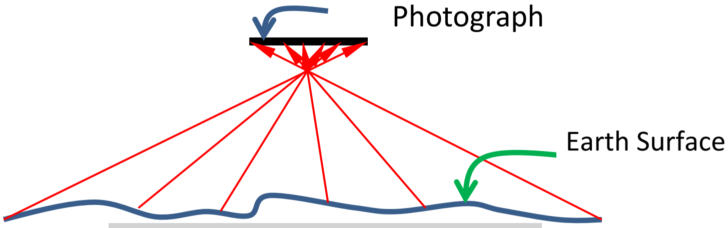
1. Photography scale

The printed view on the photographs cannot be used directly to infer the measurements because the view that is represented is perspective projection, which means all the rays intersect at a point. In order to be able to measure dimensions on the photographs, they should be planimetric or ortho.

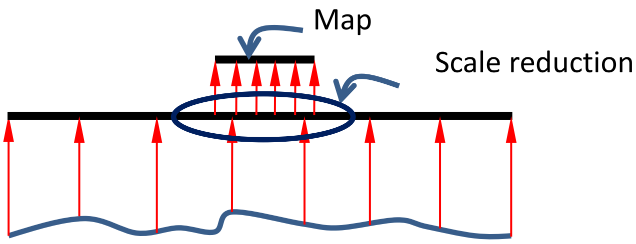
- In a **perspective** view, all light rays reflected from the Earth's surface pass through a single point at the centre of the camera lens. Therefore, the scale varies in perspective views.



Perspective projection (photograph)

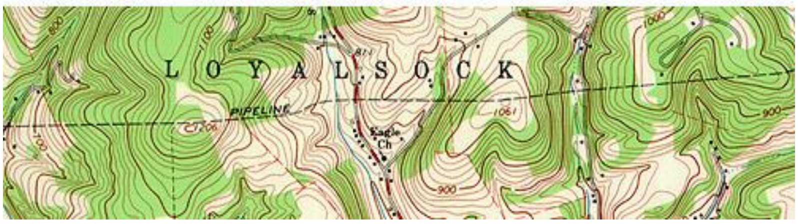
- By contrast, a planimetric (plan) view looks as though every position on the ground is being viewed from directly above. Therefore, the scale is constant over all the images.

In plan views, scale is consistent everywhere (if we overlook variations in small-scale maps due to map projections). Topographic maps are said to be **planimetrically correct**.



Map projection

Compare the map and photograph in the below figures. Both figures show the same gas pipeline, which passes through hilly Terrain. Note the deformation of the pipeline route in the photo relative to the shape of the route on the topographic map. **The deformation in the photo is caused by relief displacement.** The photo would not serve well on its own as a source for topographic mapping.



2. Types of scale for the aerial photography

Scale is what determines the relationship between the objects imaged on a photograph and their corresponding in the real world (i.e., the ground). This information also determines the amount of detail shown on a photograph or a map. The absence of scale makes it impossible to relate the size or distance between objects in a photograph to their actual sizes or distances on the ground.

The scale of the photograph depends on:

- The variation in elevation
- The flying height above ground features and,
- The effects of relief and tilt.

2.1.Vertical Photographs Scale,

A photograph may only present a uniform scale comparable to a map if the camera is perfectly vertical at the time of exposure and the Terrain is perfectly flat. Thus, the scale of vertical photograph imagery can be calculated over the flat surface by using equation 3-1

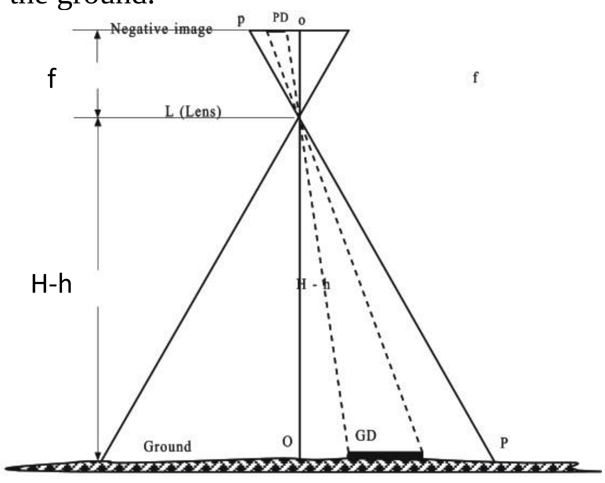
$$scale = \frac{image\ distance}{Object\ distance} = \frac{PD}{GD} \dots\dots Equ. (3-1)$$

From the geometry below figure, the scale of the vertical aerial photograph can be calculated using equation (3-2).

$$scale = \frac{f}{H-h} \dots Equ. (3-2)$$

Where:

- F is the focal length
- H, is the height above mean sea level
- h, is the elevation of the ground.



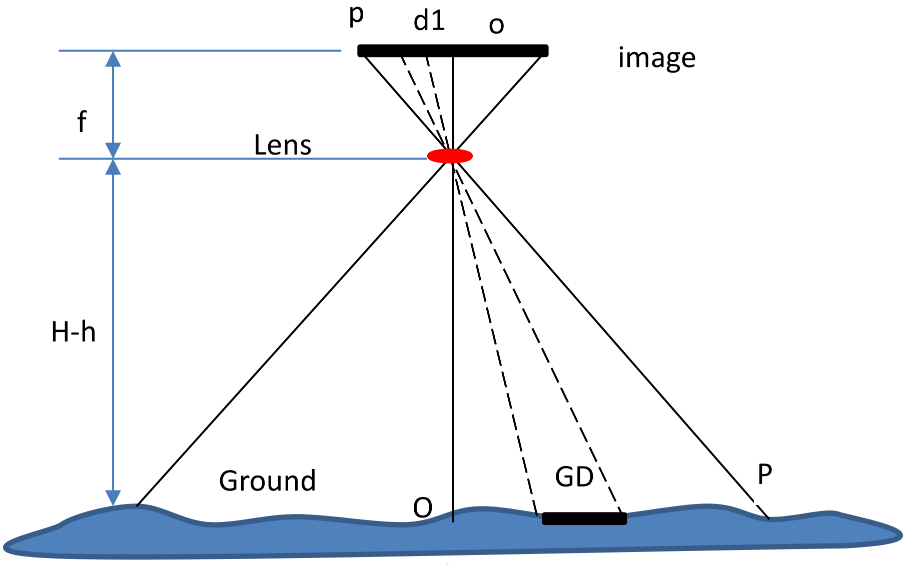
Example 1: A vertical photograph is taken at an altitude of 800m above mean sea level. The Terrain is flat and has an elevation of 237m. The camera focal length is 50mm. What is the scale?
Solution: From the equation (3-2) for the aerial photograph,

$$scale = \frac{f}{H-h} = \frac{0.050m}{800m - 237\,m} = 8.8810 * 10^{-5} = \frac{1}{11,260}$$

This can be expressed as 1:11,260. Usually, the scale is a nominal value rounded to two or three significant figures 1:11,260 ≈ 1:11,300.

Note: Since the condition of the vertical photographs and flat surface are seldom fulfilled, the photographic scale is usually considered for individual points, "which is referred to as *point scale"* or the average of a set of points, "which is referred to as *average scale"*.

2.2. **A point scale,** the most accurate type of scale, is the scale at a point with a specific elevation on the ground. This suggests that every moment on a vertical photograph at a different elevation will have a different scale (below figure). Therefore, a photograph taken over a rugged terrain will display a varying range of scales associated with the variations in elevation of the ground.



$$scale\ PS_p = \frac{f}{H - h_p} \ldots Equ.(3-3)$$

where:
PS_P is the photo scale at point P,
f is the focal length of the camera used to take a photograph,
H is the flying height of the aircraft above *MSL*, the distance between *MSL* and the lens,
h_P is the elevation of point P above *MSL*, and
$H\text{-}h_P = H_P$ is the flying height of the aircraft above point P.

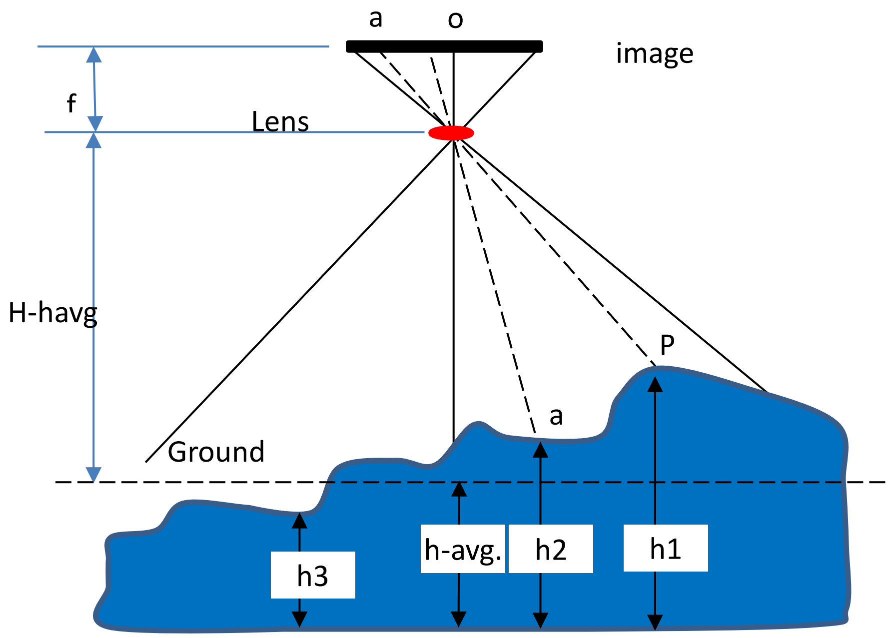
Example 2: A 305-mm focal length camera was used to take photographs from 4000 m above MSL. Find the scale of a point that is 800 m above MSL and the scale of a point that is at the MSL.
***Solution*:**

$$scale\ PS(1) = \frac{f}{H - h_p} = \frac{0.305\ m}{4000\ m - 800\ m} = 1{:}\,10490 \approx 1{:}\,10500$$

$$scale\ PS(2) = \frac{f}{H - h_p} = \frac{0.305\ m}{4000\ m} = 1{:}\,13110 \approx 1{:}\,13100$$

2.3. ***Average scale***

It is also referred to as the *nominal scale* because the average project scale may not represent the actual scale of each photograph or each point on a photograph. Unlike the point scale, which is specific to a single point on the ground, the average scale may be determined for the entire project area, a set of photographs, a single photograph, a portion of a photograph, or between two points on a photograph. Although the average scale is not as accurate as the point scale, it is commonly used.



This scale may be expressed as:

$$scale\ PS_{av} = \frac{f}{H - h_{av}} = \frac{f}{H_{av}} \dots eq\ (3-4)$$

Where:

PS_{av} is the average scale of the area considered (project, set of photographs, etc.),

h_{av} is the average elevation of the area, and

$H\text{-}h_{av} = H_{av}$ is the aircraft's flying height above the area's average elevation.

Or

$H\text{-}h_D = H_D$, the calculated scale will give at the datum by taking the flight above the datum.

Example 3: A photograph was taken from a flying height of 4500 m above sea level with a 152.4-mm focal length camera. If the average elevation of the photograph is 580 m above MSL, what is the average scale of the photograph? What would be the scale of a road junction that is at 520 m above *MSL*?

***Solution*:** Using equation (3-5), the scale of the photograph for the area is:

$$(1) scale\ PS_{av} = \frac{f}{H - h_{av}} = \frac{0.1524\ m}{4500\ m - 520\ m} = 1{:}\,26120 \approx 1{:}\,26100$$

$$(2) scale\ PS_{ap} = \frac{f}{H - h_{ap}} = \frac{0.1524\ m}{4500\ m - 580\ m} = 1{:}\,25720 \approx 1{:}\,25700$$

Example4: A 152.4-mm focal length was used to take photographs from 4000 m above *MSL* over an area that has an average elevation of 560 m above *MSL*. Find the average scale of the area. Compare this scale to that of a plane 300 m below the average elevation and that of a ridge 250 m above the average elevation.

***Solution*:** Using equation (3-4), the average scale of the area is:

$$(1) scale\ PS_{av} = \frac{f}{H - h_{av}} = \frac{0.1524\ m}{4000\ m - 560\ m} = 1{:}\,22570$$

and using equation (3-3), the scales at the plane and at the ridge are, respectively:

$$(2) scale\ PS_{300} = \frac{f}{H - h_{300}} = \frac{0.1524\ m}{4000\ m - 260\ m} = 1{:}\,24540$$

$$(3) scale\ PS_{750} = \frac{f}{H - h_{750}} = \frac{0.1524\ m}{4000\ m - 810\ m} = 1{:}\,20932$$

Therefore, an average scale of 1:22570 would be too large for the plane (whose actual scale is 1:24540) and too small for the ridge (whose actual scale is 1:20932).

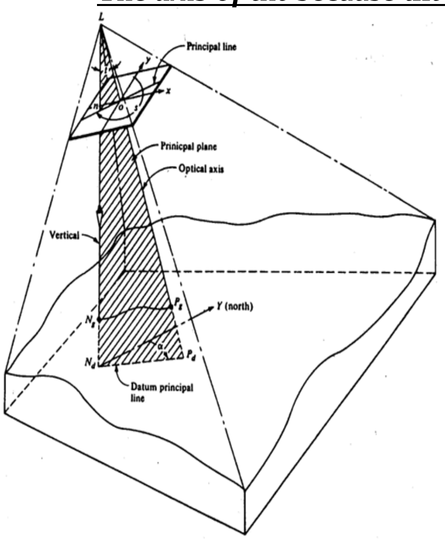
Determining the scale of a tilted photograph

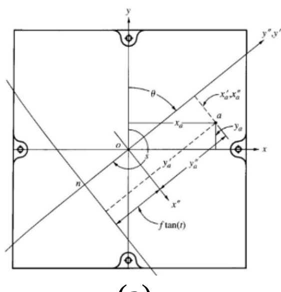
The scale determination on a tilted photograph differs from that on a vertical picture. Furthermore, the scale on a tilted photograph is increasingly altered as the distance from the ***principal point*** increases.

Due to the effect of tilt, only the point scale is effective and meaningful.

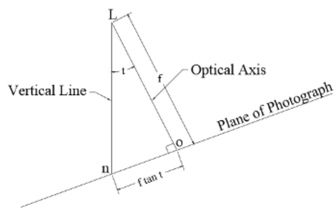
The scale of a point on a tilted photograph requires the followings:

- Flying height,
- The focal length of the camera,
- The elevation of the point,
- The position of the point with respect to the ***principal line*** and**,**
- The ***axis of tilt*** because tilt occurs in the direction of the principal line.





(a)



(b)

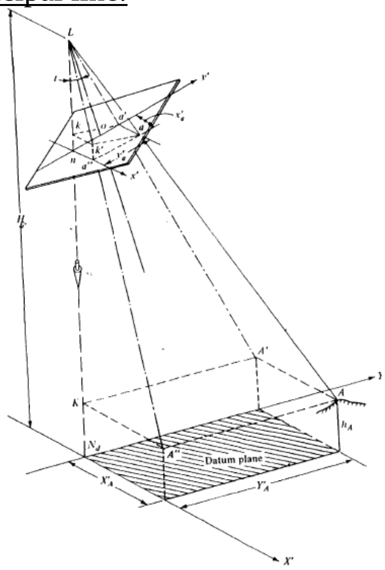


Fig-1: Geometry of a tilted photograph showing

Fig-2:(a) Auxiliary coordinate system (b)principle plane of a tilted photo

Fig-3: Scale of tilted photo and ground coordinate system

For any point in the tilted photo, the conversion from xy fiducial system to the x'y' tilted system can be obtained from

1- rotation about the principle point through the angle θ:

$$\theta = S - 180^{o} \quad \text{.....equ. (3-5)}$$

2- the coordinate of the point after translation of or origin

$$x_a' = x_a . \cos\theta - y_a \sin\theta \ldots\ldots equ. (3-6)$$

$$y_a'' = x_a . \sin\theta + y_a \cos\theta \ldots\ldots equ. (3-7)$$

By adding translation distance to the y'', as shown in fig2b, the final coordinates of the tilted photo will be

$$x_a' = x_a . \cos\theta - y_a \sin\theta \ldots\ldots equ. (3-8)$$

$$y_a' = x_a . \sin\theta + y_a \cos\theta + \text{f tan t} \ldots\ldots equ. (3-9)$$

From figure-3, the scale of the tilted photo will be:

$$scale\ of\ Pt_a = \frac{aa'}{AA'} = \frac{La'}{LA'} = \frac{Lk}{LK} \ldots\ldots equ. (3-10)$$

but

$$Lk = Ln - kn = \frac{f}{\cos t} - y_a' . \sin t \ \ldots\ldots equ. (3-11)$$

and

$$LK = H - h_A \ \ldots\ldots equ. (3-12)$$

Substituting 3-7 and 3-8 in 3-6 the scale will be

$$Stp = \frac{\frac{f}{cost} - y' . \sin t}{H - h} \ \ldots\ldots \text{equ. (3-13)}$$

Example 5: A tilted photo is taken with a 152.4mm focal length camera from a flying height of 3500m above the datum. Tilt and swing are 2°30' and 218°' respectively. Point A has an elevation of 450m above the datum, and its image coordinates concerning the fiducial axis system and xa=-7.2cm and ya=8.7cm. What is the scale of point a?

Solution

From equations 3-5,

θ=218°-180° =38°

from equation 3-9

$y_a' = -7.2 . \sin 38 + 8.7 \cos 38$ + 152.4/10* tan 2.5=3.09cm

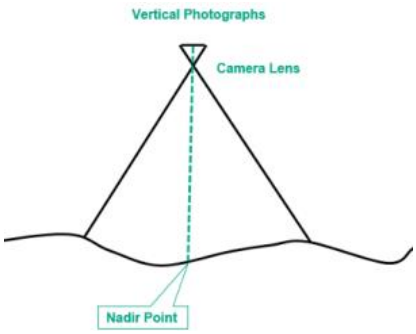
For the scale, and calculation, use the equation 3-13

$$Stp = \frac{\frac{152.4}{cos2.5} - 3.09 * 10 . \sin 2.5}{(3500 - 450) * 1000} = \frac{1}{171721} = \frac{1}{170{,}000}$$

3. **Relief displacement (Photogrammetry):** the shift or displacement in the photographic position of an image caused by the relief of the object.

The below figure shows the effect of the relief displacement on the buildings ad ground.

Probably it can be noticed that, except for the features on the nadir point, the buildings are perceived under a certain angle and that this angle changes depending on location in the image. The following figure illustrates how this came about.

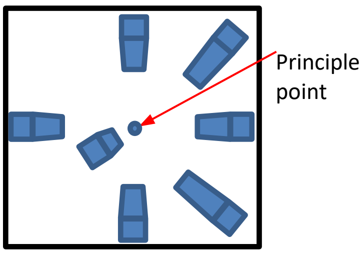


Nadir, in aerial photography, the point on the ground is vertically beneath the perspective centre of the camera lens.

Illustration of an aerial image deformation according to the distance from the nadir point, i.e. the centre of the image.

Buildings with the same height at a different location on the images, showing different relief displacement, underlie the following principles:

- Objects will tend to lean outward, i.e. be radially displaced.
- The taller the object, the greater the relief displacement.
- The further the object is from the principal point, the greater is the radial displacement.



In summary, the magnitude of the displacement in the image between the top and the bottom of an object is its relief displacement and is related to:

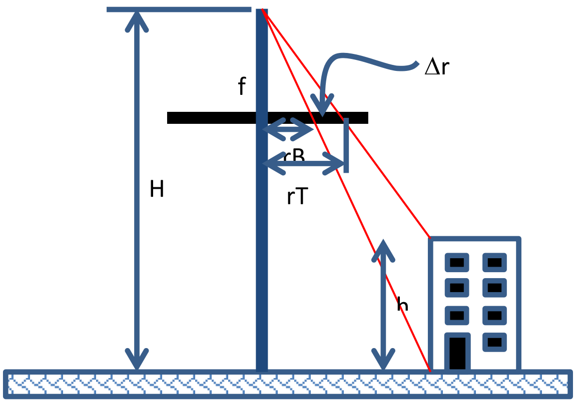
- The height of the object
- The distance from the nadir point

To derive an expression for the relationship between object height and relief displacement using geometry that is shown in the below figure:

$$\frac{r_B}{D} = \frac{f}{H} \quad , \quad D = \frac{H\, r_B}{f}$$

and

$$\frac{r_T}{D} = \frac{f}{H-h} \quad , \quad D = \frac{r_T(H-h)}{f}$$



By equating both equations with respect to D, the following will be obtained:

$$D = \frac{H\, r_B}{f} = \frac{r_T\,(H-h)}{f}$$

$$Hr_T - hr_T - H\, r_B = 0$$

$$H(r_T - r_B) = hr_T$$

$$\frac{H\Delta r}{r_T} = h$$

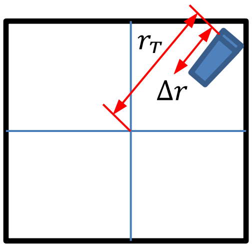
The last equation tells us that the flying height above the base of the object (H) times the relief displacement in the photograph (Δr) is divided by the radial distance from the principal point to the top of the object (r_T) is equal to the height of the object (h).
In conclusion, from the above discussion, if the flying height is known, it is possible to calculate the height of any object in a photograph.

Example: The flying height above the base of the building shown in the following figure is 500 m for a vertical photograph H. When measuring the image, the relief displacement of the building (Δr) is 4 mm, and the radial distance from the principal point to the top of the object (rT) is 75 mm.

What is the height of the building?

$$\frac{H\Delta r}{r_T} = h$$

$$500m\,\frac{4mm}{75mm} = 26.7m$$



Calculating Object Heights

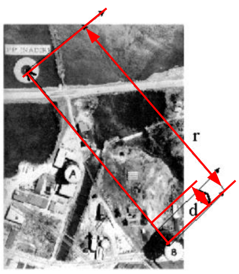
- Object heights can be determined as follows:
  - calculate flight altitude (H) by multiplying the RF denominator by the focal length of the camera
  - h = d * H / r

where:

h = Object height

d = length of object from base to top

r = distance from nadir to top of object



Example: Calculating object height from relief displacement

Photo Relief displacement for Tank B, d = 9.5 mm

Radial distance from P.P to top of Tank B = 127 mm

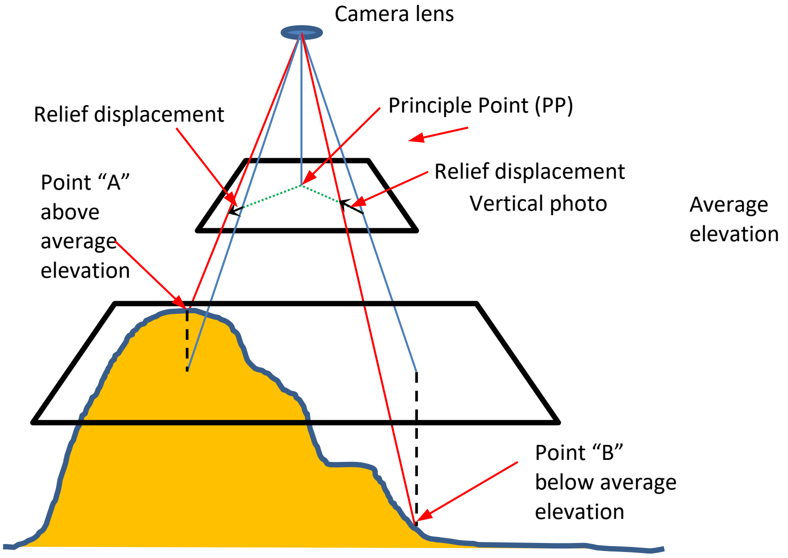
Flying Height above terrain, H = 914 m

h = d * H / r = (9.5 mm * 914 m) / 127 mm

= 68.3 m = 68 m

The relief displacement specified to be (as shown in the below figure);

- The higher the elevation of an object, the farther the object will be displaced from its actual position away from the **principal point** of the photograph (the point on the ground surface that is directly below the camera lens).
- Conversely, the lower the elevation of an object, the more it will be displaced toward the **principal point**.



Relief displacement is scale variation on aerial photographs caused by variations in terrain elevation.

The terrain elevation increases, flying height decreases and photo scale increases. As terrain elevation decreases, flying height increases and photo scale decreases.

Relief Displacement

Note: **Relief (Dic.):** “is the difference of elevations on a surface”.