



## Department of Mathematics-College of Science

### Salahaddin University/Erbil

**Subject: Measure theory**

**Course Book – (M.Sc. Level) – College of Science002 --  
Measure Theory Course book-M.Sc-2023-2024**

**Lecturer's name: Ibrahim O. Hamad**

**Academic Year: 2023-2024 -- Second Semesters**

له خوارهوه خشتهی ریژهی نههاتن رون دهکهینهوه له بابتهی Modern Geometry که پنیوسته پابهند بن پی ی:

ناگاداری کوتایی	ناگاداری سههتایی	وشیارکردنهوه	ژماره ی کاتژمیرهکان له ههفتهیهکدا (تیوری)
6	4	2	2

## Course Book

<b>1. Course name</b>	Measure Theory
<b>2. Lecturer in charge</b>	Ibrahim Othman Hamad
<b>3. Department/ College</b>	Mathematics / Science
<b>4. Contact</b>	e-mail : <a href="mailto:ibrahim.hamad@su.edu.krd">ibrahim.hamad@su.edu.krd</a> Tel: (optional) 07504630477
<b>5. Time (in hours) per week</b>	For example Theory: 2 Practical:
<b>6. Office hours</b>	Wensday and Thursday G-A(8:30 – 10:30), G-B(10:30 – 12:30)
<b>7. Course code</b>	
<b>8. Teacher's academic profile</b>	<p><b>Education:</b></p> <p>PhD</p> <p><b>Date:</b> 25-5-2007</p> <p><b>Title:</b> Generalized Curvature and Torsion in Nonstandard Analysis</p> <p><b>Place of Attainments:</b> Mathematics Department, College of Science, University of Salahaddin\Erbil, Hawler (Erbil), Kurdistan Region, Iraqi.</p> <p><b>Supervisor:</b> Professor Dr. Tahir Hassan Ismail</p> <p><b>Supervisor Address:</b> Mathematics Department, College of Computer Science and Mathematics, University of Mosul, Mosul, Iraq. Email: <a href="mailto:tahir_hs@yahoo.com">tahir_hs@yahoo.com</a></p> <p><b>M. Sc.</b></p> <p><b>Date:</b> 2-8-2000</p> <p><b>Title:</b> A Nonstandard Study on The Taylor Series Development</p> <p><b>Place of Attainments:</b> Math. Dept.-College of Sci.-Univ. of Salahaddin\Erbil-Iraq.</p> <p><b>Supervisor:</b> Professor Dr. Tahir Hassan Ismail,</p> <p><b>B. Sc</b></p>

	<p><b>Date:</b> 27-6-1992</p> <p><b>Place of Attainments:</b> Math. Dept.-College of Sci.-Univ. of Salahaddin\Erbil-Iraq Title of the Graduation Project: Number of Limit Cycles of Nonlinear Autonomous Homogeneous System of Degree Three Academic records</p> <p>2009 – Present : Assistant (Associate) Professor, 2004 – 2007: PhD student, 2005 – 2009: Lecturer, 2000 – 2005: Assistant Lecturer 1998 – 2000: M.Sc Student, 1993 – 1998: Assistant Researcher in Math. Dept.</p> <p><b>Supervising</b> 1 Ph.D + 2 M.Sc. +2 M.Sc.(under Supervision)</p> <p><b>Committees Membership and Positions</b></p> <ol style="list-style-type: none"><li>1. Member of several scientific and other department and college committees.</li><li>2. Member of the College Scientific Promotion committee.</li><li>3. 2014 Organizer of CIMPA-KURDISTAN-IRAQ research school, Inverse problems: Theory and applications, University of Salahaddin, Erbil, Kurdistan-Iraq, May 5-14, 2014, <a href="http://www.cimpa-icpam.org/spip.php?article564">http://www.cimpa-icpam.org/spip.php?article564</a> Visiting.</li><li>4. 2011 Institute de Mathematiques de Jussieu - Paris, France; for a period one month.</li><li>5. 2006 (INSPERM),University of Putra - Kuala Lumpur, Malaysia, for a period 6 month as a second part of PhD research.</li></ol> <p><i>unfortunately, we were unable to in compliance with the above invitations because of the politic situation of Iraq at that time</i></p> <p><b>Conferences and Schools:</b></p> <ol style="list-style-type: none"><li>1. Research School CIMPA UNESCO – EGYPT, Recent Development in the Theory of Elliptic Partial Differential Equations 25/1-3/2/2009. Alexandria, Egypt.</li><li>2. CIMPA-UNESCO-IPM School, Repres. Theory of Algebras,15-25-6/, 2008.Tehran, Iran.</li><li>3. First Iraq-French Mathematics Conferenc, Cooperation with College of Science 14 -18/11/2009. Hawler (Erbil) - Kurdistan Region, Iraq.</li></ol>
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	<p>4. The Second Conference on Mathematical Sciences (CMS'2008) 22-23/10/2008. Jordan, Zarqa.</p> <p>5. The Second International Conference of Mathematics 26-30/10/2008. Syria, Aleppo.</p> <p>6. International Congress "Nonstandard Methods and Applications in Mathematics- NSM" 25-31/5/ 2006, Pisa,Italy.</p> <p><i>unfortunately, we were unable to in compliance with the above invitations because of the politic situation of Iraq at that time</i></p> <p><b>Publications:</b> Papers: 20 papers in local and international journals</p> <p><b>Books:</b> Generalized Curvature and Torsion in Nonstandard Analysis, ( Nonstandard Technical Treatment for Some Differential Geometry Concepts), LAP Lambert Academic Publishing ISBN 978-3-8443-0763-4, 140 Pages.</p> <p><b>Articles</b></p> <ul style="list-style-type: none"> <li>• The Development Project of the Educational Program in Iraq (In Arabic), Journal of Afaq Al- Terbewiyha, Issued by the Ministry of Education – Iraqi Kurdistan Region, No.3(2004), pp 107-114.</li> <li>• Some Notes, About The Integration Subject in the Mathematics Book of 6th Secondary School (In Kurdish) Journal of Assoy Parwardayi, Issued by the Ministry of Education – Iraqi Kurdistan Region, No.38(2003), pp 58-63.</li> </ul> <p><b>Courses Taught</b></p> <p>1. <i>1993-1998, as an Assistant Researcher:</i></p> <p><i>(Theoretical):</i> O.D.E, probability, &amp; Statistics, Euclidean and Non Euclidean Geometry. <i>(Tutorial):</i> Euclidean and Non Euclidean Geometry y, Statistics, O.D.E, P.D.E, Topology. <i>(Computer Laboratory):</i> Basic, Fortran, and Pascal Language.</p>
<p><b>9. Keywords</b></p>	<p>Sigma-algebras, outer measure, inner measure, Lebesgue Measure, Measurable Functions, Borel Sets</p>
<p><b>10. Course overview:</b></p> <p>This course offers an in-depth introduction to measure theory, a crucial area of mathematical analysis with wide-ranging applications in probability, statistics, and various branches of mathematics. The topics covered include the construction and properties of measures, measurable functions, the Lebesgue measure, and convergence theorems. The course also introduces the concept of zero sets and the Radon-Nikodym Theorem.</p>	

**11. Course objective:**

- Understand the concept and construction of measures.
- Construct and understand measures and sigma-algebras
- Define and work with measurable functions.
- Integrate functions using the Lebesgue measure.
- Apply the major convergence theorems.
- Analyze properties of measures, including sigma-algebras and measurable sets.
- Understand and apply the concept of zero sets.
- Develop and present rigorous mathematical proofs in the context of measure theory.

**12. Student's obligation**

Students and their obligations throughout the academic year, is the attendance and completion of all tests, exams, assignments.

**13. Forms of teaching**

Magic board and discussion and allow leg students to write some problems on the board and assignments and I give hard copy of my lecture notes to students before coming lecturer time.

**14. Assessment scheme**

The students are required to do two closed book exams during of the study year. The exam has 20 marks, attendance, classroom activities with quizzes 10 marks, assignments 10 marks. Report and siminar 10 marks. There will be a final exam on 50 marks with final Exam 50 marks.

**15. Student learning outcome:**

By the end of this course, students will be able to:

- 1- Construct Measures and Sigma-Algebras: Understand the fundamental concepts of measure theory, including the construction and properties of sigma-algebras and measures.
- 2- Work with Measurable Functions: Define and manipulate measurable functions, demonstrating an ability to analyze their properties and behaviors.
- 3- Apply the Lebesgue Integral: Integrate functions using the Lebesgue integral, and compare it to the Riemann integral, understanding its advantages and applications.
- 4- Convergence Theorem, Fatou's Lemma, and the Dominated Convergence Theorem to solve problems and prove results in measure theory.
- 5- Utilize Convergence Theorems: Apply key convergence theorems such as the Monotone.
- 6- Apply the Radon-Nikodym Theorem: Understand and apply the Radon-Nikodym Theorem in various contexts, including its implications for absolute continuity and the existence of densities.
- 7- Develop Rigorous Proofs: Construct clear, concise, and rigorous mathematical proofs related to measure theory, demonstrating logical reasoning and effective communication of complex ideas.

**16. Course Reading List and References:**

**References**

- 1) Method of Real Analysis by R. Goldberg. **3<sup>rd</sup> Ed. 2020**
- 2) Mathematical Analysis by T. Apostol. **2<sup>nd</sup> Ed. 2002**
- 3) Real Analysis by H. Royden. **4<sup>th</sup> Ed. 2010**
- 4) Principles of Mathematical Analysis by W. Rudin. **3<sup>rd</sup> Ed 1976**
- 5) Measure, Integration, and a Primer on Probability Theory. **2020**
- 6) Measure Theory and Nonlinear Evolution Equations. **2022**
- 7) Measure-Theoretic Calculus in Abstract Spaces. **2024**

**17. The Topics:**

**Chapter I System of Sets**

- 1.1 Introduction.
- 1.2 Elementary family.
- 1.3 Semi-rings.
- 1.4 Rings.
- 1.5  $\sigma$ - rings.
- 1.6 Semi algebra.
- 1.7 Algebra.
- 1.8  $\sigma$ - algebra.
- 1.9 Borel algebra and Borel Sets.

**Chapter II Introduction to Measure**

- 2.1 Some Basic Question about Measure.
  - 2.1.1 What is Measure Theory?
  - 2.1.2 Why Measuring?
  - 2.1.3 How are We Measuring?
- 2.2 Measure of Sets in  $\mathbb{R}$ .
  - 2.2.1 Length and Distance.
  - 2.2.2 Measure of Open and Closed Intervals.
  - 2.2.3 Measure of Open and Closed Sets.

**Chapter III Abstract Measure.**

- 3-1 Inner and Outer Measure of Bounded and Unbounded Sets.
- 3-2 Some Type of Measures.

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<p>3-3 Properties of the Inner and Outer Measure.  3-4 <math>F_\sigma</math> and <math>G_\delta</math> Sets.  3-5 The Class of Measurable Set.  3-6 Translation Invariance of Measure  3-7 Measure Space, <math>\mathcal{L}^p</math> and <math>L^p</math> Spaces.</p> <p><b>Chapter IV Measurable Sets &amp; Functions</b></p> <p>4-1 Measurable Functions.  4-2 The Space of Measurable Functions.  4-3 Properties of Measurable Functions.  (Equivalent fun, inverse image of measurable map and Borel function).  4-4 Complete Measure and Approximation by Simple Functions.  4-5 Egoroff, Lusin's, and Dominated Convergence Theorems.  4-6 Convergency of Sequence of Measurable Functions.</p> <p><b>Chapter V Applications of Measure.</b></p>	
<p><b>19. Examinations:</b>  Compositional: In this type of exam the questions usually starts with Explain how, furthermore it is like as lecture notes and contains some homework , so there will be continuing assignments of problem outside the lecture notes (note that this problem having small marks).</p>	<p><b>19. Examinations:</b>  Compositional: In this type of exam the questions usually starts with Explain how, furthermore it is like as lecture notes and contains some homework , so there will be continuing assignments of problem outside the lecture notes (note that this problem having small marks).</p>
<p><b>20. Extra notes:</b></p>	<p><b>20. Extra notes:</b></p>
<p><b>21. Peer review</b>                      پیداچونہوہی ھاوہل</p>	<p><b>21. Peer review</b>                      پیداچونہوہی ھاوہل</p>

<b>19. Examinations:</b> Compositional: In this type of exam the questions usually starts with Explain how, furthermore it is like as lecture notes and contains some homework , so there will be continuing assignments of problem outside the lecture notes (note that this problem having small marks).	
<b>20. Extra notes:</b>	
<b>21. Peer review</b>	پیداچونہوہی ھاوہل