Ministry of Higher Education and Scientific research



# **Department of Mathematics-College of Science**

# Salahaddin University/Erbil

**Subject: Nonstandard Analysis** 

Course Book – (M.Sc. Level) – College of Education

Lecturer's name: Ibrahim O. Hamad

Academic Year: 2023-2024 -- Second Semesters

له خوار هوه خشتهی ریزهی نه هاتن رون ده که ینهوه له بابه تی Modern Geometry که پیویسته پابهند بن پی ی:

| ئاگادارى كۆتايى | ئاگاداری سەرەتايى | وشياركردناوه | ژمارەى كاتژمىرەكان لە<br>ھەفتەيەكدا(تيۆرى) |
|-----------------|-------------------|--------------|--|
| 9               | 6                 | 3            | 3  |

# **Course Book**

| 1. Course name                | Nonstandard Analysis  |  |
|-------------------------------|---|--|
| 2. Lecturer in charge         | Ibrahim Othman Hamad  |  |
| 3. Department/ College        | Mathematics / Science   |  |
| 4. Contact                    | e-mail : <u>ibrahim.hamad@su.edu.krd</u>  |  |
|                               | Tel: (optional) 07504630477   |  |
| 5. Time (in hours) per week   | For example Theory: 3<br>Practical:   |  |
| 6. Office hours               | Wensday and Thursday G-A(8:30 – 10:30), G-B(10:30 – 12:30)  |  |
| 7. Course code                |   |  |
| 8. Teacher's academic profile | Education:  |  |
|                               | PhD   |  |
|                               | Date: 25-5-2007   |  |
|                               | <i>Title</i> : Generalized Curvature and Torsion in Nonstandard Analysis  |  |
|                               | <i>Place of Attainments</i> : Mathematics Department, College of Science, University of Salahaddin\Erbil, Hawler (Erbil), Kurdistan Region, Iraqi.                  |  |
|                               | Supervisor: Professor Dr. Tahir Hassan Ismail   |  |
|                               | <i>Supervisor Address</i> : Mathematics Department, College of Computer Science and Mathematics, University of Mosul, Mosul, Iraq. Email: <u>tahir_hs@yahoo.com</u> |  |
|                               | M. Sc.  |  |
|                               | Date: 2-8-2000  |  |
|                               | <i>Title</i> : A Nonstandard Study on The Taylor Series Development   |  |
|                               | <i>Place of Attainments</i> : Math. DeptCollege of SciUniv. of Salahaddin\Erbil-<br>Iraq.   |  |
|                               | Supervisor: Professor Dr. Tahir Hassan Ismail,  |  |
|                               | B. Sc   |  |

| I |  |  |  |
|---|--|--|--|
|   | Date: 27-6-1992  |  |  |
|   | <i>Place of Attainments</i> : Math. DeptCollege of SciUniv. of Salahaddin\Erbil-<br>Iraq Title of the Graduation Project:  |  |  |
|   | Number of Limit Cycles of Nonlinear Autonomous Homogeneous System of<br>Degree Three Academic records  |  |  |
|   | 2009 – Present : Assistant (Associate) Professor,<br>2004 – 2007: PhD student,   |  |  |
|   | 2005 – 2009: Lecturer,   |  |  |
|   | 2000 – 2005: Assistant Lecturer  |  |  |
|   | 1998 – 2000: M.Sc Student,   |  |  |
|   | 1993 – 1998: Assistant Researcher in Math. Dept.   |  |  |
|   | Supervising 1 Ph.D + 2 M.Sc. +2 M.Sc.(under Supervision)   |  |  |
|   | Committees Membership and Positions  |  |  |
|   | <ol> <li>Member of several scientific and other department and college<br/>committees.</li> </ol>  |  |  |
|   | <ol> <li>Member of the College Scientific Promotion committee.</li> <li>2014 Organizer of CIMPA-KURDISTAN-IRAQ research school, Inverse problems: Theory and applications, University of Salahaddin, Erbil, Kurdistan-Iraq, May 5-14, 2014, <u>http://www.cimpa-icpam.org/spip.php?article564</u> Visiting.</li> </ol> |  |  |
|   | 4. 2011 Institute de Mathematiques de Jussieu - Paris, France; for a period  |  |  |
|   | one month.<br>5. 2006 (INSPEM), University of Putra - Kuala Lumpur, Malaysia, for a period<br>6 month as a second part of PhD research.  |  |  |
|   | <i>unfortunately,</i> we were unable to in compliance with the above invitations because of the politic situation of Iraq at that time   |  |  |
|   | Conferences and Schools:   |  |  |
|   | <ol> <li>Research School CIMPA UNESCO – EGYPT, Recent Development in the<br/>Theory of Elliptic Partial Differential Equations 25/1-3/2/2009.<br/>Alexandria, Egypt.</li> </ol>  |  |  |
|   | <ol> <li>CIMPA-UNESCO-IPM School, Repres. Theory of Algebras, 15-25-6/,<br/>2008. Tehran, Iran.</li> </ol>   |  |  |
|   | <ol> <li>First Iraq-French Mathematics Conferenc, Cooparation with College of<br/>Science 14 -18/11/2009. Hawler (Erbil) - Kurdistan Region, Iraq.</li> </ol>  |  |  |
|   |  |  |  |

|             | 4. The Second Conference on Mathematical Sciences (CMS'2008) 22-   |  |  |
|-------------|--|--|--|
|             | 23/10/2008. Jordan, Zarqa.   |  |  |
|             | <ol> <li>The Second International Conference of Mathematics 26-30/10/2008.<br/>Syria, Aleppo.</li> </ol>   |  |  |
|             | <ol> <li>International Congress "Nonstandard Methods and Applications in<br/>Mathematics- NSM" 25-31/5/ 2006, Pisa, Italy.</li> </ol>  |  |  |
|             | <i>unfortunately,</i> we were unable to in compliance with the above invitations because of the politic situation of Iraq at that time   |  |  |
|             | Publications: Papers: 20 papers in local and international journals  |  |  |
|             | <b>Books</b> : Generalized Curvature and Torsion in Nonstandard Analysis, (<br>Nonstandard Technical Treatment for Some Differential Geometry Concepts),<br>LAP Lambert Academic Publishing ISBN 978-3-8443-0763-4, 140 Pages. |  |  |
|             | Articles   |  |  |
|             | <ul> <li>The Development Project of the Educational Program in Iraq (In Arabic),<br/>Journal of Afaq Al- Terbewiyha, Issued by the Ministry of Education – Iraqi<br/>Kurdistan Region, No.3(2004), pp 107-114.</li> </ul>      |  |  |
|             | <ul> <li>Some Notes, About The Integration Subject in the Mathematics Book of<br/>6th Secondary School (In Kurdish) Journal of Assoy Parwardayi, Issued by</li> </ul>  |  |  |
|             | <ul> <li>the Ministry of Education – Iraqi Kurdistan Region, No.38(2003), pp 58-63.</li> <li>Courses Taught</li> <li>1. 1993-1998, as an Assistant Researcher:</li> </ul>  |  |  |
|             |  |  |  |
|             | (Theoretical): O.D.E, probability, & Statistics, Euclidean and Non Euclidean   |  |  |
|             | Geometry. (Tutorial): Euclidean and Non Euclidean Geometry y, Statistics,  |  |  |
|             | O.D.E, P.D.E, Topology. ( <i>Computer Laboratory</i> ): Basic, Fortran, and Pascal<br>Language.  |  |  |
| 9. Keywords | Continuity, Differentiation, Sequences and Series of Functions, Convergencey,<br>Compactness, Darbox and Riemann Integral  |  |  |
|             |  |  |  |

### 10. Course overview:

This graduate-level course provides a comprehensive introduction to nonstandard analysis, an area of mathematics that extends classical analysis by introducing infinitesimals and infinitely large numbers. The course covers the foundations of nonstandard analysis, including internal set theory, and explores various applications and related topics. Emphasis is placed on rigorous mathematical reasoning and proof techniques, preparing students for advanced research and applications of nonstandard analysis.

#### **11. Course objective:**

This course provides the essential foundation for understanding and applying nonstandard analysis, emphasizing rigorous thinking, problem-solving skills, and preparation for advanced research in mathematics

#### 12. Student's obligation

Students and their obligations throughout the academic year, is the attendance and completion of all tests, exams, assignments.

### 13. Forms of teaching

Magic board and discussion and allow leg students to write some problems on the board and assignments and I give hard copy of my lecture notes to students before coming lecturer time.

#### 14. Assessment scheme

The students are required to do two closed book exams during of the study year. The exam has 20 marks, attendance, classroom activities with quizzes 10 marks, Siminar 10 marks, article review 10 marks. There will be a final exam on 50 marks.

#### **15. Student learning outcome:**

By the end of this course, students will be able to:

- 1. Understand the foundations and principles of nonstandard analysis.
- 2. Apply internal set theory to construct and manipulate internal sets.
- 3. Utilize nonstandard techniques to solve problems in analysis.
- 4. Explore and analyze various applications of nonstandard analysis.
- 5. Develop rigorous proofs using nonstandard methods.

#### 16. Course Reading List and References: References

[1] Robinson, A.; Nonstandard Analysis 2ED, North-Holland Pub. Comp., 1974

[2] Nelson, E.; Internal set Theory, Bull. Amer. Math. Soc, Vol. 83, No. 3, 1977

- [3] Herrmann, R. A.; Nonstandard Analysis Applied to Adv. Undergraduate Mathematics-I, United States Naval Academy, 1991
- [4] Hrbacek, K.; Nonstandard Set Theory, Amer. Math. Monthly, Vol., No, 1979
- [5] Raz Kupferman, Lecture Notes in Asymptotic Methods, 2008

Lecturer's name

## <u>Ch I</u> Introduction to Nonstandard Analysis

(Weeks 1-2)

- **1.** Historical background and motivation.
- 2. Basic concepts of nonstandard analysis.
- **3.** The hyperreal number system.
- **4.** Properties of hyperreal numbers.
- **5.** Various versions of Extensions.

## **<u>Chapter II</u>** Internal Set Theory (Weeks 3-4)

- 1. Axioms of internal set theory (IST).
- 2. Standard and nonstandard elements.
- **3.** Internal and external sets
- **4.** Application of internal theory.

## <u>Ch III</u> Infinitesimals, Shadows, and s-Property (Weeks 5-8)

- **1.** Basic operations with infinitesimals and infinite number.
- 2. Notations of S-Theorems.
- **3.** Continuity and S- Continuity at a Point.
- **4.** ISome Nonstandard Results of Continuous & Monotonic Functions.
- **5.** Continuity, differentiation, and integration using infinitesimal.
- **6.** Nonstandard techniques in solving classical problem.

### <u>Ch IV</u> Fundamental Permanence Priciples (Weeks 9-10)

- **1.** The Cauchy Permanence Principle.
- 2. Robinson's Lemma.
- **3.** Fehrele Permanence Principle.
- **4.** Nonstandard Expansions.

## <u>Ch V</u> Applications of Nonstandard Analysis (Weeks 11-12)

- **1.** Applications in real analysis.
- 2. Metric Space Concepts with NSA.
- **3.** Complete and Compactness with NSA.
- **4.** Use in differential equation.
- **5.** Connections with other areas of mathematics.

| 19. Examinations:  | 19. Examinations:   |  |  |  |
|--|---|--|--|--|
| Compositional: In this type of exam the questions<br>usually starts with Explain how, furthermore it is like<br>as lecture notes and contains some homework , so<br>there will be continuing assignments of problem<br>outside the lecture notes (note that this problem<br>having small marks). | Compositional: In this type of exam the<br>questions usually starts with Explain how,<br>furthermore it is like as lecture notes and<br>contains some homework , so there will be<br>continuing assignments of problem outside<br>the lecture notes (note that this problem<br>having small marks). |  |  |  |
| 20. Extra notes:   | 20. Extra notes:  |  |  |  |
| پيداچوونهو دی هاو هٽ 21. Peer review   | پيداچوونهوه هاوه ن 21. Peer review  |  |  |  |
|  |   |  |  |  |
| 19. Examinations:  |   |  |  |  |
| Compositional: In this type of exam the questions usually starts with Explain how, furthermore it is like as lecture notes and contains some homework , so there will be continuing assignments of problem outside the lecture notes (note that this problem having small marks).                |   |  |  |  |

## 20. Extra notes:

**21.** Peer review

پيداچوونەوەى ھاوەڵ