زانكۆی سەلاحەدین - هەولێر
Salahaddin University-Erbil

Mathematical Applications in Archaeology

Research Project

Submitted to the department of (Mathematic) in partial fulfilment of the requirements for the degree of BSc. in (forth)

By:

Zaenab Hamad Mohammed

Supervised by:

Dr. Hogir Mohammed Yaseen

APR– 2025

Certification of the Supervisor

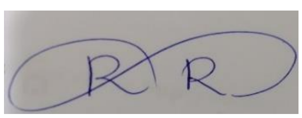
I certify that this report was prepared under my supervision at the Department of Mathematics / College of Education / Salahaddin University-Erbil in partial fulfillment of the requirements for the degree of Bachelor of philosophy of Science in Mathematics.

Signature: 

Supervisor: *Dr.Hogir Mohammed Yaseen*
Scientific grade: Lecturer

Date: 14 / 4 / 2025

In view of the available recommendations, I forward this report for debate by the examining committee.

Signature: 

Name: Assistant Prof Dr. Rashad Rashid Haji

Chairman of the Mathematics Department

Acknowledgments

With profound gratitude, I extend my endless appreciation to the Almighty for His guidance, support, and countless blessings. I firmly believe that without His direction, this work would not have been possible. I am deeply thankful to Dr. Hogir for his invaluable mentorship and unwavering support throughout my research on the mathematical applications in archaeology. His insightful guidance, from the initial stages to completion, has been instrumental in shaping this study. My sincere appreciation also goes to Dr. Rashad, Head of the Mathematics Department, for his continuous assistance and encouragement throughout this academic journey. I would like to take this opportunity to express my heartfelt thanks to my colleagues and friends for their unwavering support. Their encouragement and willingness to share their expertise on various topics have been truly invaluable. Lastly, I extend my deepest gratitude to my family for their unconditional support and generosity. Their encouragement has been a constant source of strength, especially throughout my pursuit of a bachelor's degree.

Abstract

In this work, we study some mathematical applications in archaeology. First, we study age estimation. Then, we study mathematics in ancient Egyptian archaeology. In addition, we study Egyptian methods of arithmetic. Furthermore, we study advancements in geometry by the Egyptians. In all applications, we solve many examples that illustrate the applications.

Table of Contents

Introduction

Mathematics plays an important role in studying ancient Egyptian archaeology. It helps researchers understand how pyramids and temples were designed and provides useful methods for learning about the daily lives of ancient people. One major use of math is in estimating the age of human bones and teeth, which helps scientists learn about the health and lifestyle of past populations. By using mathematical formulas and calculations, researchers can study features like dental wear and bone strength to determine a person's age (Good man, 2016) .

The ancient Egyptians developed effective methods for arithmetic that were essential for governance and construction. They used symbol replacement for basic operations, repeated doubling for multiplication, and unit fractions for division, often relying on tables to simplify calculations. These techniques, recorded in the Rhind Papyrus, which contributed to their civilization's success (Burton, 2011). The ancient Egyptians advanced in geometry to solve practical problems like land surveying after the Nile's floods. They developed formulas for measuring areas and volumes, including an approximation for a circle's area and the volume of a truncated pyramid (Scriba & Pete, 2015).

In this work, we study some mathematical applications in archaeology. This work consists of three chapters and is organized as follows. In Chapter One, we examine age estimation from bones and teeth in archaeological mathematics, number recording by the Egyptians, and Egyptian hieratic numeration. In Chapter Two, we explore addition and subtraction, Egyptian arithmetic, and the writing of fractions. In the final chapter, we study areas and volumes. Additionally, we solve many examples to illustrate these applications.

CHAPTER ONE

Mathematics in Ancient Egyptian Archaeology and Age Estimation

Mathematics is crucial in archaeology, especially for studying ancient Egypt, where it helps estimate age from skeletal remains and analyze numerical systems. Advanced techniques like protein racemization, radiocarbon dating, and statistical models improve accuracy in determining age at death, even for cremated remains [10]. We also study the development of Egyptian numeration, from hieroglyphs to hieratic and demotic scripts, emphasizing the decimal system's role in administration and economy. This work highlights how mathematical methods enhance our understanding of ancient Egyptian society and its lasting impact (Burton, 2011).

1.1. Age Estimation from Bones and Teeth in Archaeological Mathematics:

Determining age at death is a fundamental aspect of forensic identification, particularly for unidentified bodies. Traditional methods rely on assessing skeletal and dental structures, typically providing age ranges rather than exact years. More advanced techniques, such as protein racemization and radiocarbon dating, with radiocarbon dating capable of determining a probable birth year by analyzing specific proteins and tissues. Accurate age estimation is crucial for forensic and archaeological investigations, contributing to a comprehensive biological profile alongside factors like sex, height, and ancestry (Michael , Latham, & Stanly, 2010).

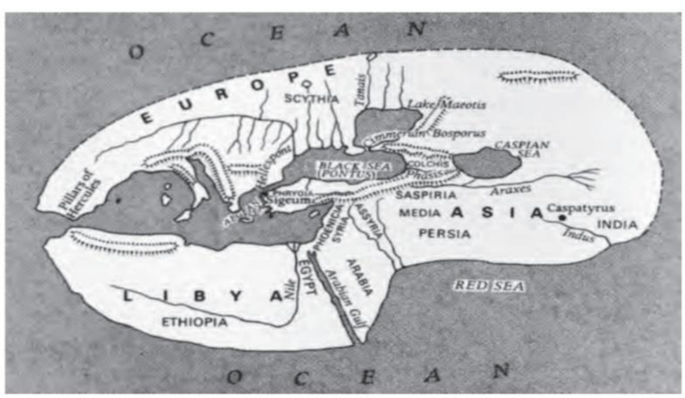
Skeletal and dental tissues, known for their durability, are commonly used for age estimation, aiding in birth year estimation and identification in forensic cases. While traditional methods focus on skeletal traits, recent advancements in biochemical, isotope, heavy metal, and radiocarbon analyses have expanded available techniques. This study evaluates root transparency as a method for estimating age at death, applying Bang and Ramm's statistical formulas to a modern Euro-American sample and introducing updated regression equations. Additionally, it examines the potential of the human sacrum for age estimation, emphasizing its value in a multifactorial approach. Given that different bones provide varying age-related information, Rining methods enhances both precision and reliability, ensuring accurate assessments in diverse forensic and archaeological contexts (Chiara & Niels , 2014).

To estimate the age of cremated bones at different preservation stages, the Kerley and Ubelaker method was applied, incorporating the lamina fundamentalis interna as an additional criterion. Research on Roman cemetery remains indicates that after cremation at 450–650°C, age assessment primarily depends on counting central Haversian canals, though with limited precision. Analysis of modern postmortem bones supports that Kerley's four criteria, along with the lamina fundamentalis interna, enhance accuracy in age estimation. For adults, age is categorized in 20-year intervals, while in children, it is determined by dental development and epiphyseal fusion. The timing of epiphyseal fusion is influenced by hormonal factors and varies by sex, typically occurring between ages 18–20 in females and 20–22 in males. Histomorphometric methods further contribute to refining age determination (M, B, M, & M, 2017) .

1.2. Number Recording of the Egyptians:

The History of Herodotus 1.2.1:

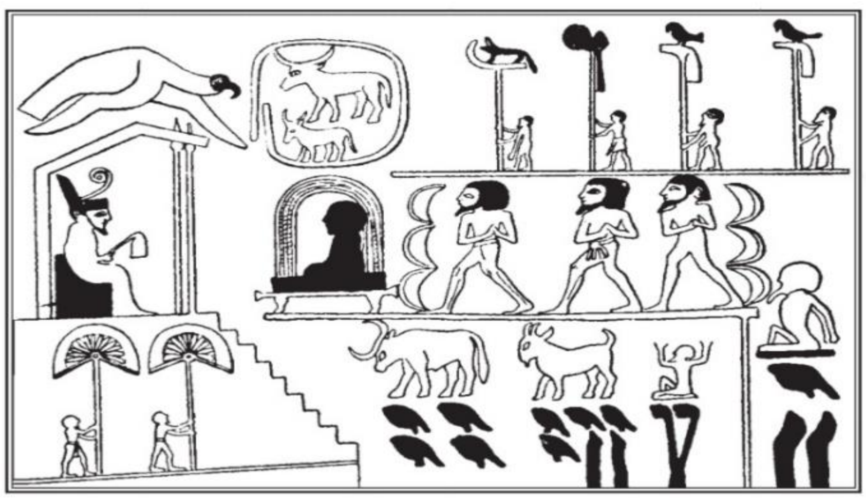
Herodotus, often referred to as the "Father of History," was a Greek historian born in Halicarnassus around 485 B.C. After being exiled due to political conflicts, he embarked on extensive travels across regions such as the Black Sea, Asia Minor, and Egypt, gathering information for his historical writings. Around 443 B.C., he settled in Thurium, where he devoted his later years to composing The History of Herodotus, the most comprehensive Greek prose work of its era. His accounts, blending historical events with cultural and ethnographic insights, were highly esteemed and publicly recited in Athens. While he did not always verify his sources, he presented multiple viewpoints on historical events, encouraging readers to interpret them independently (Burton, 2011).



The habitable world according to Herodotus. *(From* Stories from Herodotus *by B. Wilson and D. Miller. Reproduced by permission of Oxford University Press.)*

Figure 1.1

Herodotus was particularly fascinated by Egypt, extensively documenting its civilization and emphasizing the Nile's crucial role in its prosperity. He famously described Egypt as "the gift of the river," highlighting how its geography and annual floods fostered a stable and enduring society. By 3100 B.C., scattered agricultural communities along the Nile had unified under Menes, establishing a centralized government that lasted for millennia, with 32 dynasties ruling until 31 B.C. The surrounding deserts shielded Egypt from invasions, ensuring its long-lasting stability and enabling the development of an advanced administrative system that relied on early writing and numerical methods. Artifacts such as the Narmer Palette, which recorded military victories and censuses, along with religious texts like the Book of the Dead, reveal Egypt's sophisticated approach to record-keeping and governance, which played a vital role in its legacy as one of history's most influential civilizations [4].



This scene is taken from the great stone macehead of Narmer, which J. E. Quibell discovered at Hierakonpolis in 1898. There is a summary of the spoil taken by Narmer during his wars, namely "cows, 400,000, goats, 1,422,000, and captives, 120,000."

Scene reproduced from the stone macehead of Narmer, giving a summary of the spoil taken by him during his wars. *(From* The Dwellers on the Nile *by E. W. Budge, 1977, Dover Publications, N.Y.)*

Figure 1.2

Egyptian hieroglyphics, a picture-based writing system, used symbols to represent concrete objects, often retaining their original meanings. Near the Pyramid of Gizeh, archaeologists discovered hieroglyphic number symbols, confirming that the Egyptians employed a decimal system based on powers of ten. The number one was denoted by a single vertical stroke or staff, while a heelbone sign symbolized ten. This system likely emerged from the human tendency to count using fingers, a common trait among ancient societies (Burton, 2011).

For increasing values, the Egyptians used unique pictographs: a curved rope represented 100, a lotus flower stood for 1,000, a bent finger signified 10,000, a tadpole denoted 100,000, and a figure raising both hands in astonishment symbolized 1,000,000. The sign for 10,000,000 is thought to depict a rising sun. This well-structured numerical notation enabled sophisticated record-keeping, which was crucial for Egypt's administrative, economic, and engineering advancements, reinforcing its status as a pioneering civilization in governance and mathematics.

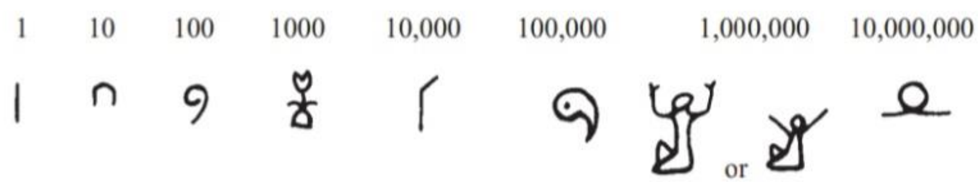


Figure 1.3

In the Egyptian hieroglyphic system, numbers were expressed additively, where the total value was the sum of the individual symbols, and each symbol could be repeated up to nine times to represent larger quantities. Typically, the writing direction was from right to left, with larger units placed first and smaller units following in order of importance. However, in some instances, the larger units were written on the left, with the symbols turned to face the direction of writing.

to indicate our number

$$1 \cdot 100{,}000 + 4 \cdot 10{,}000 + 2 \cdot 1000 + 1 \cdot 100 + 3 \cdot 10 + 6 \cdot 1 = 142{,}136.$$

Figure 1.4

To save lateral space, the symbols were often arranged in two or three rows, one above the other. Each symbol represented a distinct power of 10, meaning the numerical value of a grouping of symbols remained the same regardless of the order in which they were written. This method allowed for flexibility in the presentation of numbers while maintaining clarity in the value. For example,

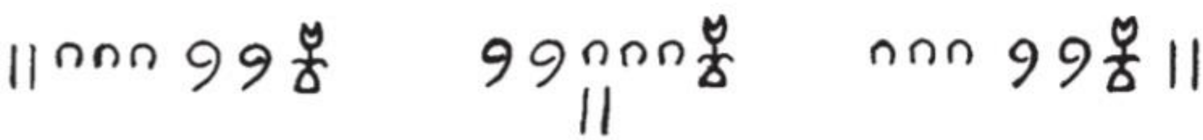


Figure 1.5

The Egyptian system was not a "positional system," meaning that the value of a symbol did not change based on its position within the number. For example, a grouping of symbols that included a series of hieroglyphs for 1,000, 200, and 30 would consistently represent the number 1232, regardless of the arrangement of the symbols within the grouping. This distinction highlights the unique nature of the Egyptian numerical system (Burton, 2011).

1.3. Egyptian Hieratic Numeration:

Writing in ancient times was initially limited to inscriptions on stone or metal, making it difficult to record lengthy texts. The Egyptians solved this issue with the invention of papyrus, made from the papyrus plant found in the Nile Delta. Thin strips of the plant were arranged in layers, soaked, pounded, and dried, creating a durable writing surface. This new material allowed for longer scrolls, which could be rolled up when not in use, and facilitated the use of ink and a brush like pen for writing.

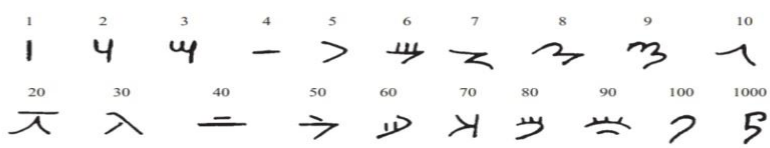


Figure 1.6

With papyrus, writing became more efficient and versatile. Egyptian priests developed a quicker, cursive writing style called "hieratic," which was less pictorial than traditional hieroglyphs and better suited to pen and ink. Over time, this style evolved into "demotic," a shorthand system that made writing even faster. Both hieratic and demotic scripts employed a system where symbols represented collections of like symbols, streamlining the process compared to the repetitive nature of hieroglyphics. The shift from hieroglyphic to demotic script also influenced Egyptian numeration. In hieroglyphics, numbers were represented by repetitive symbols, but in hieratic and demotic scripts, a ciphering method emerged, where a single mark was used to represent a group of symbols. This made numerical representation faster and more concise, marking a significant advancement in the development of written numeration. The innovation of ciphering was as important to numeration as the positional principle adopted by the Babylonians (Burton, 2011).

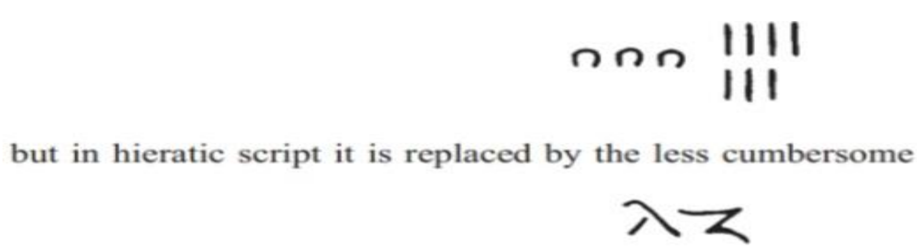


Figure 1.7

CHAPTER TWO

Egyptian Methods of Arithmetic

The ancient Egyptians developed efficient arithmetic techniques for addition, subtraction, multiplication, and division, crucial for administration and construction. Addition and subtraction involved symbol replacement, while multiplication used repeated doubling. Division, treated as reverse multiplication, often required fractions, primarily unit fractions represented by hieroglyphs. They relied on reference tables to break fractions into sums of unit fractions. These methods, documented in texts like the Rhind Papyrus, showcase their practical and innovative approach to mathematics, supporting their advanced society [5] .

2.1. Addition and subtraction:

Addition and subtraction were relatively straightforward in the Egyptian number system. For addition, the process involved gathering the symbols and replacing every ten identical symbols with the next higher-value symbol. This method would have been used by the Egyptians to add numbers such as 345 and 678 (Burton, 2011) :

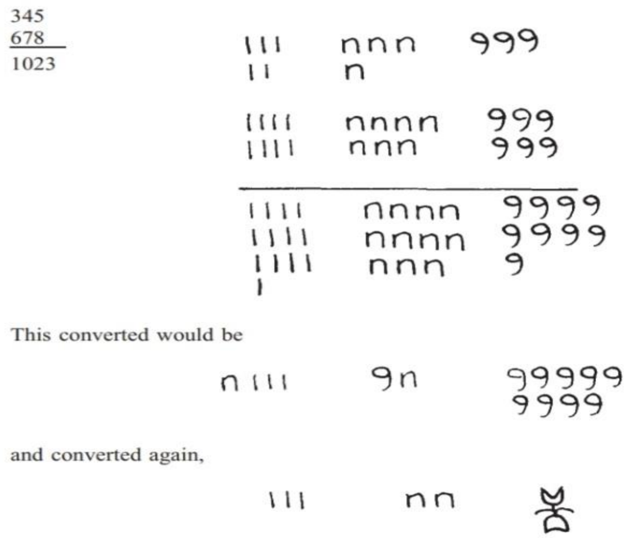


Figure 2.1

Subtraction was carried out using the same method but in reverse. In some cases, a technique called "borrowing" was applied, where a symbol representing a larger number was replaced with ten lower-value symbols to facilitate subtraction. Although the Egyptians had numerical symbols, they lacked a standardized notation for arithmetic operations. However, in the well-known Rhind Papyrus, dating to around 1650 B.C.,the scribe used specific hieroglyphs to indicate addition and subtraction, resembling a person's legs moving forward and backward (Burton, 2011):

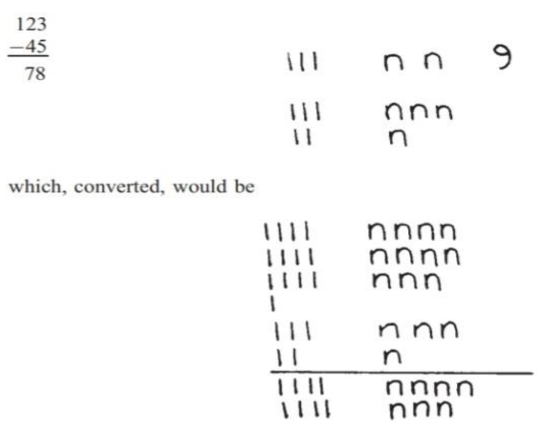


Figure 2.2

2.2. Egyptian Arithmetic:

2.2.1 Early Egyptian Multiplication:

The Rhind Papyrus initially presents itself as a comprehensive study of knowledge but is actually a practical mathematical guide. It contains 85 problems that illustrate the additive nature of Egyptian mathematics, where multiplication was performed through repeated doubling and summing relevant values (Burton, 2011) .

The Egyptian multiplication method relied on repeated doubling. To multiply two numbers, A and B, a scribe would start with the pair (1×B). They would then continuously double both numbers in the pair until the first value exceeded A. By identifying which powers of 2 sum to A, the scribe would then add the corresponding multiples of B to determine the final result (Katz, 2009) **.**

For example, to multiply 18 by 62, the number 62 would be doubled successively, and the necessary values would be added together to reach the result, doubling thus:

1	62
2	124
4	248
8	496
16	992

At this point, we stop doubling since the next step would result in a multiplier of 62 exceeding 18. Since 18 can be expressed as 2 + 16, we mark these corresponding multipliers to indicate they should be summed. Thus, the calculation for 18 multiplied by 62 would be represented as follows.

1	62
✔2	124
4	248
8	496
✔16	992
Totals 18	1116

By summing the numbers in the right-hand column corresponding to the marked multipliers, the Egyptian mathematician would arrive at the correct result, 1116. This follows the calculation: 1116 = 124 + 992, which is equivalent to (2 + 16) × 62, confirming that 18 × 62 = 1116.

If 18 had been used as the multiplicand and 62 as the multiplier, the calculation would have been organized in the following manner.

1	**18**
✔2	**36**
✔4	**72**
✔8	**144**
✔16	**288**
✔32	**576**
Totals 62	**1116**

Because 62 = 2 + 4 + 8 + 16 +32, one has merely to add these multiples of 18 to get, again, 1116.

A second multiplication method involved successively doubling one factor and halving the other. This approach would create a table like the following:

12	35
6	70
3	140
1	280

Students often wonder how the Egyptians handled halving odd numbers. For example, half of 3 is 1.5, not 1, but the table only shows whole numbers. The Egyptians simply disregarded the fractional part when using this method. The next step is to add the numbers in the second column that align with odd numbers in the first column. This results in 140 + 280, gives the correct answer. Thus, the Egyptians clearly understood the distinction between odd and even numbers, and they had two effective methods for multiplying whole numbers. A useful exercise for students is to demonstrate the accuracy of both Egyptian multiplication techniques (Good man, 2016)

2.2.2 Division:

The method of multiplication by doubling and adding works because any positive integer can be represented as a sum of unique powers of 2, such as 1, 2, 4, 8, 16, and so on. While the ancient Egyptians may not have formally proven this principle, their confidence in it was likely based on repeated practical use. This technique, also known as Russian multiplication due to its historical use among Russian peasants, was beneficial as it eliminated the need for memorizing multiplication tables (Burton, 2011).

Similarly, Egyptian division can be seen as the reverse of multiplication, where the divisor is repeatedly doubled until it equals the dividend. For example, to solve 91 ÷ 7, they would double 7 incrementally until reaching 91, effectively determining the quotient through this accumulation process.

	1	7	✓
	2	14	
	4	28	✓
	8	56	✓
totals	13	91	

By identifying that 7 + 28 + 56 equal 91, one sums the corresponding powers of 2, specifically 1 + 4 + 8, resulting in 13, which represents the quotient. The Egyptian division method is advantageous for teaching, as it does not introduce division as a completely new operation but rather as an extension of multiplication (Burton, 2011).

For division, the Egyptians more or less reversed their multiplication technique. Since we just saw that 35×12=420, we know that 420 divided by 35 is 12. The Egyptians would make their reliable doubling table to start.

1	35
2	70
4	140
8	280
16	560

They could stop at 560 since it exceeds 420, or even at 280, as it is more than half of 420, though that would require additional mental calculation. Once they reached a sufficiently large number through doubling, the next step was to subtract the largest value on the list that is less than or equal to the starting number. In this case, for 420, the largest applicable number is 280, so 420 − 280 = 140. Then, they subtract the largest number on the list that is less than or equal to 140, which is 140 itself. Since 280 corresponds to 8 and 140 corresponds to 4, adding these values gives the final quotient of 12.

$$\begin{array}{r} 420 \\ \underline{280} \\ 140 \\ \underline{140} \\ 0 \end{array}$$

The idea is that 420 consists of a certain number of 35s. By breaking it down, we see that 420 includes 280 (which is eight 35s) and 140 (which is four 35s). Adding these together shows that 420 is made up of twelve 35s (Good man, 2016).

Division was not always straightforward, and fractions were often necessary. For example, when dividing 35 by 8, the scribe would first double the divisor, 8, until further doubling would surpass the dividend, 35. Then, to account for the remainder, he would begin halving the divisor. The process would be recorded as follows:

	1	8	
	2	16	
	4	32	✓
	$\frac{1}{2}$	4	
	$\frac{1}{4}$	2	✓
	$\frac{1}{8}$	1	✓
totals	$4 + \frac{1}{4} + \frac{1}{8}$	35	

Doubling 16 results in 32, leaving a remainder of 35 - 32 = 3. To make up for this, you first halve 8 to get 4, then halve 4 to get 2, and finally halve 2 to get 1. By adding the fractions for the fourth and the eighth, the remainder of 3 is covered. Therefore, the final quotient is 4 + 1/4 + 1/8 (Burton, 2011).

2.3. Writing Fractions:

Five two-part statements from the Egyptian Middle Kingdom have recently revealed overlooked historical details, puzzling scholars for over a century. These statements, now translated into modern base-10 fractions, have uncovered new scribal arithmetic definitions, shedding light on an ancient Horus-Eye problem. The solution involved dividing a Hekta (a unit of volume) by rational numbers between 1/64 and 64, with 36 statements from 1650 BC confirming the resolution of this problem. Scribes later converted these two-part statements into one-part ones, revealing secondary methods and theoretical roles for arithmetic definitions in ancient Egyptian mathematics (Millo, 2006).

Fractions were written by placing the hieroglyph for "part," represented by a mouth (𓂋), above the denominator. For example, 1/7 was (𓂋IIIIIII) written as (seven parts) and 1/10 was (𓂋∩) (ten parts). The only fraction with a numerator other than 1 was 2/3, written was (𓂋II). Special hieroglyphs were used for the fractions 1/2, 1/4, 1/8, 1/16, 1/32, and 1/64, and these were based on the myth of Osiris. After Osiris was killed by Seth, his son Horus (the falcon) avenged his father's death but lost his eye in the process. The god Thoth, who was associated with magic, gathered and restored the eye's fragments. The unique feather markings around Horus' falcon eye became hieroglyphs symbolizing well-being, and when broken into parts, they represented specific fractions. The Egyptians recognized that these fractions summed to only 63/64 rather than a complete 64/64 or unity, so they attributed the missing 1/64 to Thoth's magic (Bob & Hoyt, 2008) **.**

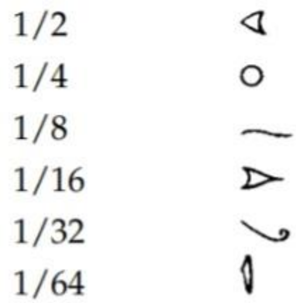
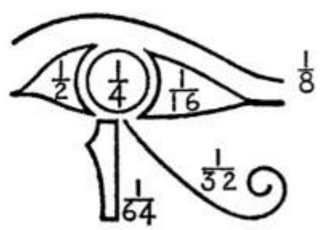


Figure 2.3

The Egyptians applied mathematics solely to practical tasks, such as measuring fields, tracking grain quantities, and distributing supplies among workers, rather than for theoretical exploration.



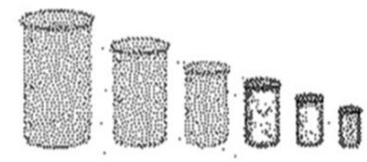
This drawing shows how hieroglyphic symbols for various common fractions were derived from different sections of the Eye of Horus.

Figure 2.4

Solving these problems required established standards for size, weight, and volume that could be applied in their calculations (Bob & Hoyt, 2008) .

Fractions have been used in mathematics for over 4,000 years, though modern notation is more recent. Early civilizations broke objects into smaller parts for measurement, leading to systems of weights and measures that improved accuracy. This practice continues today in units like ounces, inches, and liquid measurements. Initially, fractions were primarily unit fractions (numerator of 1), with more complex fractions expressed as sums of unit fractions (e.g., 3/5 as ½ + 1/10). Egyptians simplified notation by marking unit fractions with a dot or oval over the numeral to distinguish them from whole numbers. For example,

Figure 2.5



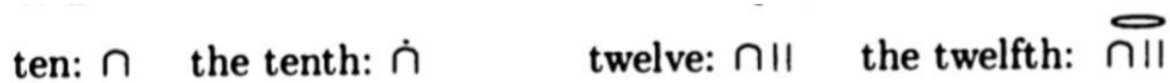


Figure 2.6

Although recording "parts" was simple, working with them proved challenging (Berlinghoff & Gouvea, 2004).

The Egyptians primarily used unit fractions (fractions with a numerator of 1), with the exception of 2/3, which had its own symbol. Other fractions like 1/2 and 1/4 also had distinct symbols. Unit fractions were represented in hieroglyphics by the integer symbol with a mark above, while in hieratic, a dot was used. Although dealing with non-unit fractions was often necessary, the Egyptians didn't view fractions in the same way we do. Instead of using non-unit fractions, they expressed them as sums of unit fractions. For instance, in a problem from the Rhind Mathematical Papyrus, when dividing 6 loaves of bread among 10 men, the solution was given as 1/2 + 1/10. This method, while cumbersome compared to our modern fractions like 3/5, made division clear and easy to follow. This system of representing fractions as sums of unit fractions was used throughout the Mediterranean for over 2,000 years (Katz, 2009) .

To aid in breaking fractions into unit fractions, the Egyptians likely relied on reference tables, many of which were memorized. The Rhind Papyrus contains a table for fractions with a numerator of 2 and an odd denominator between 5 and 101. This table, which makes up about one-third of the 18-foot roll, is the most detailed arithmetic table found in surviving ancient Egyptian papyri. The scribe first selected a decomposition for 2/n, then verified its accuracy by multiplying the chosen values to get 2, but the method for deriving the decomposition itself is not explained. Fractions of the form 2/n, where the denominators are divisible by 3, all adhere to the same general pattern. Now $\frac{2}{3k} = \frac{1}{2k} + \frac{1}{6k}$ and typical of these entries is 2/15 (the case k = 5), which is given as $\frac{2}{15} = \frac{1}{10} + \frac{1}{30}$

If we ignore the representations for fractions of the form 2/(3k), then the remainder of the 2/n table reads as shown herewith (Burton, 2011).

$2/5 = 1/3 + 1/15$

$2/7 = 1/4 + 1/28$

$2/11 = 1/6 + 1/66$

$2/13 = 1/8 + 1/52 + 1/104$

$2/17 = 1/12 + 1/51 + 1/68$

$2/19 = 1/12 + 1/76 + 1/114$

$2/23 = 1/12 + 1/276$

$2/25 = 1/15 + 1/75$

$2/29 = 1/24 + 1/58 + 1/174 + 1/232$

$2/31 = 1/20 + 1/124 + 1/155$

$2/35 = 1/30 + 1/42$

$2/37 = 1/24 + 1/111 + 1/296$

$2/41 = 1/24 + 1/246 + 1/328$

$2/43 = 1/42 + 1/86 + 1/129 + 1/301$

$2/47 = 1/30 + 1/141 + 1/470$

$2/49 = 1/28 + 1/196$

$2/51 = 1/34 + 1/102$

$2/53 = 1/30 + 1/318 + 1/795$

$2/55 = 1/30 + 1/330$

$2/59 = 1/36 + 1/236 + 1/531$

$2/61 = 1/40 + 1/244 + 1/488 + 1/610$

$2/65 = 1/39 + 1/195$

$2/67 = 1/40 + 1/335 + 1/536$

$2/71 = 1/40 + 1/568 + 1/710$

$2/73 = 1/60 + 1/219 + 1/292 + 1/365$

$2/77 = 1/44 + 1/308$

$2/79 = 1/60 + 1/237 + 1/316 + 1/790$

$2/83 = 1/60 + 1/332 + 1/415 + 1/498$

$2/85 = 1/51 + 1/255$

$2/89 = 1/60 + 1/356 + 1/534 + 1/890$

$2/91 = 1/70 + 1/130$

$2/95 = 1/60 + 1/380 + 1/570$

$2/97 = 1/56 + 1/679 + 1/776$

$2/101 = 1/101 + 1/202 + 1/303 + 1/606$

Since the first translation of the papyrus was released, mathematicians have attempted to decipher the method the scribe used to create this table (Burton, 2011).

CHAPTER THREE

Advancements in Geometry by the Egyptians

The ancient Egyptians made significant advancements in geometry, driven by practical needs such as land surveying for taxation after the Nile's annual floods. Their geometric methods, rooted in observation, included formulas for calculating areas of shapes like quadrilaterals and circles. Notably, they approximated the area of a circle using a formula equivalent to $\pi \approx 3.1605$. In volume calculations, they accurately determined the volume of a truncated pyramid, as evidenced by the Moscow Mathematical Papyrus, showcasing their geometric intuition. While their techniques were often approximate, they were sufficient for construction and administrative tasks, reflecting their innovative and practical approach to mathematics (Burton, 2011).

3.1. Area:

Egyptian geometry likely emerged from the need to resurvey land for taxation after the Nile's annual floods. The term "geometry," meaning "earth measurement" in Greek, reflects this practical purpose. According to Herodotus, King Sesostris allocated land equally and adjusted taxes based on river erosion. Egyptian surveyors, known as "rope-stretchers" by the Greeks, used knotted ropes for measurement, and their expertise was recognized even by philosophers like Democritus. Their mathematical texts contained practical, observation-based rules for calculating areas and volumes, which, though approximate, were sufficient for land measurement and construction (Burton, 2011).

Calculating the area of quadrilaterals, trapezoids, and triangles is one of the most basic geometric computations. An approximate formula for any quadrilateral with given sides is**:**

$$A = \frac{a+c}{2} \times \frac{b+d}{2}$$

This formula estimates the area by averaging the lengths of opposite sides. Interestingly, it simplifies to a triangle when one of the sides is zero, or rather, it is excluded, as the ancient Egyptians did not acknowledge zero as a number (Scriba & Pete, 2015).

One of the most remarkable achievements of the ancient Egyptians in two-dimensional geometry was their method for calculating the area of a circle, as demonstrated in Problem 50.

Example: - The problem presents a circular field with a diameter of 9 khet and instructs that 1/9 of the diameter (or 1 khet) should be subtracted, leaving 8 khet. Squaring this value gives 64, meaning the area is 64 set at of land. The Egyptian scribes' approach can be summarized as follows: subtract 1/9 of the diameter from the total diameter and then square the result. Mathematically, this is expressed as:

$$A = \left(d - \frac{d}{9}\right)^2 = \left(\frac{8d}{9}\right)^2$$

Where d represents the diameter of the circle. When compared to the actual formula for the area of a circle,

$$A = \frac{\pi d^2}{4}$$

We get

$$\frac{\pi d^2}{4} = \left(\frac{8d}{9}\right)^2$$

Solving for π gives

$$\pi = 4\left(\frac{8}{9}\right)^2 = 3.1605\ldots$$

This suggests that the Egyptians approximated π=3.1605... in their calculations (Burton, 2011).

The formula A = $(8d/9)^2$ **f**or the area of a circle may have originated from an approximation using an octagon. The Rhind Papyrus suggests a method where an octagon is formed by cutting off four isosceles triangles from a square of side 9 units. The scribe may have assumed that this octagon closely approximates the area of the inscribed circle because the portions inside and outside the circle balance out.

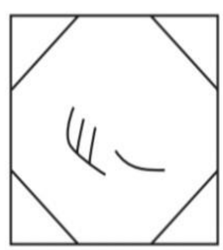


Figure 3.1

By calculating the octagon's area, we get A = 63, which is close to the value obtained when d = 9 in the formula $(8d/9)^2$. This suggests that the formula may have been derived as an approximation based on this geometric approach (Burton, 2011).

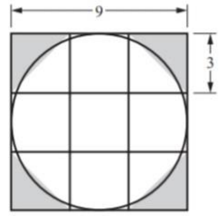


Figure 3.2

3.2. Volumes:

The ancient Egyptians primarily calculated the volumes of cuboid and cylindrical containers. While the construction of the pyramids suggests they might have understood the volume formula for pyramids, there is no concrete evidence. In 1900, Max Dehn proved that deriving this formula precisely is impossible without using a limit process [1] .

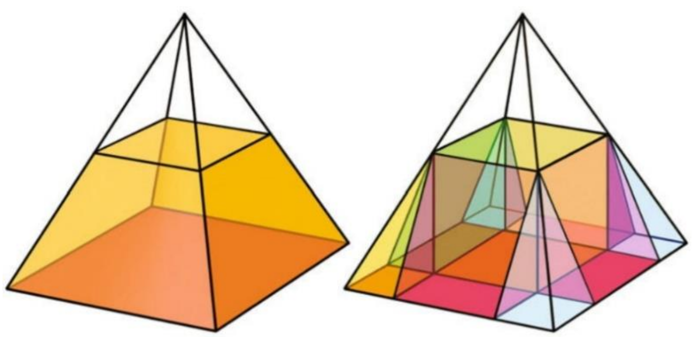


Figure 3.3

Despite this, Problem 14 of the Moscow Mathematical Papyrus (c. 1850 B.C.) correctly states the volume formula for a truncated square pyramid (frustum)**:**

$$V = \frac{h}{3}(a^2 + ab + b^2)$$

where V is the volume, h is the height, and a and b are the side lengths of the base and top squares, respectively [4].

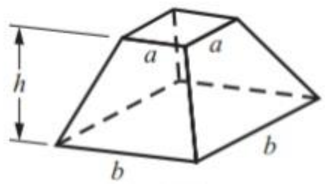


Figure 3.4

Here is an example of how to calculate the volume of a truncated pyramid. Given a truncated pyramid with a vertical height of 6, a base side length of 4, and a top side length of 2:

1. Square the base side (4), which gives 16.
2. Double the base side (4), resulting in 8.
3. Square the top side (2), giving 4.
4. Add 16, 8, and 4 together, which gives 28.
5. Take 1/3 of the height (6), resulting in 2.
6. Multiply 28 by 2, which gives 56.

The final result is 56, confirming the correct volume (Burton, 2011).

The papyrus explains the formula through a step-by-step example rather than as a general rule, but historians still regard this as a significant mathematical accomplishment. Some researchers believe the Egyptians may have also known the formula for a complete pyramid (Burton, 2011):

$$V = \frac{h}{3} \times a^2$$

And they might have guessed the constant 1/3 by analogy with the area formula for triangles. However, the truncated pyramid formula is too precise to be a guess and likely came from geometric insights or early algebraic reasoning (Burton, 2011) .

Egyptian texts don't provide strong evidence that this formula was widely applied. Instead, they sometimes approximated the volume by treating the frustum as a cuboid with an average base area (Scriba & Pete, 2015):

$$B = \frac{1}{2} \times (a^2 + b^2)$$

which leads to an inaccurate volume formula:

$$V = \frac{h}{2} \times (a^2 + b^2)$$

Historian Kurt Vogel suggested that the Egyptians may have recognized this error and corrected it by using a median area unit:

$$B = (a^2 + ab + b^2)/3$$

Through this adjustment, they arrived at the correct volume formula, possibly through an intuitive generalization. Interestingly, if the top surface area is set to zero ($b^2 = 0$), the truncated pyramid formula reduces to the correct volume formula for a full pyramid (Scriba & Pete, 2015).

Figure 3.5

References

B. B., & H. H. (2008). *Daily Life Of The Ancient Egytians.* (S. Edition, Ed.)

Berlinghoff, W. P., & Gouvea, F. Q. (2004). *Math through the age.*

Burton, D. M. (2011). *The History of Mathematics.* (S. edition, Ed.) University of New Hampshire .

C. V., & N. L. (2014). Age estimation of skeletal remains : Principle method. *Research and Report in Forensic Medical Science* , 3-9.

Katz, V. J. (2009). *A History of Mathematics An Introduction* . (3. Edition, Ed.) Addison Wesley.

M. f., Latham, K. E., & S. R. (2010). *AGE ESTIMATION OF THE HUMAN SKELETON* . Charles C Thomas publisher.

M. G. (2006). An ancient Egyptian problem and its innovative arithmetic solution . *Bulletin of the introduction to the early development mathematics* .

M. K. (1952). *An Introduction to the Early Development of Mathematics* . USA: simultaneously in canada.

M. W., B. S., M. D., & M. S. (2017). Determining human age at death using cremated bone Microstructure. *Journal of Scientific*, 785-791.

Scriba, C. J., & P. S. (2015). *5000 Years of Geometry : Mathematics in history and culture* .

پوختە

لەم کارەدا لە بەکارهێنانی بیرکاری لە شوێنەوارناسیدا دەکۆڵینەوە. سەرەتا ئێمە لە خەملاندنی تەمەن دەکۆڵینەوە. پاشان، بیرکاری لە شوێنەوارناسی میسری کۆن دەخوێنین. جگە لەوەش ئێمە لە شێوازەکانی ژمێریاری میسری و پێشکەوتنەکانی ئەندازەیی لەلایەن میسرییەکان دەکۆڵینەوە. لە هەموو جێبەجێکردنەکاندا چەندین نموونە شیکاردەکەین کەجێبەجێکردنەکان ڕووندەکاتەوە.