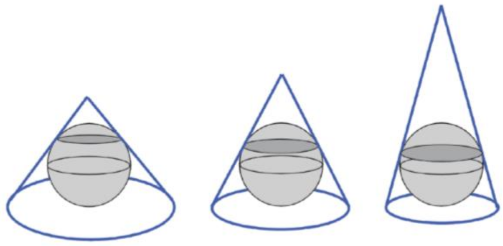
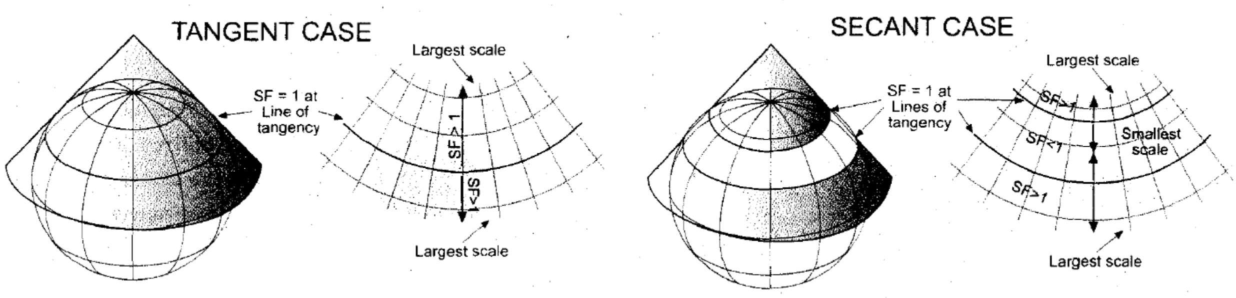
Lecture -10 Conic Projections

A conic projection is formed by bringing a cone into contact with the sphere or the ellipsoid, such as Lambert Conformal conical projection.



Standard parallel of the projection, is the parallel of latitude that tangent the sphere, It is either one tangent or two tangents in the case of the secant



There are three important classes of conic projections:

The equidistant (or simple), the conformal and the equal-area.

10-1 The conic Equidistant:

- A conic equidistant projection preserves the scale factor along a meridian (kM = 1). The parallels are then equally spaced arcs of concentric circles. The scale factor along a parallel of latitude is given as a function of latitude φ:

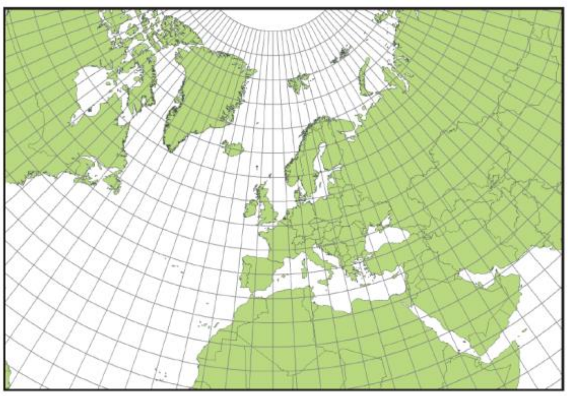
$$k_p = \frac{(G-\varphi)n}{\cos\varphi}$$

where

$$G = \frac{\cos\varphi_1}{n} + \varphi_1$$

$$n = \frac{\cos\varphi_1 - \cos\varphi_2}{\varphi_2 - \varphi_1}$$

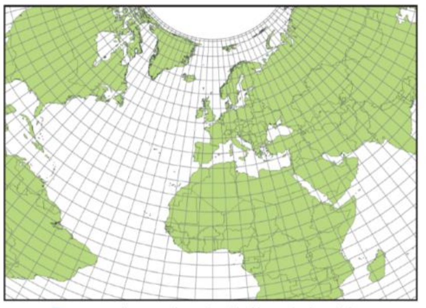
- $\phi 1, and\ \phi 2$ are two standard parallels. If only one standard parallel is used, then n will be equal to $sin\phi 1$
- The below figure is conic equidistant projection, standard parallels at 20° and 60° north.



  - The parallel in equidistant is equally spaced,
  - It is neither confromal nor equal-area, but north-south scale along all meredians is correct,
  - In the small area, the projection can be satesfctary copromise for errors in shape, scale and area.

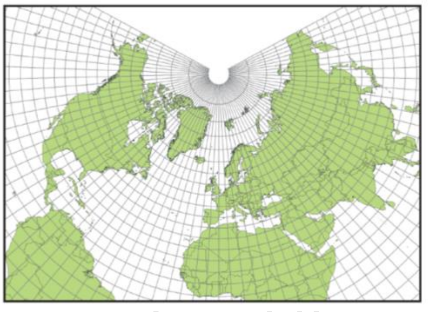
10-2 The conic Equal Area

- Such as Albers Equal-Area conic projection, with standard parallels at 20^o and 60^o north
- Parallels are unequally spaced arcs of concentric circles, more closely spaced at the north and south edge of the map.
- Meridians are equally spaced radii of the same circles, cutting parallels at right angles.
- There is no distortion in scale along two standard parallels,
- One of the most commonly projection used in the united state



10-3 Lambert Conformal Conic

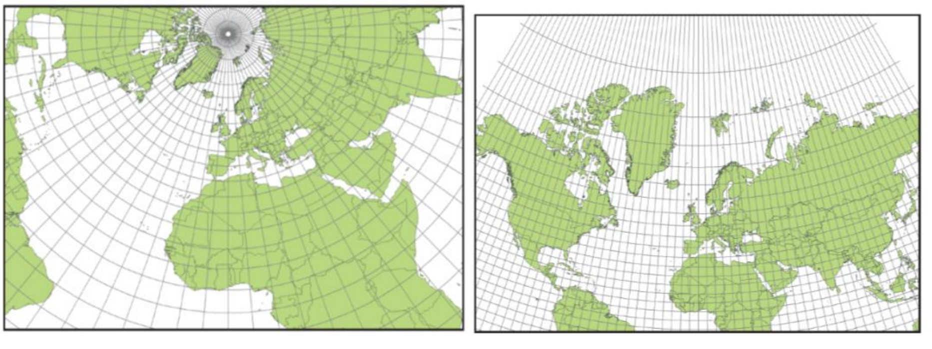
The conformal version of the conic projection is usually named after Lambert, who first developed it in 1772 (Snyder 1987). The full name is the Lambert Conformal Conic(LCC), as shown below (standard parallels at 20o and 60o north)



This is an extremely widely used projection, and it is probably true to say that the LCC and the Transverse Mercator methods between them account for 90% of base map projections worldwide.

A LCC with a standard parallel at the equator would effectively be the same as a Mercator projection, with the meridians
parallel and never reaching the infinite pole;

One with a standard parallel at 90° would be the same as a polar stereographic projection, the below figure shows the extreme parameter values when standard parallel at 80^o (Left) and 10^o (right).



- The parallels are unequally spaced arcs of concentric circles, more closely spaced near the centre of the map.
- Meridians are equally spaced radii of the same circles, thereby cutting parallels at right angles.
- Pole in same hemisphere as standard parallel is a point; other pole is at infinity.
- Used for maps of countries and regions with predominant east-west expanse.

10-4 Plotting Conic Equidistant Projections

The below formula are used to find the coordinates of the ground based on conic developed surface:

$$x = R(G - \emptyset)\sin\theta ----(11-1)$$

$$y = R(G - \emptyset o) - R(G - \emptyset)\cos\theta ----(11-2)$$

$$\theta = n(\lambda - \lambda o)$$

$$G = \frac{\cos\phi 1}{n} + \phi 1$$

$$n = (\cos\phi 1 - \cos\phi 2)/(\phi 2 - \phi 1)$$

$\phi o, \lambda o$: the latitude and longitude for the origin of the rectangular coordinates.

$\phi 1, \phi 2$: Standard parallels

If $(\lambda - \lambda o)$ exceeds the range ±180, then 360o should be added or subtracted.

In the case of one standard parallel $\phi 1$ is required then n=sin $\phi 1$,

$$\mathrm{k} = (\mathrm{G} - \phi)\mathrm{n}/\cos\phi$$

Example:

To find the Orthographic (sphere) projection coordinate, the following data are given:

radius of sphere R=1.0 unit,

Standard parallels: $\emptyset 1$:29^o 30’ N. lat.; $\emptyset 2$:45^o 30’ N. lat.;

Origion: $\emptyset o$=23^o N. lat. ; $\lambda o = 96^o$W. long.

Required to find the point coordinate of $\emptyset$=35^o N. lat. and λ=75^{o}W. long.

Solution:

$$n = (\cos 29.5 - \cos 45.5)/[(45.5 - 29.5) \times \pi/180] = 0.60678583$$

$$\theta = 0.60678583(-75 - (-96)) = 12.7424921$$

$$G = \frac{\cos 29.5}{0.60678583} + 29.5 \times \frac{\pi}{180} = 1.9492438$$

$$x = 1.0(1.9492438 - 35 \times \pi/180)\sin 12.7424921 = 0.2952057$$

$$y = 1.0(1.9492438 - 23 \times \pi/180) - 1.0\left(1.9492438 - 35 \times \frac{\pi}{180}\right)\cos 12.7424921 = 0.2424021$$

10-5 Non-geometric projection methods

- Some countries are specified to not give a suitable projection using already mentioned developable surface (i.e. cylinder, plane or cone), due to it is shape or location.

- With the advent of modern computing power and using more complex mathematics, it is now feasible to create projections in which the central line follows a feature of interest yet the desired equal area or conformal projection property is retained.

- for example, it is applied on New Zealand to produce Map Grid which was developed in the early 1970s, the problem with the shape of New Zealand is that it does not fit neatly into any of the categories that already identified: it is not predominantly north-south, nor spread east-west along a particular latitude, nor long and thin and oriented obliquely to the meridians.

- The solution adopted was closest in concept to an Oblique Mercator projection but, rather than being developed with respect to a straight line at a given azimuth, the lines of constant scale factor were complex curves that followed the approximate trend of the main islands of New Zealand.

- The most recent development of those discussed here was the Snake projection developed by (Iliffe *et al.* 2007) for use on railways and pipelines. As shown in the figure, it is applied to the West Coast Main Line from London to Glasgow