1. Stereo Vision

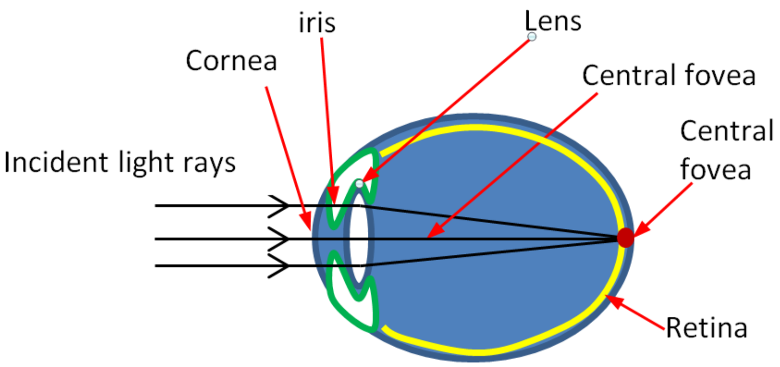
The human eyes view an object in stereo, this help to measure the depth of perception of the object's distance from the eye.

The person is possible to view objects using two types of systems:

- Binocular vision is viewing objects with both eyes simultaneously, and the perception of depth through binocular vision is called stereoscopic viewing.
- Monocular vision is used when viewing with only one eye, and judging the distance with one eye is called monoscopic.

1.1. Human eye

- The human eye and camera function in the same manner.
- Incident light passes through the corona. It is passed through the lens. The lens is connected to the eye by muscles which helps to move the lens so the optical axis can be directed to the object, similar to the camera.
- Similar to the aperture of the camera, the iris controls the light entering the eye.
- The corneas and the lens refracts the lights, so they focus and hit on the retina.
- The part of the intersection of the retina and the optical axis called the Central fovea will lead to producing a sharp image.

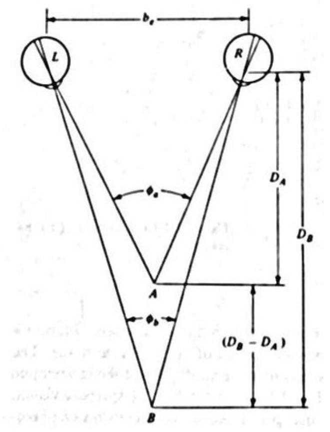


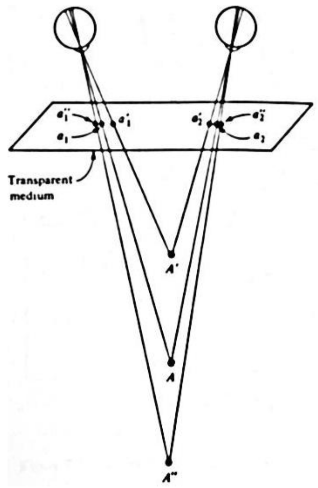
Cross section of the human eye

By illustrating the physiology of the human eye, it is possible to understand the phenomena of depth perception.

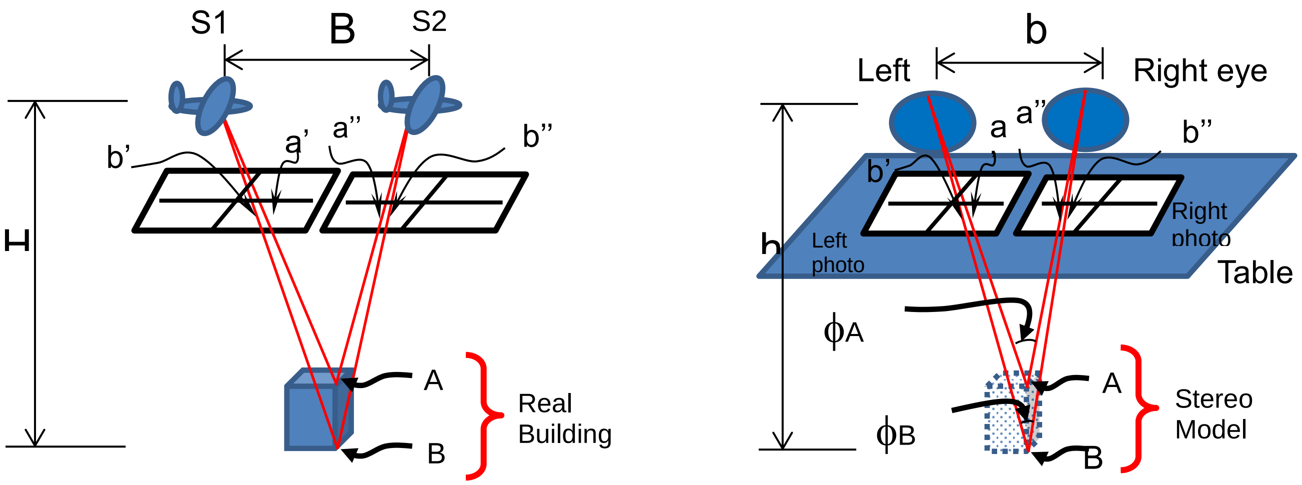
1.2. Stereoscopic Depth Perception

- Stereoscopic viewing is constructed by looking at the object with binocular vision.
- During looking at a certain point with both eyes and focusing on a point, the optical of the two eyes converge on the point, and the created angle is called **"parallactic".**
- The closer the object produces, the large angle and vice versa. The distance between left and right is called eye base "b", and the value for the average adult is about 63-69mm.
- The parallactic angle at point A is ϕ_a and at point B is ϕ_b., ϕ_a is greater than ϕ_b. The brain automatically associates the distances to points A and B from these.





For the building viewing using photogrammetry:



(Left image) photograph from two exposure statins with buildings in common (right photo) viewing the building stereoscopically

1.3. Stereo pairs

Two images of the same area, taken from different points of view, are called a **stereopair**.

In a stereopair, there is always an overlap (see red squares), where stereovision is possible because of the present Parallax. Outside these regions, you are not able to see in 3D.

Stereopair of College of Engineering Campus, 2012, Source College of engineering.

1.4. Viewing systems

To get these two images merged together in a 3D image, the observer has to see the pair of images through a viewing system called a stereoscope.

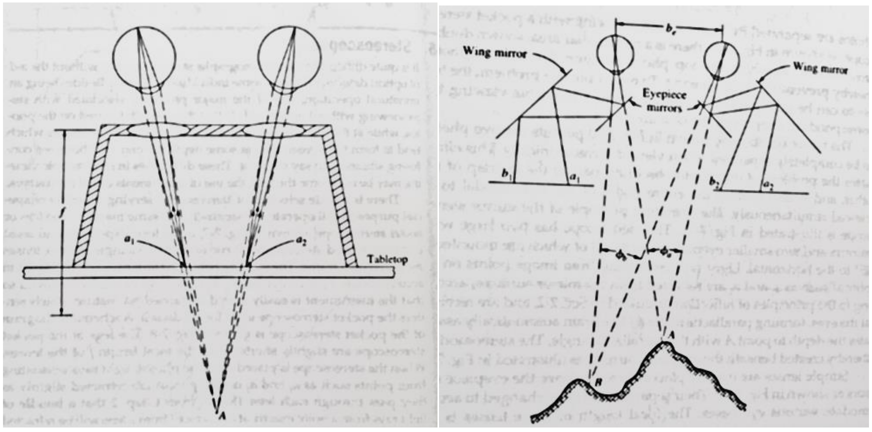
A stereoscope is a tool that assures that each eye sees only one of the stereo pairs.

Each eye thus sees a different image of the same scene. Each image is taken from a slightly different point of view.

When the brain merges the two images together, the observer experiences a 3D vision of the scene. This can be realized manually or digitally.

Manual vs digital stereoscopes. (Source: Ghent University)

The technique used to view the image stereoscopically is shown below. The left image represents viewing using pocket stereoscope, and the right image shows the viewing stereoscopically using a mirror stereoscope.



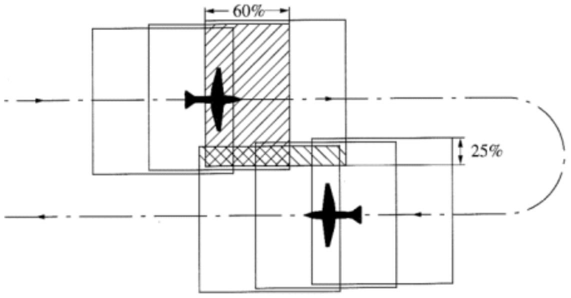
1.5. How do you obtain Stereopairs?

a-terrestrial photography

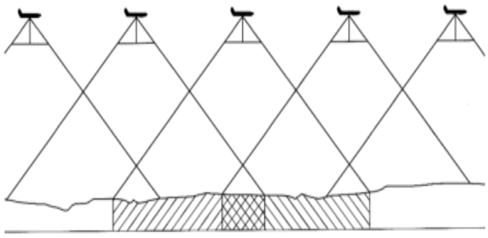
it is also called close-range photography. In this type, a regular camera is used to take photographs of the object. It is specified to deal with small and medium objects. The tilt can be controlled, and it is specified to be less expensive than an aerial camera

b- Aerial photography

Scientists first create a flight map when they want to map out a specific area. Then, flight lines are laid out on this flight map with a spacing that allows the photographs to cover a common strip of ground.



Aerial photography, view from above. Overlap is shown in striped.

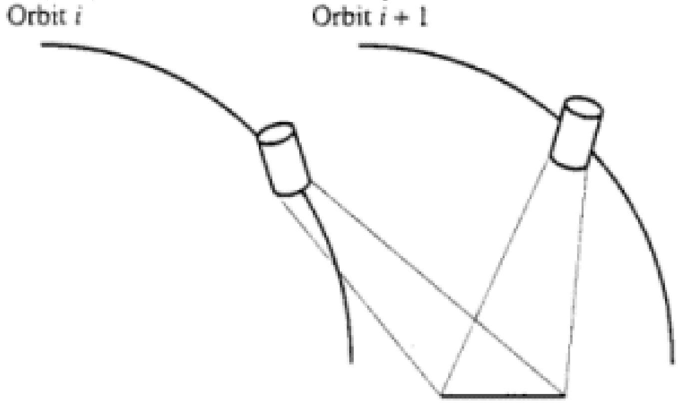


Aerial photography side view. Overlap is shown in striped.

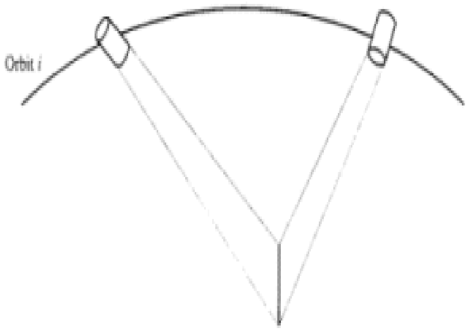
c-Satellites

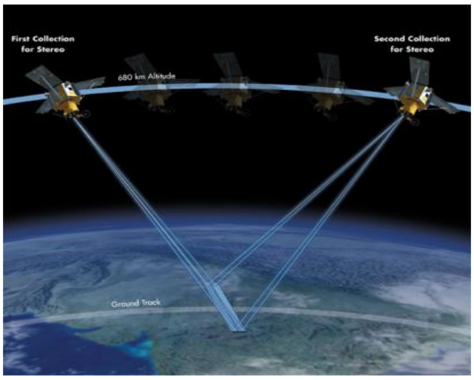
Satellites can make stereoscopic images in two ways:

- A satellite takes one image sideways when it is located in orbit (*i*) around the Earth. Then, when it is one orbit further (*i*+1) it makes another image of the same area.



- A satellite takes an image forward. Then, a few seconds later, it makes an image of the same area backward.





As mentioned earlier, the overlap area produces stereoscopic vision. Moreover, the Parallax is generated at that area, which can be implemented to measure the distance.

2. Parallax

The Parallax is the apparent displacement in the position of an object, for a frame of reference, caused by a shift in the observer's position.

To understand the Parallax, the below is a little experiment; a pencil has been held, and later the right eye is closed, and the left eye is left open to look at the object. The right image represents the object from the right eye and close the left. The produced difference in location is called Parallax, and it is used to measure the elevations on the photographs.

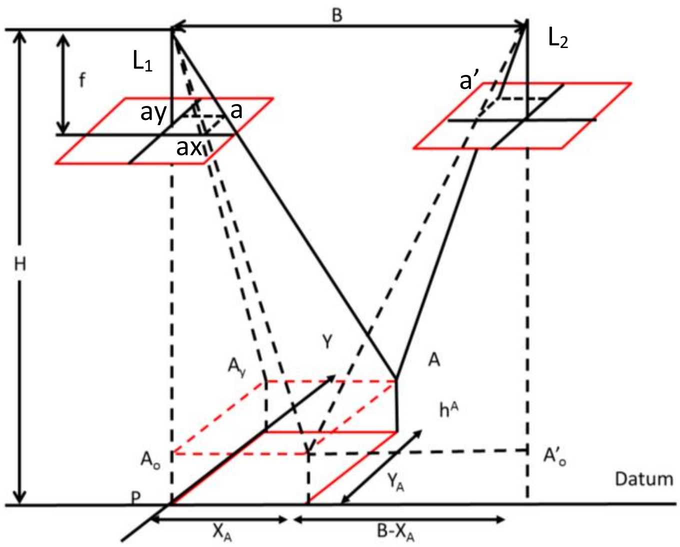
Left eye

Right eye

During image capturing, the aircraft produces an overlap area. This causing produces a parallax called **stereo parallax**, or **x-parallax** or simply **Parallax,** as shown in the below images.

2.1. Parallax equation

The X,Y and Z coordinate of a point can be measured using the measurements of their parallaxes.



The above figure will be used in order to derive the parallax equation.

Point A on the object space is represented by a and a'.

The planimetric position of point A on the ground is X_A and Y_A, It is the elevation above the datum is h_A.

By equating similar triangles of the above figure the values of points h_A, X_A and Y_A can be derived.

L_1oa_y and $L_1A_oA_y$,

$$\frac{Y_A}{H-h_A}=\frac{y_a}{f} \longrightarrow Y_A=\frac{y_a}{f}(H-h_A)-----(1)$$

And equating similar triangles L_1oa_x and $L_1A_oA_x$,

$$\frac{X_A}{H-h_A}=\frac{x_a}{f} \longrightarrow X_A=\frac{x_a}{f}(H-h_A)------(2)$$

And from similar triangles $L_2o'a'_x$ and $L_2A'_oA_x$,

$$\frac{B-X_A}{H-h_A}=\frac{-x_a}{f} \longrightarrow X_A=B+\frac{x'_a}{f}(H-h_A)------(3)$$

By equating the equations (2) and (3) and simplifying it:

$$h_A=H-\frac{Bf}{x_a-x'_a}------(4)$$

Substituting P_a for x_a-x'_a into the above it is possible to get

$$h_A=H-\frac{Bf}{\mathrm{P}_a}-----(5)$$

Substituting equation (5) into equations (1) and(2) and reducing it:

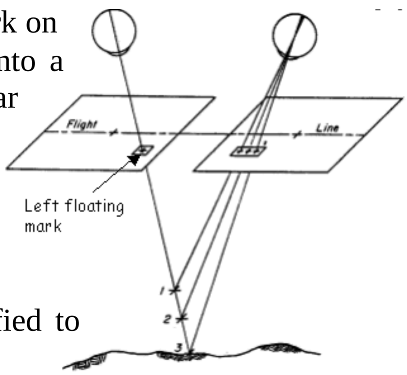
$$X_A=B\frac{x_a}{\mathrm{P}_a}-----(6)$$

$$Y_A=B\frac{y_a}{\mathrm{P}_a}-----(7)$$

2.2. Measuring the Parallax

The Parallax of a point can be measured during viewing the photos stereoscopically with the advantage of speed and accuracy. The ***floating mark*** principle can be utilized to measure the stereoscopic Parallax.

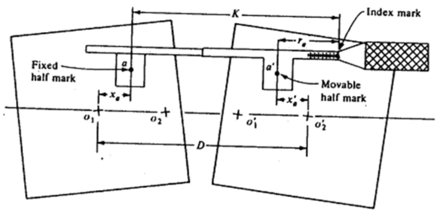
While viewing the photos' stereo model using a mirror-stereoscope, a mark attached to a piece of glass will be located on one point in the right photo and the other mark on the left photo. These marks will be shifted until they fuse together into a single mark that appears in the stereo model and lies at a particular elevation, as shown in the below figure. The device used to measure the Parallax is called the ***parallax bar.***



The floating mark may be moved about the stereo model from point to point as the terrain varies in elevation.

2.3. Parallax bar constants.

For Parallax measuring manually, a parallax bar is used. It is specified to measure up to micrometres.



The below equation is used to determine the Parallax

$$P_a=C+r_a------(8)$$

Where:

Pa parallax of the object point on the ground.

r_a is the micrometre reading on the parallax bar.

C is the parallax bar constant.

Before using the Parallax on the photos, the parallax bar constant C is determined. From the below figure C is the constant which is obtained from D-K, using the below formula,

$$C = p - r \quad ------(9)$$

To calculate the constant C, the following steps are used:

1-Distance b and b' are measured.

2-The floating mark is fused on the left principle point, and the measurement micrometre reading r_{o1} is taken and from it the value $C_1 = b' - r_{o1}$

3-The floating mark is fused on the left principle point, and the measurement micrometre reading r_{o1} is taken and from it the value $C_2 = b \quad - r_{o2}$

Generally, the values C1 and C2 are slightly different. This is due to tilting in the photograph, unequal flying height, paper shrinkage, and, measurement.

Example-1: A pair of overlapping vertical photographs were taken from a flying height of 4,045ft above sea level; with a 152.4mm-focal camera. The airbase was 1,280 ft. with the photos properly oriented, and parallax bar readings of 12.57mm and 13.04mm were obtained with the floating mark set o principle points o_1 and o_2, respectively. On the left photo b was measured as 93.73mm, and on the right photo b' was measured as 93.30mm. Parallax bar readings of 10.96mm and 15.27mm were taken on points A and B. Also, the x and y photo coordinates of points A and B measured with respect to the flight axis on the left photo were x_a=53.41mm, y_a=50.84mm, x_b=88.92mm, and y_b=-46.69mm. Calculate the elevations of points A and B and the horizontal length of line AB.

Solution:

Using equation (9) to find the parallax bar constant

C_1=b'-r_{o1}=93.30-12.57=80.73mm

C_2=b-r_{o2}=93.730-13.04=80.69mm

$$C = \frac{80.73 + 80.69}{2} = 80.71mm$$

Using equation 8 ,

$$P_a = C + r_a = 8071 + 10.96 = 91.67mm$$
$$P_b = C + r_b = 80.71 + 15.27 = 95.98mm$$

Using equation 5,

$h_A = H - \frac{Bf}{P_a} = 4{,}045 - \frac{1{,}280(152.4)}{91.67}$ =1,917 ft above sea level

$h_b = H - \frac{Bf}{P_b} = 4{,}045 - \frac{1{,}280(152.4)}{95.98} = 2{,}012$ ft above sea level

Using equations 6 and 7,

$X_A = B\frac{x_a}{P_a} = 1280.\frac{53.41}{91.67} = 746$ ft

$X_B = B\frac{x_b}{P_b} = 1280.\frac{88.92}{95.98} = 1{,}186$ ft

$Y_A = B\frac{y_a}{P_a} = 1280.\frac{50.84}{91.67} = 710$ft

$Y_B = B\frac{y_b}{P_b} = 1280.\frac{-46.69}{95.98} =$ -623ft

The horizontal length of line AB is,

$$AB \quad = \sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2}$$

$=\sqrt{(1{,}186 - 746)^2 + (-623 - 710)^2}$=1,404ft

2.4. using stereoscopic Parallax for the Elevation difference

The variance in the Parallax is caused by the difference in the elevation between one point and another. In the below figure the elevation of control point C, whose elevation h_c above datum is known. And it is required to calculate the elevation of point A.

From the parallax equation, sec 5.6 (lecture-5) the elevation ha is obtained using $h_A = H - \frac{Bf}{P_a}$. by rearranging the equation, it is possible to obtain:

$$p_c = \frac{Bf}{H - h_C} - - - - - - (8)$$

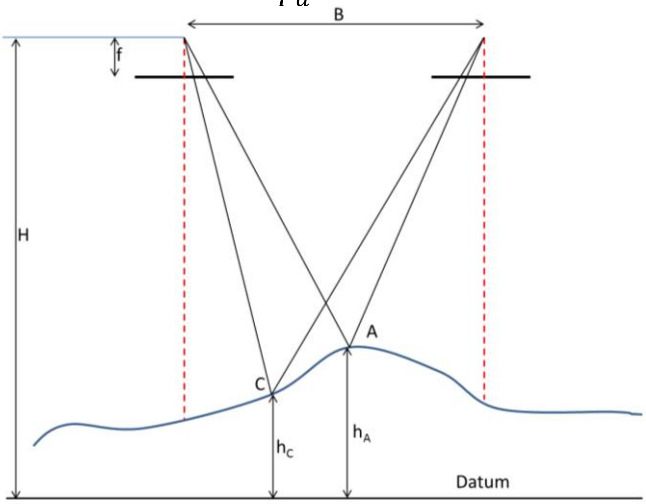
$$p_a = \frac{Bf}{H - h_A} - - - - - - (9)$$

By subtracting eq.(8) from eq.(9), the difference of Parallax will be obtained, $P_a - P_c$, and rearranging the equations:

$$p_a - p_c = \frac{Bf(h_A - h_c)}{(H - h_A)(H - h_c)} - - - - (10)$$

Let $P_a - P_c = \varDelta P$, the difference in Parallax, and from equation (9) substituting $H - h_A$, into equation (10) the following will be obtained:

$$h_A = h_C + \frac{\Delta p(H - h_c)}{p_a} - - - - (10)$$



Example 2: In example 1, an additional parallax bar reading of 11.89 was taken on control point C, whose elevation is 1,938 ft above sea level. Calculate the elevations of points A and B of that example using parallax difference Eq.10.

Using equation 8,

$$p_c = C + r_c = 80.71 + 11.89 = 92.6mm$$

For point A,

$$\varDelta p = p_a - p_c = 91.67 - 92.60 = -0.93mm$$

Using equation 10,

$$h_A = 1{,}938 + \frac{(-0.93)(4{,}045 - 1{,}938)}{91.67} = 1{,}917\ ft\ above\ sea\ level$$

For point B,

$$\varDelta p = p_b - p_c = 95.98 - 92.60 = 3.38mm$$

Using equation 10,

$$h_B = 1{,}938 + \frac{3.384(4{,}045 - 1{,}938)}{95.98} = 2{,}012\ ft\ above\ sea\ level$$

Note compare the obtained value that was obtained in example 1.

If a number of control points used in the overlap area, using equation 10 permits elevation of unknown points to be most accurately determined from the parallax difference of the nearest control point.

H.W.1 the photo coordinate of the images for a and b, which are of two points A and B measured on vertical photographs taken at a flying height of 1600m from a camera having a lens of focal length 150mm, are as follows:

Points	Coordinates	
	x (mm)	y(mm)
a	- 46.35	- 48.2
b	+38.48	+41.62

The Elevation of A and B above the mean sea level is 140m and 220m, respectively and the distance between the exposure stations are 350m. Determine the distance AB.

3. Error Evaluation

The obtained results (i.e. coordinates) contain an error. These errors arise due to different sources and must be evaluated. They obtained due to different sources such as:

1. Locating and marking the flight lines on photos,
2. orienting stereo pairs for parallax measurement,
3. Parallax and photo coordinate measurements,
4. Shrinkage or expansion of photographs,
5. Unequal flying heights for the two photos of stereo pair,
6. Tilted photographs,
7. Errors in ground control,
8. Other errors of lesser consequence such as camera lens distortion and atmospheric refraction distortion.

Example 3: compute the resulting error in the elevation of a point computed via photogrammetry. Suppose the random error in H were ±2m, in B were ±2m, and ±0.1mm in Pa. using the value of f=152.4 mm and B=390 m.

Solution:

the partial derivation of the basic equation $h_A = H - \frac{Bf}{\mathrm{P}_a}$, respect to the H,B and Pa is

$$\frac{\partial h_A}{\partial H} = 1 \qquad \frac{\partial h_A}{\partial B} = -\frac{f}{\mathrm{P}_a} \qquad \frac{\partial h_A}{\partial \mathrm{P}_a} = \frac{Bf}{(\mathrm{P}_a)^2}$$

Using statistical error propagation, the estimated error is:

$$\sigma_{hA} = \pm\sqrt{\left(\frac{\partial h_A}{\partial H}\right)^2 \sigma_H^2 + \left(\frac{\partial h_A}{\partial B}\right)^2 \sigma_B^2 + \left(\frac{\partial h_A}{\partial \mathrm{P}_a}\right)^2 \sigma_{Pa}^2}$$

Thus, the error in the hA will be:

$$\sigma_{hA} = \pm\sqrt{(1)^2(2)^2 + \left(\frac{-152.4}{91.7}\right)^2 (2)^2 + \left(\frac{390(152.4)}{91.7^2}\right)^2 (0.1)^2} = \pm 3.9m$$