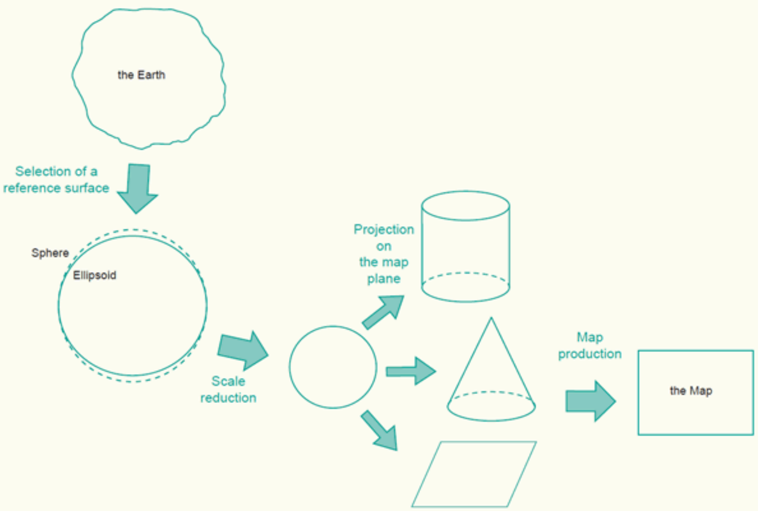
1. The process of representing the shape on a map.

Earth shape: The Earth is specified to have an irregular shape due to containing plains, mountains and valleys. Therefore, it is difficult to compute a mathematical model to represent its surface, and it is more suitable to use a **shape close to sea level.**

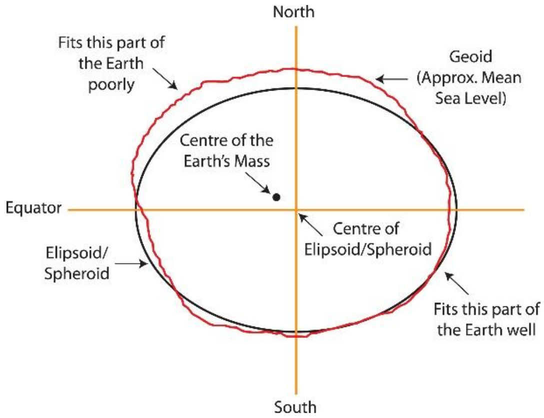


a) Reference surfaces

Two main reference surfaces (or Earth figures) are used to approximate the shape of the Earth:

- Geoid (for the height), and,
- Ellipsoid (for horizontal coordinates).

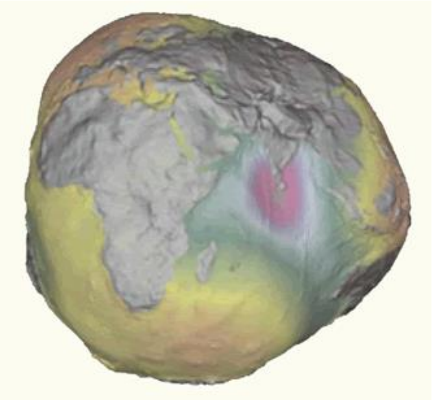
a.1 The Geoid, is the true shape of the Earth. It is specified as an Equipotential surface at mean sea level and is used for measuring heights represented on maps. The starting point for measuring these heights are mean sea level points established at the coastal place. These points represent an approximation to the Geoid.



- The most common reason for needing to know the form of the Geoid is when transforming between different heights systems.

- The shape of the Earth is given by the form of a surface that is everywhere perpendicular to the direction of gravity (such a figure is termed an equipotential surface). The force and direction of gravity are affected by irregularities in the density of the Earth's crust and mantle. It, therefore, follows the form of an equipotential surface.

- In many situations, it is necessary to know the Geoid's form, either globally or for a specific region. This can be derived (with a great deal of effort) from gravity observations, observations of satellite orbits, satellite altimetry over the oceans, and other data sources.



a.2 The Ellipsoid or Spherical,

Because of the irregularities caused by local changes in density, the Geoid remains a difficult surface to compute across.

A reference ellipsoid (or an ellipsoid) is a mathematically defined surface that approximates the Geoid, the more accurate figure of the Earth, or other planetary body.

- The Geoid is approximately spherical, with a radius of about 6370 km. For many low-accuracy or small-scale mapping applications, a sphere is a satisfactory model of the Earth.

- A rather better approximation of the shape of the Earth is an *ellipsoid of revolution*, often more conveniently called *an ellipsoid.* This surface is formed by an ellipse that has been rotated about its shortest (the minor) axis or by 'squashing' a sphere at the poles. This *flattening* of the poles is slight, about 22 km .

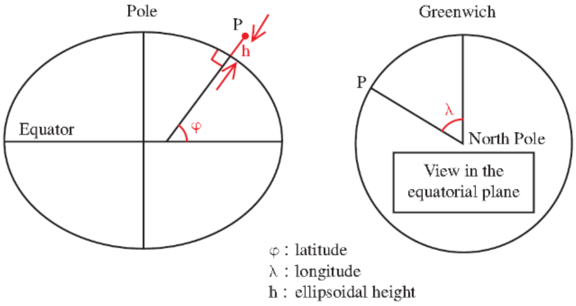
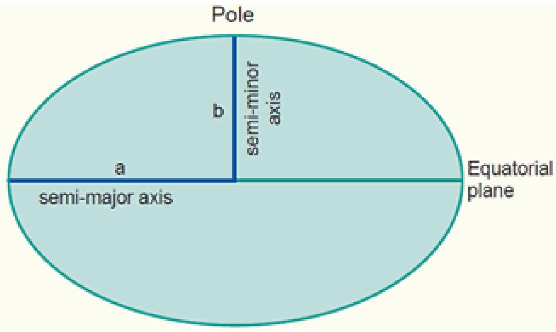


Figure 2.5 Ellipsoidal coordinate system.

a.3 <u>Ellipsoid specifications:</u>

- Providing a relatively simple mathematical figure of the Earth.
- It is used to measure locations, the latitude (ϕ) and longitude (λ), of points of interest.
- These locations on the ellipsoid are then projected onto a mapping plane.

There are many different ellipsoids defined in the world. Some well-known is the WGS84, GRS80, Bessel, or the Clarke 1880 ellipsoid.



A cross-section of an ellipsoid is used to represent the Earth's surface, defined by its semi-major axis a and semi-minor axis b.

Where: "a" is the semi-major axis, "b" the semi-minor axis. From these two parameters, it is possible to derive further parameters. Thus:

Flattening, "f", is defined as $f = \frac{a-b}{a}$

But often expressed as inverse flattening, 1/f.

Eccentricity, *e*, is defined as $e^2 = \frac{a^2-b^2}{a^2}$

Furthermore, e, f, and b can be related to each other as:

$$e^2 = 2f - f^2$$

$$\sqrt{1-e^2} = (1-f) = \frac{b}{a}$$

Thus, an ellipsoid can be completely defined using the two parameters (a and b) or (a and f) or (a and e) and the remaining parameters can be found as necessary.

Many hundred estimations of the best-fitting ellipsoidal model of the Earth have been made over the past two centuries. The below table shows the ellipsoids parameter that is used in modern surveying.

Example:

Using the defining parameters, what are the first eccentricities of the international 1924 ellipsoid (using semi-axis a and b) and GRS1980 ellipsoid (using f)?

Solution

From the above table, For the Clarke 1866 ellipsoid, using the above equation, the value will be

$$e^2 = \frac{a^2 - b^2}{a^2} = \frac{6378388^2 - 6356912^2}{6378388^2} = 0.0067226532$$

⇨ $e = 0.0819917874$

For the GRS80 ellipsoid, using the above equation

$$e^2 = 2\left(\frac{1}{298.257}\right) - \left(\frac{1}{298.257}\right)^2 = 0.006694385$$

⇨ $e = 0.0818192215$

a.4 Ellipsoid's Parameters

Ellipsoid name	Semi-major axis (a)	Flattening (f)	Comment
GRS 1980	6 378 137 m	1 / 298.257222101	The international standard.
International 1924	6 378 388 m	1 / 297.0	A former international standard.
GRS 1980 Authalic Sphere	6 371 007 m	0	An authalic sphere is one with a surface area equal to the surface area of the ellipsoid.
WGS 84	6 378 137 m	1 / 298.257223563	Used by the GPS satellite navigation system.

The difference between GRS 1980 and WGS 84 is It can be noticed the difference is insignificant and can be considered equivalent.

$$\Delta f = f_{GRS80} - f_{WGS84} = 16 \times 10^{-12}$$

Since the horizontal coordinates will be projected onto a mapping plane, the reference surface for horizontal coordinates requires a mathematical definition and description, which is called **Map projection.**

a.5 Types of ellipsoids

1.1. Local ellipsoids

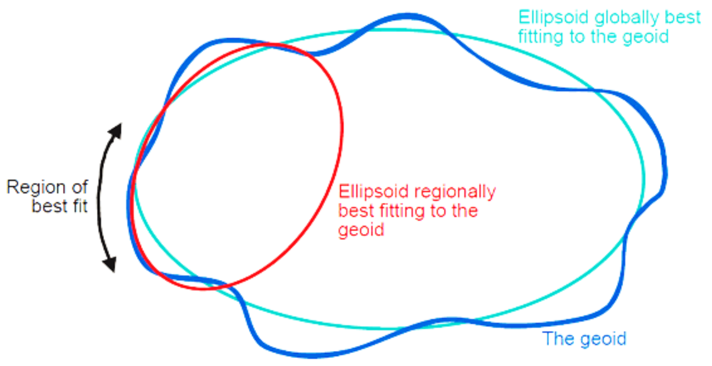
Local ellipsoids have been established to fit the Geoid (mean sea level) well over an area of local interest, which in the past was never larger than a continent.

Many different ellipsoids have been defined in the world.

This meant that the differences between the Geoid and the reference ellipsoid could effectively be ignored, allowing accurate maps to be drawn in the vicinity of the datum.

1.2. Global ellipsoids

With increasing demands for global surveying, work is proceeding to develop ***global reference ellipsoids***.
Global ellipsoids approximate the Geoid as a mean Earth ellipsoid. In contrast to local ellipsoids, which apply only to a specific country or localized area of the Earth's surface.



1.3. Defined Global ellipsoids

Different global ellipsoids have been defined. The International Union for Geodesy and Geophysics (IUGG) plays a central role in establishing these reference figures.

- In 1924, the general assembly of the IUG in Madrid introduced the ellipsoid determined by Hayford in 1909 as the ***International ellipsoid***
- At the general assembly 1967 of the IUGG in Luzern, the 1924 reference system was replaced by the ***Geodetic Reference System 1967*** (GRS 1967).
- The Geodetic Reference System 1967 was used in the planning of new geodetic surveys.
- Later, the above, it was replaced by the ***Geodetic Reference System 1980*** (GRS80) ellipsoid.
- For all practical purposes, the GRS80 and WGS84 can be considered identical

Name	a(m)	b(m)	f
International (1924)	6378388.	6356912.	1 : 297.000
GRS 1967	6378160.	6356775.	1 : 298.247
GRS 1980 and WGS84	6378137.	6356752	1 : 298.257

1.4. The local horizontal datum

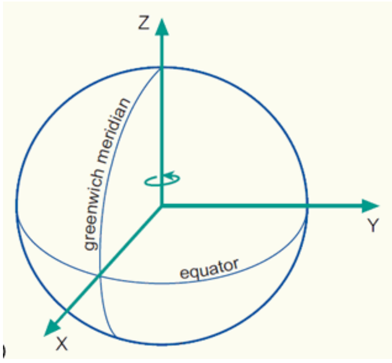
Generally, ellipsoids have varying position and orientations.
The Geoid is used to position and orient the ellipsoid by adopting a latitude (ϕ) and longitude (λ) and ellipsoidal height (h) of a so-called fundamental point and an azimuth to an additional point.
N.B.: the term ***horizontal datum*** and ***geodetic datum*** are being treated as equivalent and interchangeable words.

Datum	Ellipsoid	Datum shift (m)* (Dx,	Dy,	Dz)
Alaska (NAD-27)	Clarke 1866	-5	135	172
Bahamas (NAD-27)	Clarke 1866	-4	154	178
Bermuda 1957	Clarke 1866	-73	213	296
Central America (NAD-27)	Clarke 1866	0	125	194
Bellevue (IGN)	Hayford	-127	-769	472
CampoInchauspe	Hayford	-148	136	90
Hong Kong 1963	Hayford	-156	-271	-189
Iran	Hayford	-117	-132	-164

* *positions compared to WGS84*

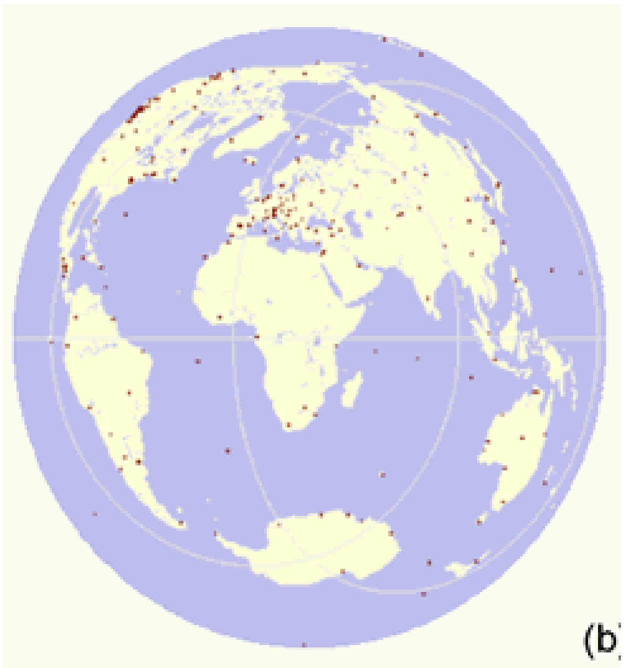
1.5. The global horizontal datum

Local horizontal datums have been established to fit the Geoid well over the area of local interest.
In the past the horizontal datum was limited to a region and it is never extended the continent.
With increasing demands for global surveying activities are underway to establish global reference surfaces.
The motivation is to make geodetic results mutually comparable and to provide coherent results also to other disciplines like astronomy and geophysics.



The most important global (or geocentric) spatial reference system for the GIS community is the ***International Terrestrial Reference System*** (ITRS). It is a three-dimensional coordinate system with a well-defined origin (the centre of mass of the Earth) and three orthogonal coordinate axes (*X*,*Y*,*Z*). (figure - right).

The ITRS is realized through the **International Terrestrial Reference Frame** (ITRF), a distributed set of ground control stations that measure their position continuously using GPS (figure (b) right).
Constant re-measuring is needed because of the involvement of new control stations and on-going geophysical processes (mainly tectonic plate motion) that deform the Earth's crust at measurable global, regional and local scales.
The ***World Geodetic System of 1984*** (WGS84) datum has been refined on several occasions and is now aligned with the ITRF to within a few centimetres worldwide. The Global Positioning System (GPS) uses the WGS84 as its reference system.
Global horizontal datums, such as the ITRF2000 or WGS84, are also called **geocentric datums** because they are geocentrically positioned with respect to the centre of mass of the Earth.



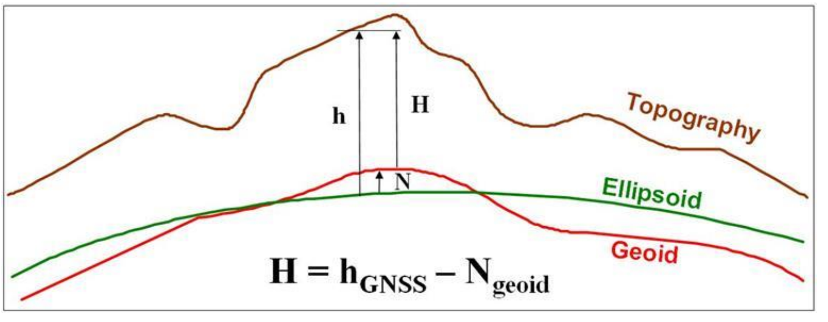
b) Geoid separation (N), or geoid undulation, is the height of the Geoid relative to a given ellipsoid of reference.

- Geoid modelling is a viable alternative for realising a long-term vertical datum.
- The Geoid is a stable surface that can be determined accurately all across any territory.
- It is defined in relation to a reference ellipsoid (geoid height), making it compatible with space-based positioning technologies (e.g., GNSS, satellite radar altimetry).

The below equation is used to find the orthometric height:

$$\mathbf{H = h - N}$$

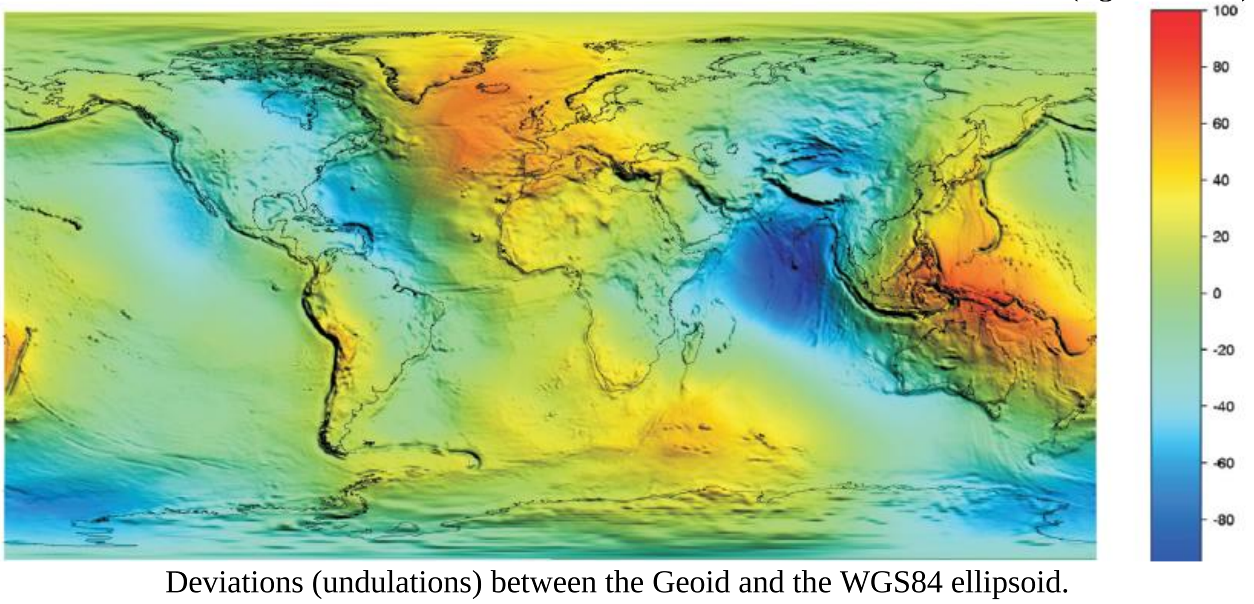
Where:
(H) Orthometric heights
(h) the ellipsoidal heights determined from GNSS technologies, and,
(N) Geoid separation



the relationship between the Geoid and ellipsoid

c) Properties of Geoid undulation:

- A mass deficiency exists, and the Geoid will dip below the mean ellipsoid.
- Conversely, the Geoid will rise above the mean ellipsoid, where a mass surplus exists.
- These influences cause the Geoid to deviate from a mean ellipsoidal shape by up to +/- 100 meters.
- The biggest presently known undulations are the minimum in the Indian Ocean with N = -100 meters and the maximum in the northern Atlantic Ocean with N = +70 meters (figure below).



Deviations (undulations) between the Geoid and the WGS84 ellipsoid.