1. Coordinate systems

Two primary references coordinate systems will be considered in photogrammetry:

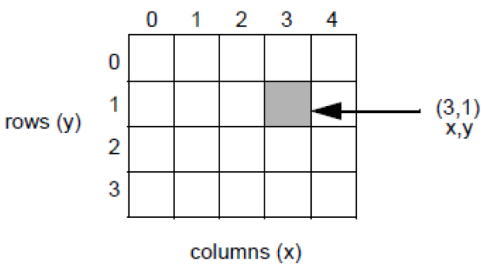
- Image-space coordinate system and object space coordinate system.
- These coordinates will be regarded as cartesian and right-handed.
- Different coordinates systems can be transformed into cartesian such as ellipsoidal object space coordinates expressed as latitude, longitude and height $(\emptyset, \lambda, h)$, which might be transformed into a local space rectangular (LSR) topocentric system or universal space rectangular (USR) geocentric system.
- The location of a pixel in a file or on a displayed or printed image is expressed using a coordinate system.

Generally, in some software, the primary coordinate systems used are:

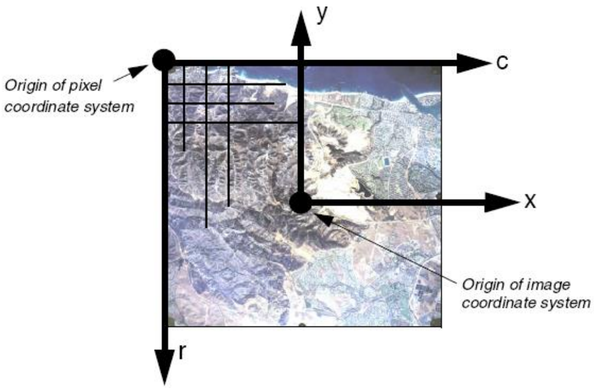
1.1. Pixel or File Coordinate System

Pixel coordinate system or sometimes called file coordinates (data file) of a digital image are defined in a pixel coordinate system, it is specified as:

- A pixel coordinate system is usually a coordinate system with its origin in the upper-left corner of the image,
- The x-axis points to the right, the y-axis points downward, and the units in pixels are shown by axes c and r in the figure below.
- These file coordinates (*c*, *r*) can also be considered the pixel column and row number, respectively.



1.2. Image Coordinate System



An image coordinate system or an image plane coordinate system is usually defined as a 2D coordinate system occurring on the image plane. It is specified as follows:

- Its origin is in the image centre. It is also referred to as the principal point.
- In aerial photographs, the principal point is defined as the intersection of opposite fiducial marks as illustrated by axes x, and y seethe figure.
- Image coordinates are used to describe positions on the film plane.

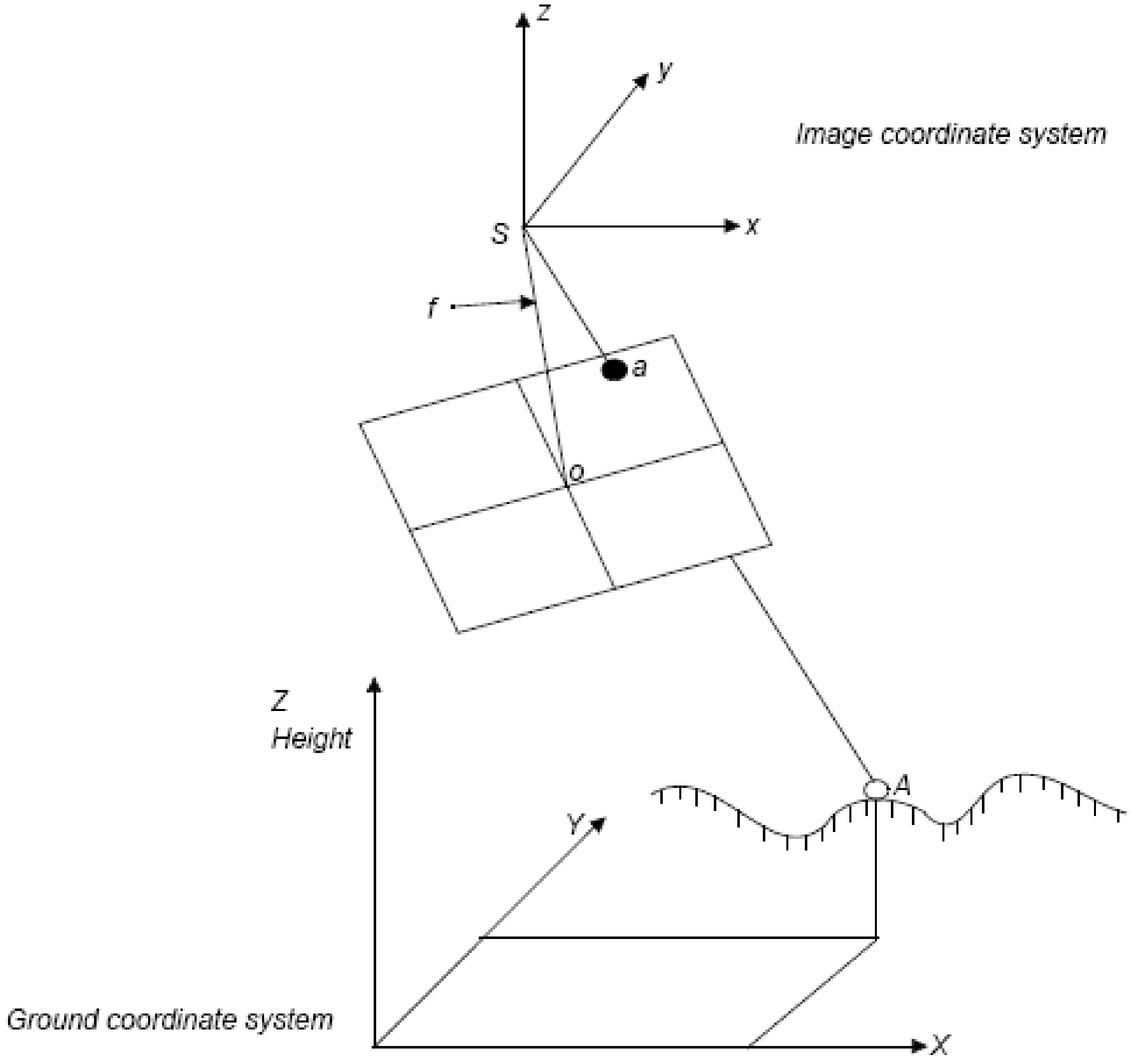
Image coordinate units are usually millimetres or microns.

1.3. Image Space Coordinate System

- An image space coordinate system, as shown in the fig below, is identical to image coordinates, except that it adds a third axis (z).
- The origin of the image space coordinate system is defined at the perspective centre *S* as shown in the figure below.
- Its x-axis and y-axis are parallel to the x-axis and y-axis in the image plane coordinate system.
- The z-axis is the optical axis; therefore, the z value of an image point in the image space coordinate system is usually equal to the focal length of the camera (f).
- Image-space coordinates are used to describe positions inside the camera and usually use units in millimetres or microns.

1.4. Ground Coordinate System

- A ground coordinate system (or object space) is usually defined as a 3D coordinate system that utilizes a known geographic map projection.
- Ground coordinates (X, Y, Z) are usually expressed in feet or meters.
- The Z value is the elevation above mean sea level for a given vertical datum.

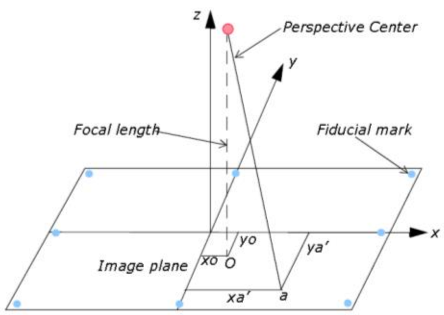


2. ***Interior orientation,*** is the process that defines the internal geometry of a camera or sensor as it existed at the time of data capture.

The variables associated with image space are defined during the process of interior orientation. Interior orientation is primarily used to transform the image pixel coordinate system or other images coordinate measurement system into the image space coordinate system.

The below figure illustrates the variables associated with the internal geometry of an image captured from an aerial camera, where *o* represents the principal point, and *a* represents an image point.

The internal geometry of a camera is defined by specifying the following variables: **principle point, focal length, fiducial marks and lens distortion.**

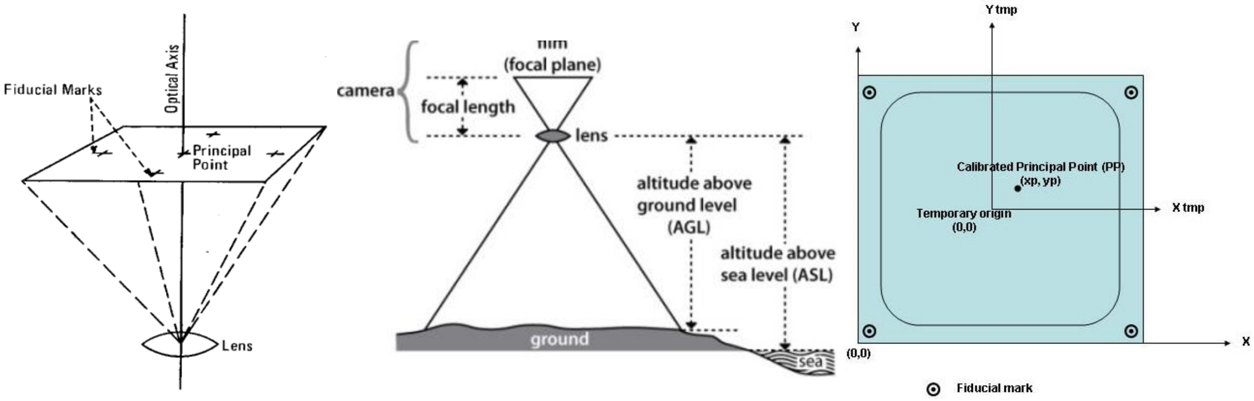


The internal geometry of a camera is defined by specifying the following variables:

2.1. Principal Point and Focal Length

A principal point is mathematically defined as the intersection of the perpendicular line through the perspective centre of the image plane. The length from the principal point to the perspective centre is called the focal length (Wang, Z., 1990).

- The intersection of the fiducial marks is called the fiducial centre, FC. This point is close but rarely coincides with the principle point(PP).
- The image plane is commonly referred to as the focal plane. For wide-angle aerial cameras, the focal length is approximately 152 mm, or 6 inches.
- For some digital cameras, the focal length is 28 mm.



- Prior to conducting photogrammetric projects, the focal length of a metric camera is accurately determined or calibrated in a laboratory environment.
- In the laboratory, this is calibrated in two forms: the principal point of autocollimation and the principal point of symmetry, which can be seen from the camera calibration report.
- Most applications prefer to use the principal point of symmetry since it can best compensate for the lens distortion.

Example: a nominal coordinate system is defined on a photograph with respect to the fiducial marks, with origin at FC. By camera calibration, either in a laboratory or by self-calibration in a block adjustment, the principal point is determined to be at location PPA, With coordinates $(x_o, y_o)=(0.015, -0.005)$. If a point P has coordinates (x,y)=(75.542,26.381) with respect to FC, what are its coordinates with respect to PPA?

Solution:

$$x' = x - x_o = 75.542 - 0.015 = 75.527$$
$$y' = y - y_o = 26.381 + 0.005 = 26.386$$

2.2. Lens Distortion

Lens distortion deteriorates the positional accuracy of image points located on the image plane, as shown in the below figure:

Details of the original image Details of the undistorted image

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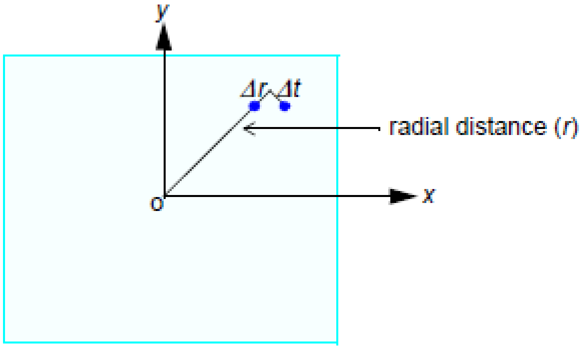
Two types of radial lens distortion exist ***radial*** *and* ***tangential lens distortion*.**

Lens distortion occurs when light rays passing through the lens are bent, thereby changing directions and intersecting the image plane at positions deviant from the norm. For example, the figure below illustrates the difference between radial and tangential lens distortion.



Radial vs. Tangential Lens Distortion

- Radial lens distortion causes imaged points to be distorted along radial lines from the principal point *o*.
- The effect of radial lens distortion is represented as Δr, and it is also commonly referred to as symmetric lens distortion.
- Tangential lens distortion occurs at right angles to the radial lines from the principal point.
- The effect of tangential lens distortion is represented as Δt. Since tangential lens distortion is much smaller in magnitude than radial lens distortion, it is considered negligible.
- The effects of lens distortion are commonly determined in a laboratory during the camera calibration procedure.
- The effects of radial lens distortion throughout an image can be approximated using a polynomial. For example, the following polynomial is used to determine coefficients associated with radial lens distortion:

$$\Delta r = k_1 r^1 + k_2 r^3 + k_3 r^5 + k_4 r^7 \text{------}(1)$$

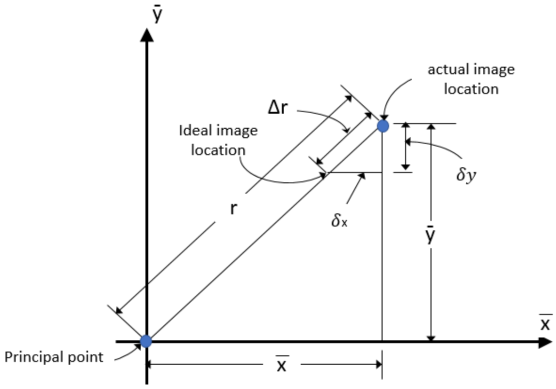
Where:

- Δr represents the radial distortion along a radial distance *r* from the principal pt.
- Coefficients (k_1, k_2, k_3 and k_4) are obtained from calibration reports computed using statistical techniques.

The below procedure is used to calculate the correct location of the x, y position:
1-convert the fiducial coordinates x and y, to coordinates x and y, to coordinates $\bar{x}$ and $\bar{y}$, relative to the principle point, using

$$\bar{x} = x - x_p \quad --- (2)$$
$$\bar{y} = y - y_p \quad --- (3)$$
$$r = \sqrt{\bar{x}^2 + \bar{y}^2} \quad --- (4)$$

2- Using equations 7-3 to find the Δr



3-The corrections δx and δy for the x and y components are calculated

$$\frac{\Delta r}{r} = \frac{\delta x}{\bar{x}} = \frac{\delta y}{\bar{y}} \quad --- (5)$$

From which

$$\delta x = \bar{x}\frac{\Delta r}{r} \quad --- (6)$$
$$\delta y = \bar{y}\frac{\Delta r}{r} \quad --- (7)$$

4- the corrected coordinates x_c and y_c Are calculated from:

$$x_c = \bar{x} - \delta x \quad --- (8)$$
$$y_c = \bar{y} - \delta y \quad --- (9)$$

Example: An older USGS camera calibration report specifies the calibrated focal length f=153.206mm and coordinates of the calibrated Principe point as x_p=0.008mm and y_p=-0.001mm. Using the calibration values, compute the corrected coordinates for an image point having coordinates x=62.579mm, y=-90.916mm relative to the fiducial axes. The computed k values using the least square method are k_1=0.2296, k_2=-35.89, k_3=1018, and k_4=12,100.

Solution: Compute the distance from the principle point to the image point using (2) and (3).

$$\bar{x} = x - x_p = 62.579 - 0.008 = 62.571mm = 0.062571m$$
$$\bar{y} = y - y_p = -80.916 - (-0.001) = -80.915mm = -0.080915m$$
$$r = \sqrt{\bar{x}^2 + \bar{y}^2} = \sqrt{0.062571^2 + (-0.080915)^2} = 0.1023m$$

Given the value for r and the k coefficients, compute Δr

$$\Delta r = (0.2296)(0.1023) + (-35.89)(0.1023)^3 + (1018)(0.1023)^5 + (12{,}100)(0.1023)^7$$
= -0.0021mm

Compute δx and δy:

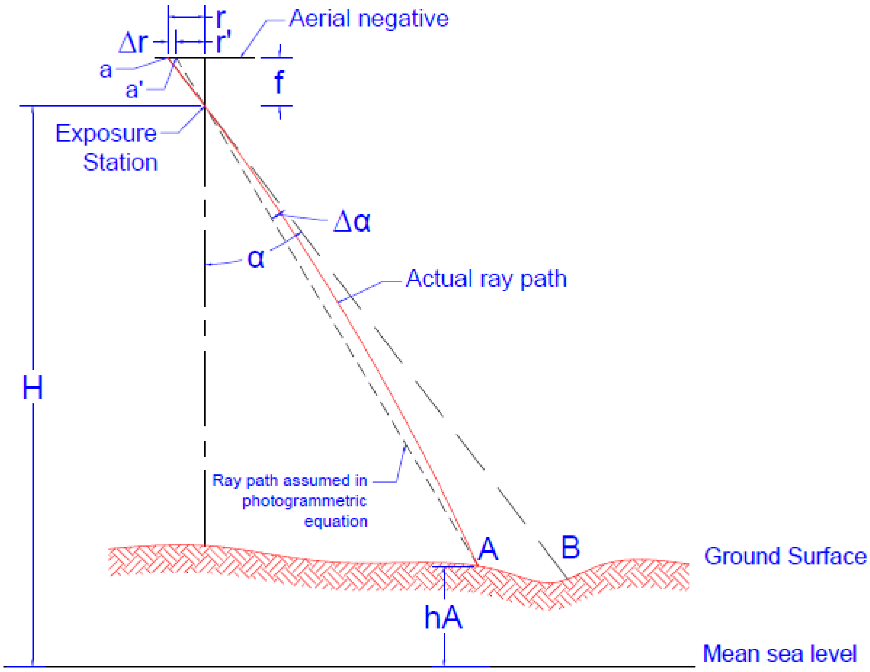
$$\delta x = 0.062571\frac{-0.0021}{0.1023} = -0.0013mm$$
$$\delta y = -0.080915\frac{-0.0021}{0.1023} = 0.0017mm$$

Compute corrected coordinates xc and yc :

x_c=62.571-(-0.0013)=62.572mm
y_c=-80.915-0.0017=-80.917mm

2.3. Corrections for Atmospheric Refraction

Due to the density of the atmosphere the light rays do not travel in straight lines, but rather they are bent according to Snell's law, as shown in the below figure:



Photogrammetric equations assume that light rays travel in straight paths. If the fraction were ignored, the light ray would appear to be coming from B instead of A. Therefore, corrections should be applied to the image coordinates.

The angular distortion due to the atmospheric refraction can be calculated from the below equation;

$$\Delta\alpha = K \tan\alpha ----(10)$$

Where K depends on the flying height above mean sea level and the elevation of the object point. Various formula are available to calculate the K value, and the one that is used in photogrammetry is:

$$K = (7.4 \times 10^{-4})(H-h)[1-0.02(2H-h)] ----(11)$$

Where:

H: is the flying height of the camera above the mean seal level in kilometres,

H0: is the elevation of the point above mean seal level in kilometres,

K in degrees.

The procedure for calculating the atmospheric correction to image coordinate on the vertical photo is as follows:

1-compute radial distance:

$$r = \sqrt{x^2+y^2} ----(12)$$

2-from above figure, find α

$$\tan(\alpha) = \frac{r}{f} ----(13)$$

3- from equ.(10), the value of $\Delta\alpha$ can be obtained.

$$\Delta\alpha = K\frac{r}{f} ----(14)$$

4-The corrected radial distance (r') can be determined,

$$r' = f\tan(\alpha - \Delta\,\alpha) ----(15)$$

5- Finally the change in radial distance can be calculated

$$\Delta r = r - r' ----(16)$$

6- The value of δx, and δy can be calculated

$$\delta x = \bar{x}\frac{\Delta r}{r} ----(17)$$

$$\delta y = \bar{y}\frac{\Delta r}{r} ----(18)$$

7-finally the corrected image coordinates x and y are computed from

$$x_c = \bar{x} - \delta x \;--- (19)$$

$$y_c = \bar{y} - \delta y \;--- (20)$$

Example: A vertical photograph taken from a flying height of 3500 m above mean sea level contains the image an of object point A at coordinates (concerning thee fiducial system) x_a=73.287 mm and y_a= -101.307 mm. If the elevation of object point A is 120m above mean sea level and the camera had a focal length of 153.099 mm, compute the x’ and y’ coordinates of the point, corrected for atmospheric refraction.

Solution:

Compute r by Eq. 12

$$r = \sqrt{73.287^2 + (-101.307)^2} = 125.036\,mm$$

Express tan α by Eq. 13 and solve for α

$$\tan\alpha = \frac{125.036}{153.099}$$

$$\alpha = 39.2386^o$$

Compute K by Eq. 11

$$K = (7.4 \times 10^{-4})(3.5 - 0.12)[1 - 0.02(2(3.5) - 0.12)] = 0.0022^o$$

Compute Δα by Eq. 14

$$\Delta\alpha = 0.0022^o\left(\frac{125.036}{153.099}\right) = 0.0018^o$$

Compute r' by Eq. 15

$$r' = 153.099\tan(39.2386^o - 0.0018^o) = 125.029\,mm$$

Compute Δr by Eq. 16

$$\Delta r = 125.036 - 125.029 = 0.008\,mm$$

Compute $\delta x\ and\ \delta y$ by Eqs. (17) and (18)

$$\delta x = 73.287\left(\frac{0.008}{125.036}\right) = 0.005\,mm$$

$$\delta y = -101.307\left(\frac{0.008}{125.036}\right) = -0.006\,mm$$

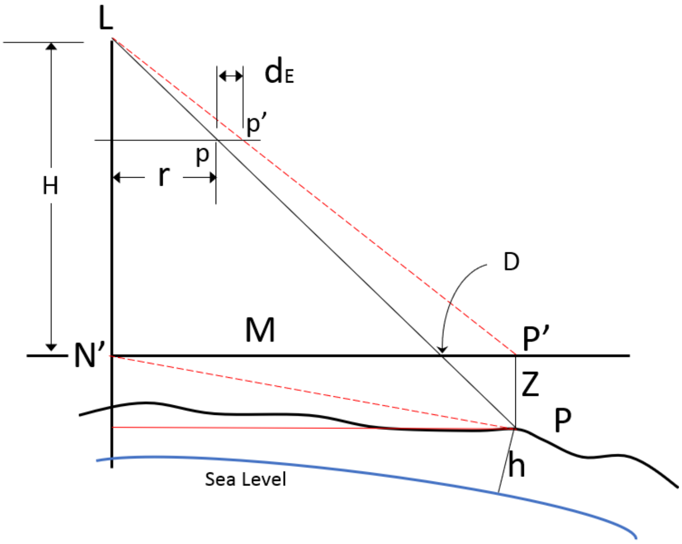
Subtract the coordinates δx and δy from x and y, respectively to obtain corrected coordinates x' and y'.

$$x' = x - \delta x = 73.287 - 0.005 = 73.282\ mm$$

$$y' = y - \delta y = -101.307 - (-0.006) = -101.301\ mm$$

2.4. Corrections for Earth Curvature

The earth curvature correction for vertical photography is straightforward.
As shown in the figure the point P located on the earth surface is projected on the plane is P'



The correction due to the earth's curvature $d_E = \frac{r^3 H}{2f^2 R} - - - (21)$

The curvature correction value d_E is in mm
r is the image coordinate radial distance
H is the flying height in m
F is the focal length in mm, and,
R is the earth's curvature.

Example:
Given a vertical frame aerial photograph taken from an altitude of 3000m with a 152-mm lens, what is the earth curvature correction for a point at image coordinate x=59.043mm, y=72.392mm? Assume the terrain elevation is 300m, and the radius of the earth is R=6371km.

$$r = \sqrt{59.043^2 + 72.392^2} = 93.417mm$$

$$d_E = \frac{r^3 H}{2f^2 R} = \frac{93.417^3 * (3000 - 300)}{2 * 152^2 * 6{,}371{,}000} = 0.007mm$$

$$\delta x = \bar{x}\frac{\Delta r}{r} = 59.043\frac{0.007}{93.417} = 0.004mm$$

$$\delta y = \bar{y}\frac{\Delta r}{r} = 72.392\frac{0.007}{93.417} = 0.005mm$$

$$x_c = \bar{x} + \delta\text{x} = 59.043 + 0.004 = 59.047mm$$

$$y_c = \bar{y} + \delta\text{y} = 72.392 + 0.005 = 72.397mm$$