FORCE, STRESS, DEFORMATION AND STRAIN

Force (F) is a vector quantity. Force can cause a body in a state of rest to accelerate, and can cause a moving body to accelerate or decelerate or change direction.

F = ma (force = mass × acceleration) ………. Newton's Second Law of Motion.

►There are **two** different **types** of forces.

1- **Body forces**: Forces that result from action of a field at every point within the body are called **body forces.**

2-**Surface force**: Forces that act on a specific surface area in a body are called **surface force.**

Stress represented by the symbol **σ** (sigma), is defined as the force per unit area [*A*], **or**

$\sigma = F/A$

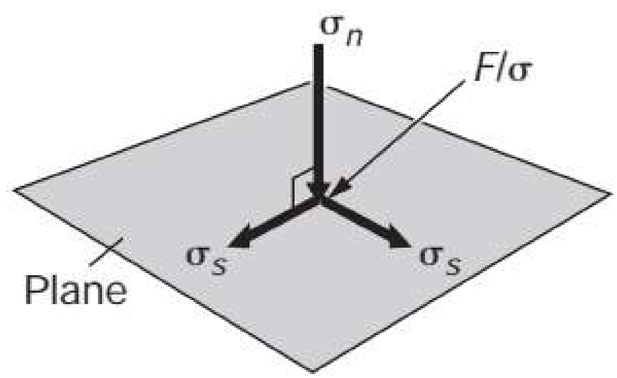
Two –Dimensional stress: Normal stress and Shear stress

Stress acting on a plane **is a vector quantity**, it has **magnitude** and **direction**.

Stress on a plane it can be resolved into:

(i) The **vector component** normal to the plane is called the **normal stress**, (**σn**) (*sometimes just the symbol **σ** is used*);

(ii) **(ii)** the **vector component along** the plane is the **shear stress (σs)** (*sometimes the symbol **τ** (tau) is used*)

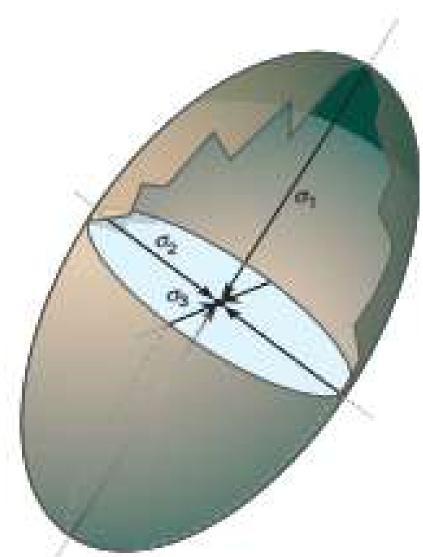


Three –Dimensional stress: Principal planes and Principal stresses

Principal stress: The **normal stress** on a plane with zero shear stress; **three principal stresses** exist, **σ1 ≥ σ2 ≥ σ3**. These principal stresses have two properties: (1) they are *orthogonal* to each other, and (2) they are *perpendicular* to three planes; these planes are called the **principal planes of stress**.

The **σ1, σ2, and σ3** are the **major, intermediate, and minor principal stress**, respectively.

Stress ellipsoid: imaginary geometric representation of stress; the axes of the stress ellipsoid are the principal stresses.



Isotropic stress All three principal stresses have equal magnitude (describes a sphere).

Anisotropic stress At least one principal stress has a magnitude unequal to the other principal stresses (describes an ellipsoid).

Applying aforementioned procedure for the faces normal to x, **y** and **z,** we obtain a total of nine stress components.

In the direction of	**x:**	**y:**	**z:**
stress on the face normal to **x:**	**σxx**	**σxy**	**σxz**
stress on the face normal to **y:**	**σyx**	**σyy**	**σyz**
stress on the face normal to **z:**	**σzx**	**σzy**	**σzz**

*σ*xx, *σ*yy, **and** *σ*zz **are normal stress components and the other six are shear stress components** (*σ***xy** and *σ***yx**, *σ***yz** and *σ***zy**, and *σ***xz** and *σ***zx**). If these components were unequal, the body would move.

The nine components of stress put into a matrix (second-order tensor) known as the **stress tensor** or **stress matrix.**

$$\begin{vmatrix} \sigma 11 & \sigma 12 & \sigma 13 \\ \sigma 21 & \sigma 22 & \sigma 23 \\ \sigma 31 & \sigma 32 & \sigma 33 \end{vmatrix}$$

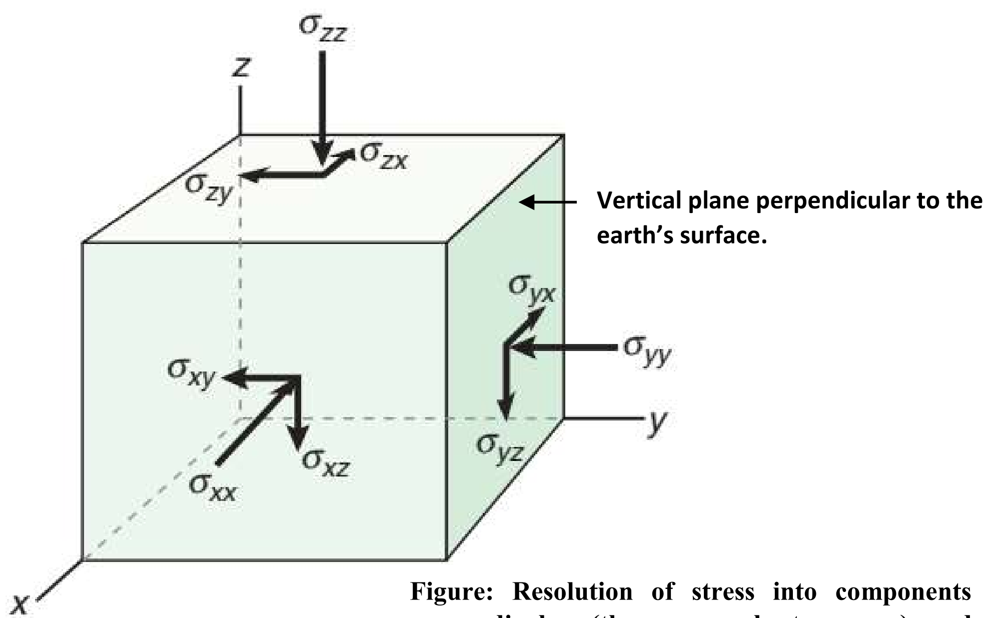


Figure: Resolution of stress into components perpendicular (three normal stress, σn) and components parallel (six shear stresses, σs) to the three faces of a small cube, relative to the reference system x, y, and z.

The normal stresses **σ**11, **σ**22 and **σ**33 occupy the diagonal while the off-diagonal terms represent the shear stresses.

The **σm** is called the **mean stress**; **σm**= (σ11+σ22+σ33)/3.

► Differential stress (σd) = (σ1 - σ3).

►Static stress as σ1 = σ2 = σ3.

►Confining pressure as σ2 = σ3 for the conditions σ1 > σ2 = σ3.

Deformation

Deformation describes the collective displacements of points in a body; in other words, it describes the complete transformation from the initial to the final geometry of a body. This change can include a translation (movement from one place to the other), a rotation (spin around an axis), and a distortion (change in shape).

Three stages of deformation

If an object is subjected to stresses it passes through three stages of deformation.

1-Elastic deformation; if the stress is withdrawn, the *body returns* to its *original shape and size*. Or **Elastic Deformation** Mechanisms by which elastic behavior occurs, namely the bending and stretching, without breaking, of chemical bonds holding atoms or molecules together.

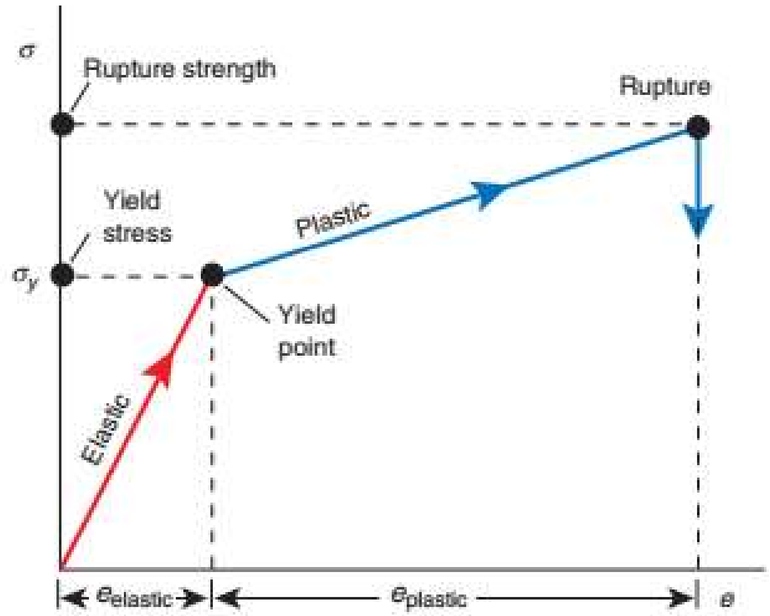


Figure 6.9 Stress–strain curve for elastic–plastic deformation.

There is always a limiting stress, called the ***elastic limit (yield point)***; if this is exceeded, the object does not return to its original shape. Below the elastic limit, the deformation obeys ***Hooke's law*** which states that strain is proportional to stress.

2- Plastic deformation; if the stress exceeds the elastic limit, the deformation is plastic; that is, the specimen only *partially* returns to its *original shape* even if the stress is removed. Or **Plasticity Deformation** mechanism that involves progressive breaking of atomic bonds without the material losing coherency.

3- Rupture; When there is continued increase in the stress, one or more fractures develop, and the specimen eventually fails by rupture (fracture). Or **Fracturing** deformation mechanism by which a rock body or mineral loses coherency by simultaneously breaking many atomic bonds.

Strength: Stress that a material can support before failure.

Yield stress: Stress at which permanent strain occurs.

*Based on the aforementioned relation **two main types of rocks**/substances are distinguish.*

I/Brittle substances/rocks are those that rupture before any significant plastic deformation takes place.

II/Ductile substances/rocks are those that undergo a large plastic deformation before rapture. After the elastic limit has been exceeded, ductile/plastic substances undergo a long interval of plastic deformation, and in some instances they may never rupture.

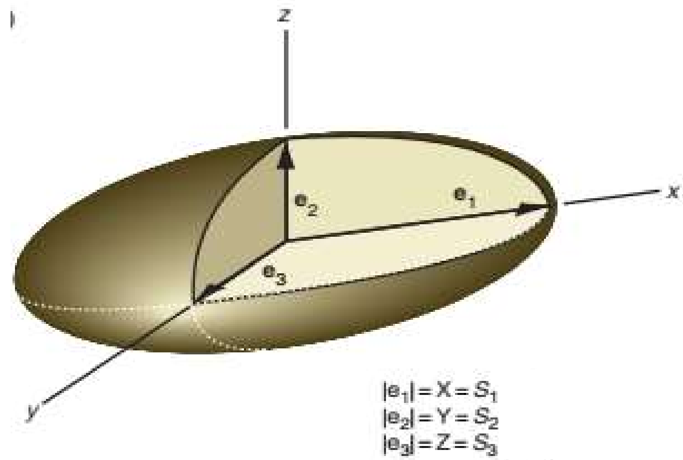
Rheology is the study of the mechanical properties of solid materials as well as fluids and gases. The name derives from the Greek word "rheo", which means "to flow". Rheology deals with the flow of rocks, while **rock mechanics** primarily deals with the way rocks respond to stress by brittle faulting and fracturing.

Strain

Strain describes the changes of points in a body relative to each other; so, it describes the distortion of a body.

The strain ellipsoid

The strain ellipsoid is the deformed shape of an imaginary sphere with unit radius that is deformed along with the rock volume. The strain ellipsoid has three mutually orthogonal planes of symmetry, the principal planes of strain, which intersect along three orthogonal axes that are referred to as the principal strain axes. Their lengths (values) are called the principal stretches. These axes are commonly designated X, Y and Z, but the designations **S1, S2** and **S3** as well as **e1, e2** and **e3** are also used.



Factors controlling mechanical behavior of materials

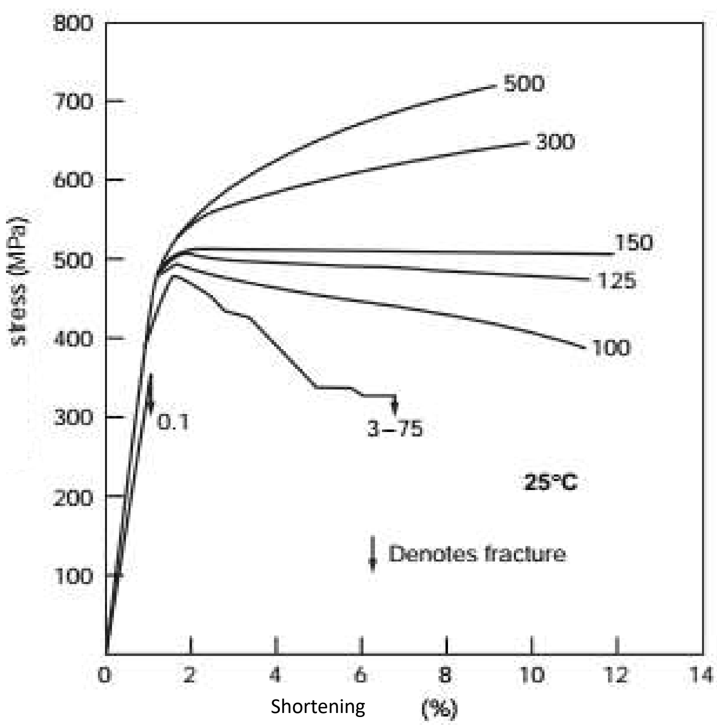
The mechanical response of rocks to stress is different for different conditions .

a) Confining pressure

Figure illustrate the behaviour of Solenhofen Limestone .Seven separate experiments are shown at confining pressure of 0.1,3-75,100,125,150,300 and 500 MPa.

- increase in confining pressure increases the elastic limit and the ultimate strength.

- These experiments indicate that rocks exhibiting very little plastic deformation near the surface of the earth may be very plastic under high confining pressure.



b) Temperature

Figure 2.4 shows six tests run on Yule Marble.The uppermost curve is that obtained at 300°C temperature,whereas lowest curve is that obtained at tempreture of 800 °C .

- It is apparent that **plastic deformation is less common near the surface of the earth**, where the confining pressure and tempreture are low,than it is at greater depths,where higher temperatures and greater confining pressure increase possibility of plastic deformation.

- An increase in temperature lowers the yield stress or weakens the rock.

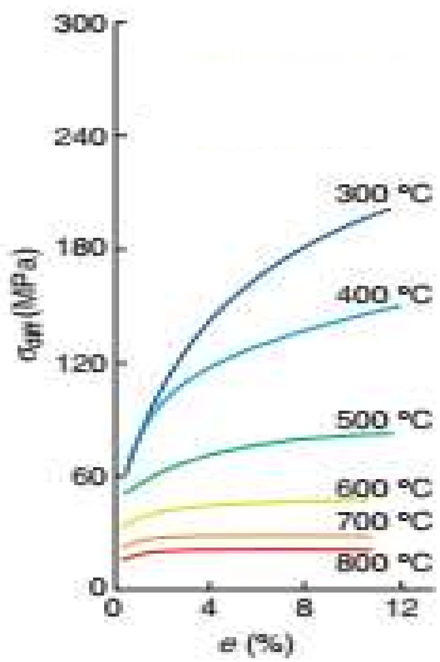


Figure 2.4 Stress-shortening (strain) curves for Yule marble.

c) Time

An analysis of the effects of time is concerned with such subjects as creep and strain rate.

Creep is the plastic deformation of a material that is subjected to a persistent and constant stress.Or Creep refers to the slow continuous deformation with the passage of time .

i) ***In a short-time experiment***, Solenhofen Limestone under atmosheric pressure and room temperature has a strength of 2560 kg/cm^2 .

ii) In a long-time experiment,Solenhofen Limestone subjected to a commpressive stress of 1400 kg/cm2-half the value of the strength**(i).**

At the *end of one day*,it has been shortened about *0.0006 %*; after **10 days** about **0.011 %**; after *100 days* about *0.016 %*;and after ***400 days*** a little more than **0.019 %**.

Result:After long continued stress the rocks become much weaker.

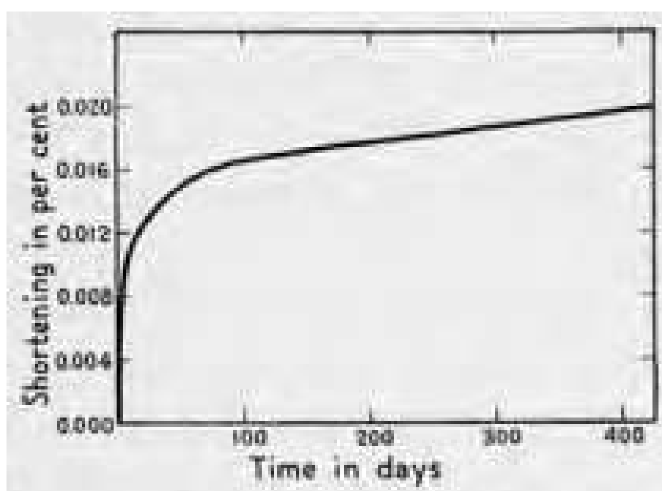


FIG. 10—Creep Curve for Solenhofen Limestone Under a Stress of 1400 kg./cm.^{2} (After D. T. Griggs.)

As soon as the load is applied, the rock experiences an elastic deformation. This is followed by three kinds of mechanical response, called primary, secondary, and tertiary creep.

Result: **The initial elastic strain and primary creep** are due to initial loading and **are not particularly time-dependent**. The **amount of strain accommodated by the rock during secondary and tertiary creep** is **a function of the time**.

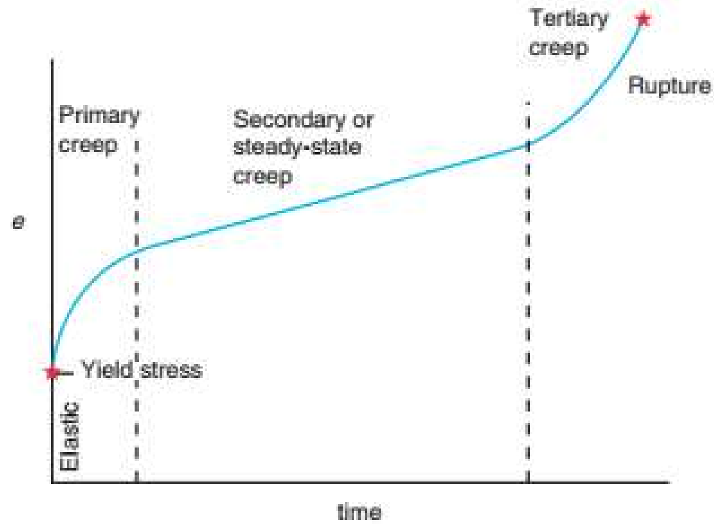


Figure 6.13 Strain–time curve for creep experiment. After initial elastic deformation, three types of creep can be defined. See text for discussion.

d) Solution

The solutions dissolve old minerals and precipitate new ones.Under such conditions the mechanical properties of rock are greatly modified .

Experiments have been performed on alabaster-gypsum with solution present.In all cases compressive stress was 205 kg/cm^2 (less than half the normal elastic limit of 480 kg/cm^2) ,and tempreture 24°C.

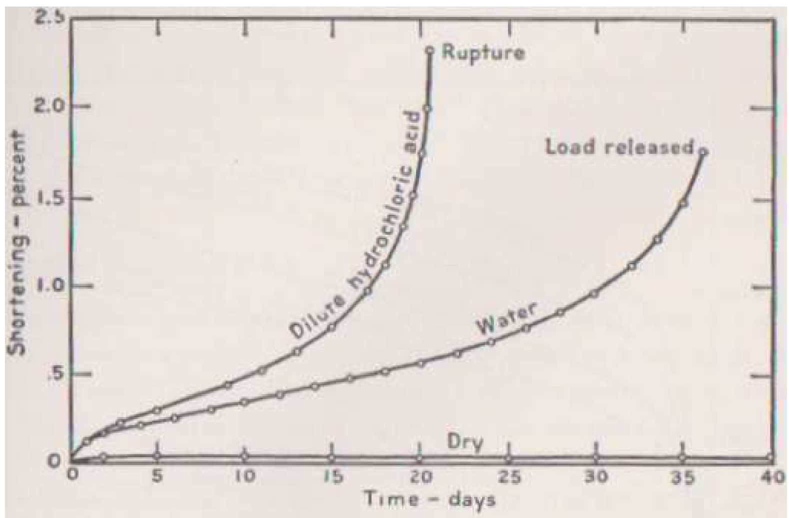


Fig. 14. Effect of solutions on deformation of alabaster. (After D. T. Griggs.)

Results: *Lowest curve* : a *dry specimen* within a few days the specimen *had shortened about 0.03%* , but there was no further shortening even after 40 days.

Intermediate curve (water had access to the alabaster)had shortened 1 % at the end of 30 days and 1.75 percent by the end of 36 days,when the load was released.

Highest curve(HCl had access to the alabaster) shortened 2% before rupturing at the end of 20 days.

- Using of different solution in the alabaster rock gives different shortening*(deformation;strength)* values under similar conditions.

- **Reacting solutions lower the strength of the rocks**.

e) Pore pressure

The pore pressure weakens the rock. Normally the strength of a rock increase at depth because of the increase in confining pressure. But with increasing pore pressure the effective confining pressure decreases. Moreover, with increasing pore pressure the rocks are less coherent.

f) Anisotropy and inhomogeniety

Isoropic material is rocks whose mechanical properties were uniform in *all directions*. Rocks that show bedding,banding, or foliation are not isotropic.

The **strength** of such rocks would depend upon the **orientation** of the applied **forces** to the **planar structures of rock**. Yule marble specimens under confining pressure 10,000kg/cm^2 and room temperature show great plastic deformation.The **solid lines** represent **compression**(*compressive stress*),in which *strain is shortening parallel to the axis of the cylinder* .Under *compression* the *cylinder perpendicular to the foliation is stronger than the cylinder parallel to the foliation*.The **broken lines** represent tests under **tension** (*tensile stress*) ,in which *strain is lengthening parallel to the cylinders*.Under tension the cylinder parallel to the foliation is much stronger than the cylinder perpendicular to foliation.

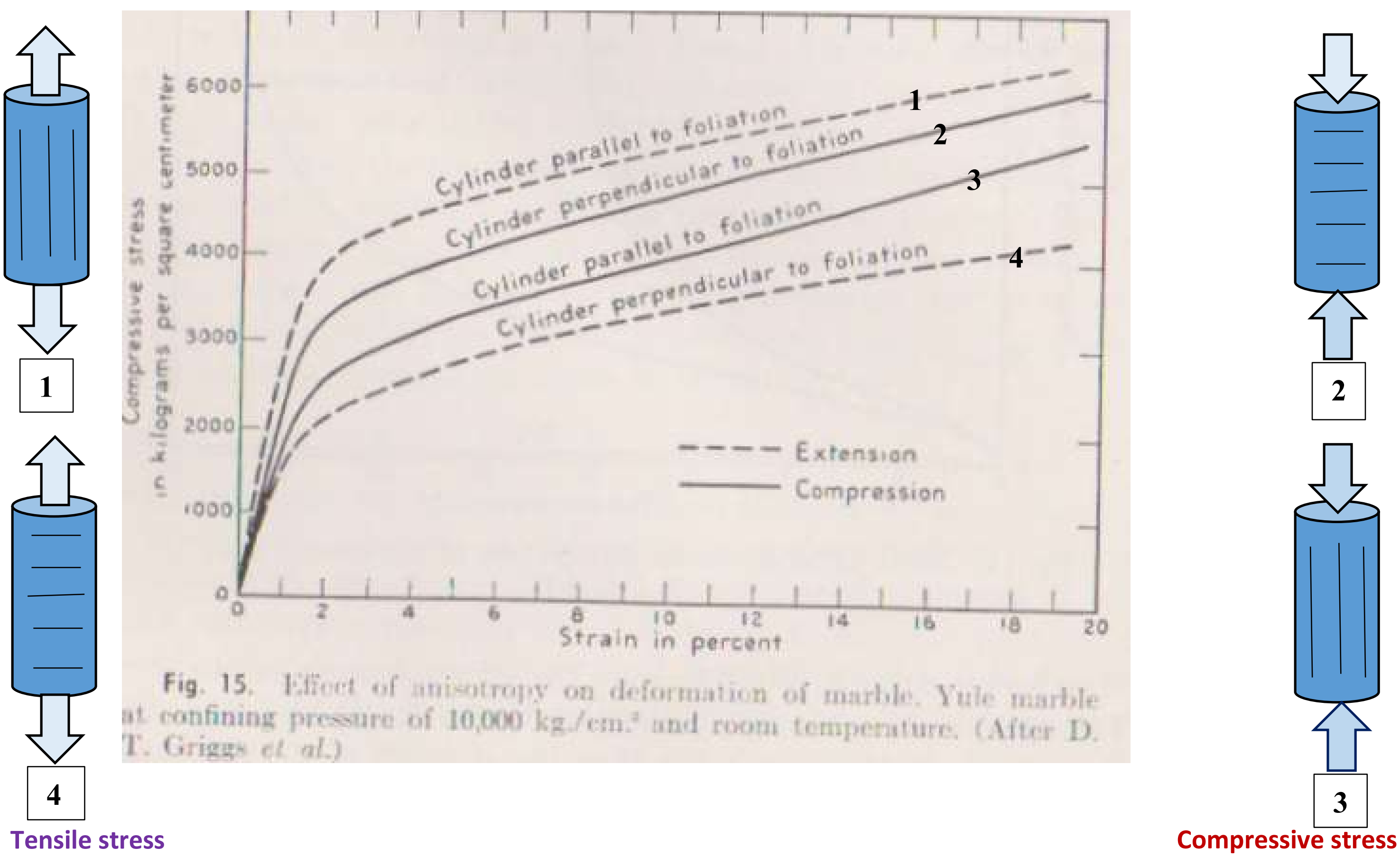


Fig. 15. Effect of anisotropy on deformation of marble. Yule marble at confining pressure of 10,000 kg./cm.^{2} and room temperature. (After D. T. Griggs *et al.*)

Q.1/Compare between (a) Coaxial and Non-coxial strain;(b) Homogeneous and Heterogeneous strain.