam at a distance $z = 10\,m$ from the minimum beam radius is given by,

$$W_z = 3 \times 10^{-4}\,m \left[1 + \left(\frac{632.8 \times 10^{-9}\,m \times 10\,m}{\pi \times (3 \times 10^{-4}\,m)^2} \right)^2 \right]^{1/2}$$

$$=$$

and the wavefront curvature of the HeNe Gaussian laser beam at a distance $z = 10$ from the minimum beam radius is given by,

$$R_z = 10\,m \left[1 + \left(\frac{\pi \times (3 \times 10^{-4}\,m)^2}{632.8 \times 10^{-9}\,m \times 10\,m} \right)^2 \right]$$

$$=$$

Chapter (4): Spectroscope of the Laser Light

4.1 Line width properties

Atoms in either upper or lower levels of the laser medium will not interact with a perfectly monochromatic beam. This is the fact that all spectral lines have a finite wavelength or frequency spread, i.e., (fluorescent or spectral linewidth). This can be seen in both emission ~~and~~ absorption, and if we measured the emission of a typical spectral source as a function of frequency, we will get bell-shaped curve illustrated in Fig. (4.1).

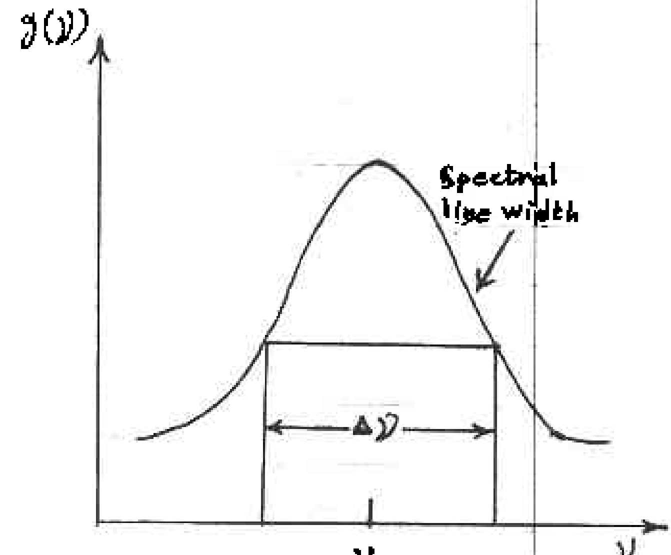


Fig. (4.1)

The precise shape of the curve is given by the lineshape function $g(\nu)$, which represents the frequency distribution of the radiation in a given spectral line. The precise form of $g(\nu)$ which is normalized so that the area under the curve is unity, depends on the particular mechanisms causing the spectral broadening.

The most important mechanisms are collision (or pressure) broadening as a homogeneous broadening and Doppler broadening as nonhomogeneous broadening. The shape of the emission linewidth will be either Lorentzian for homogeneous broadening or Gaussian for nonhomogeneous broadening, as illustrated in Fig. (4.2), a comparison of Nd: glass (nonhomogeneous broadening with Gaussian shape) and Nd: YAG (homogeneous broadening with Lorentzian shape).

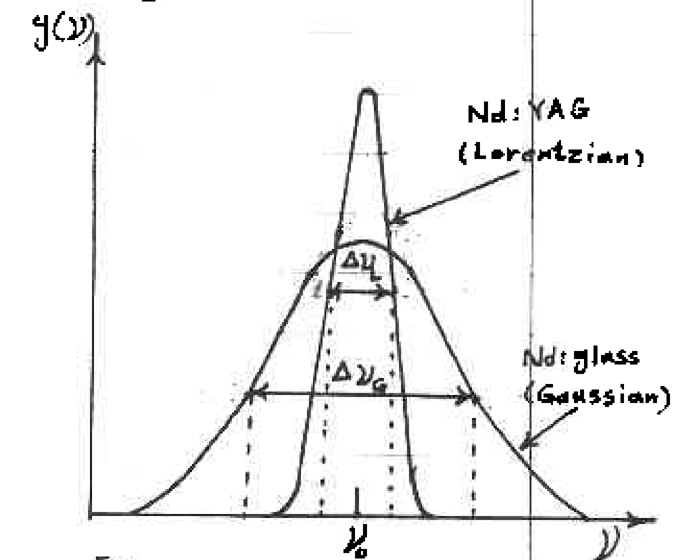


Fig. (4.2)

4.2 Homogenous Broadening

A line-broadening mechanism is referred to as homogenous when it broadens the line of each individual atom, and therefore the whole system, in the same way.

The most important homgeneous line broadening wich is collision broadening in a gas (such as gas laser), It is due to collision of an atom with other atoms, ions, free electrons, or the walls of the resonator. In a solid, it is due to the interaction of the atom with the phonons of the crystal lattice (such as a solid-state laser material), (Fig. 4.3)

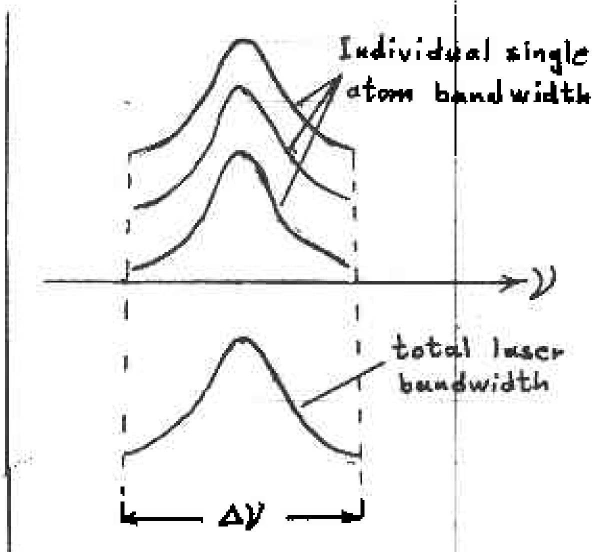


Fig.(4.3)

4.3 Nonhomogeneous Brodening

A line-broadening mechanism is referred to as nonhomogenous when it leads to the atomic resonance frequencies being distributed over a band of frequencies and therefore results in a broadened line for the system as a whole without broadening the line of individual atoms, (Fig. 4.4)

Doppler broadening results from the differences in frequency measured for the radiation emitted from atoms as they travel away from or towards an observer,

The observed frequencies will be in the range $\nu^{*} = \nu\left(1 \pm \frac{v}{c}\right)$

where v is the velocity of the atom along the direction of observation and

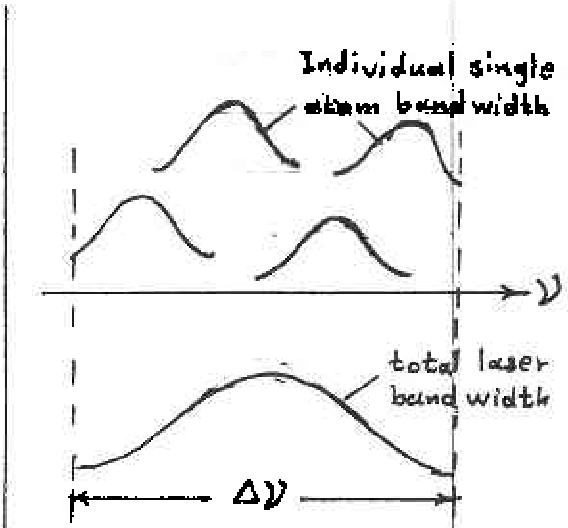


Fig.(4.4)

c is the speed of light. The spectral halfwidth $\Delta\nu$ is given by

$$\Delta\nu = \frac{2\nu v}{c}$$

4.4 Longitudinal Mode Selection Methods

If the laser gain bandwidth is broader than the longitudinal mode spacing $\Delta\nu = c/2\eta L$, where L is the distance between the end mirrors of the cavity and η is the index of refraction of the gain medium, then it is possible to have more than one longitudinal mode lasing within the cavity. Sometimes it is desirable to have only a single longitudinal mode lasing within the cavity, and it may not be possible to shorten the cavity spacing L in order to produce a single mode according to the formula. However, it is still possible to insert within the laser cavity an additional Fabry-Perot cavity that serves as an extra frequency-selective loss element.

This additional element is typically an optically transmissive material consisting of two parallel surfaces, both coated to achieve a specific reflectivity. The values of the mirror reflectivities are designed to give the desired additional loss at frequencies other than the desired laser frequency, thus quenching the gain at all modes except the desired one. Such a fixed Fabry-Perot device is shown in Fig. (4.5) and is known as an etalon. It is typically a thick piece of quartz with optical surfaces that are especially flat, parallel, and of very high quality. Since the frequencies at wich the Fabry-Perot resonances occur are proportional to the spacing between the mirrors, the etalon can be rotated as shown in the figure to tune the transmission to the desired frequency.

In lasers having a broad gain bandwidth - such as a dye laser, a Ti: Al_2O_3 laser, or lasers with multiple single-frequency transitions such as an Argon ion laser - it is often desirable to tune or select any specific laser wavelength over that gain bandwidth without changing the cavity mirrors. A simple means of providing such a wide range of tunability is to install either a dispersive element (such as a prism) within the cavity or a variable-frequency high reflector (such as a diffraction grating) as one of the mirrors of the cavity. Tuning is accomplished by rotating the prism or the grating. Simple examples of these two cavity arrangements are shown in Fig. (4.6). The diffraction grating has higher dispersion than a prism and therefore offers more precise wavelength selection.

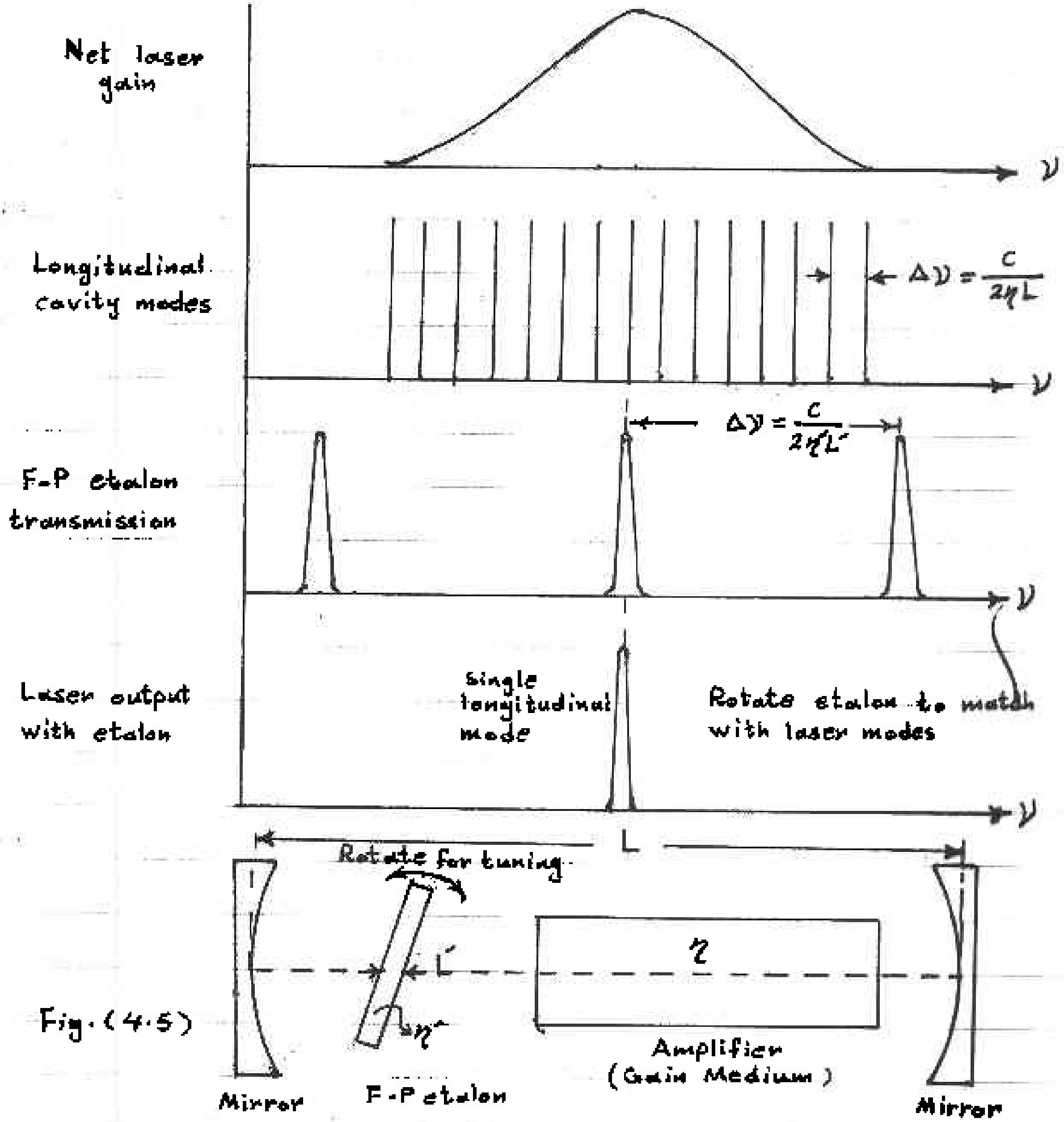

Net laser gain
ν
Longitudinal cavity modes
$\Delta\nu = \frac{c}{2\eta L}$
ν
F-P etalon transmission
$\Delta\nu = \frac{c}{2\eta' L'}$
ν
Laser output with etalon
Single longitudinal mode
Rotate etalon to match with laser modes
ν
L
Rotate for tuning
L'
η
η'
Mirror
F-P etalon
Amplifier (Gain Medium)
Mirror

Fig. (4.5)

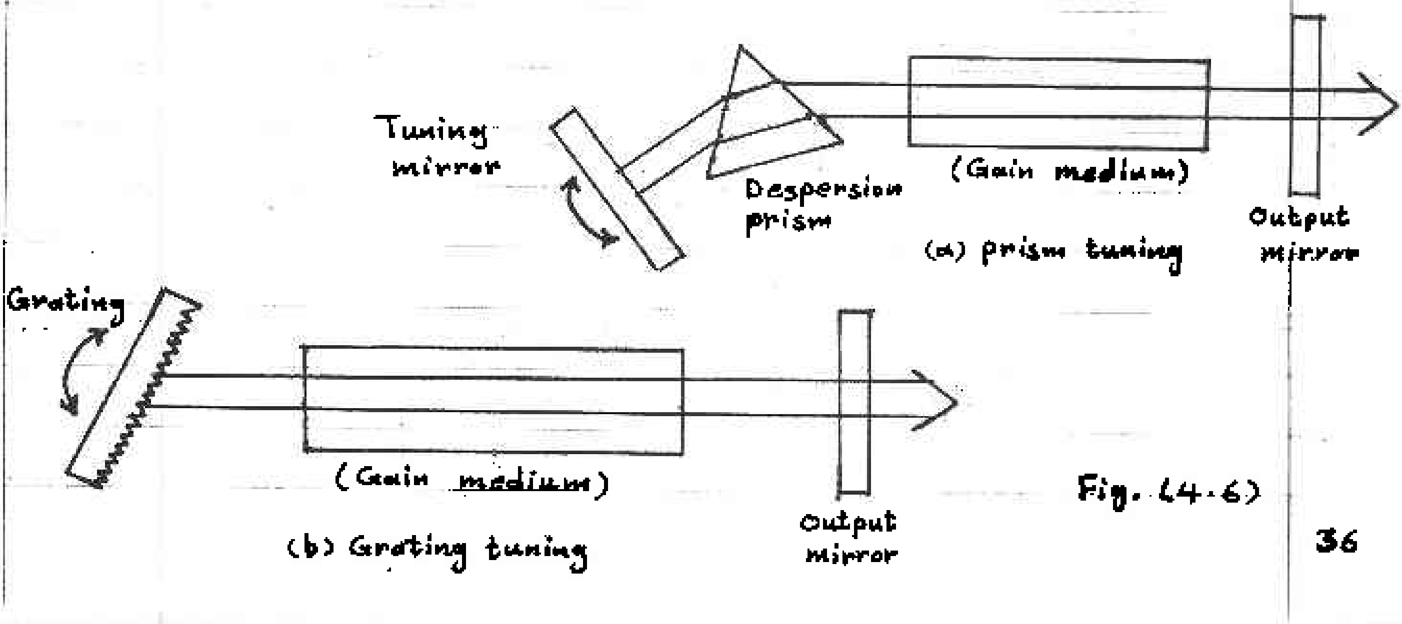

Tuning mirror
Despersion prism
(Gain medium)
Output mirror
(a) prism tuning
Grating
(Gain medium)
Output mirror
(b) Grating tuning

Fig. (4.6)

Problems chapter (4): Spectroscope of the Laser Light

4.1 Calculate the spectral broadening due to the Doppler effect in the carbon dioxide (CO_2) laser ($\lambda = 10.6\ \mu m$) assuming that the temperature of the pumping discharge is 400 k. The relative atomic masses of carbon and oxygen are 12 and 16.

Solution:

The average thermal velocity of the carbon dioxide molecules along the laser axis is given by $\frac{1}{2} M v^2 = \frac{1}{2} kT$,

that is $v = (kT/M)^{1/2}$,

where M is the mass of a carbon dioxide molecule and k is Boltzmann's constant.

The spectral broadening is given by $\Delta\nu = \frac{2\nu v}{c} = \frac{2v}{\lambda}$ or $\Delta\nu = \frac{2}{\lambda}\left(\frac{kT}{M}\right)^{1/2}$

From the Avogadro constant (N_A),

$$M = \frac{44}{6.022 \times 10^{26}} = 7.31 \times 10^{-26}\ kg$$

$$\therefore\ \Delta\nu = \frac{2}{\lambda}\left(\frac{kT}{M}\right)^{1/2} = \frac{2}{10.6 \times 10^{-6}\ m}\left(\frac{1.381 \times 10^{-23}\ J\ K^{-1} \times 400\ K}{7.31 \times 10^{-26}\ kg}\right)^{1/2}$$

$$\Delta\nu = 51.9\ MHz$$

4.2 Determine the frequency difference between successive maxima for a Fabry - Perot etalon in HeNe laser in which the mirrors are separated by 0.01 m.

Solution;

Because the mirrors are located in a gas, the index of refraction (η') is essentially unity, Thus the frequency difference $\Delta\nu$ is given by,

$$\Delta\nu = \frac{c}{2\eta' L'} = \frac{3 \times 10^{8}\ m\ s^{-1}}{2 \times 1 \times 0.01\ m} = 1.5 \times 10^{10}\ Hz$$

Hence the spacing between successive maxima for a F-P etalon is about 10 times the typical emission linewidth of a Doppler_broadened gas laser emission linewidth.

Chapter (5): High Power Techniquies

5.1 Q-Switching

Q-switching is a technique for obtaining short, intense pulses of radiation from lasers. It involves deliberately introducing a time-dependent loss into the cavity. With a high loss present the gain due to the population inversion can reach quite large values without laser action occurring. The high loss thus prevents laser action while energy is being pumped into the excited state of the medium. If, when a large population inversion has been achieved, the cavity loss is suddenly reduced, laser oscillation can then begin and build-up rapidly. All the available energy is emitted in a single, large pulse. This quickly depopulates the upper lasing level to such an extent that the gain is reduced below threshold and laser action stops.

A word about the nomenclature: Q-switching refers to changing the Q (quality factor) value of the cavity. The Q value is inversely proportional to the energy dissipated per cycle. In a high-loss situation therefore Q is small, while when the loss is removed Q switches to higher values.

We may imagine Q-switching to be carried out by placing a closed shutter within the laser cavity, thereby isolating the cavity from the laser medium. After the laser has been pumped the shutter is opened, thus restoring the Q of the cavity.

There are two obvious requirements for effective Q-switching:

(a) The pumping rate must be faster than the spontaneous decay rate of the upper lasing level, otherwise it will not be possible to build up a sufficiently large population inversion.

(b) The Q-switching mechanism must operate rapidly compared to the build-up of the laser oscillation, otherwise the latter will build up only gradually and a longer pulse will be obtained with lower peak power. In practical terms this means that ideally the switch should operate within a nanosecond or so.

5.1.1 Mechanical Q-Switching

Mechanical Q-switching involves rotating one of the cavity mirrors at a very high angular velocity. The optical losses within the cavity will be large except for the brief interval during each rotation when the mirrors are very nearly parallel, (Fig. 5.1). Just before this point is reached a trigger mechanism initiates the flashlamp pumping. As the mirrors are not quite parallel, the population inversion can build up without laser action starting. When the mirrors became parallel Q-switching occurs and the Q-switched pulse then develops.

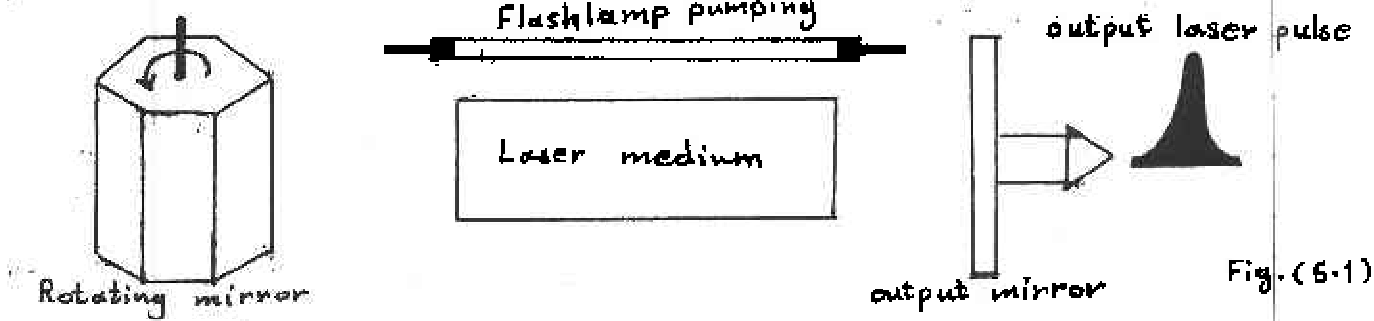


Fig. (5.1)

The mirror rotation speed involved may be as high as 100 rev s^{-1}, and this implies that the cavity could be Q-switched every 10^{-3} s.

5.1.2 Acousto-Optic Q-Switching

The acousto-optic effect is the change in the refractive index of a medium caused by the mechanical strains which accompany an acoustic wave as it travels through the medium. The acoustic wave sets up a diffraction grating which can then be used to deflect a laser beam. The frequencies of acoustic wave used in practice are ultrasonic (in the range of 50 MHz).

Fig. (5.2) illustrates the mode of operation of an acousto-optic Q-switch.

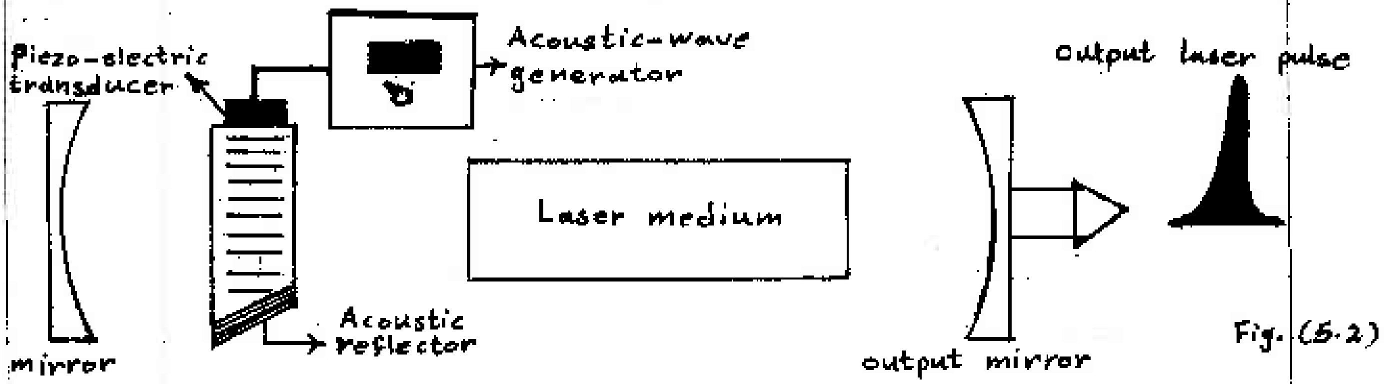


Fig. (5.2)

when the acoustic wave is present a significant fraction of the beam energy in the cavity is diffracted out of the cavity, thus introducing an additional loss mechanism and reducing the cavity Q value. When the acoustic wave is turned off diffraction ceases and the Q value returns to its former high level.

5.1.3 *Electro-* optic Q-switching

The electro-optic Q-switching which is based on the pockels effect is one of the mast useful techniques tat can be used. This effect concerns the behavior of polarized light as it passes through certain electro-optic materials (crystals) which are subject to electric fields. Thus if we take such a crystal and allow laser light which is a plane polarized along a particular crystal direction to pass through it, then provided an electric field of appropriate magnitude is applied along the beam direction, the laser beam will emerge with its plane of polarization rotated through 90°. With no electric field applied, however, there is no rotation. If such a crystal is placed inside a laser cavity together with a polarizer as in Fig. (5.3), the electro-optic crystal can be used as a Q-switch.

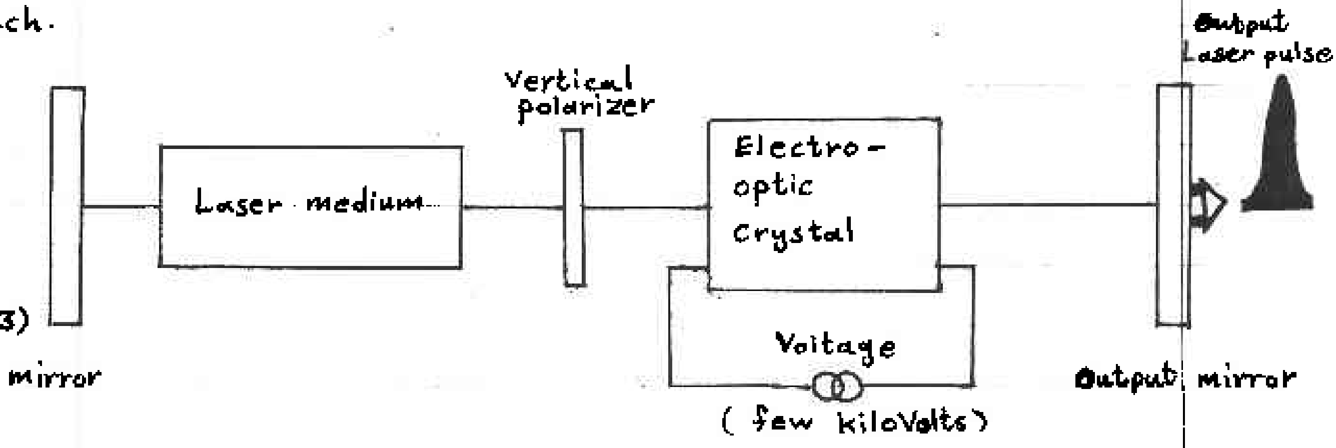


Fig.(5-3)

The laser light emerging from the lasing medium, will pass through the polarizer and hence become plane polarized. It then traverses the electro-optic crystal twice before returning to the polarizer. If the plane of polarization of the beam has been rotated through 90°, however, it will be unable to pass through the polarizer and the shutter will be effectively closed. The electro-optic Q-switching time is in the order of nanoseconds or less

5.1.4 passive Q-Switching

passive Q-switching relies on the action of so-called saturable absorbers, which are materials (often dye solutions) whose absorption decreases with increasing irradiance. The high irradiance excites the solution dye molecules from ground state to upper state, thus reducing the ground-state population. Since the amount of absorption exhibited by the dye depends on the number of molecules in the ground state, increasing the total amount of irradiation leads to the overall absorption to fall. In saturable absorbers, the molecules absorb radiation so strongly that appréciable numbers can be excited to the upper state, and significant changes made in the absorption of the dye.

If the ground and excited state populations are almost equal, then the absorption becomes very small; in this case the dye is said to saturated.

If such a dye cell is placed inside a laser cavity, the dye cell can be used as a Q-switch. Fig. (5.4). Initially, at low light intensity, the dye is opaque, thereby preventing laser action and allowing a large build-up of population inversion. However at high light intensity, the dye's excited states become populated and the dye begins to bleach. In a very short time the dye can be switch from being highly absorbing to being almost transparent, thus allowing the formation of a giant laser pulse.

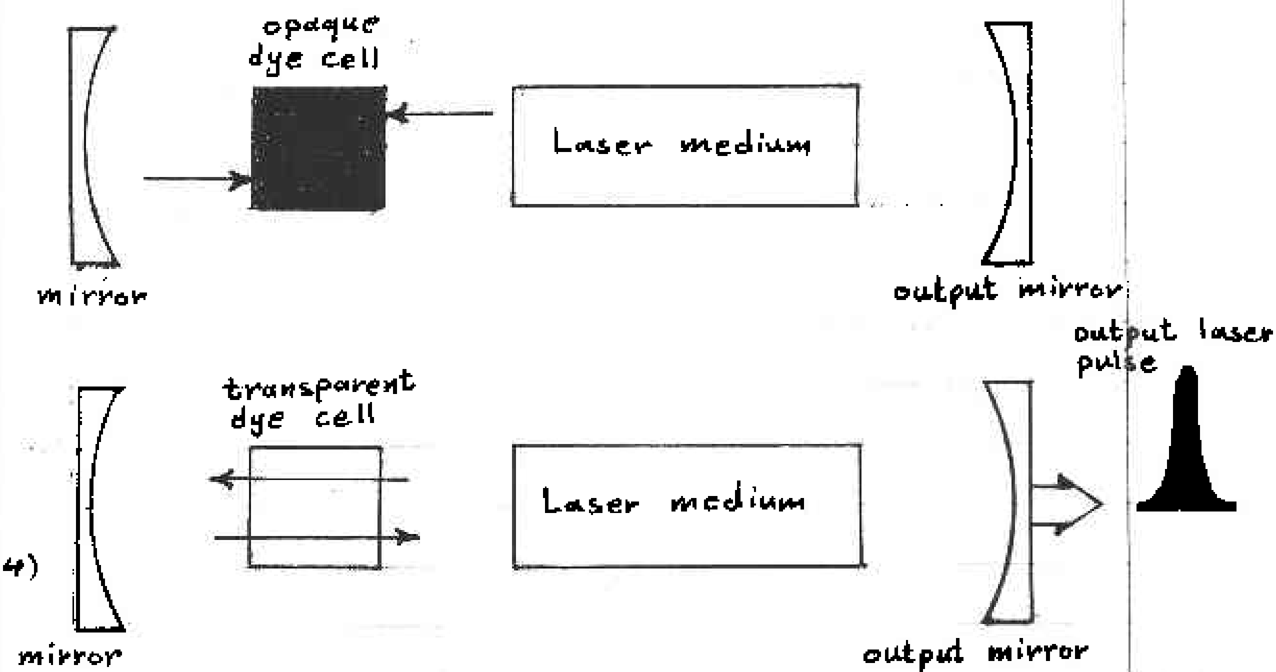


Fig. (5.4)

5.2 Mode-Locking

The technique of mode-locking allows the generation of laser pulses of ultra-short duration (from a few tens of femtoseconds to a few tens of picoseconds). Mode-locking refers to the situation where the cavity longitudinal modes are made to oscillate with a comparable amplitude and with locked phases. Mode-locking is achieved by combining in phase a number of distinct longitudinal modes of a laser, all having slightly different frequencies. The result of phase-locking condition, is that the oscillating longitudinal modes interfere to produce a train of evenly spaced giant laser pulses, Fig. (5.5).

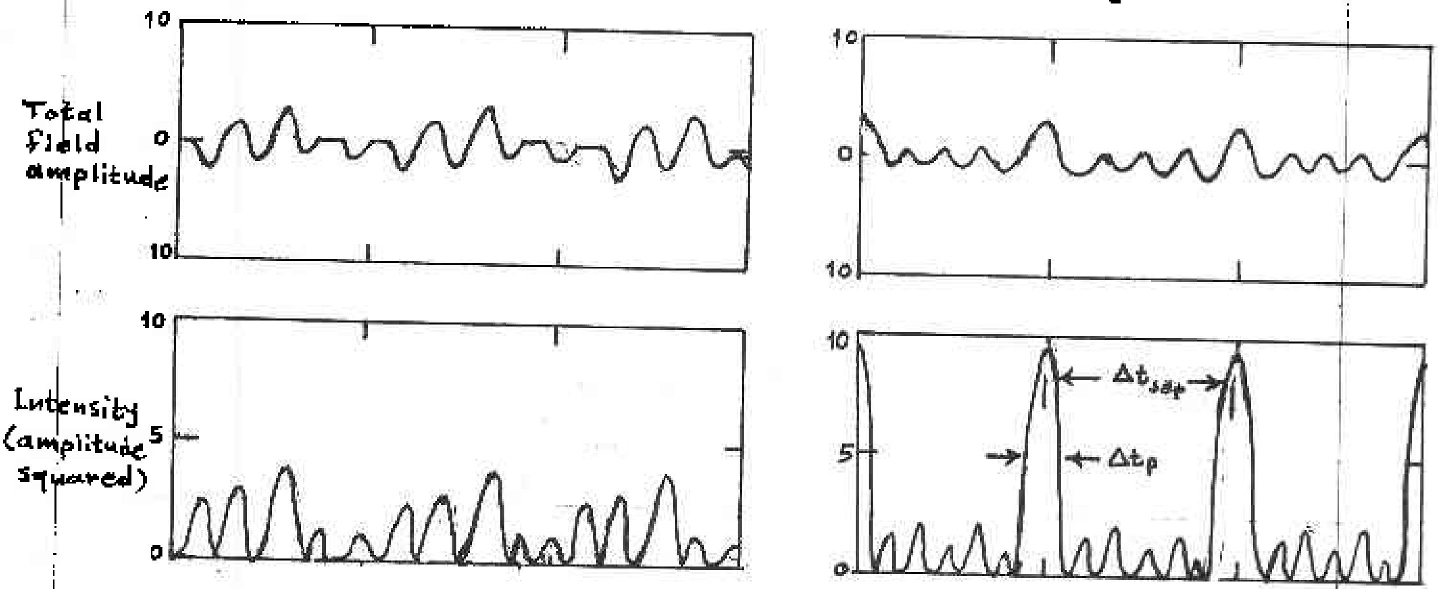


Fig.(5.5) (a) Amplitude and intensity of the sum of three out-of-phase waves added together. (b) Amplitude and intensity of the sum of three-in-phase waves added together.

The mode-locked pulse width (FWHM) is given by the expression

$$\Delta t_p = \frac{2\eta L}{M c} = \frac{2\pi}{\Delta\omega M} = \frac{1}{\Delta\nu M} = \frac{1}{\text{gain bandwidth}} \qquad ---- (5.1)$$

Where c is the velocity of the laser beam in a medium having index of refraction η and L is the separation between the cavity mirrors. M is the number of longitudinal modes of equal amplitude oscillating simultaneously in the cavity.

The predicted pulse width of each mode-locked pulse can be as short as short as the reciprocal of the bandwidth of the emission line, which is just $m\Delta\nu$.

The mode-locked pulses are separated in time (the time differences between two successive maxima) is given by the expression

$$\Delta t_{sep} = t_{n+1} - t_n = \frac{2(n+1)\pi}{\Delta\omega} - \frac{2n\pi}{\Delta\omega} = \frac{2\pi}{\Delta\omega} = \frac{2\pi}{2\pi\Delta\nu} = \frac{1}{\Delta\nu} = \frac{2nL}{c} \qquad (5.2)$$

5.2.1 Active Mode-locking

Acousto-optic switches (modulators) are very useful mode-locking switches for low-gain cw lasers. The switch consists of a quartz crystal with a piezoelectric transducer attached to one side. The crystal is located at one end of the laser cavity near the laser mirror. An RF signal (20-50 MHz) is applied to the transducer; this introduces an acoustic wave within the quartz crystal. This acoustic wave produces a periodic loss within the quartz crystal, due to Bragg reflection, at twice the applied frequency of the RF signal. When the RF signal reduces to zero electric field, the loss not present, the shutter is effectively open, the mode phases are locked, and short pulse passes through the shutter.

5.2.2 Passive Mode-locking

Saturable absorbers switches are very useful mode-locking switches for high-power lasers such as Nd:glass and ruby. The switch consists of a saturable absorber cell (dye solutions) which is located at one end of the laser cavity near the laser mirror. When a saturable absorber is used to mode lock a laser, then the laser is simultaneously Q-switched. The result is the production of a periodic train of mode-locked pulses, a series of narrow (10 ps), mode-locked pulses contained within an envelope which may be several hundred nanosecond long. The peak power within the individual pulses may be enormous because of their very short duration.

Problems chapter (5) : ~~(~~High Power Techniques)

5-1 Compute the pulse width Δt_p and the separation between pulses Δt_{sep} for the mode-locked Nd: YAG laser where the fluorescent linewidth is 1.1×10^{11} Hz and the laser rod of refractive index (1.82 for YAG) is 0.1 m long. Assume that the laser mirrors are very close to the ends of the rod.

Solution:

The maximum number of longitudinal modes for the Nd: YAG laser with fluorescent linewidth (spectral width) 1.1×10^{11} Hz is,

$$m = \frac{2\nu_m \eta L}{c} = \frac{2 \times 1.1 \times 10^{11}\,\text{Hz} \times 1.82 \times 0.1\,\text{m}}{3 \times 10^{8}\,\text{ms}^{-1}} \simeq 133 \quad \text{modes}$$

The pulse width Δt_p is.

$$\Delta t_p = \frac{2\eta L}{mc} = \frac{2 \times 1.82 \times 0.1\,\text{m}}{133 \times 3 \times 10^{8}\,\text{ms}^{-1}} \simeq 10\ \text{ps}$$

The pulses separation is,

$$\Delta t_{sep} = \frac{2\eta L}{c} = \frac{2 \times 1.82 \times 0.1\,\text{m}}{3 \times 10^{8}\,\text{ms}^{-1}} \simeq 1.3\ \text{ns}$$

5.2 Compute the mode-locked pulse width Δt_p and the separation between pulses Δt_{sep} for a HeNe laser operating at 632.8 nm with a mirror cavity spacing of $L = 0.5$ m. Assume that the HeNe laser longitudinal modes will lase over the FWHM emission linewidth of 1.5×10^{9} Hz, and refractive index for HeNe gas, $\eta \cong 1$.

Answer: ($\Delta t_p = 6.67 \times 10^{-10}$ s , $\Delta t_{sep} = 3.33 \times 10^{-9}$ s).

Chapter (6) : Type of Lasers

At the heart of each laser is an active medium which exhibits optical gain over a narrow range of wavelengths, and in fact the name given to a laser is invariably that of the active medium employed. Lasers are divided into many types depending on the physical state of the active medium (solid, liquid, and gas) employed. Lasers often have a number of features in common such as general energy-level schemes and pumping mechanisms.

6.1 Solid state lasers.

6.1.1 The Ruby Laser (Cr^{3+} : Al_2O_3)

The ruby laser is of historical interest since it was the first successful laser to be made. The active medium is aluminum oxide (Al_2O_3) with about 0.05% by weight of chromium as an impurity. The active ions are chromium, Cr^{3+}, which replace aluminum ions in the lattice. Fig. (6.1) shows the three-energy level system of the ruby laser. Optical pumping is due to the excitation of Cr^{3+} ions to the broad upper energy levels (denoted $^4F_1$ and $^4F_2$ in spectroscopic notation). These excited states give up a part of their energy to the thermal oscillations of the ruby crystal lattice and fall comparatively fast from the states $^4F_1$ and $^4F_2$ onto metastable (having relatively long lifetimes) levels denoted as 2A and E. These latter levels are the upper laser level and lasing transitions are from these levels to the laser lower (ground) level $^4A_2$.

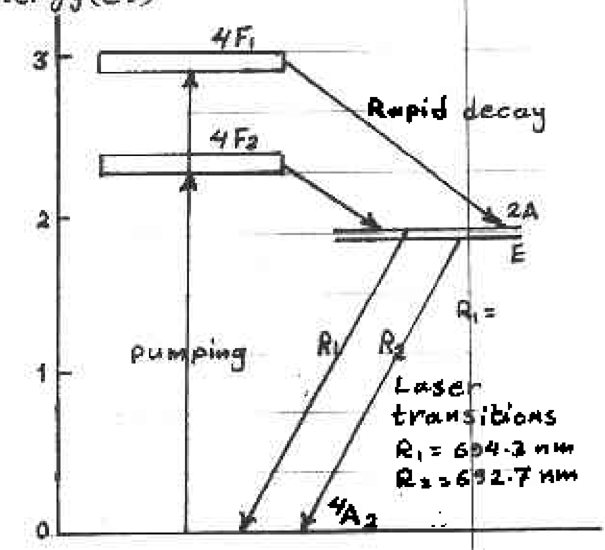


Fig. (6.1) : The three-level system of the ruby laser.

Optical pumping for ruby laser as pulsed operation can be obtained by pumping with a flashtube. A high-pressure mercy arc lamp is often used, whose output matches the ruby absorption bands.

6.1.2-The Nd:YAG Laser

The neodymimum (Nd^{3+}) doped yattrium aluminium garnet (YAG) laser is the most popular type of solid state laser. The Nd^{3+} ions replace yttrium ions in the lattice, with a maximum possible doping level of about 1.5%. Fig.(6.2) shows the energy-levels involved in laser action.

The Four-energy level system with the lasing transition taking place between the upper energy level $^4F_{3/2}$ and the lower energy level $^4I_{11/2}$, (The lower energy level $^4I_{11/2}$ is suifficiently far above the ground energy level $^4I_{9/4}$ to be practically empty at room temperature). The initial and final energy levels are split into 2 and 6 crystal field levels respectively, so that several lasing wavelengths are possible. The most powerful of these (60% of the emitted energy) occurs at 1.064 μm, and this is usually the one used.

Since it is four-energy level system the optical pumping requirements are modest and for pulsed operation can be met with a fairly simple flashlamp and reflecting cavity.

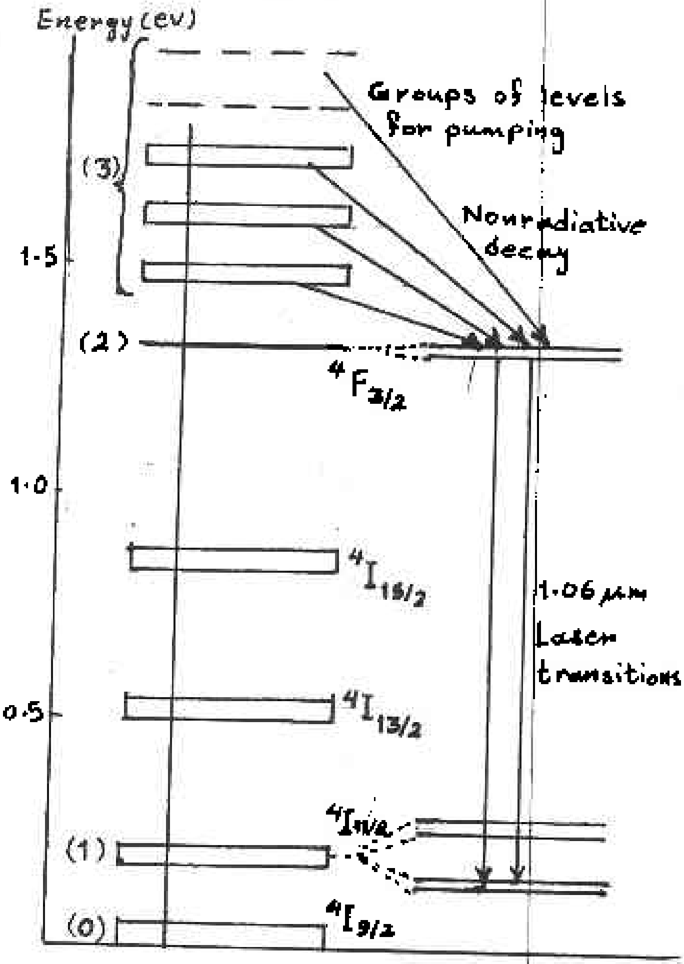


Fig.(6.2) Energy-level of Nd:YAG laser.

The total energy output during a single flashtube pulse can range from 0.01 J to 100 J with pulse repetition frequencies up to 300 Hz. The average power during a pulse can be quite large, thus a 10 J pulse lasting for some 0.5 ms implies an average power of some 2×10^4 w. It is possible, to compress the total energy into a single spike by the Q-switching technique, power of up to 10^8 W may then be achieved. Continuous wave (CW) operation is also possible using quartz-halogen lamp as a pumping source.

6.2- Gas Lasers.

These lasers operate with rarefied gases as their active media (at pressure of 10 to 10 torr) excited by an electric discharge.

6.2.1- The HeNe laser

The HeNe laser is one of the commonest of all types of laser. The active medium is a mixture of helium and neon in the ratio of about ten parts of He to one part of Ne. A schematic HeNe atomic laser is shown in Fig.(6.3). The gas, at fairly low pressure, is contained within a glass discharge tube with anode and cathode at either end. If the cavity mirrors are outside

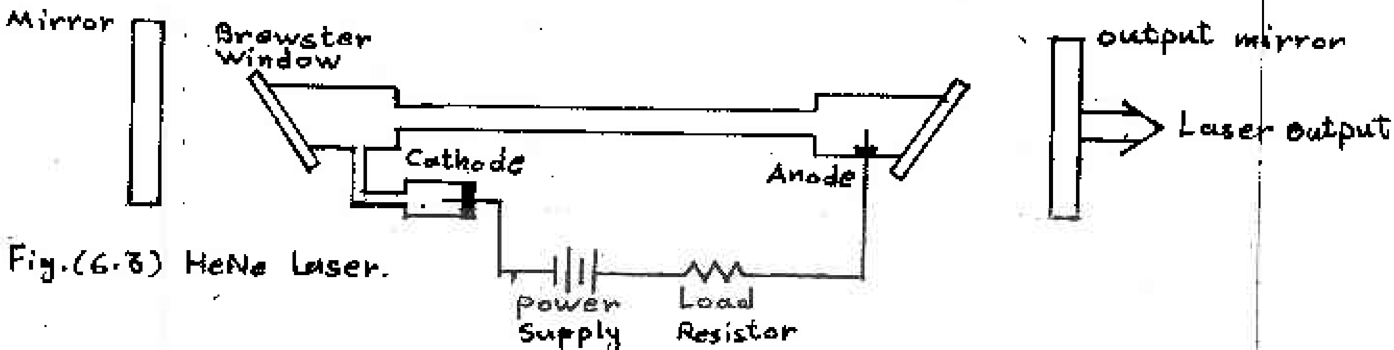


Fig.(6.3) HeNe Laser.

the tube, then Brewster windows may be used at the ends of the tube to minimize reflection losses. Voltage of a few kilovolts applied across the tube then initiate a gas discharge. The free electron in the discharge collide with the atoms and pump them to an excited state.

The HeNe laser is a four-energy level system where the lasing transitions take place between the energy levels of Ne. Several different transition are possible starting from two groups of levels 3S and 2S as shown in Fig.(6.4). Direct electron transitions to these levels are not efficient and an intermediary must be used. The energy levels of the excited He atoms are quite close in energy to the upper lasing energy level of Ne. The excited He atoms collide with the Ne and give up their energy in the process. The laser transition at 632.8 nm is the common used one.

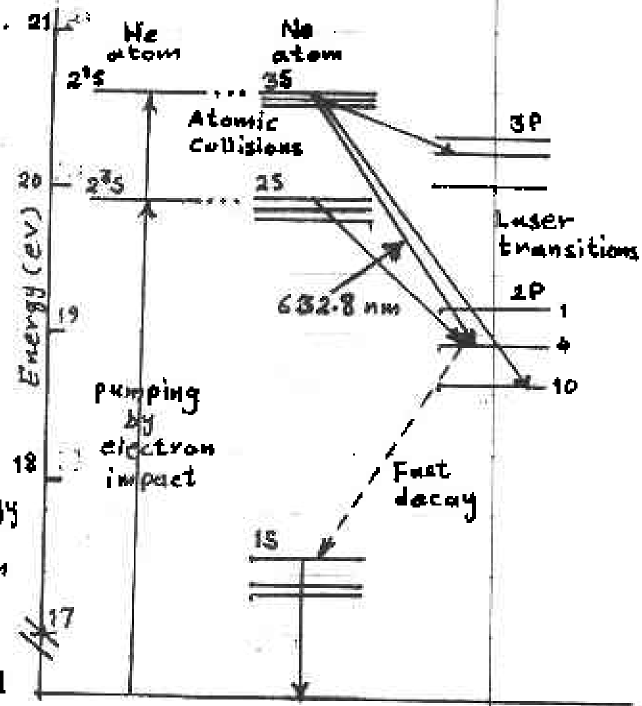


Fig.(6.4) Energy levels of HeNe Laser.

6.2.2- The Carbon Dioxide Laser

The carbon dioxide (CO_2) laser is the most important laser of its class and terms of technological applications. It exhibits both high efficency (up to 40%) and high power output (as 10 kW).

The lasing transitions involved are resulting from the quantization of the vibrational and rotational energy of the CO_2 molecule. There are three modes of vibrational oscillation which are designated as symmetric, bending, and asymmetric, as shown in Fig.(6.5). Eeach of these vibration is quantized.

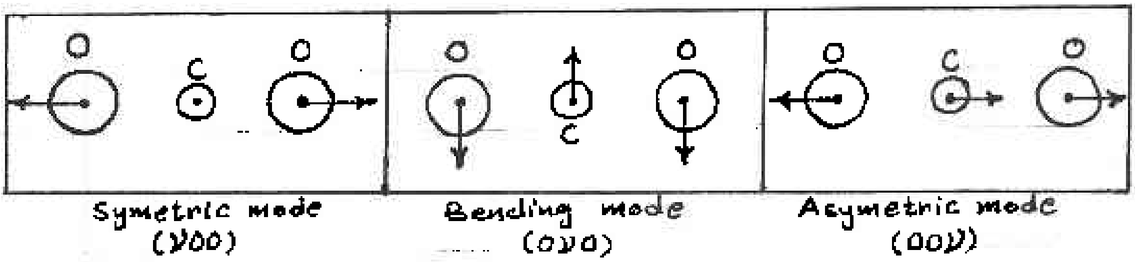


Fig.(6-5)

As well as vibrating, the whole molecule can rotate about its center of mass. The fundamental rotational energy quanta are, however, much smaller than the vibrational quanta and as a result the vibrational energy levels are split into a number of closely spaced rotational sublevels. Fig.(6.6) shows the energy-levels of CO_2.

In the CO_2 Lase, excitation is a two-step process, with N_2 involved as the intermediary. The first vibrational levels of N_2 are close to the (001) vibrational levels of CO_2. The latter from the initial levels of a large number of laser transitions with the strongest at 10.6 μm.

Low overall pressures (about 10 torr) are generally used for CW operation but short, high-energy pulse lasers operate at higher pressures.

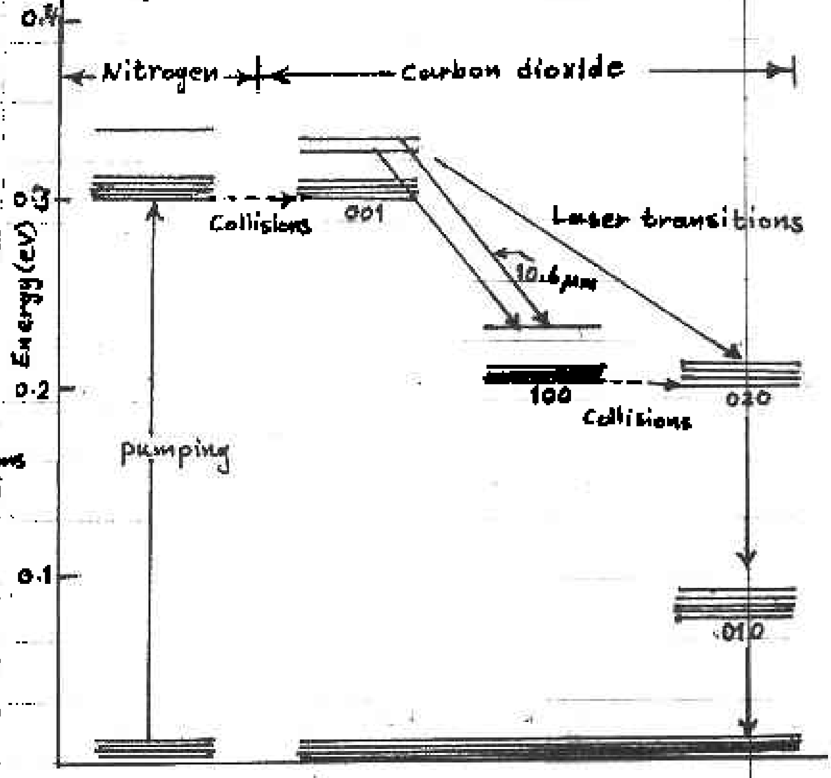
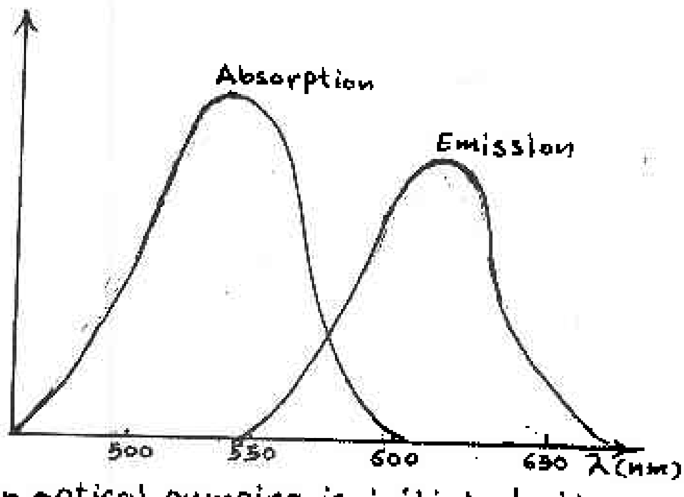


Fig.(6.6) Energy-levels of CO_2 laser

6.3 - Liquid Dye Lasers

Dye lasers, often referred to as organic dye lasers, are produced in liquid gain media, The dyes have very broad emission and gain spectrums that lead to tunable laser output and short-pulse (mode-locked) laser output. The laser gain medium consists of strongly absorbing and emitting organic dyes dissolved in a liquid solvent. A typical dye laser can operate over a wavelength range of 30-40 nm. The solutions of organic dyes are strongly fluorescent; that is they absorb radiation over a certain band of wavelengths and emit over another band situated at somewhat longer wavelengths. The energy difference between absorbed and emitted photons ultimately appears as heat. A typical example is rhodamine 6G in ethanol, whose absorption and emission spectra are shown in Fig.(6.7).

The energy-level diagram for a typical dye laser is shown in Fig.(6.8), which indicates the singlet series of levels S_0 and S_1 and the triplet series of T_1 and T_2. Since S_1 is a very broad energy level, the range of absorbing wavelengths (the absorption spectrum) that can pump population into that state S_1 (the upper laser level) is also very broad.



Fig(6.7)

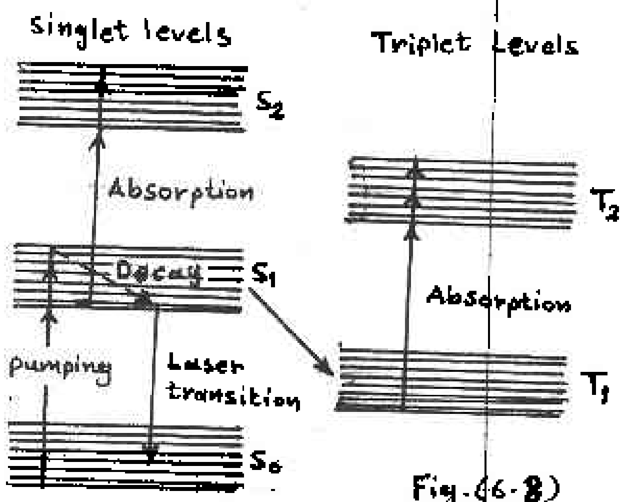


Fig.(6.8)

After optical pumping is initiated, the population within the excited singlet state S_1 rapidly relaxes within 10^{-12} to 10^{-13} second, to the lowest levels of S_1, Lasing action can take place between these levels and those near the top of S_0. The S_1 to S_0 transition is often very rapid (~ 10 ns), necessitating intense pumping to achieve population inversion. Since the transitions are between bands of energy levels a range of lasing transitions is possible.

6.4 Semiconductor Lasers

Semiconductor laser devices are small and efficient with typical dimensions of less than a millimeter. They operate on wavelengths ranging from 0.6 to 1.55 μm, depending upon the materials of the laser medium. Certain semiconductor materials are capable of producing population inversion when connected to an energy source. The principle of operation of a semiconductor laser like GaAs p-n junction is shown in Fig (6.9), where the semiconductor valance band, V, and conduction band, C, separated by the direct energy gap, E_g. If the semiconductor is at T = 0K, then the valance band will be completely empty. Excitation of electrons is provided when a forward-bias voltage of a few volts is applied across the semiconductor layers. This means that there is then a population inversion between the valance and conduction bands. The electrons in the conduction band fall back into the valance band and recombine with holes, emitting a photon at the junction in a process called recombination radiation. The process of stimulated emission of radiation will produce laser oscillation when the semiconductor is placed in a suitable resonator and the appropriate threshold conditions are fulfilled. Optical resonator can be done by polishing the end faces perpendicular to the plane of the p-n junction.

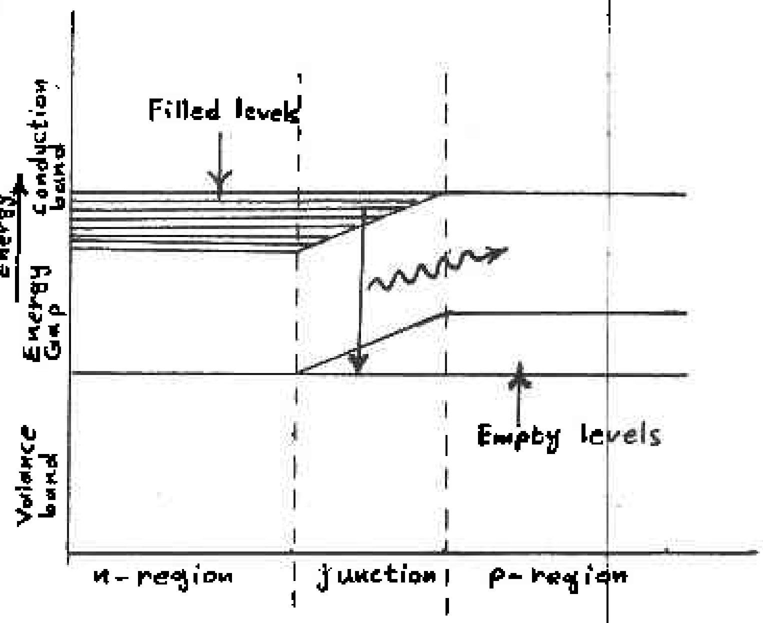


Fig. (6.9)

GaAs lasers are efficient sources of coherent radiation and are noted for their small size. A typical GaAs laser at room tempearture has an operating wavelength of 0.905 μm, produces 6 W of peak radiant power and requires an applied voltage of about 1.5 V, with efficiency of about 10%.