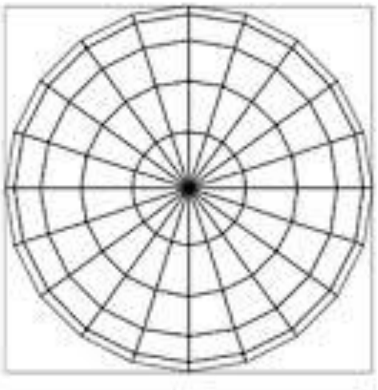
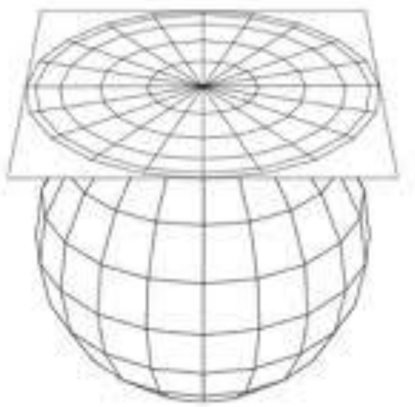
1. Azimuthal Projections

An azimuthal projection is formed by bringing a plane into contact with the sphere or ellipsoid and formulating a set of rules for transferring features from one surface to the other.





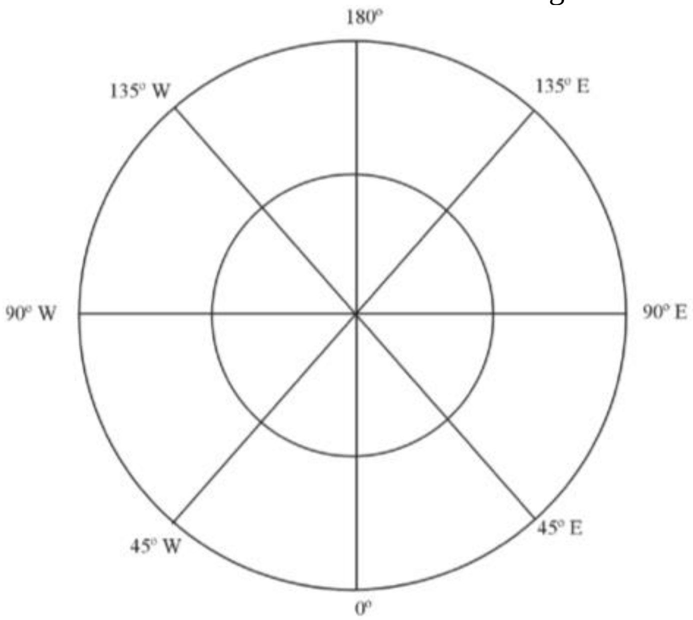
The properties preserved can be distance, area, shape, or others.

The scale factor distortion will be circularly symmetric because the point of contact between a sphere and a plane is a single point, therefore, it is particularly suited to small 'circular' features on the surface of the Earth.

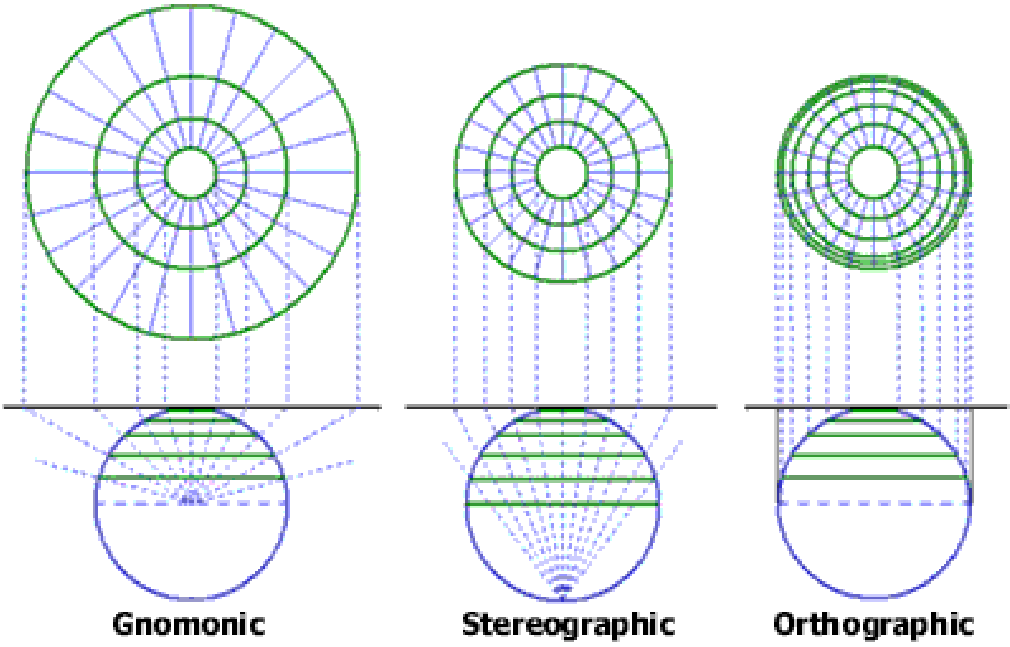
A special case of the azimuthal Projection is where the point of contact is one of the poles. This is referred to as a *polar projection*. This has the rather obvious application of mapping the Polar Regions.

The polar projections are formed by taking the meridians off the sphere and placing them on the plane. The amount of distortion of the meridians will be a function of the type of Projection, with the distortion of the parallels following in consequence.

The general form of the polar Projection will therefore be a set of meridians radiating from the pole with no distortion of the angle at the centre. This is shown in the below figure:



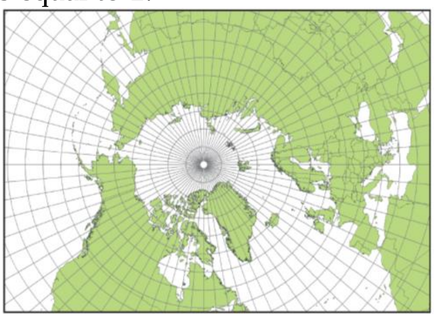
Generally, it can be classified into three types based on the type of Projection: Gnomonic, Stereographic and, Orthographic



1.1. Azimuthal equidistant projections

The azimuthal equidistant is formed by keeping the scale factor equal to 1 in the radial direction from the Projection's centre.

In the case of the polar equidistant, an example of which is shown in the below figure, this means that the scale factor on the meridians, kM, is equal to 1.

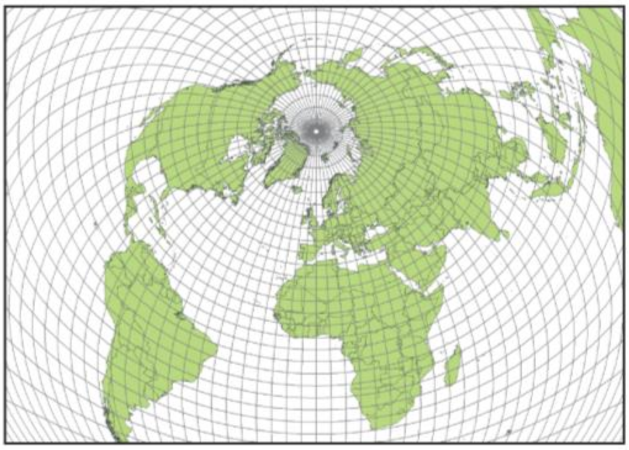


The scale factor along a parallel, kP, is given as a function of latitude, φ, by:

$$k_P = \frac{\frac{\pi}{2} - \varphi}{\cos \varphi}$$

Thus increases from 1 at the pole to 1.02 at 70° and 1.09 at 50°.

The figure below is another example, and it shows an azimuthal equidistant projection centred on London. All distances from London are correct when measured from the map; all other distances will be too long.



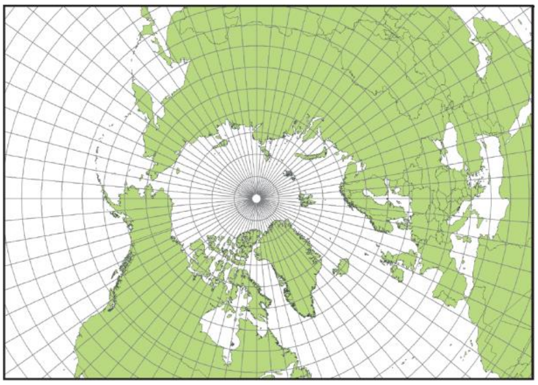
The formulas used for equal distant are

$$x = R(\pi/2 - \phi)\sin(\lambda - \lambda_0)$$
$$y = -R(\pi/2 - \phi)\cos(\lambda - \lambda_0)$$
$$k = (\pi/2 - \phi)/\cos\phi$$

1.2. Azimuthal equal-area

The azimuthal equal-area Projection is formed in a similar way to the azimuthal equidistant, except that the scale factor of the lines radial from the centre is set to the inverse of the scale factor in the perpendicular direction.

An example of the equal area for the pole is shown :

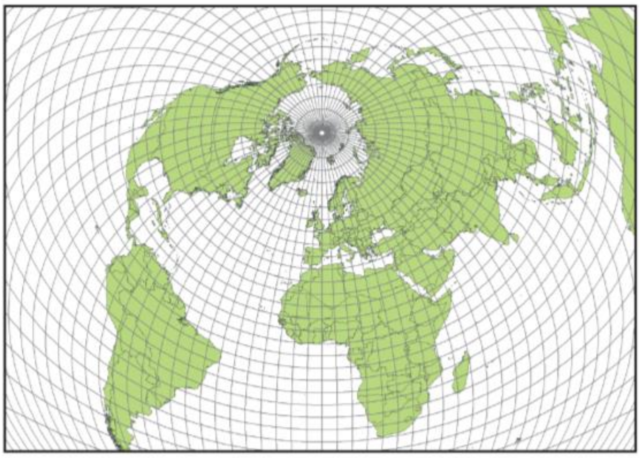


This leads to scale factors of:

$$k_M = \cos(45° - \varphi/2)$$

$$k_P = \sec(45° - \varphi/2)$$

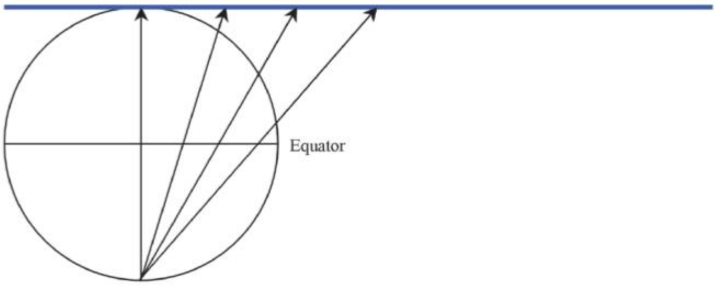
The ellipsoidal formulae are more complex than the above equations. A projection based on the ellipsoidal form developed by Lambert is used for European statistical mapping and other purposes where true area representation is required.



1.3. Stereographic

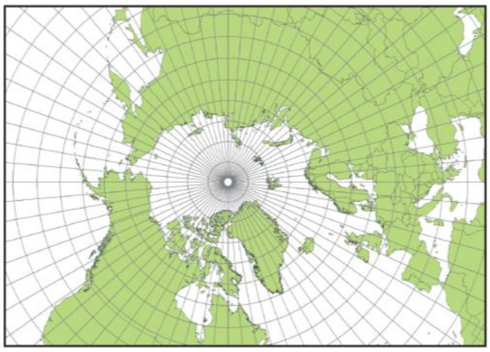
The conformal version of the azimuthal Projection is termed the *stereographic.*

This is for historical reasons since this Projection can be constructed graphically by projecting all points from a 'viewing point' on the opposite side of the Earth from the centre of the Projection, as shown in the below figure:



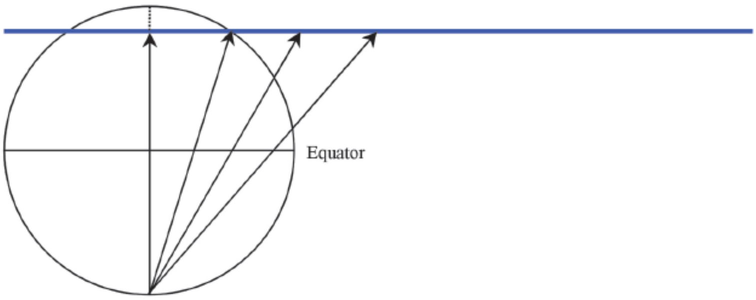
This type of Projection is sometimes used as the basis for national mapping, particularly appropriate for small, compact countries or islands.

The polar stereographic, shown in the below figure, is used as a complement to the Universal Transverse Mercator system beyond latitudes 84°N and 80°S, when it is known as the Universal Polar Stereographic projection, or UPS.



In this usage, the scale at the pole (kO) is reduced to 0.994, which results in a *standard parallel* (where scale is true) of 81°06'52.3".

The above type is *secant* case of the formation of a stereographic projection is illustrated in the figure:



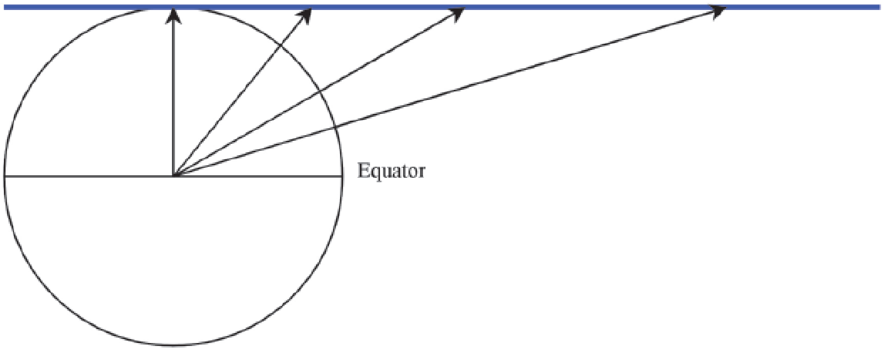
In general, the scale factor for the polar aspect is given by:

$$k = k_o \sec^2(45° - \varphi/2)$$

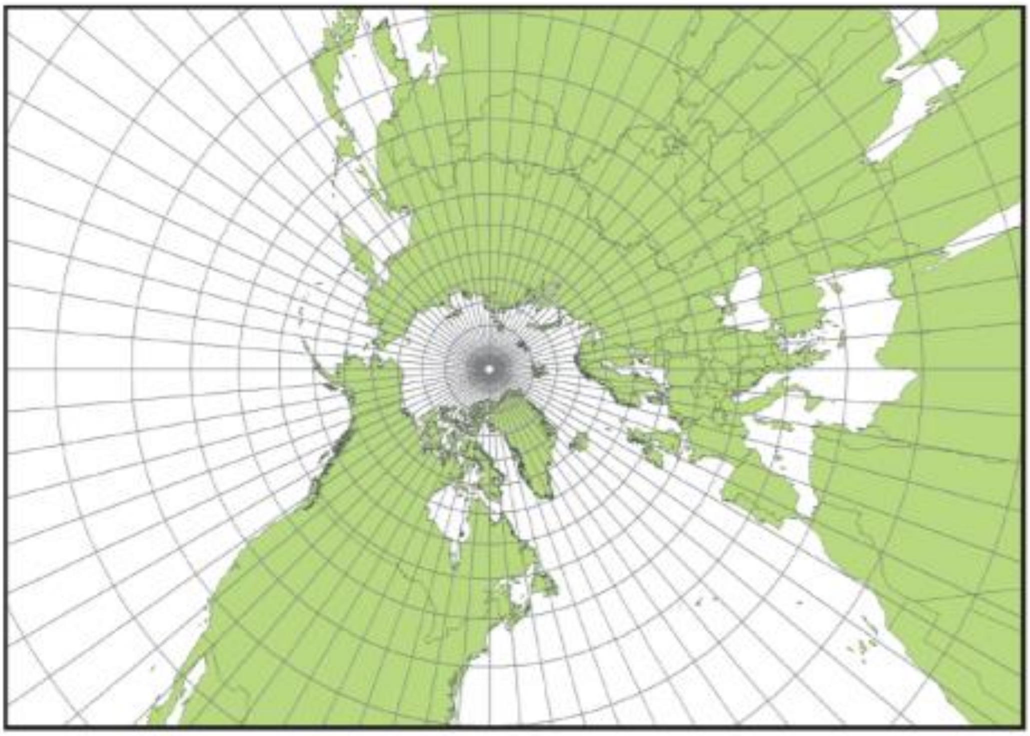
The oblique aspect of the stereographic Projection is tangential at a chosen point on the surface of the sphere or ellipsoid. If a scale factor is applied, the scale is true on (in the spherical case) a small circle centred on the origin.

1.4. Gnomonic

Gnomonic Projection is seldom used in modern applications. It is formed by projecting all points from the centre of the Earth, as shown in the below figure:



An example of the polar aspect is shown below figure (north pole projection). As would be expected, the scale factor distortion becomes extreme away from the centre of the Projection, reaching a value of 2 along the meridian when the latitude is 45° and a value of 4 at a latitude of 30°.



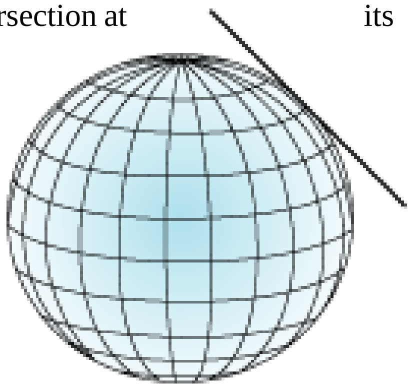
It is not possible to show an entire hemisphere with this Projection.

This type of Projection is only used when planning the shortest route between two points is required.

The only advantage of this Projection is that it is the only one where all great circles (the shortest route between two points on a sphere) are shown as straight lines.

1.5. Oblique Azimuthal equidistant projection

The oblique azimuthal equidistant Projection will show each graticule intersection at its correct distance and azimuth from the origin.



Oblique

It is possible to use trigonometry to calculate such polar coordinates for each intersection needed.

The azimuthal equidistant Projection is useful at airports, missile launching facilities, seismograph stations, for studies of radio transmissions, and others. In each case, the map is centred on the location of the airport or scientific facility

2. Plotting Azimuthal projections

Polar Azimuthal Equidistant projection, calculating the plotting coordinates

$$x = Rk' \cos(\phi) \sin(\lambda - \lambda o) ----(1)$$

$$y = Rk'[cos\phi o . sin\phi - sin\phi o . cos\phi . \cos(\lambda - \lambda o)] ----(2)$$

<u>Where</u>

$$k' = c/\sin(c)$$

$$\cos(c) = [\sin\phi o . \sin\phi + cos\phi o . cos\phi . \cos(\lambda - \lambda o)]$$

For the north pole ($\phi o = 90$) the equations will

$$x = R\left(\frac{\pi}{2} - \phi\right) \sin\,(\lambda - \lambda o)$$

$$y = -R\left(\frac{\pi}{2} - \phi\right) \cos\,(\lambda - \lambda o)$$

$$k = \left(\frac{\pi}{2} - \phi\right) \cos\,(\phi)$$

<u>Example:</u>

To find the equidistant (sphere) projection coordinate, the following data are given:
Radius of sphere R=3.0 unit,
Centre of azimuthal:
$\emptyset o$=40^{o} N. lat.
$\lambda o = 100$W. long.
Required to find the point coordinate of $\emptyset$=20^{o} S. lat. and λ=100E. long.

<u>Solution:</u>

The value of the x and y are:
$\cos(c) = [\sin 40 \sin(-20) + cos40 . \cos\,(-20) . \cos(100 + 100)]$=-0.8962806

C=153.6733925

$$k' = \left(153.6733925 \times \frac{\pi}{180}\right) / \sin(153.6733925) = 6.0477621$$

$$x = 3.0 \times 6.0477621 \cos(-20) \sin(100 + 100) = -5.8311398 \text{ units}$$

$$y = 3.0 \times 6.0477621[\cos 40 \sin(-20) - \sin 40 \cos(-20) \cos(100 + 100)] = 5.5444634\ units$$

2.1. Plotting Orthographic project.

For the calculation of the coordinate of the point on 2D plane using azimuthal orthographic Projection, the following formula will be used:

$$x = R\cos\phi\sin(\lambda - \lambda o) ----(3)$$

$$y = R\,[\cos\phi o\sin\phi - sin\phi o.cos\phi.\cos(\lambda - \lambda o)] ----(4)$$

$$h' = \cos(c) = [\sin\phi o\sin\phi + cos\phi o.cos\phi.\cos(\lambda - \lambda o)]$$

$k' = 1.0$

Where:
$\emptyset o$ is the latitude of the centre point and the origin if the Projection
λo is the longitude of the centre point
h' is the scale factor of the line radiating from the centre
k' is the scale factor in the direction perpendicular to a line radiating from the centre
c is the angular distance of the given point from the centre of the Projection.

<u>In the case of the north pole, the value of $\emptyset o$ is 90</u>

$$y = R[-cos\phi.\cos(\lambda - \lambda o)]$$

$$h \quad = sin\phi$$

<u>Regarding the south pole, the value of $\emptyset o$ is -90, then</u>

$$y = R[cos\phi.\cos(\lambda - \lambda o)]$$

$$h \quad = -sin\phi$$

<u>Example:</u>
To find the Orthographic (sphere) projection coordinate, the following data are given:
radius of sphere R=1.0 unit,
Centre of azimuthal:
$\emptyset o$=40^{o} N. lat.
$\lambda o = 100$W. long.
Required to find the point coordinate of $\emptyset 1$=30^{o} N. lat. and $\lambda 1$=110W. long.

<u>Solution:</u>
First, the point will be checked whether it in the beyond viewing or not
$\cos(c) = (\sin 40)(\sin 30) + (\cos 40)(\cos 30)(\cos -110 + 100)$
cos(c)=0.9747290
since the value is positive, it means the point within the view.

The coordinate of the point respect to the azimuthal Projection are:

x = 1.0 $\cos 30\sin(-110 + 100)$= -0.1503837

y = $1.0\,[\cos 40\ \sin 30 - \sin 40.\cos 30.\cos(-110 + 100)] = -0.1651911$

2.2. Stereographic projection-calculations

Mathematically, a point at a given angular distance from the chosen centre point on the sphere is plotted on the Stereographic Projection.

The value of the coordinate is obtained from the following formula:

$$x = \mathrm{R.k.} \cos\phi \sin(\lambda - \lambda o) ----(5)$$

$$y = R\,.\mathrm{k.}\,[\cos\phi o\,.\sin\phi - sin\phi o.cos\phi.\cos(\lambda - \lambda o)] ----(6)$$

$$k = \frac{2\mathrm{ko}}{1 + \sin\phi o.sin\phi + \cos(\phi o)\cos(\phi)\cos(\lambda - \lambda o)}$$

Where: K is the scale factor and it is equal in all directions since the Projection is conformal

Example:

To find the Stereographic (sphere) projection coordinate, the following data are given:
the radius of sphere R=1.0 unit,
Centre of azimuthal:
$\emptyset o$=40° N. lat.
$\lambda o = 100°$ W. long.
Ko=1.0
Required to find the point coordinate of $\emptyset$=30° N. lat. and λ=75° W. long.

Solution

$$k = 2 \times 1.0/[1 + \sin(40)\,.\sin(30) + \cos(40)\cos(30)\cos(-75 + 100))$$

k=1.0402304

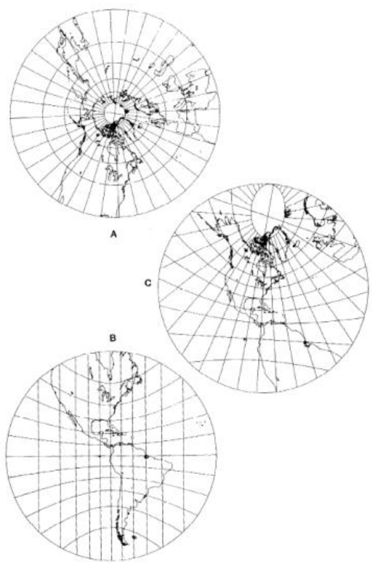
$$x = 1.0 \times 1.0402304 \times \cos 30 \times \sin(-75 + 100)$$

$$x = 0.3807224 \text{ unit}$$

$$y = 1.0 \times 1.0402304 \times [\cos 40 \sin 30 - sin40.cos30.\cos(-75 + 100)]$$

$$y = -0.1263802 \text{ unit}$$

2.3. Gnomonic Projection calculations



This type is specified to be known by Greek 2000 years ago. It is used only in spherical form. All meridians, equator and great circles are shown as straight lines. It is specified to be neither conformal nor equal-area. The sample of it is shown in the right images regarding the (A) polar aspect, (B) equator aspect, and (c)oblique aspect.

The formula used to find the projected coordinates are:

$$x = \mathrm{R.k'.} \cos\phi \sin(\lambda - \lambda o) ----(10 - 7)$$

$$y = R\,.\mathrm{k'.}\,[\cos\phi o \sin\phi - sin\phi o.cos\phi.\cos(\lambda - \lambda o)] ----(10 - 8)$$

$$k' = \frac{1}{\cos(c)} = 1/\,[\sin\phi o.\sin\phi + cos\phi o.cos\phi.\cos(\lambda - \lambda o)]$$

Example:

To find the Gnomonic (sphere) projection coordinate, the following data are given:
the radius of sphere R=1.0 unit,
Centre of azimuthal:
$\emptyset o$=40^o N. lat.
$\lambda o = 100^o$ W. long.
It is required to find the point coordinate of $\emptyset$=30^o N. lat. and λ=110^o W long.

Solution

First, check for the visibility of the point,

$$\cos(c) = sin40 \times sin30 + cos40cos30 \cos[-110 - (100)] = 0.9747290$$

Since cos(c) is positive (not zero or negative), the point is in the view and may be plotted, first calculate k' then x and y.:

$$k' = \frac{1}{0.9747290} = 1.0259262$$

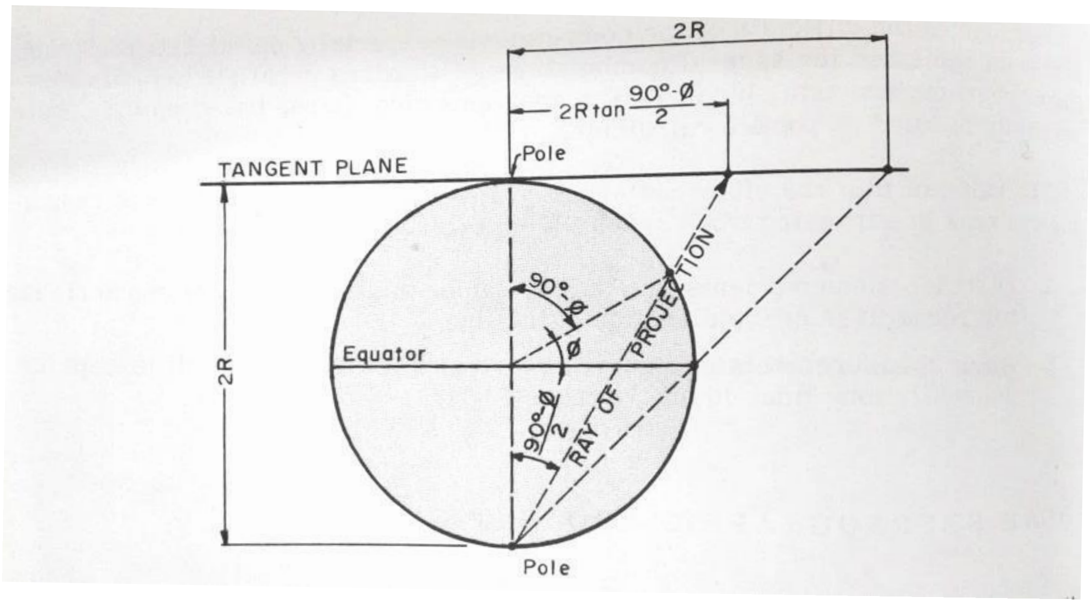
$$x = 1.0 \times 1.0259262 \cos(30) \sin[-110 - (-100)] = -0.1542826$$

$$y = 1.0 \times 1.0259262 \times [\cos(40)\sin(30) - \sin(40)\cos(30)\cos(-110 - (-100))]$$
$$= -0.1694789 unit$$

2.4. Stereographic projection scale factor:

Example-1, Verify that a stereographic projection is conformal by calculating an approximate scale factor in two perpendicular directions at $\emptyset = 60^o$

Solution: the scale factor along the 60th parallel may be determined by dividing its length on the map by its length on the globe:



$$Map\ distance = 2\pi\left(2R\frac{90-\emptyset}{2}\right)$$

Globe distance =2 πR cos Ø

$$Scale\ factor = \frac{2\tan\left[\frac{90-\emptyset}{2}\right]}{cos\emptyset}$$

$$= \frac{2\tan\left[15\right]}{cos60} = 1.0718$$

In the radial direction, consider the 1^{o} increment from $\emptyset = 59.5^{o}$ to $\emptyset = 60.5^{o}$, and divide the map distance by the globe distance.

Map distance = 2R tan $\left(\frac{90-59.5}{2}\right)$-2R tan $\left(\frac{90-60.5}{2}\right) = 2R(tan15.25 - tan14.75)$

S.F=$2R(tan15.25 - tan14.75)$/R(1*pi/180)=1.0718

Example 2: Find the scale factor along the 60th parallel of the polar gnomonic Projection. Compare it to the value for the polar stereographic in the above example;

map distance =Rtan (90- Ø)

Scale factor =$\frac{2\pi Rtan(90-\emptyset)}{2\pi Rcos60} = \frac{tan(30)}{cos60} = 1.155$