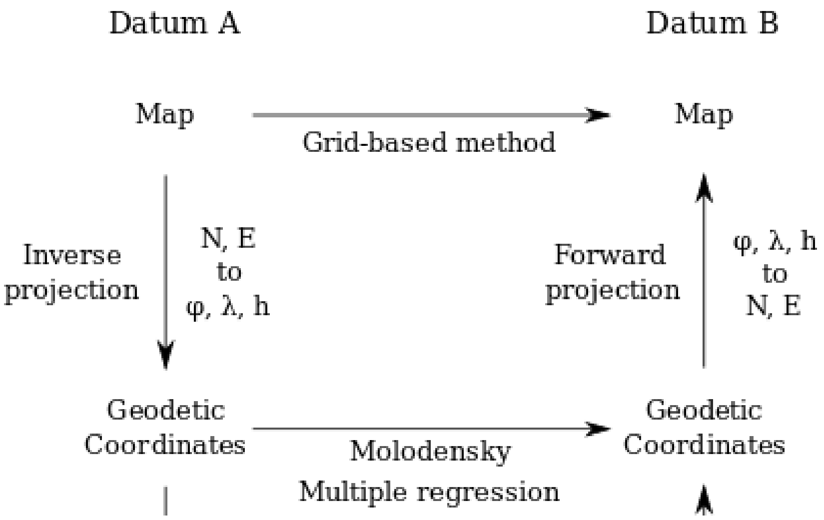
The Conversion and Transformation

Conversions Mathematical operations for changing the coordinate system from one coordinate reference system to another.

Transformations are the coordinate conversion, including the datum.



Geocentric Cartesian coordinate systems that introduce a datum make each one have a different relationship to the Earth.

Datum transformation is important for surveyors in order to integrate the different coordinate systems into one another.

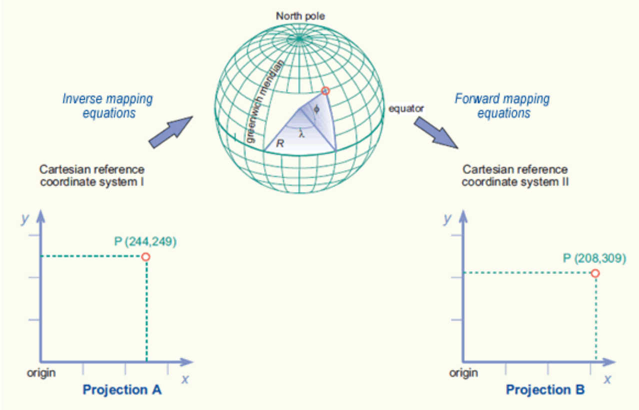
Or, usually, for mapping purposes at large and medium scales. (e.g. map and GIS users are often collecting spatial data in the field using satellite navigation technology and need to represent this data on published maps on a local horizontal datum.)

9-1 Changing Map Projection

Forward and inverse mapping equations are generally used to transform data from one map projection to another.

The ***inverse equation*** of the source projection is used first to transform source projection coordinates (*x*,*y*) to geographic coordinates (∅,λ). Next, the ***forward equation*** of the target projection is used to transform the geographic coordinates (∅,λ) to target projection coordinates (*x'*,*y'*).

The first equation takes us from projection *A* into geographic coordinates. The second takes us from geographic coordinates (∅,λ) to another map projection B. This is illustrated in the figure below.

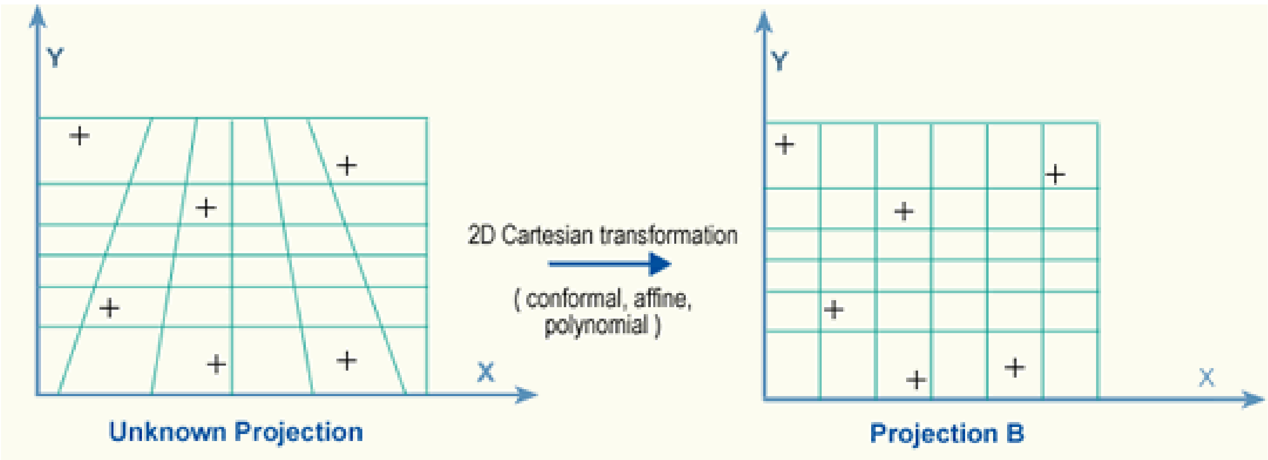


The coordinate system (map projection) of the input data must be known to use the mapping equations for a projection change.

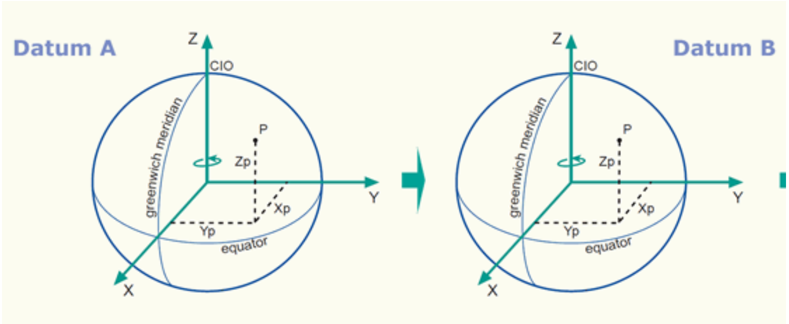
If the coordinate system of the input data is not known, it may be possible to use a 2D Cartesian transformation.

2D ground control points (GCPs) or common points are then required to determine the relationship between the unknown and the known coordinate system.

The transformation may be conformal, affine, polynomial, or of another type, depending on the geometric differences between the two map projections, as shown in the below figure:

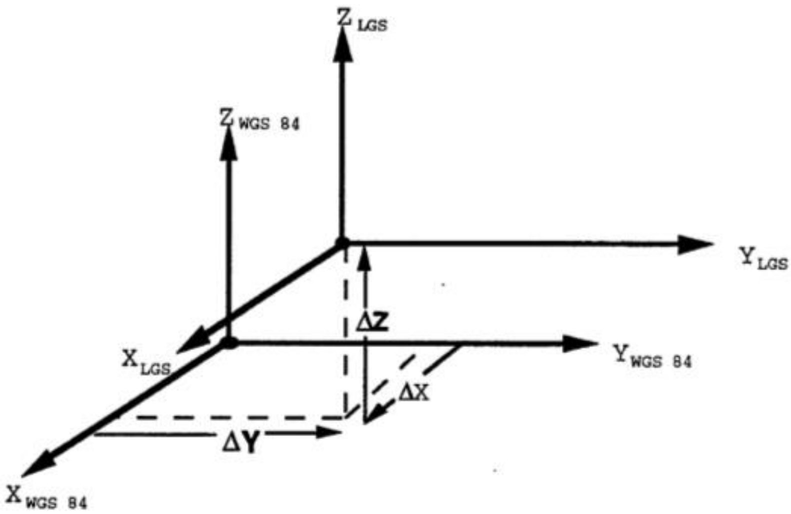


9-2 GEOCENTRIC TRANSFORMATION



9-2-1 Three parameter geocentric transformation

A simple three parameter shift – *geocentric translations* – relates two datum systems through three translations. The method applies a shift between the centres of the two geocentric coordinate systems. This shift is defined by the parameters ΔX, ΔY and ΔZ (or X_o,Y_o and Z_o).



The transformation of coordinates from the source system (X_S, Y_S, Z_S) to the target system (X_T , Y_T , Z_T) is given by:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{TARGET} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{SOURCE} + \begin{pmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{pmatrix}_{S\ to\ T}$$

Where ΔX, ΔY and ΔZ are the geocentric translation parameter values for the translation *from* the source coordinate reference system *to* the target system.

The three-parameter geocentric transformation can be exact at a single point, but in the general case it gets less accurate the larger the area to which a user attempts to apply it.

Example-1

Consider a North Sea point with coordinates derived by GPS satellite in the WGS84 geocentric coordinate reference system, with coordinates of:

X_s = 3771 793.97 m
Y_s = 140 253.34 m
Z_s = 5124 304.35 m

whose coordinates are required in terms of the ED50 coordinate reference system which takes the International 1924 ellipsoid. The three parameter geocentric translations method's parameter values from WGS84 to ED50 for this North Sea area are given as dX = +84.87m, dY = +96.49m, dZ = +116.95m.

Applying the quoted geocentric translations to these, we obtain new geocentric values now related to ED50:

X_t = 3771 793.97 + 84.87 = 3771 878.84 m
Y_t = 140 253.34 + 96.49 = 140 349.83 m
Z_t = 5124 304.35 + 116.95 = 5124 421.30 m

Example-2

This example combines the geographic/geocentric conversion of section 2.2.1 above with the geocentric translation method.

Consider a North Sea point with coordinates derived by GPS satellite in the WGS84 geographical coordinate reference system, with coordinates of:

latitude φ_s = 53°48'33.82"N,

longitude λ_s = 2°07'46.38"E,
and ellipsoidal height h_s = 73.0m,

whose coordinates are required in terms of the ED50 geographical coordinate reference system which takes the International 1924 ellipsoid. The three parameter geocentric translations method's parameter values from WGS84 to ED50 for this North Sea area are given as dX = +84.87m, dY = +96.49m, dZ = +116.95m.

The WGS 84 geographical coordinates first convert to the following geocentric values using the formulas from section 2.2.1:

X_S = 3771 793.97 m
Y_S = 140 253.34 m
Z_S = 5124 304.35 m

Applying the quoted geocentric translations to these, we obtain new geocentric values now related to ED50:

X_t = 3771 793.97 + 84.87 = 3771 878.84 m
Y_t = 140 253.34 + 96.49 = 140 349.83 m
Z_t = 5124 304.35 + 116.95 = 5124 421.30 m

Using the reverse conversion given in section 2.2.1 above, these convert to ED50 values on the International 1924 ellipsoid as:

latitude φ_t = 53°48'36.565"N,
longitude λ_t = 2°07'51.477"E,
and ellipsoidal height h_t = 28.02 m,

9-2-2 Seven parameter geocentric transformation

it is called Helmert transformation. It relates two datum systems through a rotation, an origin shift and a scale factor.

The transformation is expressed with seven parameters:

- three rotation angles (a,b,g);
- three origin shifts (ΔX,ΔY and ΔZ); and,
- one scale factor (s).

An example, the ITRF (X,Y,Z) coordinates of the given point in the state of Baden-Württemberg are transformed to the Potsdam datum (is a geodetic datum defined in Germany and established in 1983):

X (ITRF) = **4,156,939.96m**

Y (ITRF) = **671,428.74m**

Z (ITRF) = **4,774,958.21m**

The 7 parameters for transforming the point from ITRF to the Potsdam datum are given as follow:

α=+1.04sec, b=+0.35sec, γ=-3.08sec, ΔX=-581.99m, ΔY=-105.01m, ΔZ=-414.00m and $s=1-(8.3 * 10^{-6})=0.9999917$.

This set of parameters provided by the federal mapping organization of Germany was calculated using common points distributed throughout Germany.

Solution:

Applying these parameters to the given point results in the following Potsdam (X,Y,Z) coordinates:

X (Potsdam) = 4,156,305.34m

Y (Potsdam) = 671,404.31m

Z (Potsdam) = 4,774,508.25m

The two sets of transformed coordinates agree at a level of a few meters. The difference in X is ***0.38m***, in Y around ***2.57m***, and in Z around ***0.04m***.

These differences can be explained because of the different sets of transformation parameters. In a different country, the agreement could be at the level of centimetres or tens of meters and this depends primarily on the quality of implementation of the local horizontal datum.

Applicability

The 7-parameter transformation methods are most commonly encountered when transforming data acquired in a modern system such as GPS to a national coordinate reference system.

Errors in the order of one, two, or more metres might be encountered, depending on the extent of the area covered and the age of the original survey.

9-2-3 Ten parameter geocentric transformation

It is called the ***Molodensky-Badekas 10-parameter*** transformation relates two datum systems through a rotation, an origin shift and a scale factor. This is the same as for the Helmert transformation methods, but instead of deriving the rotations about the origin of the geocentric coordinate system, they are derived at a location within the points used in the determination of the parameters.

- Three additional parameters, the coordinates of the rotation point (*Xp*,*Yp*,*Zp*), are then required.
- The transformation is therefore expressed with 10 parameters:
- Three rotation angles (a,b,g); three origin shifts (ΔX,ΔY and ΔZ); one scale factor (*s*); and, the coordinates of the rotation point (*Xp*,*Yp*,*Zp*).
- Given in the source geocentric coordinate system.
- Compared to the Helmert transformation, the Molodensky-badekas usually provides a better approximation, but the transformation is not reversible.

9-3 Transformations between geographic coordinate reference systems

Datum transformations via the geographic coordinates directly relate the ellipsoidal latitude (ϕ) and longitude (λ), and possibly also the ellipsoidal height (h), of both datum systems. This is illustrated in the figure below.

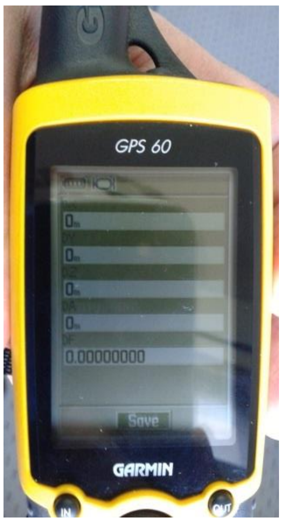
9-3-1- geographic offsets. It relates both datum systems with only two parameters, the difference in the geographic latitude ($\Delta\phi$) and the difference in the geographic longitude ($\Delta\lambda$).
The ellipsoidal height (h) is mostly not included. The method is only used for purposes where low accuracy can be tolerated. The equation is:

Example:
A position with coordinates of 38°08′36.565″N, 23°48′16.235″E referenced to the 2D coordinate reference system (EPSG CRS code 4120) is to be transformed t system (EPSG CRS code 4121). Transformation parameters from Greek to GGRS8

$$d\varphi = -5.86''$$
$$d\lambda = +0.28''$$

Then $\varphi_{GGRS87} = 38°08'36.565''N + (-5.86'') = 38°08'30.705''N$

and $\lambda_{GGRS87} = 23°48'16.235''E + 0.28'' = 23°48'16.515''E$



9-3-2- ***Molodensky*** and ***Abridged*** ***Molodensky*** transformation

A set of formulae that transforms from one set of ellipsoidal coordinates to another is due to Molodensky (Stansell 1978).

The Molodensky method and transformation parameter values for selected WGS 84 to local transformations are often incorporated into basic hand-held GPS receivers. Some receivers allow the user to enter their parameter values for Δa, Δf, ΔX, ΔY and ΔZ.

The standard equations directly relate ellipsoidal latitude and longitude coordinates and ellipsoidal height of two datums by deriving the geographic coordinate offsets.

$$\varphi_t = \varphi_s + d\varphi$$
$$\lambda_t = \lambda_s + d\lambda$$
$$h_t = h_s + dh$$

where

$$d\varphi'' = (-dX.\sin\varphi.\cos\lambda - dY.\sin\varphi.\sin\lambda + dZ.\cos\varphi + [a.df + f.da].\sin2\varphi) / (\rho . \sin1'')$$
$$d\lambda'' = (-dX.\sin\lambda + dY.\cos\lambda) / (\nu.\cos\varphi . \sin 1'')$$
$$dh = dX.\cos\varphi.\cos\lambda + dY.\cos\varphi.\sin\lambda + dZ.\sin\varphi + (a.df + f.da).\sin^2\varphi - da$$

$$da = a_t - a_s$$
$$df = f_t - f_s = 1/(1/f_t) - 1/(1/f_s).$$

Example:

For a North Sea point with coordinates derived by GPS satellite in the WGS84 geographical coordinate reference system, with coordinates of:

latitude φ_s = 53°48'33.82"N,
longitude λ_s = 2°07'46.38"E,
and ellipsoidal height h_s = 73.0m,

whose coordinates are required in terms of the ED50 geographical coordinate reference system which takes the International 1924 ellipsoid.

The three geocentric translations parameter values ***from*** WGS84 ***to*** ED50 for this North Sea area are given as dX = +84.87m, dY = +96.49m, dZ = +116.95m.

Ellipsoid Parameters are:

WGS 1984	a = 6378137.0 metres	1/f = 298.2572236
International 1924	a = 6378388.0 metres	1/f = 297.0

Then

da = 6378137 – 6378388 = –251
df = 0.003352811 - 0.003367003 = -1.41927E-05

whence

$d\varphi$ = 2.545"
$d\lambda$ = 5.097"
dh = – 44.98 m

ED50 values on the International 1924 ellipsoid are then:

latitude φ_t = 53°48'36.565"N
longitude λ_t = 2°07'51.477"E
and ellipsoidal height h_t = 28.02 m

Because ED50 is a geographical 2D coordinate reference system the height is dropped to give:

latitude φ_t = 53°48'36.565"N
longitude λ_t = 2°07'51.477"E

9-3-3-Reversibility

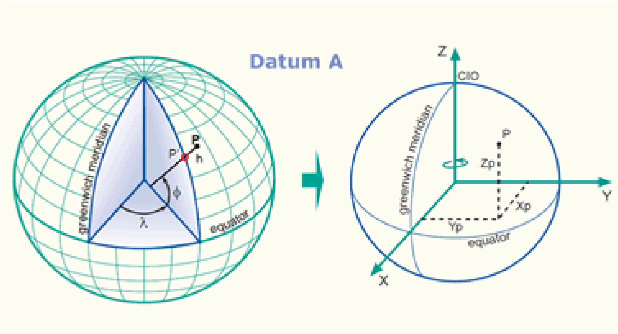
The transformation methods that have been mentioned earlier (Geocentric and geographic) are reversible, and it is possible to find the source by reversing the equation, as shown below:

$$\begin{pmatrix} \varphi \\ \lambda \\ h \end{pmatrix}_{Source} = \begin{pmatrix} \varphi \\ \lambda \\ h \end{pmatrix}_{Target} - \begin{pmatrix} \Delta\varphi \\ \Delta\lambda \\ \Delta h \end{pmatrix}$$

9.4 Coordinate system conversion

The coordinates of a point can be converted from one coordinate system to another. For instance, the ***Geographic coordinates*** (ϕ,λ) of a point P' (figure below) concerning the selected reference surface, they are also called **geodetic coordinates or ellipsoidal coordinates,** can be converted to or from Geocentric Cartesian coordinates using a set of formulas. As shown below:

9.4.1-In the case of required the Cartesian coordinates given the geodetic coordinates



$X = (v + h)\cos \emptyset \cos \lambda \ldots$ equ.(1)

$Y = (v + h)\cos \emptyset \sin \lambda \ldots$ equ.(2)

$Z = \{(1 - e^2)v + h\}\sin \emptyset \ldots$ equ.(3)

$v = \frac{a}{(1-e^2 sin^2 \phi)^{1/2}} \ldots$ equ.(4)

Where:

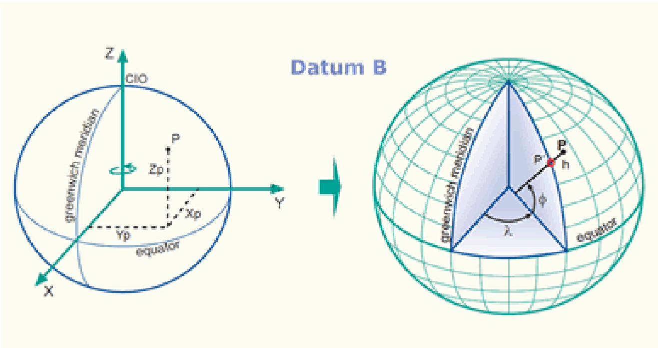
φ is the latitude, positive north;

λ is the longitude, positive east;

h is the ellipsoidal height (the height above the ellipsoid surface).

v is the radius of curvature in the prime vertical because the radius of the ellipsoid changes with the angle from the major axis

9.4.2: In the case that is required is the 3D geographical coordinates given the Cartesian coordinates:



$\tan\lambda = \frac{Y}{X}$ … equ.(5)

$\tan\phi = \frac{Z+\varepsilon\, b\, sin^3 u}{p-e^2\, a\, cos^3 u}$ … equ.(6)

$h = p\sec\phi - v$ … equ.(7)

Where:

a is the semi-major axis

b is the semi-minor axis of the ellipsoid.

$\mathbf{P} = (\mathrm{X}^2 + \mathrm{Y}^2)^{1/2}$… equ.(8)

$\tan \mathrm{u} = \frac{\mathrm{Z}}{\mathrm{P}}\frac{\mathrm{a}}{\mathrm{b}}$……….. equ.(9)

$\boldsymbol{\varepsilon} = \frac{\mathrm{e}^2}{1-\mathrm{e}^2}$…………... equ.(10)

N.B. the semi-major axis unit must be the same of the length units of the Cartesian axes.

Example: given the geodetic coordinate f the point P1, (52º39’27.253’’ N, 1º43’’4.5177’’E, H,24.70m), the coordinates are latitude, longitude and z. Find the Cartesian coordinate of the point p, if the value of the e2 and v 6.6705397616E-03 and 6.3910506260E+6, respectively.

Solution:

Using the equations 1, 2 and 3 to determine the coordinates of point P1.

Convert the latitude and longitude to radians from degrees.

Latitude conversion 52º39’27.253’’ N ➔ 0.9190479779rad

Longitude conversion 1º43’’4.5177’’E➔ 0.0299833879rad

$X = (v + h)\cos\emptyset\cos\lambda$➔

$$X = (6.3910506260E6 \quad + 24.7)\boldsymbol{cos}\,(0.9190479779)\,\boldsymbol{cos}\,(0.0299833879)$$

X =3874938.849m

$Y = (v + h)\cos\emptyset\sin\lambda$ ➔

$Y = (6.3910506260E6 \quad + 24.7)\,\mathbf{cos}(\,0.9190479779)\,\mathbf{sin}(0.0299833879)$➔

Y= 116218.624 m

$Z = \{(1 - e^2)v + h\}\boldsymbol{sin\emptyset}$➔

$Z = \{(1 - 6.6705397616\mathrm{E} - 03)6.3910506260\mathrm{E}6 + 24.7\}sin0.9190479779$➔

$$Z = 5047168.207 \text{ m}$$

H.W. prove that the above Cartesian coordinate (3874938.849m, 116218.624m , 5047168.207m), is equivlant to the geodetic coordinates (52º39’27.253” N, 1º43’’4.5177”E, 24.70m)