Arithmetic

Integers

0	zero	10	ten	20	twenty
1	one	11	eleven	30	thirty
2	two	12	twelve	40	forty
3	three	13	thirteen	50	fifty
4	four	14	fourteen	60	sixty
5	five	15	fifteen	70	seventy
6	six	16	sixteen	80	eighty
7	seven	17	seventeen	90	ninety
8	eight	18	eighteen	100	one hundred
9	nine	19	nineteen	1000	one thousand

-245	minus two hundred and forty-five
22 731	twenty-two thousand seven hundred and thirty-one
1 000 000	one million
56 000 000	fifty-six million
1 000 000 000	one billion [US usage, now universal]
7 000 000 000	seven billion [US usage, now universal]
1 000 000 000 000	one trillion [US usage, now universal]
3 000 000 000 000	three trillion [US usage, now universal]

Fractions [= Rational Numbers]

$\frac{1}{2}$	one half	$\frac{3}{8}$	three eighths
$\frac{1}{3}$	one third	$\frac{26}{9}$	twenty-six ninths
$\frac{1}{4}$	one quarter [= one fourth]	$-\frac{5}{34}$	minus five thirty-fourths
$\frac{1}{5}$	one fifth	$2\frac{3}{7}$	two and three sevenths
$-\frac{1}{17}$	minus one seventeenth		

Real Numbers

-0.067	minus nought point zero six seven
81.59	eighty-one point five nine
$-2.3 \cdot 10^6$	minus two point three times ten to the six
$[= -2\,300\,000$	minus two million three hundred thousand]
$4 \cdot 10^{-3}$	four times ten to the minus three
$[= 0.004 = 4/1000$	four thousandths]
$\pi\ [= 3.14159\ldots]$	pi [pronounced as ‘pie’]
$e\ [= 2.71828\ldots]$	e [base of the natural logarithm]

Complex Numbers

i	`i`
$3+4i$	`three plus four i`
$1-2i$	`one minus two i`
$\overline{1-2i}=1+2i$	`the complex conjugate of one minus two i equals one plus`

The real part and the imaginary part of $3+4i$ are equal, respectively, to 3 and 4.

Basic arithmetic operations

Addition:	$3+5=8$	`three plus five equals [= is equal to] eight`
Subtraction:	$3-5=-2$	`three minus five equals [= ...] minus two`
Multiplication:	$3\cdot 5=15$	`three times five equals [= ...] fifteen`
Division:	$3/5=0.6$	`three divided by five equals [= ...] zero point`

$(2-3)\cdot 6+1=-5$	`two minus three in brackets times six plus one equals mi`
$\frac{1-3}{2+4} \;=\; -1/3$	`one minus three over two plus four equals minus one thir`
$4!\ [=\ 1\cdot 2\cdot 3\cdot 4]$	`four factorial`

Exponentiation, Roots

5^2	$[=5\cdot 5=25]$	`five squared`
5^3	$[=5\cdot 5\cdot 5=125]$	`five cubed`
5^4	$[=5\cdot 5\cdot 5\cdot 5=625]$	`five to the (power of) four`
5^{-1}	$[=1/5=0.2]$	`five to the minus one`
5^{-2}	$[=1/5^2=0.04]$	`five to the minus two`
$\sqrt{3}$	$[=1.73205\ldots]$	`the square root of three`
$\sqrt[3]{64}$	$[=4]$	`the cube root of sixty four`
$\sqrt[5]{32}$	$[=2]$	`the fifth root of thirty two`

In the complex domain the notation $\sqrt[n]{a}$ is ambiguous, since any non-zero complex number has n different n-th roots. For example, $\sqrt[4]{-4}$ has four possible values: $\pm 1 \pm i$ (with all possible combinations of signs).

$(1+2)^{2+2}$	`one plus two, all to the power of two plus two`
$e^{\pi i}=-1$	`e to the (power of) pi i equals minus one`

Divisibility

The multiples of a positive integer a are the numbers $a, 2a, 3a, 4a, \ldots$. If b is a multiple of a, we also say that a divides b, or that a is a divisor of b (notation: $a \mid b$). This is equivalent to $\frac{b}{a}$ being an integer.

Algebra

Algebraic Expressions

$A = a^2$	capital a equals small a squared
$a = x + y$	a equals x plus y
$b = x - y$	b equals x minus y
$c = x \cdot y \cdot z$	c equals x times y times z
$c = xyz$	c equals x y z
$(x + y)z + xy$	x plus y in brackets times z plus x y
$x^2 + y^3 + z^5$	x squared plus y cubed plus z to the (power of) five
$x^n + y^n = z^n$	x to the n plus y to the n equals z to the n
$(x - y)^{3m}$	x minus y in brackets to the (power of) three m x minus y, all to the (power of) three m
$2^x 3^y$	two to the x times three to the y
$ax^2 + bx + c$	a x squared plus b x plus c
$\sqrt{x} + \sqrt[3]{y}$	the square root of x plus the cube root of y
$\sqrt[n]{x + y}$	the n-th root of x plus y
$\frac{a+b}{c-d}$	a plus b over c minus d
$\binom{n}{m}$	(the binomial coefficient) n over m

Indices

x_0	x zero; x nought
$x_1 + y_i$	x one plus y i
R_{ij}	(capital) R (subscript) i j; (capital) R lower i j
M_{ij}^k	(capital) M upper k lower i j; (capital) M superscript k subscript i j
$\sum_{i=0}^n a_i x^i$	sum of a i x to the i for i from nought [= zero] to n; sum over i (ranging) from zero to n of a i (times) x to the i
$\prod_{m=1}^{\infty} b_m$	product of b m for m from one to infinity; product over m (ranging) from one to infinity of b m
$\sum_{j=1}^n a_{ij} b_{jk}$	sum of a i j times b j k for j from one to n; sum over j (ranging) from one to n of a i j times b j k
$\sum_{i=0}^n \binom{n}{i} x^i y^{n-i}$	sum of n over i x to the i y to the n minus i for i from nought [= zero] to n

Inequalities

$x > y$	`x is greater than y`
$x \geq y$	`x is greater (than) or equal to y`
$x < y$	`x is smaller than y`
$x \leq y$	`x is smaller (than) or equal to y`
$x > 0$	`x is positive`
$x \geq 0$	`x is positive or zero; x is non-negative`
$x < 0$	`x is negative`
$x \leq 0$	`x is negative or zero`

Watch how these mathematical expressions are changed from Math Talk into Plain English.

$5 + 1$
five plus one

$a + 6$
a plus six

$4 - 3$
four minus three

$a - 5$
a subtract five

3×4
three times four

$5 \cdot 2$
five times two

$4x$
four x

$5y$
five times y

Congruences

Two integers a, b are *congruent* modulo a positive integer m if they have the same remainder when divided by m (equivalently, if their difference $a - b$ is a multiple of m).

$a \equiv b \pmod{m}$ $a \equiv b \ (m)$	`a is congruent to b modulo m`

Basic arguments

It follows from ... that ...

We deduce from ... that ...

Conversely, ... implies that ...

Equality (1) holds, by Proposition 2.

By definition, ...

The following statements are equivalent.

Thanks to ..., the properties ... and ... of ... are equivalent to each other.

... has the following properties.

Theorem 1 holds unconditionally.

This result is conditional on Axiom A.

... is an immediate consequence of Theorem 3.

Note that ... is well-defined, since ...

As ... satisfies ..., formula (1) can be simplified as follows.

We conclude (the argument) by combining inequalities (2) and (3).

(Let us) denote by X the set of all ...

Let X be the set of all ...

Recall that ..., by assumption.

It is enough to show that ...

We are reduced to proving that ...

The main idea is as follows.

We argue by contradiction. Assume that ... exists.

The formal argument proceeds in several steps.

Consider first the special case when ...

The assumptions ... and ... are independent (of each other), since ...

..., which proves the required claim.

We use induction on n to show that ...

On the other hand, ...

..., which means that ...

In other words, ...

$f(x)$	f of x	$\sinh(x)$	hyperbolic sine x
$g(x,y)$	g of x (comma) y	$\cosh(x)$	hyperbolic cosine x
$h(2x,3y)$	h of two x (comma)	$\tanh(x)$	hyperbolic tan x
$\sin(x)$	sine x	$\sin(x^2)$	sine of x squared
$\cos(x)$	cosine x	$\sin(x)^2$	sine squared of x; sine x, all squared
$\tan(x)$	tan x	$\frac{x+1}{\tan(y^4)}$	x plus one, all over over tan of y to the four
$\arcsin(x)$	arc sine x	$3^{x-\cos(2x)}$	three to the (power of) x minus cosine of two x
$\arccos(x)$	arc cosine x	$\exp(x^3+y^3)$	exponential of x cubed plus y cubed
$\arctan(x)$	arc tan x		

Set theory

$x \in A$	x is an element of A; x lies in A; x belongs to A; x is in A
$x \notin A$	x is not an element of A; x does not lie in A; x does not belong to A; x is not in A
$x, y \in A$	(both) x and y are elements of A; ...lie in A; ...belong to A; ...are in A
$x, y \notin A$	(neither) x nor y is an element of A; ...lies in A; ...belongs to A; ...is in A
$\emptyset$	the empty set (= set with no elements)
$A = \emptyset$	A is an empty set
$A \neq \emptyset$	A is non-empty
$A \cup B$	the union of (the sets) A and B; A union B
$A \cap B$	the intersection of (the sets) A and B; A intersection B
$A \times B$	the product of (the sets) A and B; A times B
$A \cap B = \emptyset$	A is disjoint from B; the intersection of A and B is empty
$\{x \mid \ldots\}$	the set of all x such that ...
$\mathbf{C}$	the set of all complex numbers
$\mathbf{Q}$	the set of all rational numbers
$\mathbf{R}$	the set of all real numbers

Logic

$S \vee T$	S or T
$S \wedge T$	S and T
$S \implies T$	S implies T; if S then T
$S \iff T$	S is equivalent to T; S iff T
$\neg S$	not S
$\forall x \in A \ldots$	for each [= for every] x in A ...
$\exists x \in A \ldots$	there exists [= there is] an x in A (such that) ...
$\exists!\, x \in A \ldots$	there exists [= there is] a unique x in A (such that) ...
$\nexists x \in A \ldots$	there is no x in A (such that)...

$x > 0 \wedge y > 0 \implies x + y > 0$	if both x and y are positive, so is $x + y$
$\nexists x \in \mathbf{Q} \quad x^2 = 2$	no rational number has a square equal to two
$\forall x \in \mathbf{R}\, \exists y \in \mathbf{Q} \quad \lvert x - y \rvert < 2/3$	for every real number x there exists a rational number y such that the absolute value of x minus y is smaller than two thirds

Exercise. Read out the following statements.

$$x \in A \cap B \iff (x \in A \wedge x \in B), \qquad x \in A \cup B \iff (x \in A \vee x \in B),$$
$$\forall x \in \mathbf{R} \quad x^2 \geq 0, \qquad \neg \exists x \in \mathbf{R} \quad x^2 < 0, \qquad \forall y \in \mathbf{C}\, \exists z \in \mathbf{C} \quad y = z^2.$$

Intervals

(a,b)	open interval a b
$[a,b]$	closed interval a b
$(a,b]$	half open interval a b (open on the left, closed on the right)
$[a,b)$	half open interval a b (open on the right, closed on the left)

f'	f dash; f prime; the first derivative of f
f''	f double dash; f double prime; the second derivative of f
$f^{(3)}$	the third derivative of f
$f^{(n)}$	the n-th derivative of f
$\frac{dy}{dx}$	d y by d x; the derivative of y by x
$\frac{d^2y}{dx^2}$	the second derivative of y by x; d squared y by d x squared
$\frac{\partial f}{\partial x}$	the partial derivative of f by x (with respect to x); partial d f by d x
$\frac{\partial^2 f}{\partial x^2}$	the second partial derivative of f by x (with respect to x) partial d squared f by d x squared
∇f	nabla f; the gradient of f
Δf	delta f

$\int f(x)\,dx$	integral of f of x d x
$\int_a^b t^2\,dt$	integral from a to b of t squared d t
$\iint_S h(x,y)\,dx\,dy$	double integral over S of h of x y d x d y

Small Greek letters used in mathematics

α	alpha	β	beta	γ	gamma	δ	delta
ϵ, ε	epsilon	ζ	zeta	η	eta	θ, ϑ	theta
ι	iota	κ	kappa	λ	lambda	μ	mu
ν	nu	ξ	xi	o	omicron	π, ϖ	pi
ρ, ϱ	rho	σ	sigma	τ	tau	υ	upsilon
ϕ, φ	phi	χ	chi	ψ	psi	ω	omega

Capital Greek letters used in mathematics

B	Beta	Γ	Gamma	Δ	Delta	Θ	Theta
Λ	Lambda	Ξ	Xi	Π	Pi	Σ	Sigma
Υ	Upsilon	Φ	Phi	Ψ	Psi	Ω	Omega

A. Cardinal Numbers (Counting Numbers) /'ka: dinl 'nʌmbə(r)/

/kaunting 'nʌmbə(r)/

Example:

1	one	/wʌn/
2	two	/tu:/
3	three	/θri:/
4	four	/fɔ:/
5	five	/faɪv/
6	six	/sɪks/
7	seven	/'sevən/
8	eight	/eɪt/
9	nine	/naɪn/
10	ten	/ten/
11	eleven	/ɪ'levən/
12	twelve	/twelv/
13	thirteen	/θɜ:'ti:n/
14	fourteen	/fɔ:'ti:n/
15	fifteen	/fɪf'ti:n/
16	sixteen	/sɪkst'i:n/
17	seventeen	/seven'ti:n/
18	eighteen	/eɪ'ti:n/
19	nineteen	/naɪn'ti:n/
20	twenty	/'twentɪ/
21	twenty-one	/twentɪ'wʌn/
22	twenty-two	/twentɪ'tu:/
23	twenty-three	/twentɪ'θri:/
24	twenty-four	/twentɪ'fɔ:/
25	twenty-five	/twentɪ'faɪv/
26	twenty-six	/twentɪ'sɪks/
30	thirty	/'θɜ:tɪ/
40	forty	/'fɔ:tɪ/
50	fifty	/'fɪftɪ/
60	sixty	/'sɪkstɪ/
70	seventy	/'sevəntɪ/
80	eighty	/'eɪtɪ/
90	ninety	/'naɪntɪ/
100	a hundred; one hundred	/ə 'hʌndrəd/ /wʌn 'hʌndrəd/
101	a hundred and one	/ə 'hʌndrəd ən wʌn/
110	a hundred and ten	/ə 'hʌndrəd ən ten/
120	a hundred and twenty	/ə 'hʌndrəd ən 'twentɪ/
200	two hundred	/tu: 'hʌndrəd/
300	three hundred	/θri: 'hʌndrəd/
900	nine hundred	/naɪn 'hʌndrəd/
1 000	a thousand, one thousand	/ə θ'aʊzənd/ /wʌn 'θaʊzənd/
1 001	a thousand and one	/ə 'θaʊzənd ən wʌn/
1 010	a thousand and ten	/ə 'θaʊzənd ən ten/
1 020	a thousand and twenty	/ə 'θaʊzənd ən 'twentɪ/
1 100	one thousand, one hundred	/wʌn 'θaʊzənd wʌn 'hʌndrəd/
1 101	one thousand, one hundred and one	/wʌn 'θaʊzənd wʌn 'hʌndrəd ən wʌn/
9 999	nine thousand, nine hundred and ninety-nine	/naɪn 'θaʊzənd naɪn 'hʌndrəd ən 'naɪntɪ 'naɪn/
10 000	ten thousand	/ten 'θaʊzənd/
100 000	a hundred thousand	/ə 'hʌndrəd 'θaʊzənd/
1 000 000	a million	/ə 'mɪljən/
1 000 000 000	a billion	/ə 'bɪljən/
1 000 000 000 000	a trillion	/ə 'trɪljən/

B. Ordinal Numbers/Place Numbers /'ərdinəl 'nʌmbə(r)/

Example:

1st	first	/fɜ:st/
2nd	second	/'sekənd/
3rd	third	/θɜ:d/
4th	fourth	/fɔ:θ/
5th	fifth	/fɪfθ/
6th	sixth	/sɪksθ/
7th	seventh	/'sevənθ/
8th	eighth	/eɪtθ/
9th	ninth	/naɪnθ/
10th	tenth	/tenθ/
26th	twenty-sixth	/twentɪ'sɪksθ/
27th	twenty-seventh	/twentɪ'sevənθ/
28th	twenty-eighth	/twentɪ'eɪtθ/
29th	twenty-ninth	/twentɪ'naɪnθ/
30th	thirtieth	/'θɜ:tɪəθ/
31st	thirty-first	/θɜ:tɪ'fɜ:st/
40th	fortieth	/'fɔ:tɪəθ/
50th	fiftieth	/'fɪftɪəθ/
100th	hundredth	/'hʌndrədθ/
1 000th	thousandth	/'θaʊzəndθ/
1 000 000th	miilionth	/'mɪljənθ/

- **Natural Numbers**

/'nætʃral 'nʌmbə(r)/

1,2,3,... *one, two, three, and so forth (without end).*

1,2,3,..., 10 *one, two, three, and so forth up to ten.*

11th	eleventh	/ɪ'levənθ/
12th	twelfth	/'twelfθ/
13th	thirteenth	/θɜ:'ti:nθ/
14th	fourtheenth	/fɔ:'ti:nθ/
15th	fidteenth	/fɪf'ti:nθ/
16th	sixteenth	/sɪks'ti:nθ/
17th	seventeenth	/seven'ti:nθ/
18th	eighteenth	/eɪ'ti:nθ/
19th	nineteenth	/naɪn'ti:nθ/
20th	twentieth	/'twentɪəθ/
21st	twenty-first	/twentɪ'fɜ:st/
22nd	twenty-second	/twentɪ'sekənd/
23rd	twenty-third	/twentɪ'θɜ:d/
24th	twenty-fourth	/twentɪ'fɔ:θ/
25th	twenty-fifth	/twentɪ'fɪfθ/

Natural numbers can be divided into two sets:

Odd Numbers /ɒd 'nΛmbə(r)/ and **Even Numbers** /'i:vn 'nΛmbə(r)/

- **Whole Numbers** /həʊl 'nΛmbə(r)/

  Natural Numbers + 0 *zero/o/nought.*

  /'ziərəu/ /nə:t/

- **Integers** /'intəjər/

  ..,--2,1,0,1,.. *.., negative two, negative one, zero, one, ..*

- **Rational numbers** /'ræʃnəl 'nΛmbə(r)/ are numbers that can be expressed as fraction.

- **Irrational Numbers** /i'ræʃnəl 'nΛmbə(r)/ are numbers that cannot be expressed as fraction, such as $\sqrt{2}, \pi$.

- **Real Numbers** /riəl 'nΛmbə(r)/ are made up of rational and irrational numbers.

- **Complex Numbers** /'kompleks 'nΛmbə(r)/

  Complex numbers are numbers that contain **real** and **imaginary** part.

$2+3i$ 2 is called the **real part**, 3 is called the **imaginary part**, and i is called **imaginary unit** of the complex number.

- **A Digit** /'dɪdʒɪt/ is any one of the ten numerals 0,1,2,3,4,5,6,7,8,9.

  Example:

  3 is a single-digit number, but 234 is a three-digit number.

  In 234, 4 is the units digit, 3 is the tens digit, and 2 is hundreds digit.

- **Consecutive** /kən'sekjʊtɪv/ **numbers** are counting numbers that differ by 1.

  Examples:

  83, 84, 85, 86, and 87 are 5 consecutive numbers.

  84, 85, 86, ... are successor /sək-'ses-ə(r)/ of 83.

  84 is the immediate successor of 83.

  1, 2, ..., and 82 are predecessor /'predə-ses-ə(r)/of 83.

  82 is the immediate predecessor of 83.

  36, 38, 40, and 42 are 4 consecutive even numbers.

Operation on Numbers

Addition (+) , Subtraction (-) , Multiplication (×) , Division(:)

/ə'diʃn/ /sab'træksyən/ /'maltəplə'keisyen/ /di'vi3n/

Symbols in Numbers Operation

+	added by/plus/and /ædid bai/ /plΛs/ /ənd/
-	subtracted by/minus/take away /səb'træktid bai/ /'mainəs/ /teik ə'wei/
±	plus or minus /plΛs o:(r) 'mainəs/
×	multiplied by/times /'mΛltiplaid bai/ /taimz/
:	divided by/over /di'vaidid bai/ /'əuvə(r)/

Symbols for Comparing /kəm'peə(r)ing / **Numbers**

=	is equal to/equals/is /iz "i:kwəl tu:/ /"i:kwəlz/ /iz/
≠	is not equal to/does not equal /iz not "i:kwəl tu:/ /'dΛznt "i:kwəl/
<	is less than/is smaller than /iz les thən/ /iz smŏlər thən/
>	is greater than/is more than /iz greitər thən/ /iz mə:(r) thən/
≤	is less than or equal to /iz les thən o:(r) "i:kwəl tu:/
≥	is more/greater than or equal to /iz mə:(r)/ /greitər thən o:(r) "i:kwəl tu:/
≅	is approximately equal to /iz ə'proksimatli "i:kwəl tu:/

The mathematical sentences that use symbols "=" are called **equation**, and the mathematical sentences that use symbols "<", ">", "≤", or "≥" are called **inequalities**.

Examples

$ax + b = 0$ is a linear equation.

$ax^2 + bx + c = 0$ is a quadratic equation.

$3x^3 - 2x^2 + 3 = 0$ is a cubic equation.

$\frac{a+b}{2} \geq \sqrt{ab}$ is called AM-GM inequality.

Examples

- $2 + 3 = 5$

two	*is added by* *plus* *and*	*three*	*is equal to* *equals* *is*	*five.*

2 and 3 are called **addends** or **summands**, and 5 is called **sum**. /sΛm/

- $10 - 4 = 6$

Ten	*is substracted by* *minus* *take away*	*four*	*is equal to* *equals* *is*	*six.*

10 is the **minuend**, 4 is the **subtrahend**, and 6 is the **difference**/'difrəns/

➢ 7 × 8 = 56

Seven	*is multiplied by* *times*	*eight*	*is equal to* *equals* *is*	*fifty-six*

7 is the **multiplicator**/’mΛltəplə’kətwr/, 8 is the **multiplicand**/’mΛltəplə’kənd/, and 56 is the **product**/’prodəkt/.

➢ 45 : 5 = 9

forty-five	*is divided by* *over*	*five*	*is equal to* *is*	*nine.*

45 is the **dividend**, 5 is the **divisor** /də’vaizə(r)/, and 9 is the **quotient**/’kwəuʃnt/.

Practice

1. Read out the following operations, and for every operations name each number’s function.
   a. 1,209 + 118 = 1,327
   b. 135 + (-132) = 3
   c. 2 – (-25) = 27
   d. 52 – 65 = -13
   e. 9 x 26 = 234

Saying Fraction

$\frac{1}{2}$	A/one half /ə/wΛn ha:f/
$\frac{1}{3}$	A/one third /ə/wΛn θɜ:d/
$\frac{1}{4}$	A/one quarter /ə/wΛn ‘kwɔ:tə(r)/
$\frac{5}{6}$	Five sixths/Five over six
$\frac{22+x}{7}$	Twenty-two plus x all over seven
$13\frac{3}{4}$	Thirteen and three quarters

0.3	Nought/zero/o point three
3.056	Three point o five six
273.856	Two hundred and seventy-three point eight five six

Practice

1. Read out the following fractions
   a. $\frac{2}{5}$
   b. $\frac{3}{4}$
   c. $\frac{5}{8} x \frac{1}{4} = \frac{5}{32}$
   d. $2\frac{1}{2} : \frac{9}{10} = 3\frac{2}{5}$
   e. $\frac{1}{9} - \frac{1}{8} \neq \frac{1}{24}$
   f. 13,945.614
   g. 43.554
   h. 6.9 x 2.2 = 15.18
   i. 72.4 x 61.5 = 4452.6
2. Fill the blank spaces with the right words.
   a. In the fraction seven ninths, __________ is the numerator, and _____________ is the _____________.
   b. The ______________ of two thirds and a half is four over three.
   c. An integer plus a fraction makes a _________________.

Divisibility

4 | 12

12 is ***divisible*** *by 4.*
/di'vizəbl/

12 is a ***multiple*** *of 4.*
/mʌltipl/

4 ***divides*** *12.*
/di'vaidz/

4 ***is a factor*** *of 12*
/'fæktə(r)/

15 is not divisible by 4.

If 15 divided by 4 then the quotient is 3 and **the remainder** is 3.
/thə ri’meɪndə(r)/

0 is divisible by all integers

Prime numbers /praim ’nΛmbə(r)z/

Every numbers is divisible by 1 and itself. These factors (1 and itself) are called **improper divisors**. /im’propə(r) də’vaizə(r)z/

Prime numbers are numbers that have only improper divisors.

Example:

5 is a prime number, but 9 is not a prime number or a **composite number**. /kompəzit ’nΛmbə(r)z/

Common Divisors /‘komən də’vaizə(r)z/

Example:

1,2,3,4,6, and 12 are divisors (factors) of 12.

1,3,5, and 15 are divisors of 15.

1 and 3 are ***common divisors*** *of 12 and 15.*

3 is the ***greatest common divisor*** /greitəst ‘komən də’vaizə(r)/ *of 12 and 15.*

The ***g.c.d of*** *12 and 15 is 3.*

gcd(12,15) = *3.*

Common Multiples /‘komən ‘mΛltiplz/

Example:

5,10,15,20,25, …are multiples of 5.

4,8,12,16,20,24,… are multiples of 4.

5,10,15,20 are four first multiples of 5.

4,8,12,16,20 are five first multiples of 4.

20,40,60, … are ***common multiples*** *of 4 and 5.*

20 is the ***least common multiple*** /li:st ‘komən ‘mΛltipl/ *of 4 and 5.*

The ***l.c.m of*** *4 and 5 is 20.*

How to Say Powers

x^2	x squared /ˈskweə(r)d/
x^3	x cubed /kju:bd/
x^n	x to the power of n x to the n-th power x to the n x to the n-th x upper /ˈΛpə(r)/ n x raised /reizd/ by n
$(x+y)^2$	x plus y all squared bracket /ˈbrækit/ x plus y bracket closed squared x plus y in bracket squared

Practice

A. Read out the following terms and say their values.

1. 2^6
2. $\left(\frac{2}{3}\right)^3$
3. $x^5 : x^2$
4. $(3ab)^4$

LAWS FOR POWERS

for equal exponents

➢ **First Law for Power:**

$$(ab)^n = a^n b^n$$

A product raised by an exponent is equal to product of factors raised by same exponent

$$(a/b)^n = a^n/b^n$$

For equal basis

➢ **Second Law for Powers:**

$$a^m \times a^n = a^{m+n}$$

- *The product of two powers with equal basis equals to the basis raised to the sum of the two exponents*
- *When expressions with the same base are multiplied, the indices are added*

How can we say this rule?

$$a^m : a^n = a^{m-n}$$

➢ **Third Law for Powers:**

$$(a^m)^n = a^{mn}$$

How to Say Radicals

$\sqrt{x}$	**(square) root of x**
$\sqrt[3]{y}$	**cube root of y**
$\sqrt[n]{z}$	**n-th root of z**
$\sqrt[5]{x^2y^3}$	**fifth root of (pause) x squared times y cubed** **fifth root of x squared times y cubed in bracket**

Logarithm

$$x = a^b \Leftrightarrow b = {}^a\log\ x$$

In this term, a is also called **base.**

How to Say Logarithm

$^n\log x$	**log** /lɒg/**x to the base of n** **log base n of x**
ln 2	**natural log of two** **"L N" of two**
$^5\log^2 25$	**log squared of twenty-five to the base of five** **log base five of twenty-five all squared**

Laws for Logarithm

- **First Law for logarithm:**

  The logarithm of a product is equal to the sum of the logarithm of the factors

$$^b\log(xy) = {}^b\log\ x + {}^b\log\ y$$

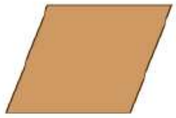
- **Second Law for logarithm:**

  The logarithm of a quotient is equal to the difference of the logarithms of the dividend and divisor

$$^b\log(x/y) = {}^b\log\ x - {}^b\log\ y$$

Parallelogram

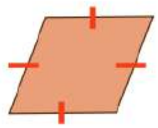
A parallelogram has 2 pairs of parallel sides.



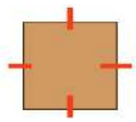
Types of Parallelogram

A rectangle is a parallelogram with 4 right angles.

A rhombus is a parallelogram with 4 sides of equal length



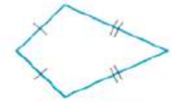
A square is a parallelogram with 4 right angles and 4 sides of equal length.



Another Types of Quadrilateral

A <u>trapezoid</u> has exactly one pair of parallel sides.

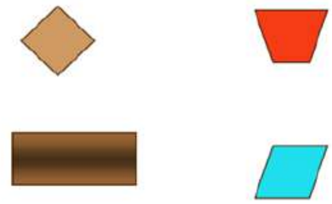
A <u>kite</u> has exactly two pairs of congruent adjacent sides.



A <u>trapezium</u> has exactly one pair of parallel sides and two right angles.

Practice

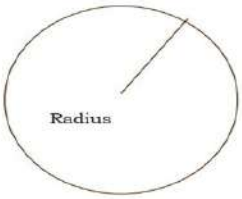
Classify the <u>quadrilaterals</u> in as many ways as possible.



Circle

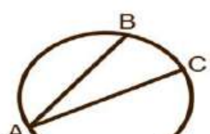
<u>Circle</u> is the set of all points in a plane that are a given distance from the center.

<u>Radius</u> (plural: **radii**) is a segment line that joins the center to a point on the circle.

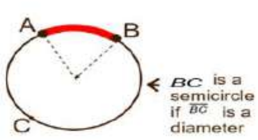


Parts of a Circle

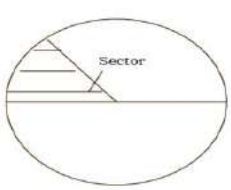
- <u>Chord</u>: a line joining two points on a circle.
  - AC and AB are chords.
- <u>Diameter</u>: a chord that passes through the circles center.
  - AC is a diameter



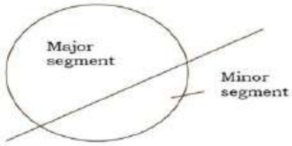
- <u>Arc</u>: two points on a circle and all the points needed to connect them.
  - minor arc AB or
  - major arc ACB or



- <u>Central Angle</u>: an angle whose vertex is at the center of the circle
- <u>Sector</u>: It is a region enclosed by two radii and arc of the circle.

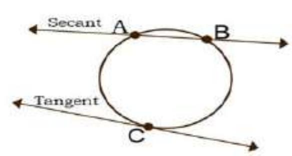


- <u>Segment</u>: It is a region enclosed by a chord and the arc joining the chord. The segment made my minor arc is called minor segment and segment by major arc as major segment.

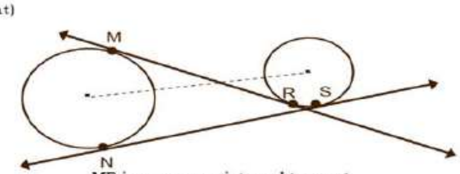


Line that Cuts the Circle

- <u>Secant</u>: a line that intersects a circle at exactly two points
- <u>Tangent</u>: a line that intersects a circle at exactly one point
  - The point of contact is called the **<u>point of tangency</u>** or point of contact.



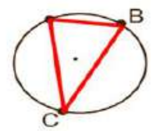
- A **<u>common tangent</u>** is a line tangent to two circles (not necessarily at the same point)



MR is a common <u>internal</u> tangent
NS is a common <u>external</u> tangent

Inscribed and Circumscribed Polygons

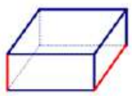
- **<u>Inscribed</u>**: A polygon is inscribed in a circle (or another polygon) if all of its vertices lie on the circle (or another polygon).
  - The circle center is the **<u>incenter</u>** of the polygon



- **<u>Circumscribed</u>**: A polygon is circumscribed about a circle if each of its sides is tangent to the circle.
  - The circle center is the **<u>circumcenter</u>** of the polygon

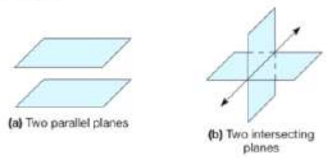
Position of Lines in Space

In plane geometry, two lines can be parallel or intersect. But in space, there is another choice. Two lines can be skew. Skew Lines are nonintersecting lines that are not parallel.



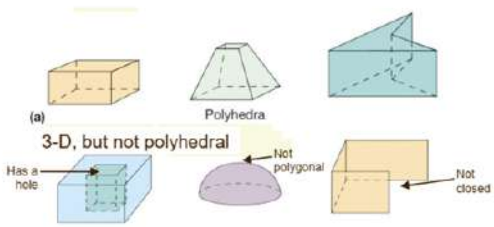
Position of Planes in a Space

In three dimensions, planes are similar to lines in two dimensions. They can be parallel, or intersect.



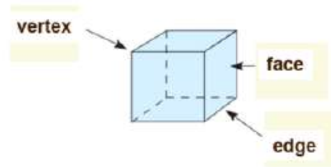
Polyhedra (Bidang Banyak)

- A ***polyhedron*** is the union of polygonal regions such that a finite region of space is enclosed without any holes in the interior.



Parts of a Polyhedra

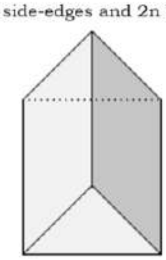
- The polygonal shapes that form the polyhedron are called its ***faces/sides***.
- The line segments where two faces meet are its ***edges***.
- A point where three or more edges meet is a ***vertex***



Types of a Polyhedra

A. Prisms

- If a polyhedron has identical polygonal faces that are opposite each other, then it is a **prism**
- The segments that connect base side and top side is called **side-edges**. The others is called **based-edges**.
- A n-sided prism has n side-edges and 2n base-edges

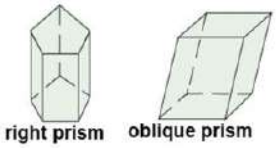


- The opposite, identical side of a prism are called its ***bases***. The other sides are parallelograms and are called the ***lateral sides***.

Symbol	Usage	Interpretation	Article
$\cup$	$A \cup B$	union of the sets A and B	Union (set theory)
$\cap$	$A \cap B$	intersection of the sets A and B	Intersection (set theory)
$\setminus$	$A \setminus B$	difference of sets A and B	Difference (set theory)
$\triangle$	$A \triangle B$	symmetric difference of sets A and B	Symmetric difference
$\times$	$A \times B$	Cartesian product of sets A and B	Cartesian product
$\dot{\cup}$	$A \dot{\cup} B$	disjoint union of sets A and B	Disjoint union
$\sqcup$	$A \sqcup B$	disjoint intersection of sets A and B	
C	A^{C}	complement of the set A	Complement (set theory)
$-$	$\overline{A}$		
$\mathcal{P}$	$\mathcal{P}(A)$	power set of the set A	Power set
$\mathfrak{P}$	$\mathfrak{P}(A)$		

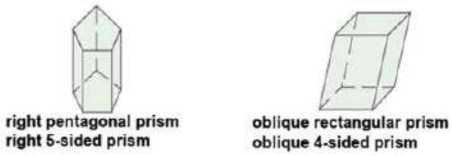
Symbol	Usage	Interpretation	Article
$=$	$a = b$	a equals b	Equality (mathematics)
$\neq$	$a \neq b$	a does not equal b	Inequality (mathematics)
$\equiv$	$a \equiv b$	a is identical to b	Identity (mathematics)
$\approx$	$a \approx b$	a is approximately equal to b	Approximation
$\sim$	$a \sim b$	a is proportional to b	Proportionality (mathematics)
$\propto$	$a \propto b$		
$\hat{=}$	$a \hat{=} b$	a corresponds to b	Correspondence (mathematics)

- If the lateral sides of a prism are rectangles, it is a ***right prism***. If not, it is called an ***oblique prism***.



Naming Prism

When naming a prism we use two main descriptors. First, we say whether it is right or oblique, then we say what type of polygon form the prism's bases or the number of its lateral sides.



Formula for Volume and Surface Area of a Prism

Generally, the formula for volume of prism is

V = Area of its base × its height

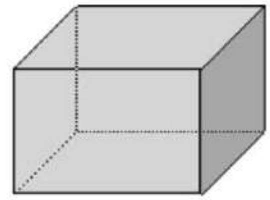
The height in this formula is the real height not the slant height.

General formula for its surface area is

A = sum of all its sides area

2. Cuboid (Rectangular Prism)

- A rectangular prism has 8 rectangular solid angles, 12 edges, equal and parallel in fours.
- It is bounded by three pairs of congruent rectangles lying in parallel planes.



Formula for Volume and Surface Area of a Cuboid

Suppose that we can name the three different edges of cuboid as length, width, and height. Then, the formula for volume of cuboid is

V = length × width × height

And for surface area is

A = 2×[(length × width) + (length × height) + (width × height)]

Pyramid

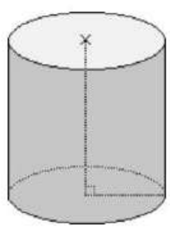
- A **pyramid** is a three-dimensional solid with one polygonal base and with line segments connecting the vertices of the base to a single point somewhere above the base.

Cylinder

- A **cylinder** is a prism in which the bases are circles or ellips.
- The volume of a cylinder is the area of its base times its height $V = \pi r^2 h$
- The surface area of a cylinder is $A = 2\pi r^2 + 2\pi rh$

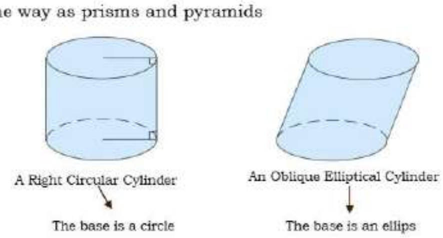
r = length of base's radius

h = height of cylinder



Types of Cylinder

- A cylinder can be right or oblique, and cylinders are named in the same way as prisms and pyramids



Cone

- A **cone** is like a pyramid but with a circular base instead of a polygonal base.
- The volume of a cone is one-third the area of its base times its height:

$V = \frac{1}{3}\pi r^2 h$

- The surface area of a cone is base surface area + curved surface area:

$A = \pi r^2 + \pi rs$

or

$A = \pi r^2 + \pi r\sqrt{r^2 + h^2}$

r = length of base's radius

h = heigth of cone

s = slant height of cone

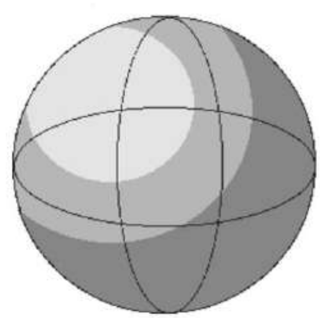
Sphere

- **Sphere** is the mathematical word for "ball." It is the set of all points in space a fixed distance from a given point called the center of the sphere.
- A sphere has a radius and diameter, just like a circle does.
- The volume of a sphere is:

$V = \frac{4}{3}\pi r^3$

- The surface area of a sphere is:

$A = 4\pi r^2$



Symbol	Usage	Interpretation	Article	LaTeX	H
$<$	$a < b$	a is less than b	Comparison (mathematics)	<	<
$>$	$a > b$	a is greater than b		>	>
$\le$	$a \le b$	a is less than or equal to b		\le, \leq	≤
$\leqq$	$a \leqq b$			\leqq	
$\ge$	$a \ge b$	a is greater than or equal to b		\ge, \geq	≥
$\geqq$	$a \geqq b$			\geqq	
$\ll$	$a \ll b$	a is much smaller than b		\ll	
$\gg$	$a \gg b$	a is much bigger than b		\gg	

Divisibility

Symbol	Usage	Interpretation	Article	LaTeX	H
$\mid$	$a \mid b$	a divides b	Divisibility	\mid	
$\nmid$	$a \nmid b$	a does not divide b		\nmid	
$\perp$	$a \perp b$	a and b are relatively prime	Relatively prime	\perp	&perp
$\sqcap$	$a \sqcap b$	greatest common divisor of a und b	Greatest common divisor	\sqcap	
$\wedge$	$a \wedge b$			\wedge	
$\sqcup$	$a \sqcup b$	least common multiple of a und b	Least common multiple	\sqcup	
$\vee$	$a \vee b$			\vee	
$\equiv$	$a \equiv b \pmod{m}$	a and b are congruent modulo m	Modular arithmetic	\equiv	&equ

Intervals

Symbol	Usage	Interpretation	Article	LaTeX	H
[ ]	$[a, b]$	closed interval between a and b	Interval (mathematics)	( ) [ ]	
] [	$]a, b[$	open interval between a and b			
( )	(a, b)				
[[	$[a, b[$	right-open interval between a and b			
[ )	$[a, b)$				
]]	$]a, b]$	left-open interval between a and b			
( ]	$(a, b]$				

Elementary functions

Symbol	Usage	Interpretation	Article	LaTeX	H
$\vert \; \vert$	$\vert x \vert$	absolute value of x	Absolute value	\vert	
[ ]	$[x]$	biggest whole number less than or equal to x	Floor and ceiling functions	[ ]	
$\lfloor \; \rfloor$	$\lfloor x \rfloor$			\lfloor \rfloor	&lfl &rfl
$\lceil \; \rceil$	$\lceil x \rceil$	smallest whole number greater than or equal to x		\lceil \rceil	&lce &rce
$\surd$	$\sqrt{x}$	square root of x	Square root	\sqrt	&rad
	$\sqrt[n]{x}$	n-th root of x	nth root		
%	$x\,\%$	x Percent	Percent	\%	

Note: the power function is not represented by its own icon, but by the positioning of the exponent as a superscript.

Complex numbers

Symbol	Usage	Interpretation	Article	LaTeX	H
$\Re$	$\Re(z)$	real part of complex number z	Complex number	\Re	
$\Im$	$\Im(z)$	imaginary part of complex number z		\Im	
$-$	$\bar{z}$	complex conjugate of z	Complex conjugate	\bar	
$*$	z^*			\ast	&low
$\vert \; \vert$	$\vert z \vert$	absolute value of complex number z	Absolute value	\vert	

Symbol	Usage	Interpretation	Article
π		pi (Archimedes' constant)	Pi
e		Euler's constant	e (mathematics)
φ		golden ratio	Golden ratio
i		imaginary unit (square root of −1)	Imaginary unit

See also: mathematical constant for symbols of additional mathematical constants.

Calculus

Sequences and series

Symbol	Usage	Interpretation	Article
$\sum$	$\sum_{i=1}^{n}, \sum_{i\in I}$	sum from $i = 1$ to n or over all elements i in set I	Summation
$\prod$	$\prod_{i=1}^{n}, \prod_{i\in I}$	product from $i = 1$ to n or over all elements i in set I	Product (mathematics)
$\coprod$	$\coprod_{i=1}^{n}, \coprod_{i\in I}$	coproduct from $i = 1$ to n or over all elements i in set I	Coproduct
( )	(a_n)	sequence of elements $a_1, a_2, \ldots$	Sequence
$\to$	$a_n \to a$	sequence (a_n) tends to limit a	Limit of a sequence
∞	$n \to \infty$	n tends to infinity	Infinity

Functions

Symbol	Usage	Interpretation	Article
$\to$	$f\colon A \to B$	function f maps from set A to set B	Function (mathematics)
	$A \xrightarrow{f} B$		
$\mapsto$	$f\colon x \mapsto y$	function f maps element x to element y	
	$x \overset{f}{\mapsto} y$		
( )	$f(x)$	image of element x under function f	Image (mathematics)
	$f(X)$	image of set X under function f	
[ ]	$f[X]$		
$\vert$	$f\vert_X$	restriction of function f to set X	Restriction (mathematics)
$\cdot$	$f(\cdot)$	placeholder for a variable as argument of function f	Free variable
-1	f^{-1}	inverse function of f	Inverse function
$\circ$	$f \circ g$	composition of functions f and g	Function composition
$*$	$f * g$	convolution of functions f and g	Convolution
$\hat{}$	$\hat{f}$	Fourier transform of function f	Fourier transform

Limits

Symbol	Usage	Interpretation	Article
$\uparrow$	$\lim_{x\uparrow a} f(x)$	limit of function f as x approaches a from below	Limit of a function
$\nearrow$	$\lim_{x\nearrow a} f(x)$		
$\to$	$\lim_{x\to a} f(x)$	limit of function f as x approaches a	
$\searrow$	$\lim_{x\searrow a} f(x)$	limit of function f as x approaches a from above	
$\downarrow$	$\lim_{x\downarrow a} f(x)$		

Elementary geometry

Symbol	Usage	Interpretation	Article
[ ]	$[AB]$	line segment between points A and B	Line segment
| |	$\|AB\|$	length of line segment between points A and B	
—	$\overline{AB}$		
→	$\overrightarrow{AB}$	vector between points A and B	Euclidean vector
∠	$\angle ABC$	angle between line segments BA and BC	Angle
△	$\triangle ABC$	triangle with vertices A, B and C	Triangle
□	$\square ABCD$	quadrilateral with vertices A, B, C and D	Quadrilateral
∥	$g \parallel h$	lines g and h are parallel	Parallel (geometry)
∦	$g \nparallel h$	lines g and h are not parallel	
⊥	$g \perp h$	lines g and h are orthogonal	Orthogonality

Vectors and matrices

Symbol	Interpretation	Article
$(v_1, \ldots, v_n)$	row vector comprising elements v_1 through v_n	Vector (mathematics and physics)
$\begin{pmatrix} v_1 \\ \vdots \\ v_m \end{pmatrix}$	column vector comprising elements v_1 through v_m	
$\begin{pmatrix} a_{11} & \ldots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \ldots & a_{mn} \end{pmatrix}$	matrix comprising elements a_{11} through a_{mn}	Matrix (mathematics)

Vector calculus

Symbol	Usage	Interpretation	Article
·	$v \cdot w$	dot product of vectors v and w	Dot product
( )	(v, w)		
⟨ ⟩	$\langle v, w\rangle$ $\langle v \mid w\rangle$		
×	$v \times w$	cross product of vectors v and w	Cross product
[ ]	$[v, w]$		
( )	(u, v, w)	triple product of vectors u, v and w	Triple product
⊗	$v \otimes w$	dyadic product of vectors v and w	Dyadic product
∧	$v \wedge w$	wedge product of vectors v and w	Wedge product
| |	$\|v\|$	length of vector v	Euclidean norm
‖ ‖	$\\|v\\|$	norm of vector v	Norm (mathematics)
ˆ	$\hat{v}$	normalized vector of vector v	Unit vector

Matrix calculus

Symbol	Usage	Interpretation	Article
·	$A \cdot B$	product of matrices A and B	Matrix multiplication
∘	$A \circ B$	Hadamard product of matrices A and B	Hadamard product (matrices)
⊗	$A \otimes B$	Kronecker product of matrices A and B	Kronecker product
T	A^{T}	transposed matrix of matrix A	Transposed matrix
H	A^{H}	conjugate transpose of matrix A	Conjugate transpose
*	A^*		
†	$A^\dagger$		
−1	A^{-1}	inverse matrix of matrix A	Inverse matrix
+	A^+	Moore–Penrose pseudoinverse of matrix A	Moore–Penrose pseudoinverse
| |	$\|A\|$	determinant of Matrix A	Determinant
‖ ‖	$\\|A\\|$	norm of matrix A	Matrix norm