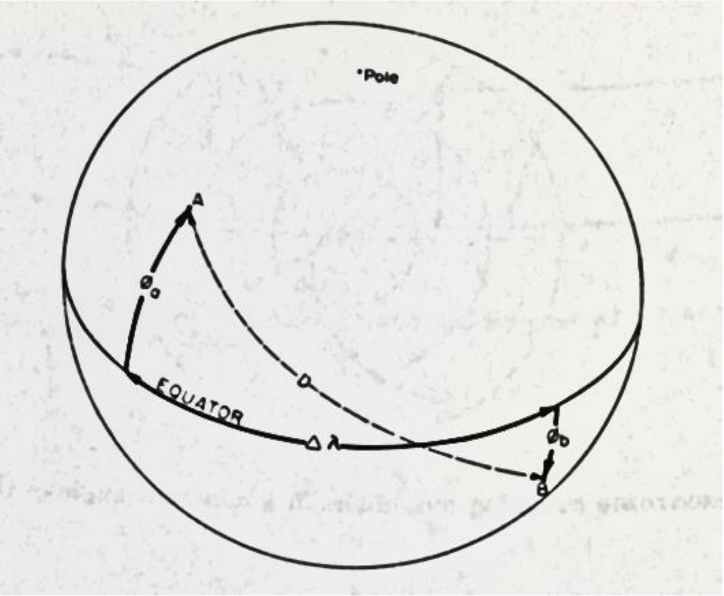
1. Great Circle:

- A great circle is a section of a sphere that contains a diameter of the globe.
- Sections of the sphere that do not contain a diameter are called small circles.
- A great circle becomes a straight line in a gnomonic projection.
- Each meridian and equator is a great circle. The shortest distance between two points on the earth's surface is a great circle route.

It is possible to calculate the shortest distance using the spherical trigonometry equation,

$$\mathbf{\cos D = (\sin \emptyset_a)(\sin \emptyset_b) + (\cos \emptyset_a)(\cos \emptyset_b)(\cos \Delta\lambda)}$$



Where:

$\emptyset_a$ Latitude at point a

$\emptyset_b$ Latitude at point b

$\Delta\lambda$ Difference of the longitude between a and b, the difference is not necessarily less than 180.

D is the arc distance (central angle) between points a and b.

Example: Find the shortest distance from Pittsburgh (USA) to Peking(China) on the globe with R=30cm.

If it is known that $\emptyset_a = 40, \lambda_a = -80, \emptyset_b = 40, and\ \lambda_b = 116.$

Solution:

$$\Delta\lambda = 360 - (116 + 80)$$

$$\cos D = (\sin \emptyset_a)(\sin \emptyset_b) + (\cos \emptyset_a)(\cos \emptyset_b)(\cos \Delta\lambda)$$

$\cos D = (\sin 40)(\sin 40) + (\cos 40)(\cos 40)(\cos 164)$=-0.1509

D=98.68^o

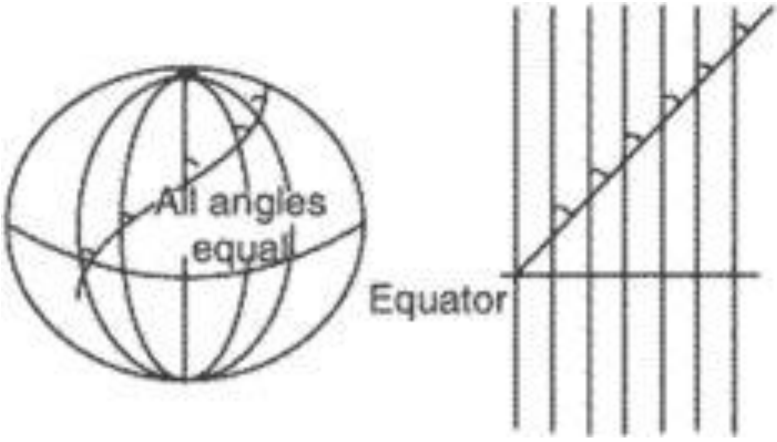
Surface Distance =(2Π30/360)98.68 ^{o}=51.7cm.

And the average length of a degree on a great circle is

Surface Distance =(2Π6370/360)98.68 ^{o}=10,971km.

And the length of a 1-second arc is about 30.88m

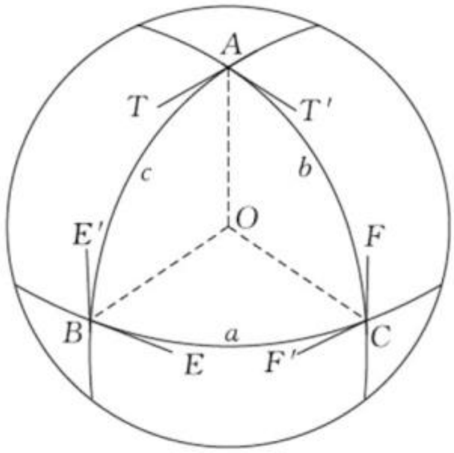
2. **Loxodrome or rhumb line,** although a great circle provides the shortest possible route between two points, it may be a challenging route to follow if navigating manually by compass. Therefore, in flying, it would be simpler to follow the route with constant bearing (on the line called follow ***Loxodrome or rhumb line***) than following the route with changing the bearing (***on great circle)***



3. Spherical triangle

It is a closed figure formed on the surface of a sphere bounded by three arcs of great circles.

The great circle is defined to be the intersection of a sphere with a plane containing the centre of the sphere



- The three arcs of great circles are the sides of the spherical triangle, denoted by lowercase letters a, b, and c.
- The spherical angles formed by the arcs of great circles are called the angles of the spherical triangle, denoted by the uppercase letters A, B, and C.
- A trihedron O-ABC is formed by connecting the vertices of the spherical triangle ABC with the centre of the sphere O.
- The radian measure of a central angle of a circle is equivalent to the length of the arc the angle subtends, which yields:

$$a = \angle BOC, b = \angle AOC, c = \angle AOB.$$

Given are:

$$A = \angle TAT', B = \angle EBE', C = \angle FCF'.$$

The Spherical Law of Sines formula:
$(\sin a / \sin A) = (\sin b / \sin B) = (\sin c / \sin C)$

The Spherical Law of Cosines formula:
$\cos a = \cos b \cos c + \sin b \sin c \cos A$

The amount by which the sum of the angles of a spherical triangle exceeds the sum of the angles of a plane triangle is called Spherical Excess and is denoted by ε:

$$\varepsilon = A + B + C - 180 ----(1)$$

The computational formula of ε is given by:

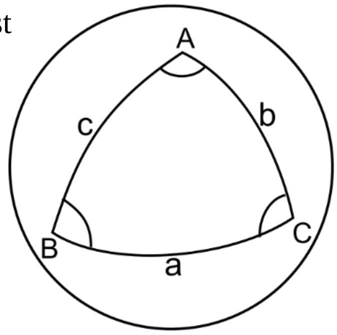
$$\varepsilon = \frac{S}{R^2} ------(2)$$

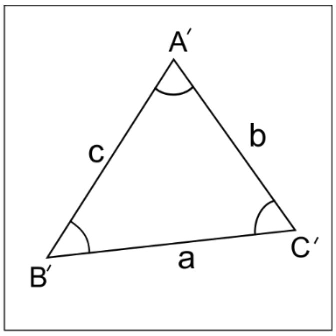
Where S denotes the area of the spherical triangle and R is the radius of the sphere.
The value of ε is in radian to convert it to second multiply by 180/pi*3600=206264.81”

Use the below formula to correct the surface angles:

$$\left.\begin{aligned} A_1 &= A_0 - \varepsilon''/3 \\ B_1 &= B_0 - \varepsilon''/3 \\ C_1 &= C_0 - \varepsilon''/3 \end{aligned}\right\},$$

Ex: Calculate the spherical excess and adjust the angles
From a=24512m, b=34930m, and, c=36621m. use R=6371km.
a_l=pi/180*a^o*R
then $a^o = 0^o\ 13'\ 13.59''$; $b^o = 0^o\ 18'\ 50.88''$,
and ,$c^o = 0^o\ 19'\ 45.63''$
Calculate surface angles (A,B,C)





$$\cos(A) = \frac{\cos(a) - \cos(b) * \cos(c)}{\sin(b) * \sin(c)}$$

$$\cos(A) = \frac{\cos(0o\ 13'\ 13.59'') - \cos(0o\ 18'\ 50.88'') * \cos(0o\ 19'\ 45.63'')}{\sin(0o\ 18'\ 50.88'') * \sin(0o\ 19'\ 45.63'')}$$

A=$39^o\ 58'\ 50.06''$

$\cos(B) = \frac{\cos(b) - \cos(a) * \cos(c)}{\sin(a) * \sin(c)}$ ➔ B=$66^o\ 17'\ 33.71''$

$\cos(C) = \frac{\cos(c) - \cos(a) * \cos(b)}{\sin(a) * \sin(b)}$ ➔ C=$73^o\ 43'\ 38.31''$

Angle	Value	Correction.	Corrected angles
A	$39^o\ 58'\ 50.06''$	-0.69	$39^o\ 58'\ 49.37''$
B	$66^o\ 17'\ 33.71''$	-0.69	$66^o\ 17'\ 33.02''$
C	$73^o\ 43'\ 38.31''$	-0.70	$73^o\ 43'\ 37.61''$
Sum	$180^o\ 00'\ 2.08''$	-2.08	$180^o\ 00'\ 00''$

Spherical excess=+2.08”
Correction =2.08”/3=0.693”