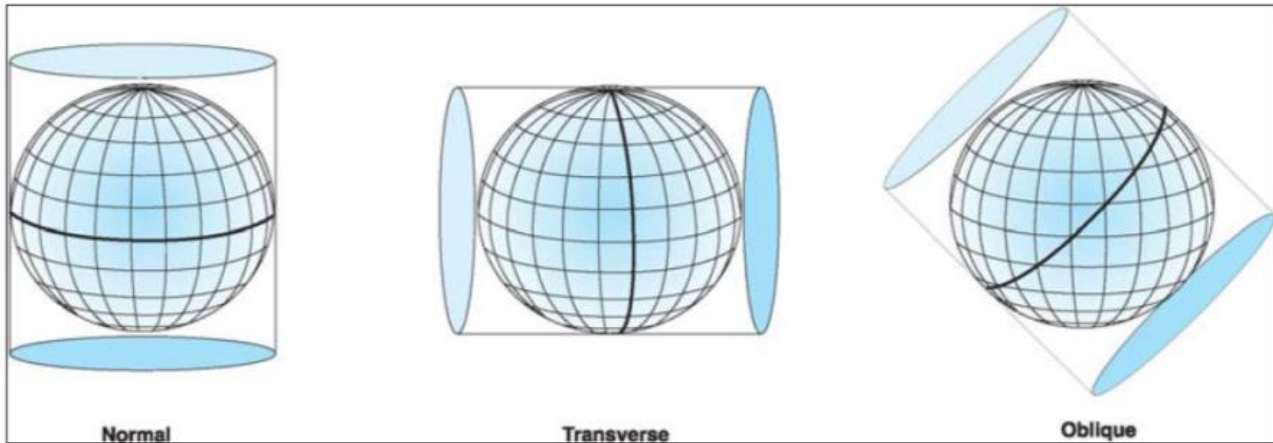


### 1. Cylindrical Projections

It is a type of projection in which a cylinder is used as a developed surface.

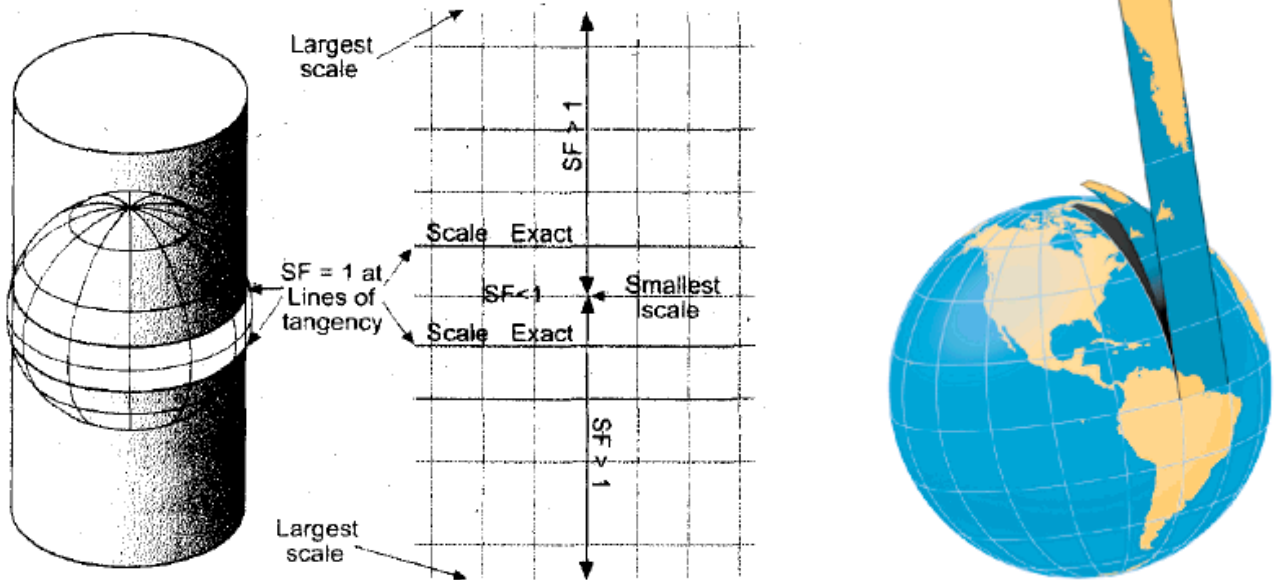
As shown in below Figure, there are three main different cylindrical map projections:

1. A normal cylinder projection has a cylinder in which the Equator is the line of tangency.
2. A transverse cylinder projection has its tangency at a meridian, and;
3. An oblique cylinder is rotated around a great circle line between the Equator and the meridians.



All cylindrical projections show the line of tangency between the cylinder and the sphere. In addition, however, there is a secancy type, as shown below.

#### SECANT CASE

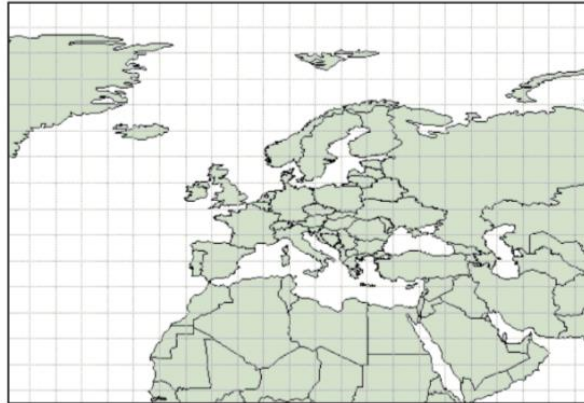


It is essential to know that, along the line of tangency and secancy, there is no distortion, and thus the lines are equidistance

Each of the above three mentioned types can be divided according to the distortions that occur during the projection:

**1.1. Equidistant cylindrical projection**

The cylindrical equidistant projection distorts parallels by the scale factor  $\sec\phi$  whilst leaving the meridians undistorted,  $k_M=1$ . It is, therefore, the case that areas have also been distorted.



- It is not used for rigorous topographic mapping because its distortion characteristics are unsuitable.
- This method has one of the most straightforward formulas available.

For the forward calculation of the Equidistant Cylindrical method:

$$X = R \cdot (\lambda - \lambda_0) \cdot \cos(\phi_0)$$

$$Y = R \cdot \phi$$

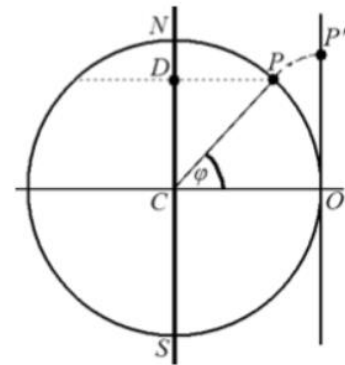
For the reverse calculation

$$\phi = Y / R$$

$$\lambda = \lambda_0 + (X / R \cos(\phi_0))$$

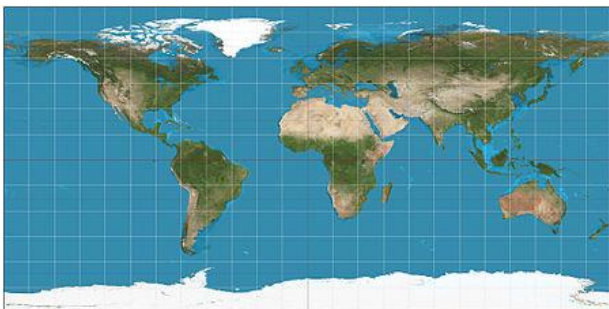
If the latitude of natural origin ( $\phi_0$ ) is at the Equator, the method is known as Plate Carrée. As shown in Figure  $OP=OP'$

$$\text{Parallel Scale Factor} = \frac{DP'}{DP} = \frac{2\pi R}{2\pi R \cos\phi} = \sec\phi$$

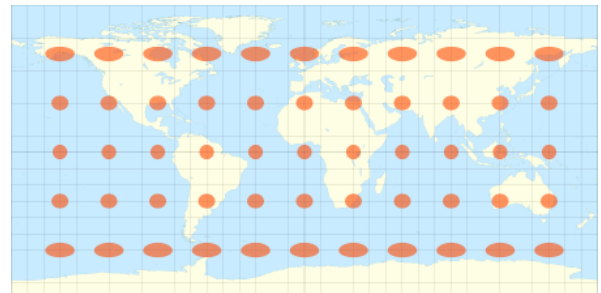


**1.1.1. Specifications of equidistance cylindrical projection**

- 1-As with all normal aspect cylindrical projections, the meridians are straight and parallel to each other.
- 2-The distances along the meridians are undistorted.
- 3-The scale along the Equator is true, but the scale of all other parallels becomes increasingly distorted towards the poles, with the extreme case of the poles being represented as straight lines.  $\sec 90^\circ = \infty$ ,
4. Consequently, the shape and the area become increasingly distorted towards the poles.
5. The flat, square appearance of the graticule leads to the French term *plate carrée*, which is sometimes also used in English for certain forms of this projection.



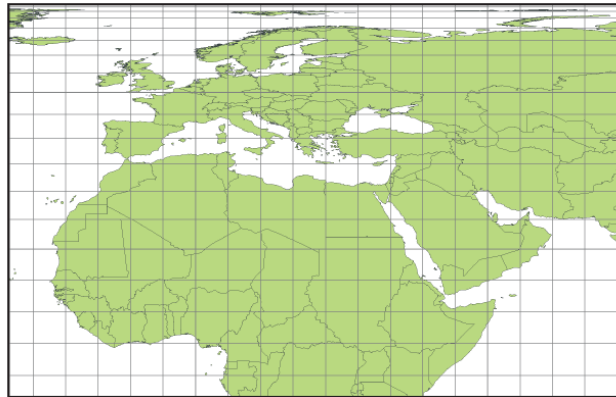
Equiarectangular projection of the world; the standard parallel is the Equator (plate carrée projection)



The equiarectangular projection with Tissot's indicatrix of deformation

### 1.2. Cylindrical equal area

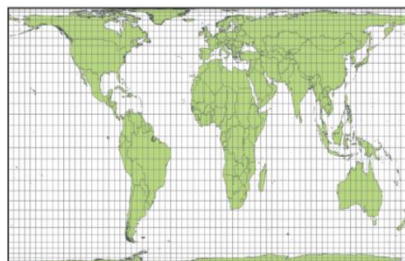
- An equal-area projection can be formed according to the rule of the equation that  $k_M k_P = 1$ .
- Then, since the parallels must be distorted by  $\sec\phi$  whatever happens (to fit onto the cylinder), this leads to the conclusion that:  $k_M = 1/k_P = \cos\phi$
- Hence, each small section of each meridian is multiplied by  $\cos\phi$  as it is 'unpeeled' and placed on the projection. See the below Figure:



### The Specifications of the equal-area cylindrical projection

1. The scale factor in the equatorial region is close to 1 for both the meridians and the parallels. This is a consequence of cylindrical projections being optimal for equatorial regions.
2. The scale factor along the meridians is no longer equal to 1. Distances cannot be measured directly off such a map. Furthermore, the correction to be applied is not straightforward, as the scale factor is a function of latitude.
3. The shape distortion is now extreme towards the poles, as in addition to the scale factor  $\sec\phi$  along the parallels, there is the distortion  $\cos\phi$  along the meridians.
4. Formulae can again be derived for converting between  $(\phi, \lambda)$  and  $(E, N)$ , which require the coordinates of the origin and the false coordinates as defining parameters.

A further refinement of this projection is the Peters Projection, often used by international organisations for displaying the countries of the world in their correct relative sizes, as shown below.



- It is specified to keep the equal-area property but to change the shape,
- It is achieved by applying a further scaling of 0.5 along the parallels and 2 along the meridians.
- The shape of features is now correct in the mid-latitudes, as opposed to the equatorial regions with the conventional form, and there is less shape distortion near the poles.

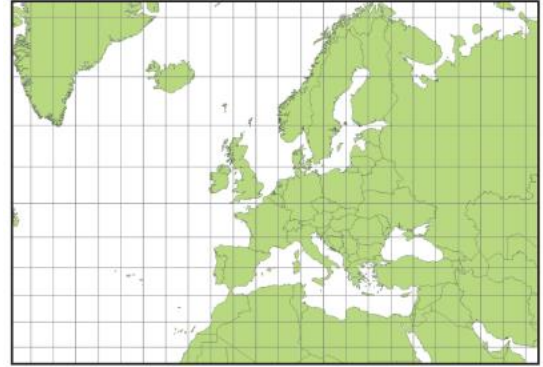
### The Mercator projection

One of the most important cylindrical projections is the conformal version, which is named *Mercator*. In this projection, it is again noted that

$k_P = \sec\phi$ , and hence from:

$k_M = k_P = \sec\phi$

This then leads to a projection such as that shown in Figure.



### The general features of Mercator projection are:

1-The scale factor at any point and in any direction is equal to  $\sec\phi$ , the secant of the latitude.

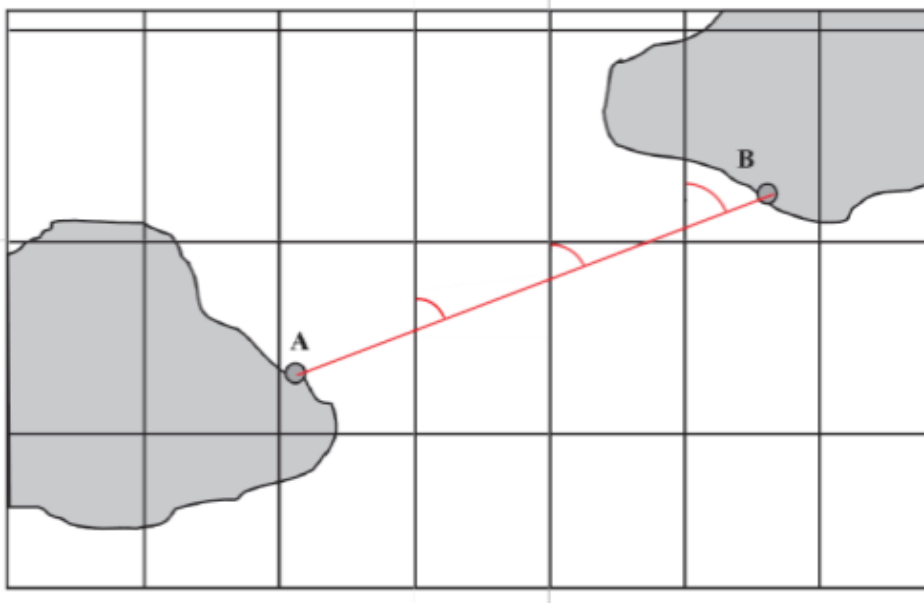
2-In consequence, the pole is now of infinite size and at an infinite distance from the Equator, and hence cannot be represented on the projection.

3-The fact that the meridians are parallel and that the angles are preserved makes this an ideal navigation projection.

- As shown in the below Figure, the line AB on the map, has a constant angle with respect to the meridians (the *azimuth from north*), which can be read directly from the map, it is termed a *rhumb line* or a *loxodrome*.
- The drawn line is the azimuth that should be followed in navigating from A to B, however, it is not the shortest route between A and B, due to the variation of scale factor within the projection. The shortest route between the two, the *great circle*, in most cases will in fact be projected as a curved line.

4. An individual map sheet (or navigation chart) will have different scale factor that varies within the map according to the range of latitudes it represents. (e.g. A chart of the English Channel, for example, might represent the range of latitudes between  $49^\circ$  N and  $51^\circ$  N. The scale factor would then vary between  $\sec 49^\circ$  and  $\sec 51^\circ$ , or 1.52 and 1.59.)

5-It is appropriate then to apply an *overall scaling* to the map so that the scale factor is on average equal or closer to 1.



**Example1:**

Plan a cylindrical equidistant projection for a map of Africa, assuming it must fit within 20cm square.

**Solution:**

- The existing map of Africa shows that it is bounded approximately, by 40° N, 35 ° S, 20 ° W, and 50° E.
- The width of 70° along the Equator and the height of 75° will be shown with a scale factor of 1.000.
- The controlling dimension will be north and south.
- Thus the largest generating globe and equating to 20cm, can be obtained from  $75/180 \cdot \pi \cdot R = 20\text{cm} \rightarrow R = 15.28\text{cm}$
- Thus the radius of 15.28cm corresponds to the following scale:

$$\text{Scale} = \frac{\text{globe radius}}{\text{earth radius}} = \frac{15.28\text{cm}}{637,000,000\text{cm}} = \frac{1}{41,690,000}$$

NB: if parallels and meridians are to be plotted at 10° intervals, the dimensions of the square will be:  
Grid spacing =  $10/75 \cdot 20\text{cm} = 2.67$  or grid spacing =  $10/180 \cdot \pi \cdot 15.28\text{cm} = 2.67\text{cm}$ .

**Example 2:** Determine the scale factor along the top edge of the map of Africa discussed in the example 1

**Solution:** As moving toward the poles, the globe distances are decreased. Therefore the scale factor will be greater than one:

$$\text{Scale factor} = \frac{\text{Map Distance}}{\text{Globe Distance}} = \frac{2.67}{2.67 \cos 40} = 1.305$$

*H.W. plan a cylindrical equidistant projection for a map of Kurdistan, assuming that it must fit within 25cm square and grid spacing equal to 1 degree, then plot the Kurdistan boundary on it by measuring 100 points from Google earth. Use R=6370km.*

*The deadline is shown on the system, submit the excel online and the plot on hardcopy. (put all map elements)*

Assume that the boundary is 39N, 32N, 41E, and 48E.

Hint: for the boundary coordinate calculations using equidistant projection, use the following procedure, for example, if geodetic lat. and long., the coordinate of the point is (37.102681, 42.381519):

$$X = R \cdot (\lambda - \lambda_0) \cdot \cos(\varphi_0)$$

$$Y = R \cdot \varphi$$

- Then  $X = 6371 \cdot (42.381519 - 41) \cdot \pi / 180 \cdot \cos(0) \cdot 1000 = 153,617.904\text{m}$
- $Y = 6371 \cdot 37.102681 \cdot \pi / 180 \cdot 1000 = 4,125,629.892\text{m}$ . This value is calculated from the Equator, in order to calculate from the latitude 32, then subtract  $(32 \cdot \pi / 180 \cdot 6371 \cdot 1000)$  from the above-calculated values.
- This means that the value of Y for plotting will be  $= 567,392.239\text{m}$

**Example 3:** find the projected coordinate using Equal area projection for the geodetic coordinates (37.102681°, 42.381519°).

**Solution:** the formula used in the equal-area projection is:

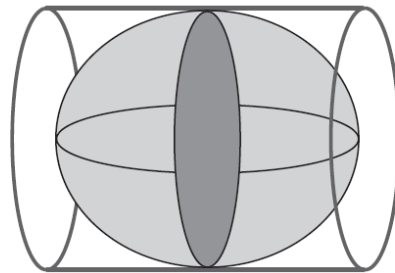
|   |   |
|---|---|
| $x = \cos(\varphi_0) \times \lambda$        | $x = \cos(0) * 42.381519 * \pi / 180 = 0.739697m$ |
| $y = \frac{\sin(\varphi)}{\cos(\varphi_0)}$ | $y = \sin(37.102681) / \cos(0) = 0.603245m$       |

The value of  $\varphi_0$  is equal to zero because the tangent line is at the Equator.

### 1.3. Cylindrical Projections- Transverse Mercator

In the earlier projections, the cylinder used in the map projection was normal. With, sometimes, the tangent line at the Equator, therefore, the area that not close to the Equator was suffering high distortion.

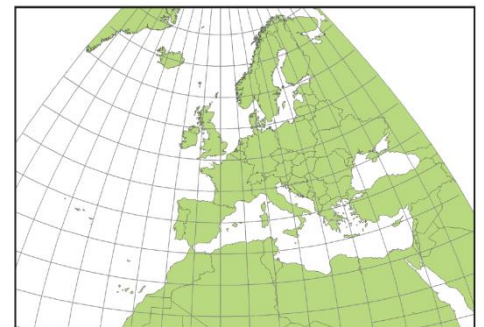
For those parts of the Earth that do not lie close to the Equator, an alternative is to turn the cylinder onto its side and make the line of contact a particular *meridian*, see below figure:



A projection so formed is termed a *transverse cylindrical* projection and can be based on any chosen longitude of origin.

Again, a set of rules can be proposed to produce equal area, equidistant, or conformal projections.

The most important of these is the Transverse Mercator projection method, an example of which is shown in Figure (right), which has been based on 0° as the longitude of origin, sometimes known as the *central meridian*.

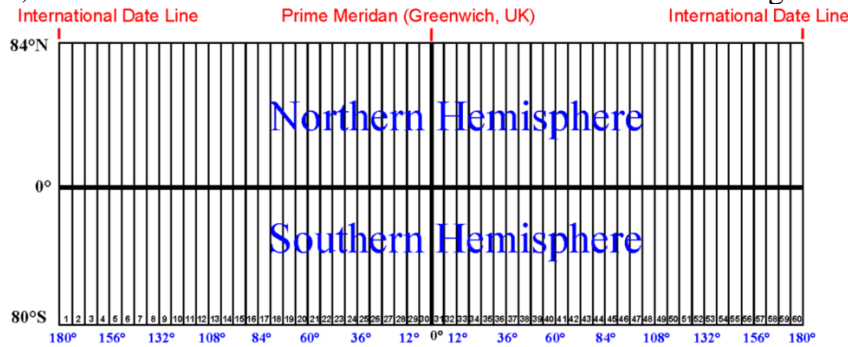


#### 1.4. The important features of this projection method are:

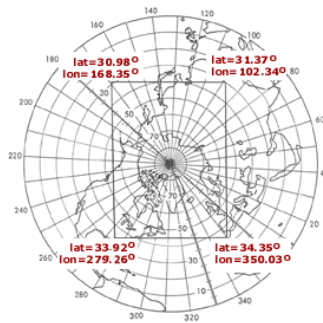
- 1-The projection is conformal.
- 2-The scale factor at each point is the same in any direction and is given by:  $k = \sec\theta$  .  
Where  $\theta$  is exactly analogous to  $\varphi$ , for a sphere can be found from the expression:  $\theta \approx \Delta\lambda \cos\varphi$
3. The meridians are no longer parallel to each other and are no longer straight lines.  
-The exception is the central meridian, which is a straight line.

### 1.5. UNIVERSAL TRANSVERSE MERCATOR (UTM)

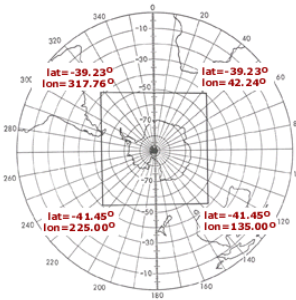
- The United States Army Corps of Engineers developed the UTM coordinate system in the 1940s.
- It is a specialised application of the Transverse Mercator projection.
- The globe is divided into 60 north and south zones, each spanning six degrees longitude.
- Each zone has its central meridian. Zones 1N and 1S start at  $-180^\circ$  W. The limits of each zone are  $84^\circ$  N and  $80^\circ$  S, with the division between north and south zones occurring at the Equator.



- The polar regions use the Universal Polar Stereographic coordinate system.

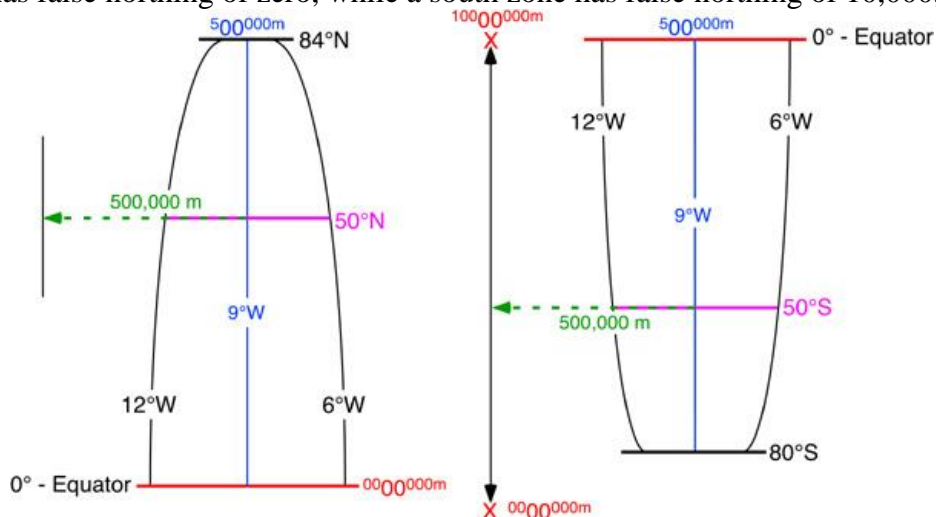


Northern Hemisphere



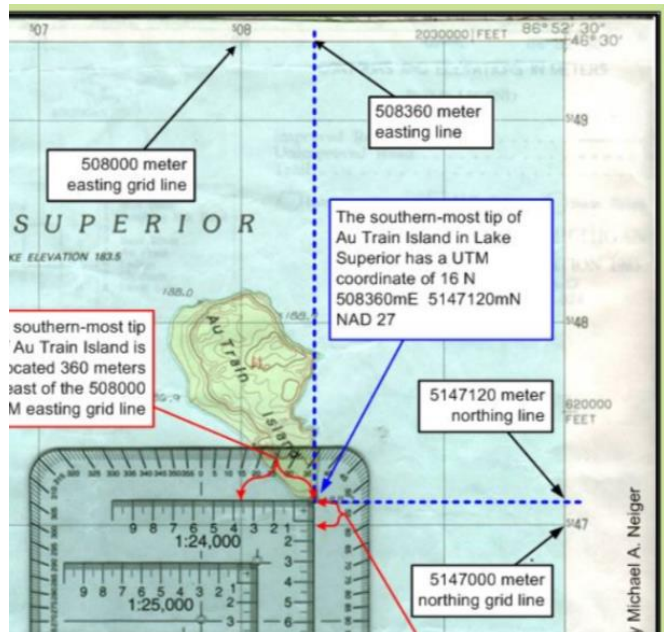
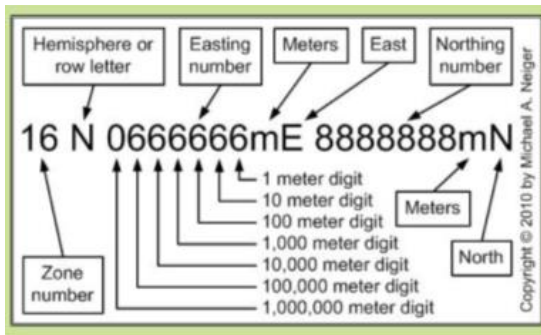
Southern Hemisphere

- The origin of each zone is its central meridian and the Equator. To eliminate negative coordinates, the coordinate system alters the coordinate values at the origin.
- The value given to the central meridian is the false easting, and the value assigned to the Equator is the false northing. A false easting of 500,000 meters is applied.
- A north zone has false northing of zero, while a south zone has false northing of 10,000,000 meters.



### 1.6. Reading the UTM maps and the values digit

To read the point value on the UTM map, first, the zone should be written then the hemisphere part should be written either north or south. Later, the value of the coordinates should be written,



In the above map is required to read a coordinate of a point on the UTM map is 16N 508360mE 5147120mN

Kurdistan is covered in zone **38N**,  
 Iraq is located in zone 37N,38N and 39N.

### 1.7. cylindrical projection- Oblique Mercator

- The final classification of cylindrical is the situation where a country or region to be mapped is longer in one direction than another but is not aligned along a meridian or parallel.
- In this situation, it is possible to formulate an oblique aspect of the Mercator projection to minimise the scale factor, as shown in below Figure:
- In defining the oblique projection method, it is necessary to specify the azimuth of the central line and the origin, false easting, and northing.
- The scale factor will now be proportional to the secant of the angular distance from the centre line.
- An example of the use of this projection method is in peninsular Malaysia.

