1. -Map Projections

A ***map projection*** is a mathematically described technique representing the Earth's curved surface on a flat map.

The obstacles in map projections:

- Distortions and cutting on the real ground when it are converted into a plane.
- There is no single method to achieve the conversion.

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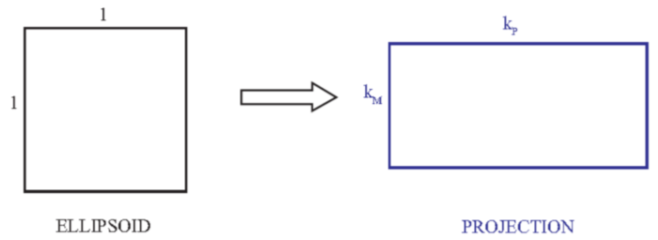
1.1. Scale factor,

The Features on the surface of a sphere or an ellipsoid suffer distortion when projected onto a plane. Therefore, precisely describing the amount of distortion using the **scale factor** is essential.

If k_P represents the scale factor along a parallel, k_M represents the scale factor along a meridian. As shown in the figure below, with a small square of dimension (1×1) on the ellipsoid's surface, the square is then projected as a rectangle of dimensions $(k_P \times kM)$.



This is provided by the definition of the *scale factor*, which is given the symbol *k* in this text. Then:

$$k = \frac{distance\ on\ the\ projection}{distance\ on\ the\ ellipsoide}$$

1.1.1. Scale factor specifications:

- Different at each point on the projection, and in many cases, will have different values in each direction.
- It can be applied to only a short distance (in theory, infinitesimally short).
- For longer lines, the relevant parameter is the integrated mean of the point scale factors along the length of the line.
- It is essential to understand that this scale factor results purely from the act of projecting to a flat surface and is, therefore, unrelated to the scale of the map (a ratio such as 1:50 000).
- The ideal value of the scale factor is 1, representing no distortion.
- It should also be emphasized that distortion of this type is not the same as an 'error' in the map, as the rules governing it are clearly defined, and the true coordinates can always be recovered if the values of the parameters of the projection are known.
- The distortion in the direction of the parallels will be different from the direction of the meridians.

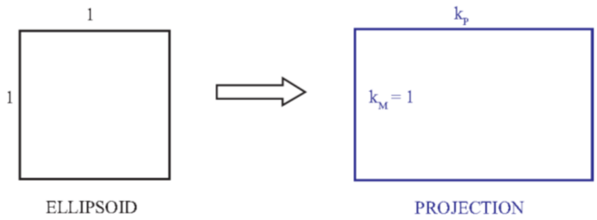
1.1.2. Types of scale factor

During projections, the ***shape, area,*** *and* ***size*** of features on the surface of the sphere or ellipsoid will differ. This is caused due to introducing distortions.

The usual approach is to attempt to preserve ***<u>one</u>*** of these, which will usually be done at the expense of all the others, and it is the most commonly used of all projections.

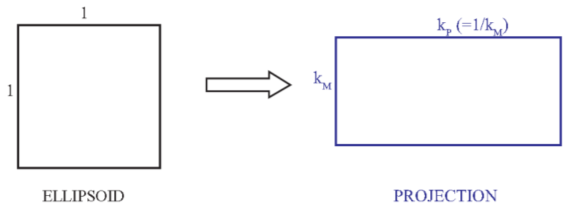
1.1.2.1. equidistant projection

- It may be required that certain distances, as measured on the sphere, should be undistorted when shown on the projection.
- It is impossible to preserve all distances, as this would then achieve the unachievable goal of an undistorted projection.
- The distances along all meridians should remain undistorted, which is the same as saying that: $k_M=1$ (the scale factor along the meridian is equal to one).
- It can be seen that there remains a scale factor along the parallels, which is not equal to one, and that both the shape and the area of the square have been distorted.



1.1.2.2. Equal-Area Projection

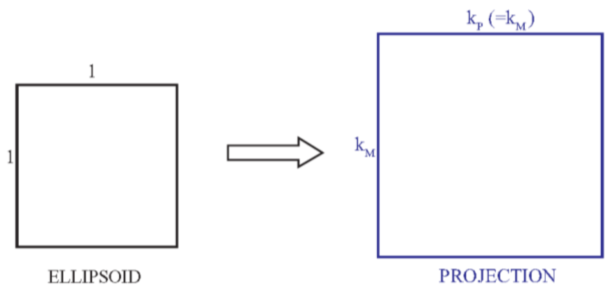
- This type of projection is used to preserve the area during the projection.
- Therefore, $K_M \cdot Kp=1$ (the area of the projected unit square is equal to one).



1.1.2.3. *Orthomorphic* projection or more commonly called *conformal* projection.

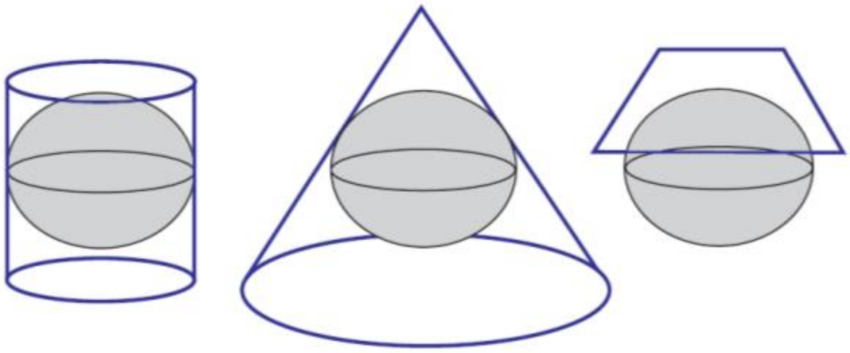
The principal classification of projections is that which preserves the *shape* of Features.

- The relationship between the scale factors is: kM=kP
- In preserving the shape, a conformal projection also preserves angles. For example, the angle between the side of the unit square and the diagonal is 45°: this is the angle that would be measured by someone observing on the ground.
- It is most significant in land surveying, as angles measured on the ground can be transferred to the projection for use in computations.
- It is one of the most frequently used and is likely to be the basis of almost all large-scale mapping.
- Other types that preserve neither shape, area, nor any distances are possible and are sometimes used.



1.2. Developable surfaces

- projections were derived by first projecting from the ellipsoid (or a sphere for a simplified model) to an **intermediate surface** that was of such a nature that it could be unravelled without distortion.



- The principal forms of these intermediate surfaces are the cone, the cylinder, and the plane itself.
- Their curvature is in one dimension only. They can be unravelled to a plane without further distortion – the surfaces are *developable*.
- Most projection methods may be classified **firstly** according to the shape of the developable surface, which is dictated primarily by the geographical area to be mapped but also by the function of the map, and **secondly** by the features on the sphere that are to be preserved on the projection.
- The scale factor distortion will be minimal in the region around the point or line of contact between the two surfaces. However, where the two surfaces are touching, the scale factor will equal 1.
- It should be noted here, however, that a developable surface is not a *necessary* step in forming a projection. It is possible to derive a set of formulae to convert geographic coordinates to grid coordinates in purely mathematical terms. In general, equations can be derived from the form:

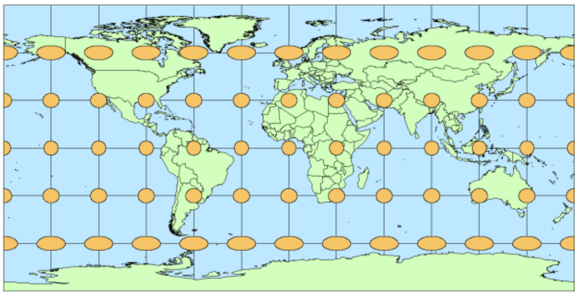
$$(E, N) = f(\emptyset, \lambda)$$

1.3. Tissot's indicatrix

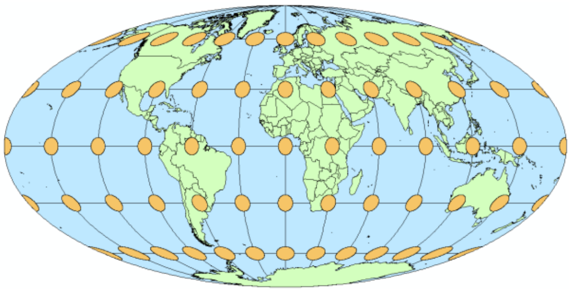
It is a tool to characterize the distortion due by projecting a spherical representation of the Earth onto a flat surface, i.e., a map projection.

Tissot's indicatrices are used on the spherical model.

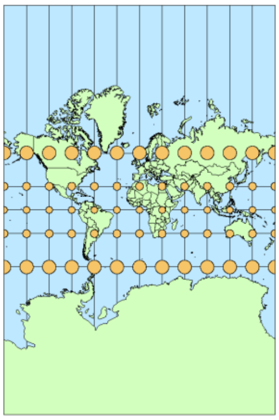
- A convenient way of showing distortion.
- The indicatrix's size, shape, and orientation will vary from one part of the map to another.



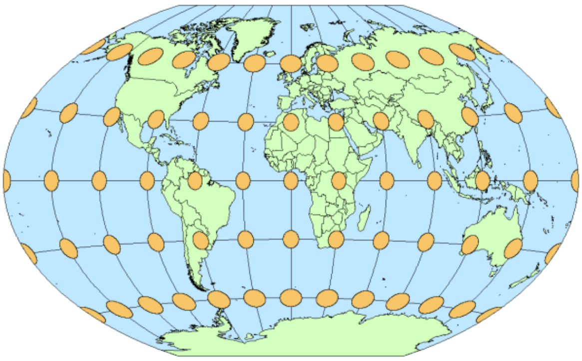
Tissot's indicatrices are used on WGS84 projections,



Tissot's indicatrices are used on Mollweide projection. All indicatrices are equal area but distorted



Tissot's indicatrices are used on the Mercator projection. All indicatrices are <u>circles</u> because a conformal projection is used.



Tissot's indicatrices are used on the Winkel Tripel projection, the indicatrices vary in <u>size, shape, and orientation</u> because a compromise projection is used.