Topics in Power System Dynamics

Chapter I
Introduction to the Course

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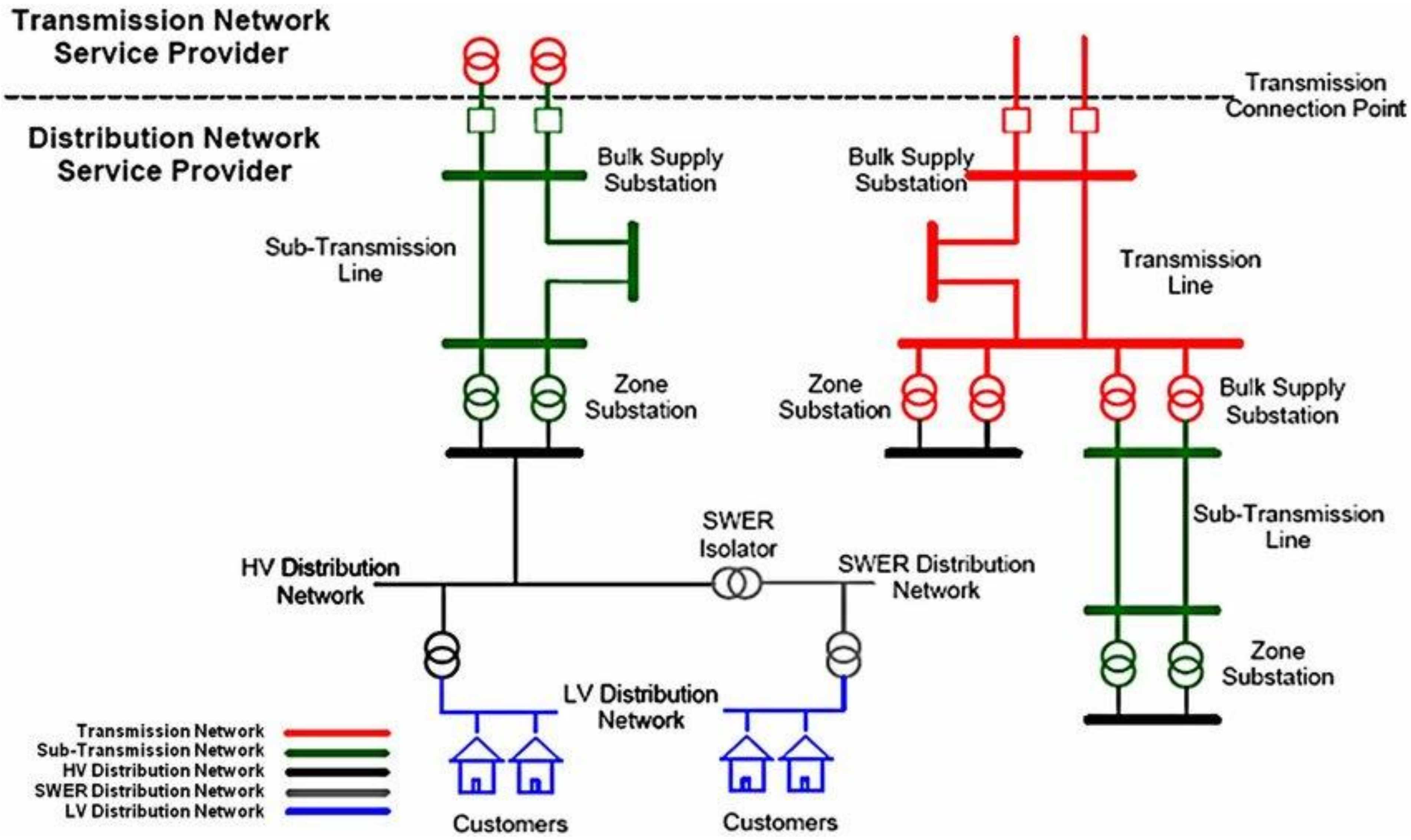
Electrical Power System



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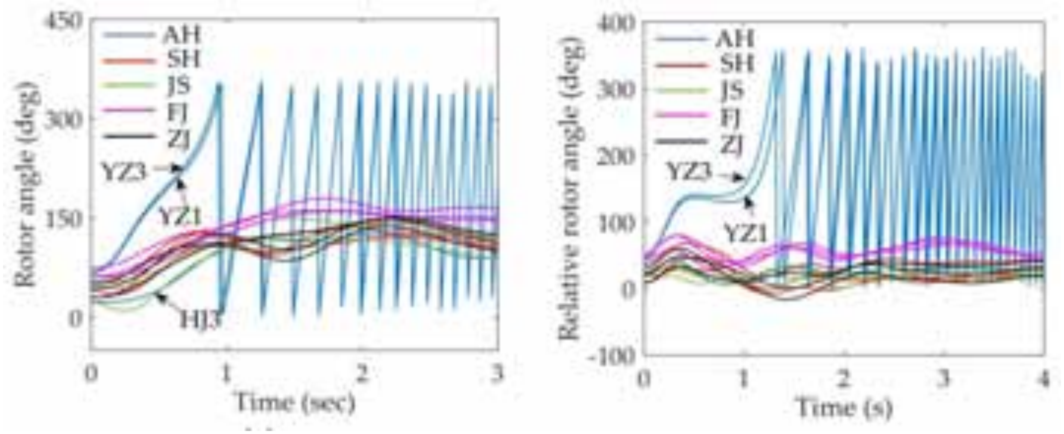
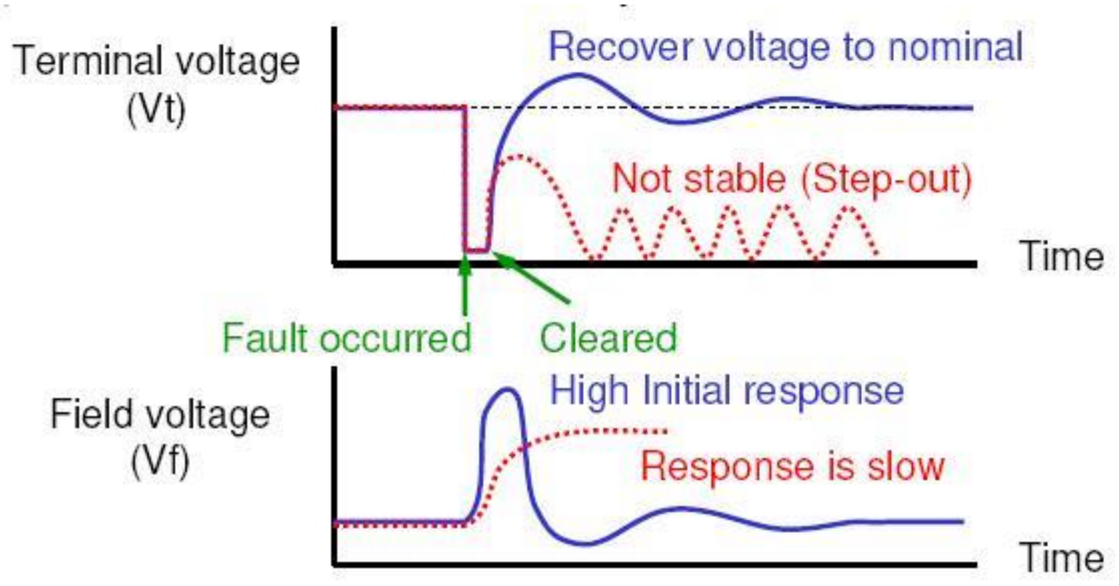
Dynamic Phenomena in Electrical Power Systems





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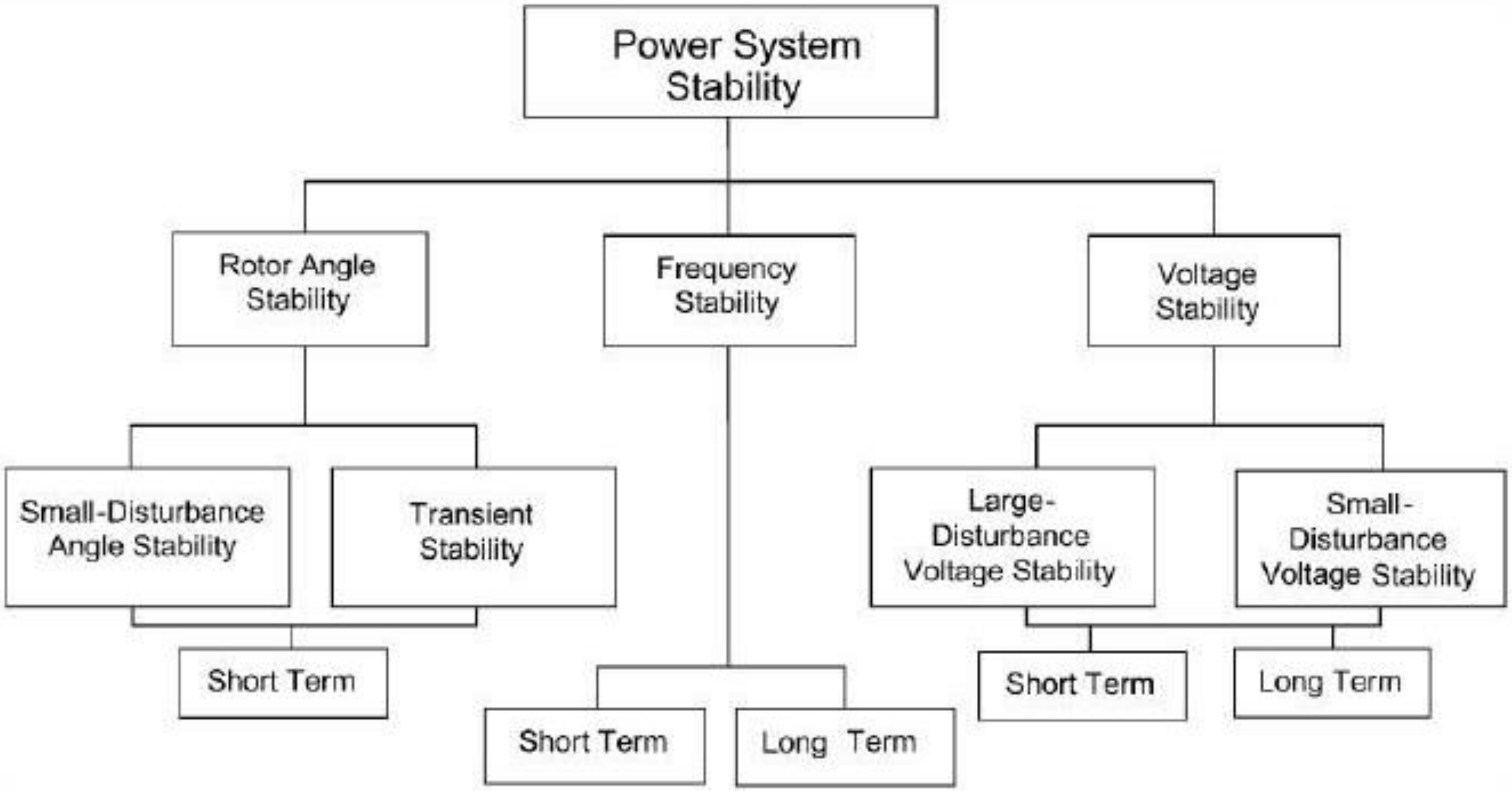
Classification of Stability in Power Systems



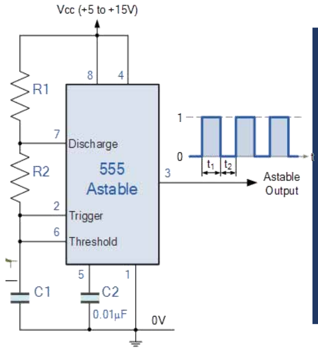
Mathematical Interpretation of Stability

Example: Second Order System subjected to a Unit Step Signal

Typical second order system may be written in the form of:

$$\frac{Y(s)}{R(s)} = \frac{w_n^{\,2}}{s^2+2wn\,\xi\,s+wn^2}$$

Input signal



The System

$$\frac{w_n^{\,2}}{s^2 + 2wn\,\xi\,s + wn^2}$$

Output signal

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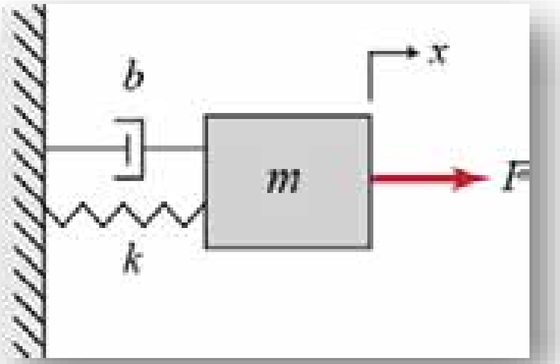
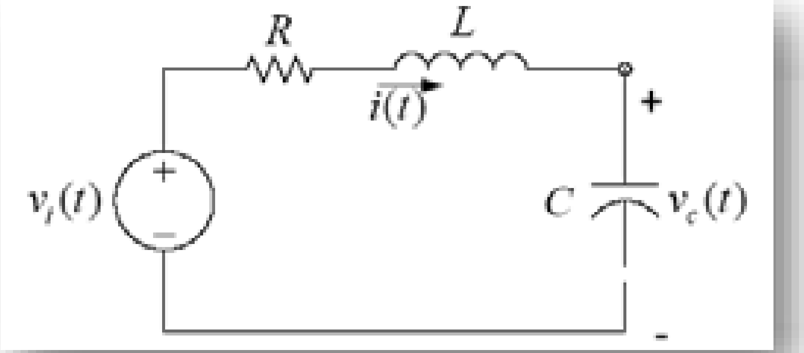
A typical Second Order System





$$\frac{X(S)}{F(s)} = \frac{1}{ms^2+bs+k}$$

$$G(s) = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Note: Compare to determine the values of ω_n (undamped natural frequency) and ξ (damping ratio)

$$\frac{Y(s)}{R(s)} = \frac{w_n{}^2}{s^2+2wn\,\xi\,s+wn^2}$$

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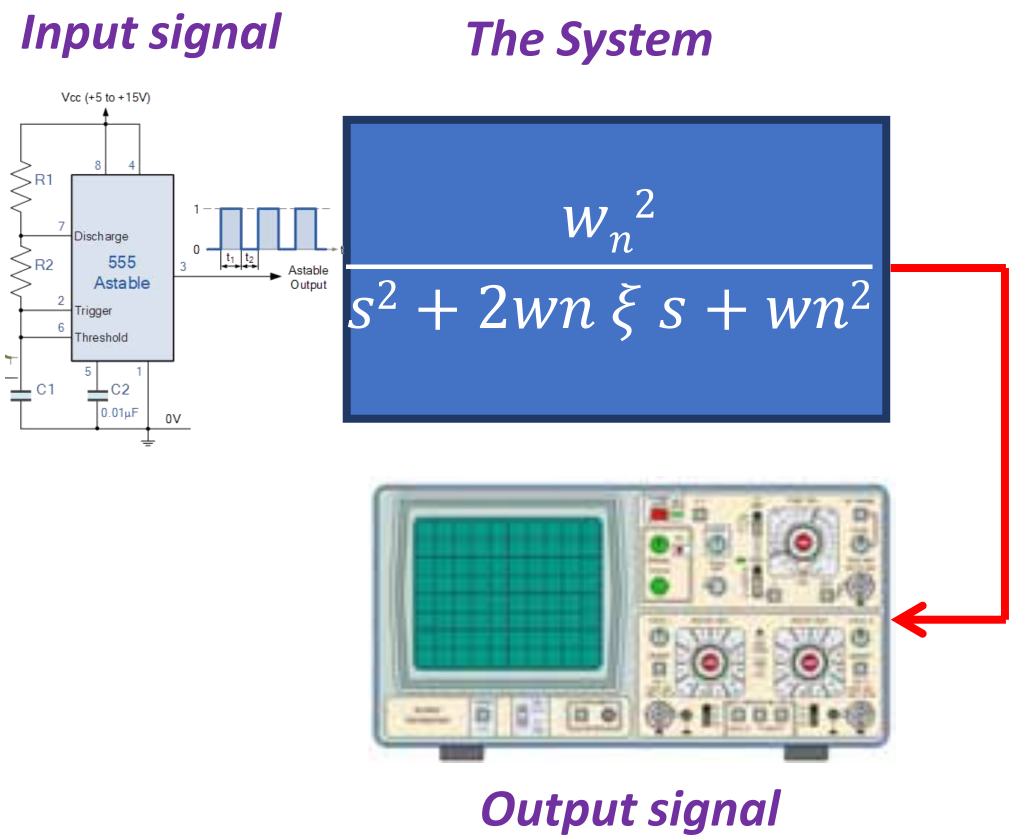
If this system is subjected to a unit step signal, R(s)=1/s then,

$$Y(s) = \frac{1}{s} * \frac{w_n^{\ 2}}{s^2+2wn\,\xi\,s+wn^2}$$

Using partial fraction expansion,

$$Y(s) = \frac{A}{s} + \frac{Bs+C}{s^2+2wn\,\xi\,s+wn^2}$$



By partial fraction expansion, A=1, hence

$$Y(s) = \frac{1}{s} * \frac{\omega_n^{\,2}}{s^2+2\omega_n\,\xi\,s+\omega_n^{\,2}} = \frac{1}{s} + \frac{Bs+C}{s^2+2\omega_n\,\xi\,s+\omega_n^{\,2}}$$

$$\frac{Bs+C}{s^2+2\omega_n\,\xi\,s+\omega_n^{\,2}} = \frac{\omega_n^{\,2}}{s(\,s^2+2\omega_n\,\xi\,s+\omega_n^{\,2})} - \frac{1}{s}$$

$$Y(s) = \frac{1}{s} - \frac{s+\xi\omega_n}{s^2+2\omega_n\,\xi\,s+\omega_n^{\,2}} - \frac{\xi\omega_n}{s^2+2\omega_n\,\xi\,s+\omega_n^{\,2}}$$

$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$

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$$Y(s) = \frac{1}{s} - \frac{s+\xi\omega_n}{(s^2+2\omega_n\,\xi\, s+\omega_n{}^2)} - \frac{\xi}{\sqrt{1-\xi^2}}\,\frac{\omega_d}{s^2+2\omega_n\,\xi\, s+\omega_d{}^2}$$

$$Y(s) = \frac{1}{s} - \frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d{}^2} - \frac{\xi}{\sqrt{1-\xi^2}}\,\frac{\omega_d}{(s+\xi\omega_n)^2+\omega_d{}^2}$$

$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$

Taking inverse Laplace transform,

$$\boldsymbol{y(t) = 1 - e^{-\zeta\omega_n t}\cos(\omega_d\, t) - \frac{\zeta}{\sqrt{1-\zeta^2}}\, e^{-\zeta\omega_n t}\sin(\omega_d t)}$$

This equation represents the dynamic response (output signal in time domain). More versions of the equation are available by mathematical re-arrangement of the equation.

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$$y(t) = 1 - e^{-\zeta\omega_n t}\cos(\omega_d\, t) - \frac{\zeta}{\sqrt{1-\zeta^2}}\; e^{-\zeta\omega_n t}\sin(\omega_d t)$$

$$y(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\{\sqrt{1-\zeta^2}\cos(\omega_d\, t) - \zeta\ \sin(\omega_d t)\}$$

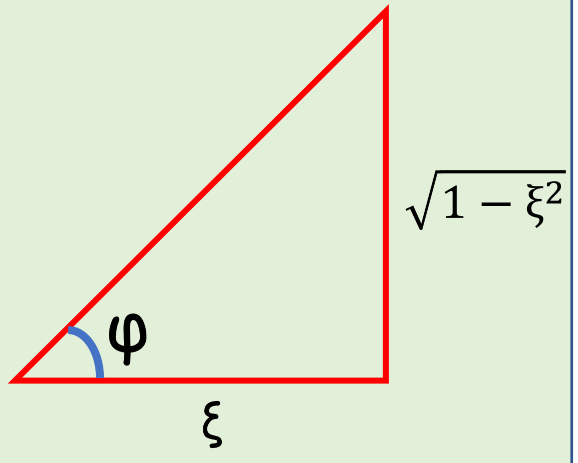
$$y(t) = 1 - \frac{e^{-\zeta\omega nt}}{\sqrt{1-\zeta^2}}\{\sin(\varphi)\ \ (\cos(\omega d\ t) - \cos(\varphi)\sin(\omega dt)\}$$

$$y(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d\, t + \varphi)$$

or,

$$y(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin\left(\omega_d\, t + \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$

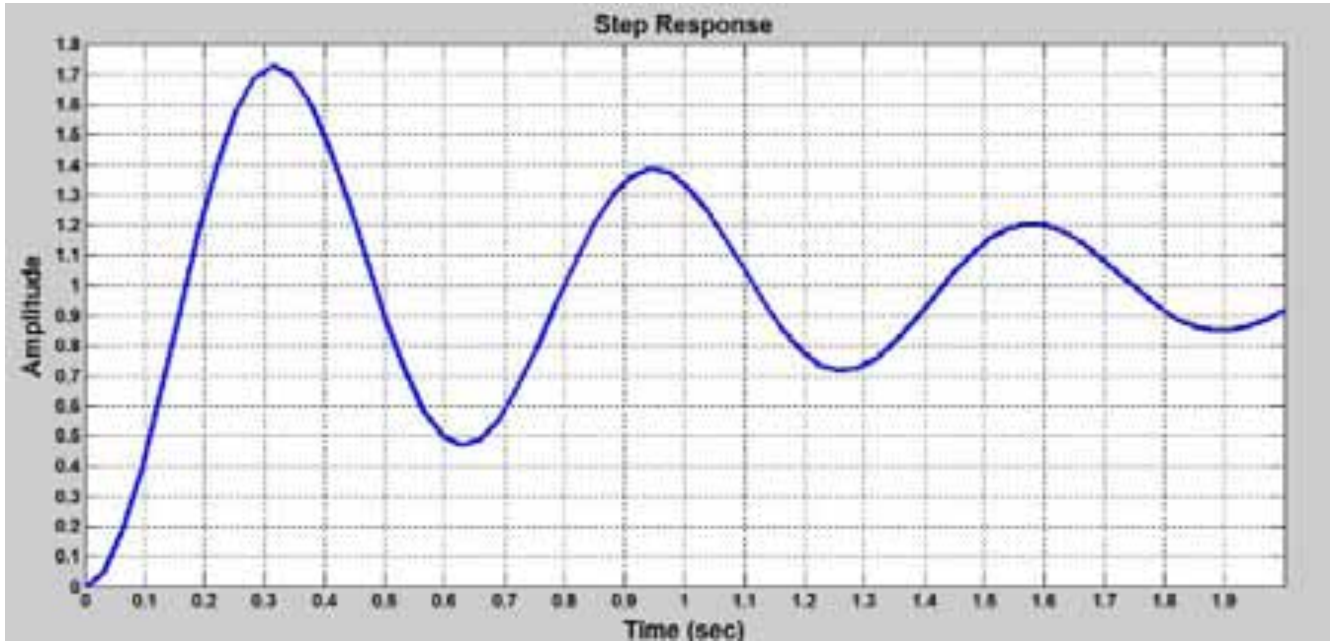
This is another version of the output signal equation



$$\varphi = tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$$

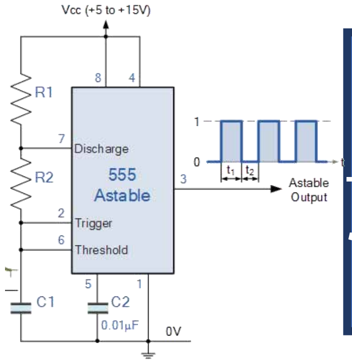
This triangle is introduced as a mathematical notation for re-arrangement of the output signal equation.

$$y(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin\left(\omega_d\, t + \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$



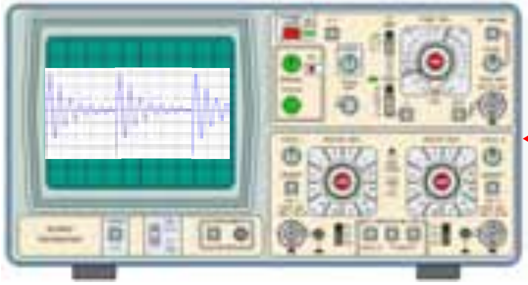
Note: The output equation is a sine wave with a variable amplitude.

Input signal



The System

$$\frac{w_n^{\,2}}{s^2 + 2wn\,\xi\, s + wn^2}$$



Output signal

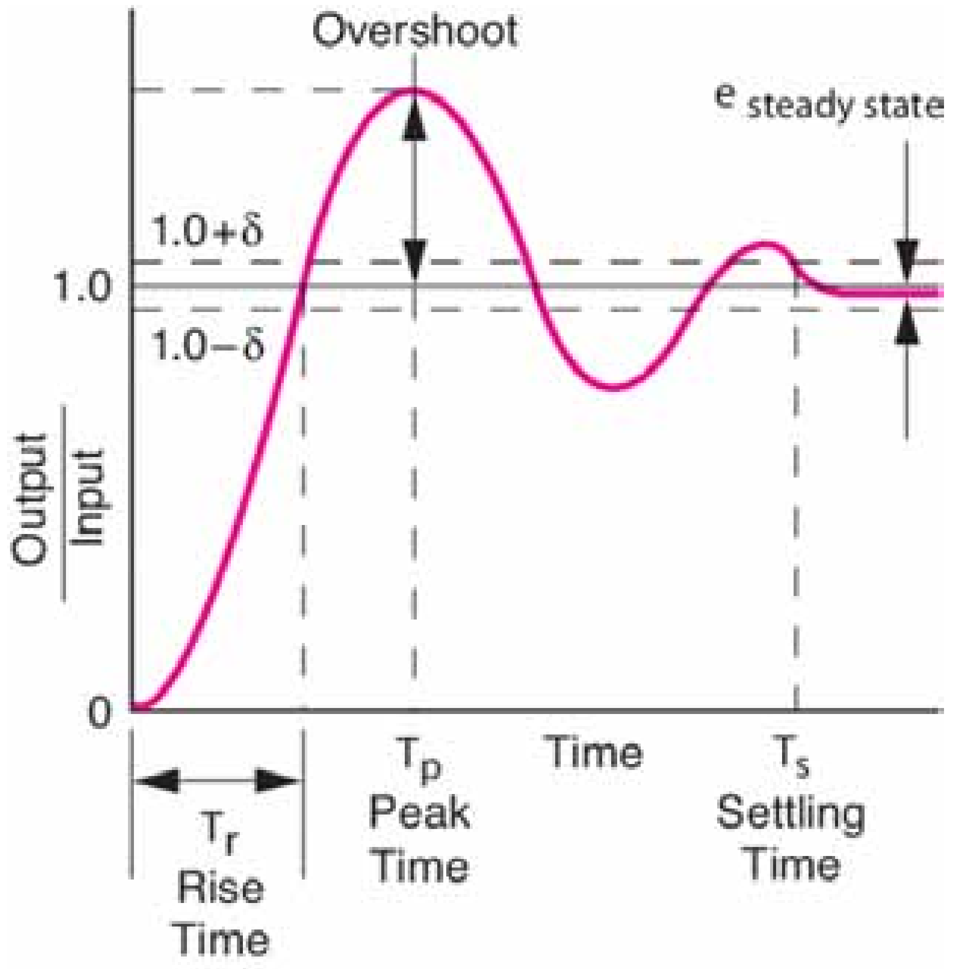
error (e) = input - response

$$= e^{-\zeta\omega_n t}\cos(\omega_d t) - \frac{\zeta}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\sin(\omega_d t)$$

Steady State error (e_{ss}) (error as time goes to infinity) is:

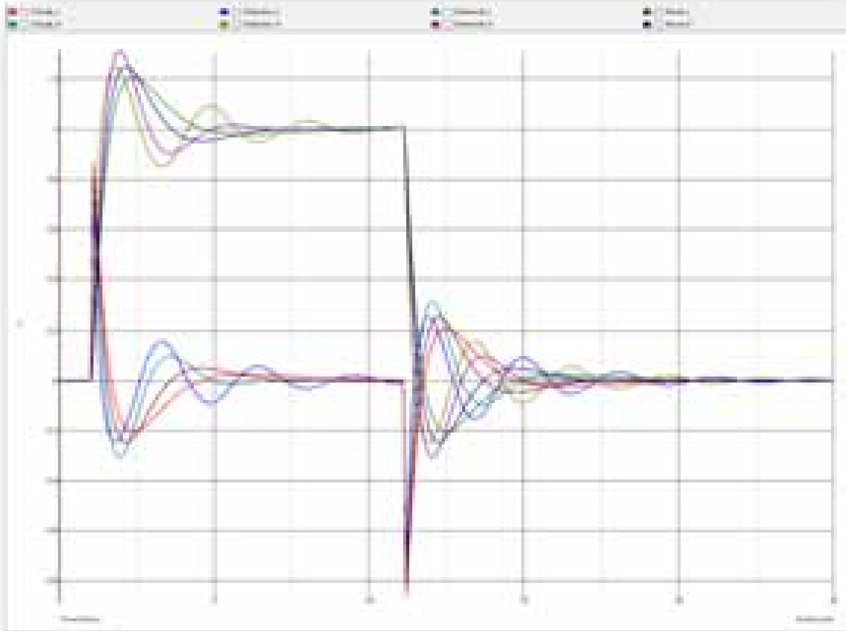
$$e_{ss} = \lim_{t\to\infty}(e) = 0$$

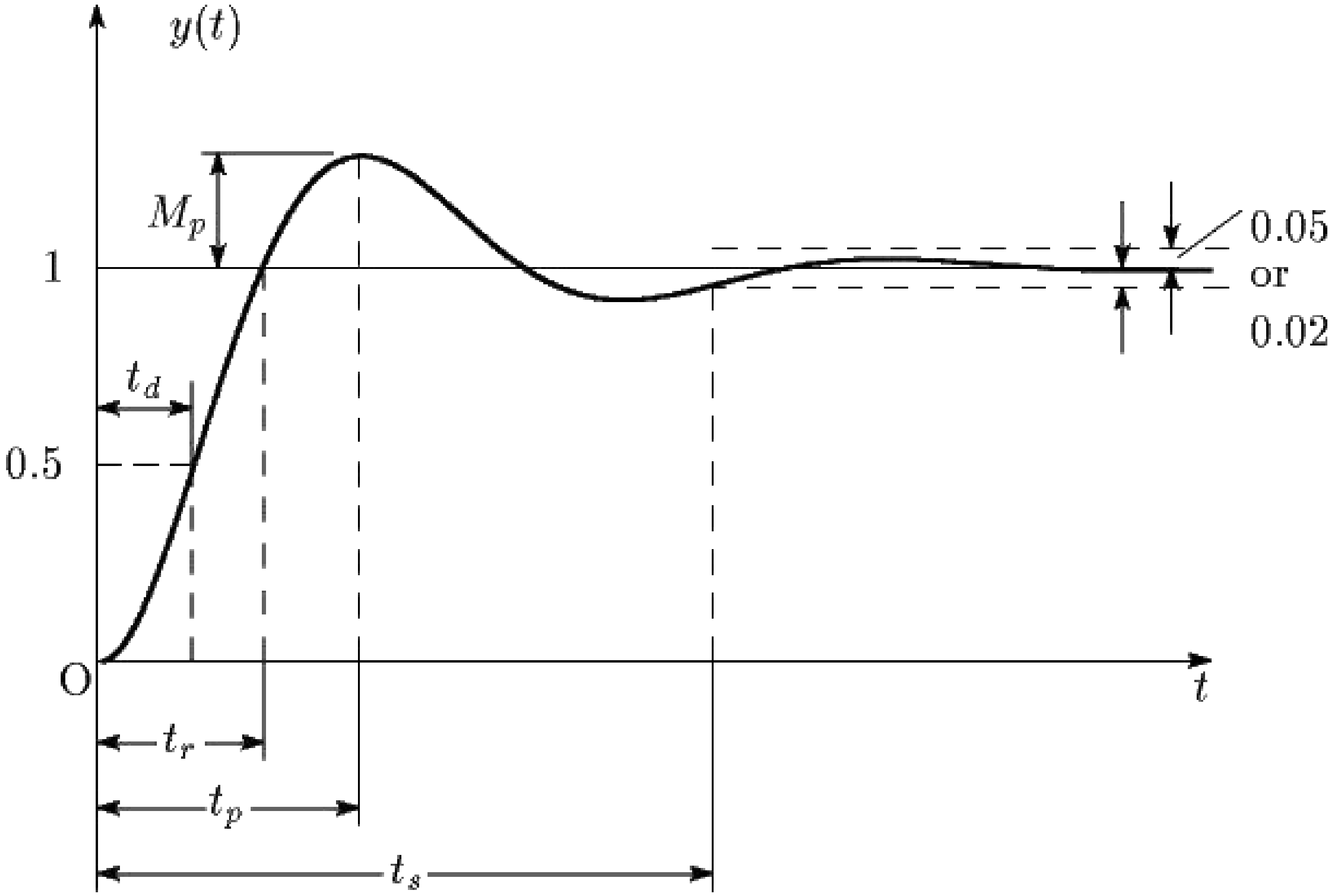
Interpretation: there is no deviation from the reference (unit step signal) at the end (t=∞).



<u>Time domain specifications</u>

(t_r, t_d, t_p, t_s, M_p)





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The effect of ξ and ω

$$\frac{Y(s)}{R(s)} = \frac{w_n{}^2}{s^2+2wn\,\xi\, s+wn^2}$$

The system is subjected to a step input yield:

$$Y(s) = \frac{1}{s} * \frac{\omega_n{}^2}{(s^2+2\omega_n\,\xi\, s+\omega_n{}^2)}$$

Using partial fraction method to find Laplace inverse of Y(s),

$$Y(s) = \frac{1}{s} + \frac{A}{(s-s_1)} + \frac{B}{(s-s_2)}$$

Taking Laplace inverse for the Y(s),

$$y(t) = 1 + A\, e^{s_1 t} + B\, e^{s_2 t}$$

Where,

$$s_1 = -\zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1}$$

$$s_2 = -\zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1}$$

$$y(t) = 1 + K\, e^{-\zeta\omega_n t}\ (e^{\omega_n\sqrt{\zeta^2-1}t} + e^{-\omega_n\sqrt{\zeta^2-1}t})$$

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Output for different values of ξ

$$y(t) = 1 + K\, e^{-\zeta\omega_n t}\ \left(e^{\omega_n\sqrt{\zeta^2-1}t}\ +\ e^{-\omega_n\sqrt{\zeta^2-1}t}\right)$$

$$\cos x = \mathrm{Re}\left(e^{ix}\right) = \frac{e^{ix} + e^{-ix}}{2}$$

$$\sin x = \mathrm{Im}\left(e^{ix}\right) = \frac{e^{ix} - e^{-ix}}{2i}$$

$$\frac{Y(s)}{R(s)} = \frac{{\omega_n}^2}{s^2+2\omega_n\,\xi\, s+{\omega_n}^2}$$

$$s_{1,2} = -\zeta\omega_n \pm wn\sqrt{\zeta^2 - 1}$$

Cases:

a. $\xi>1$, b. $\xi=0$, c. $0<\xi<1$, d. $\xi=1$,

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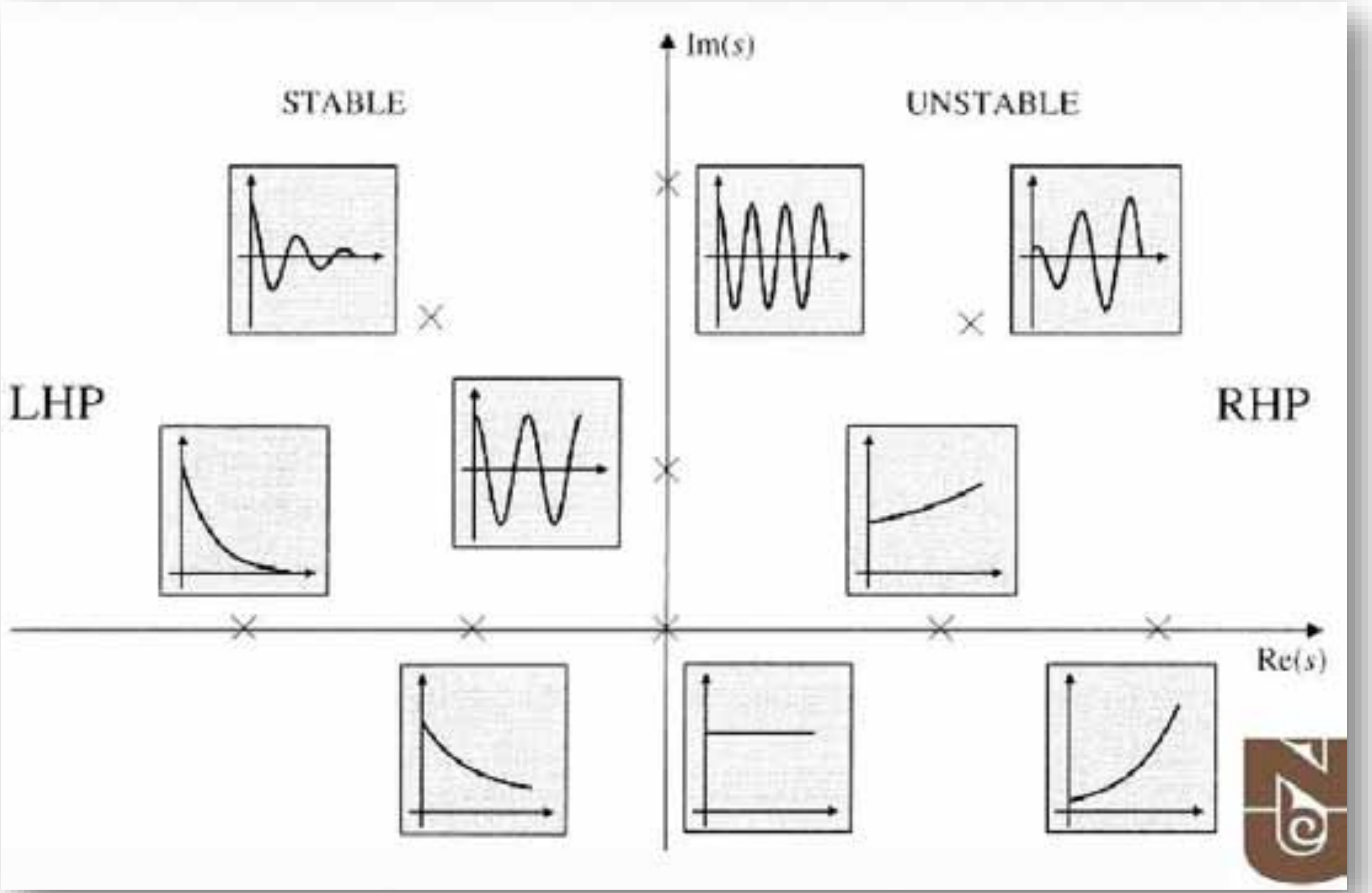
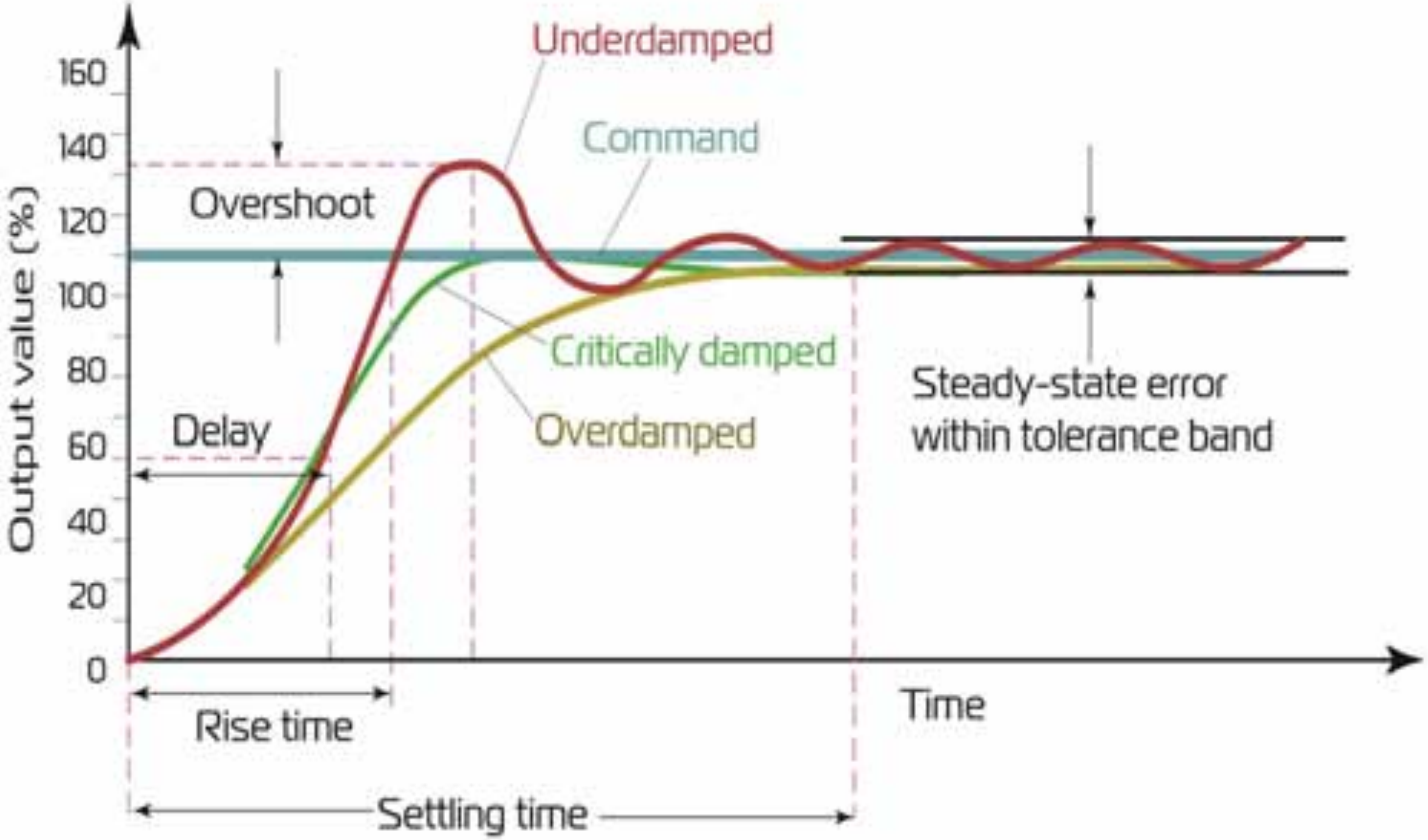
Summary and Conclusion



Response Types



Characteristic equation

$$\frac{Y(s)}{R(s)} = \frac{{\omega_n}^2}{s^2+2\omega_n\,\xi\,s+{\omega_n}^2}$$ Transfer Function

$$s^2 + 2\omega_n\zeta s + {w_n}^2=0$$ Characteristic equation

$$s_{1,2} = -\zeta\omega_n \pm wn\sqrt{\zeta^2 - 1}$$ Roots of characteristic equation

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✓ Stability (Physically)

A system is said to be stable if after the occurrence of a disturbance has the ability to restore its initial condition or to reach to a states very close to that of the original.

✓ Stability (Mathematically)

A system is said to be stable if and only if, all roots of the characteristic equation lie on the LHS of the complex plane.

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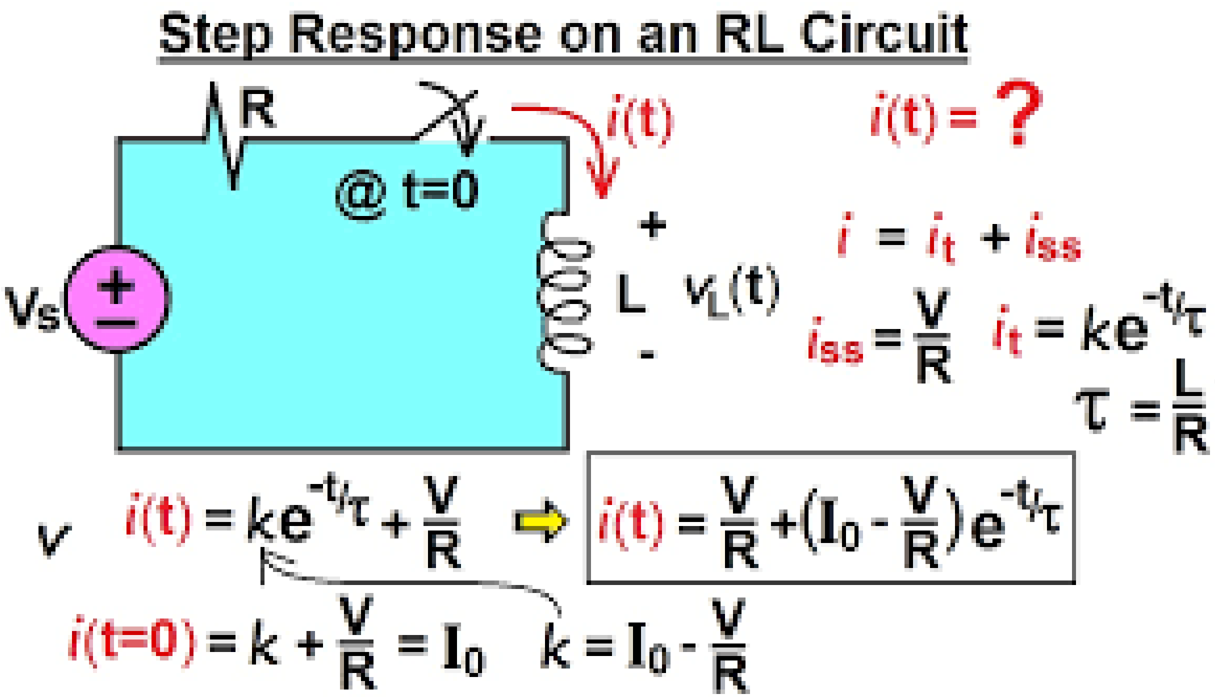
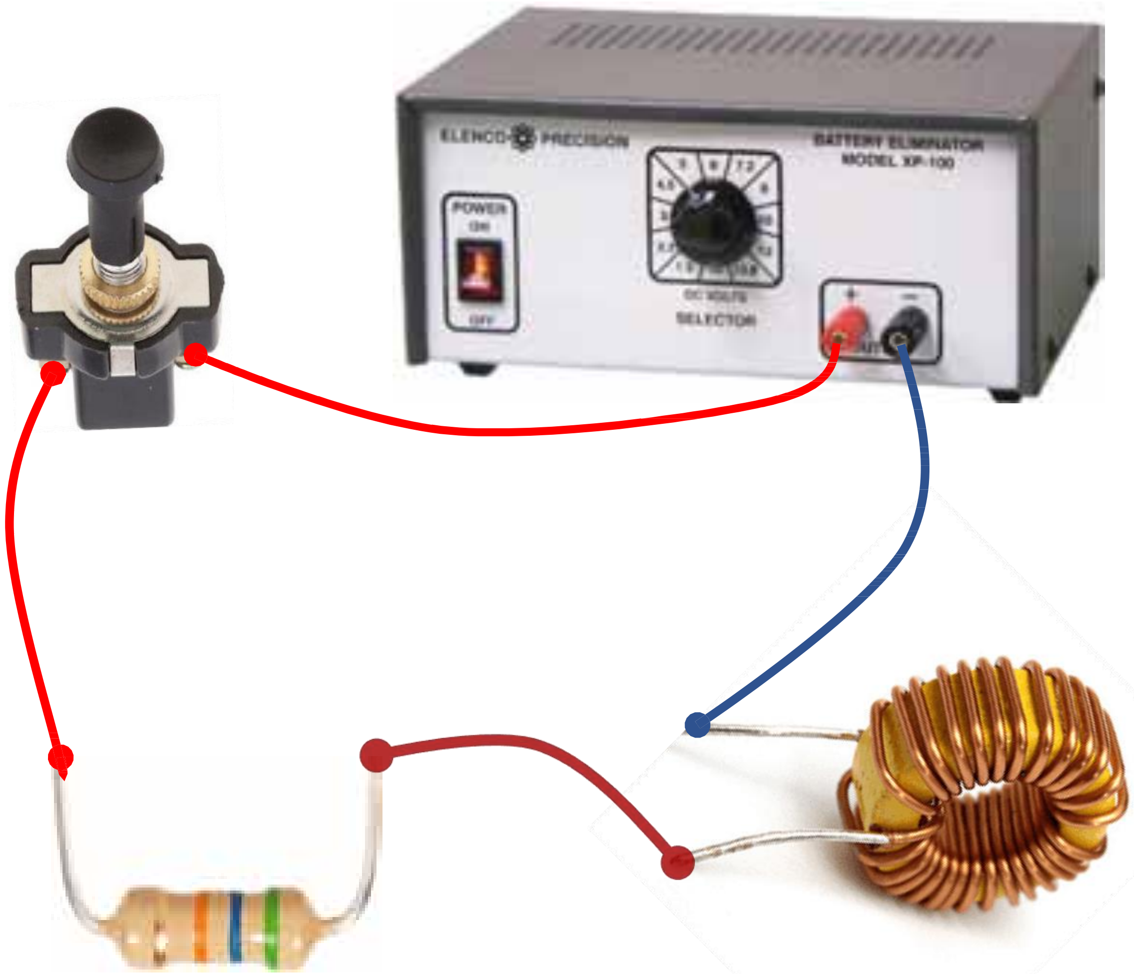
State Variable

Example: RL circuit





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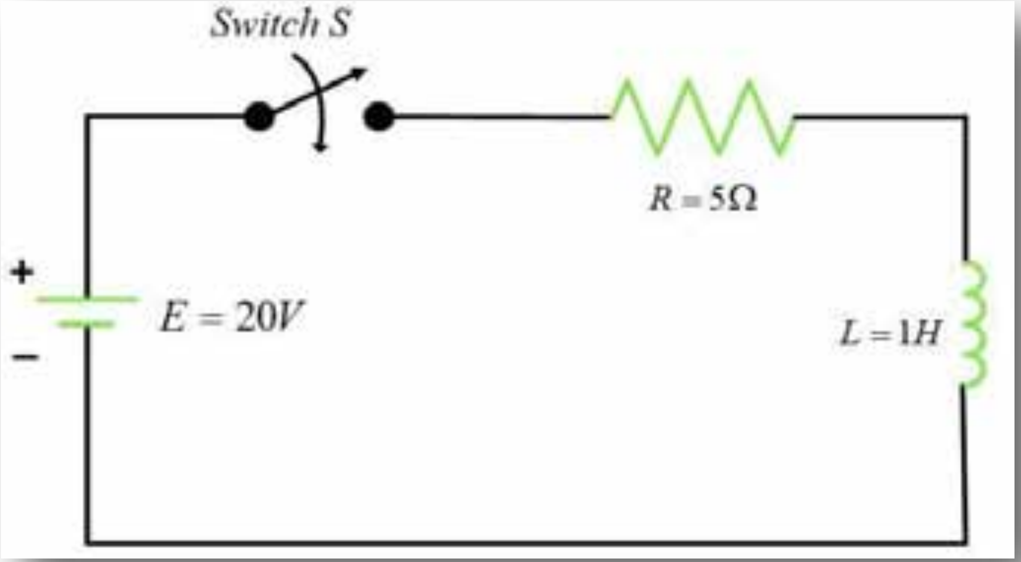
Solution of RL circuit

$$E = RI + L\frac{dI}{dt}$$

$$\frac{E}{s} = R\mathscr{L}(I) + L(s\mathscr{L}(I) - I_0)$$

$$\mathscr{L}(I) = \frac{E/L}{s(R/L + s)} + \frac{I_0/L}{R/L + s}$$

$$I(t) = \frac{E}{R} + [I_0 - \frac{E}{R}]e^{-Rt/L}$$



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State Variable

The state of a dynamic system is the smallest set of variables called (state variables) such that the knowledge of these variables at $t=t_o$ and the input applied for $t \geq t_o$ completely determine the behavior of the system for any time $t \geq t_o$.

If n state variables are needed to completely describe the behavior of a given system, then these variables can be considered the n components of a vector called state vector.

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The State Space Mathematical model

Consider a system is described by the 2nd order differential equation

$$a_1 \frac{d^2y(t)}{dt^2} + a_2 \frac{dy(t)}{dt} + a_3 y(t) = u(t)$$

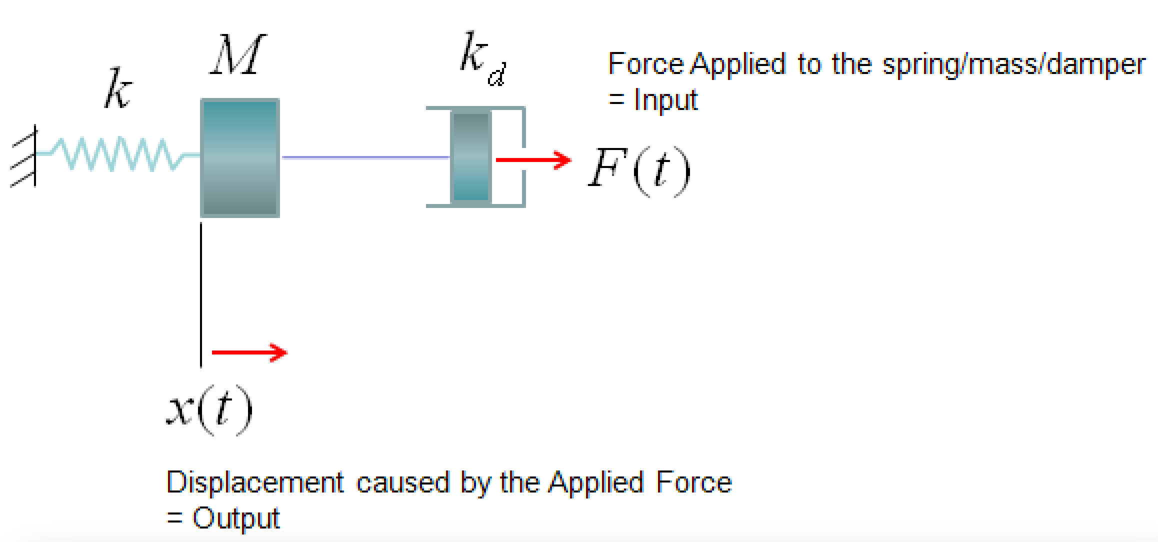
Let

$y(t) = x_1$

$\dot{y}(t) = \dot{x}_1(t) = x_2$

$\dot{x}_2(t) = \ddot{y}(t) = -\frac{a_2}{a_1} x_2(t) - \frac{a_3}{a_1} x_{1(t)} + \frac{1}{a_1} u(t)$

Example of a 2nd order system



Applied Force, Spring Force, Movement of Mass, Damping Fource

$$F(t) = kx(t) + M \frac{d^2x(t)}{dt^2} + k_d \cdot \frac{dx(t)}{dt}$$

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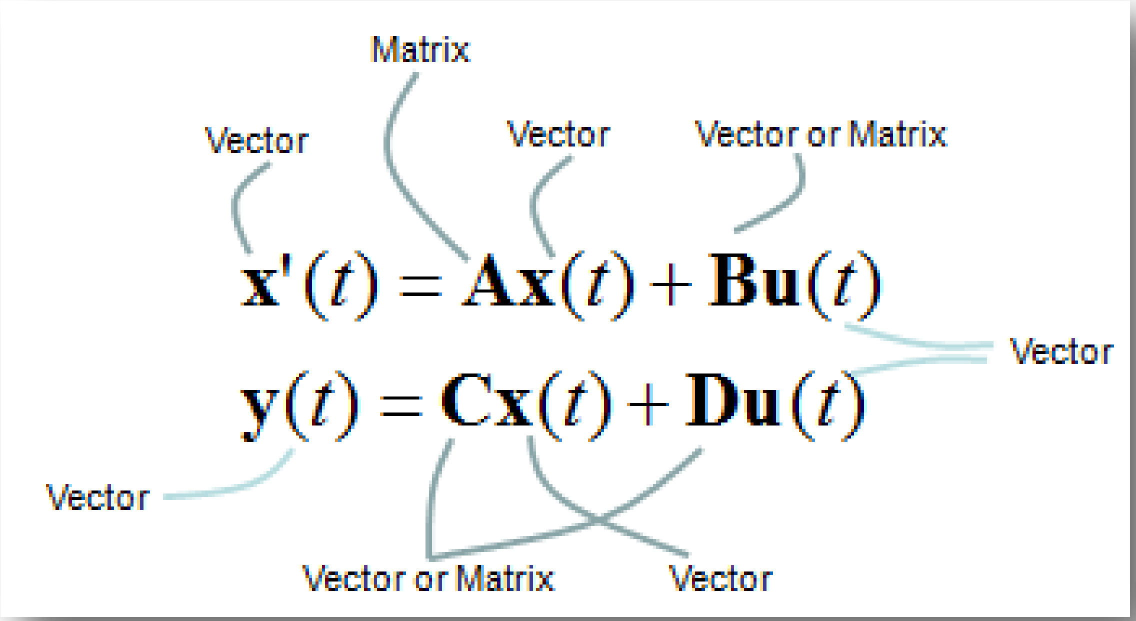
State Variable

$$\dot{y}(t) = \dot{x}_1(t) = x_2$$

$$\dot{x}_2(t) = \ddot{y}(t) = -\frac{a_2}{a_1}x_2(t) - \frac{a_3}{a_1}x_{1(t)} + \frac{1}{a_1}u(t)$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ \frac{-a_3}{a_1} & \frac{-a_2}{a_1}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{a_1}\end{bmatrix}\mathrm{u(t)}$$

$$[x_1(t)] = [1 \quad 0]\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$



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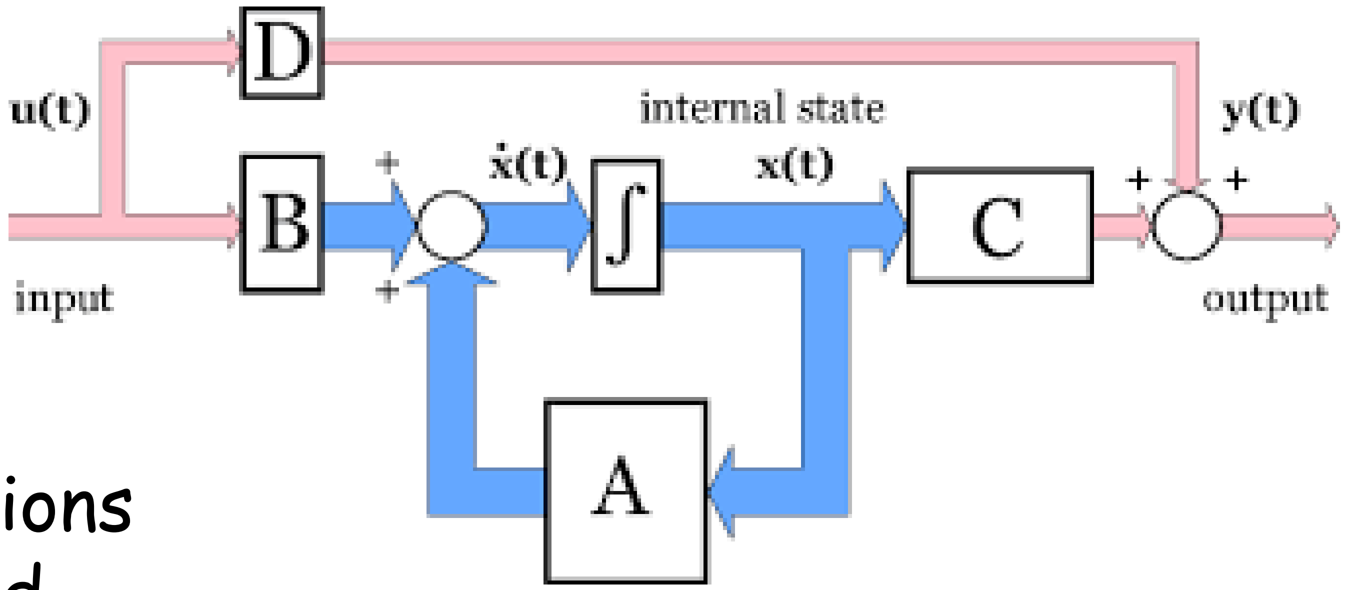
State space model

$$\dot{x}\,(t) = A\,x(t) + B\,u(t)$$

$$y(t) = c\,x(t) + D\,u(t)$$



The first and the second equations are known as state equation and output equation respectively

Example: Find the eigenvalues

$$[A]=\begin{bmatrix}-3 & 2\\ -1 & 0\end{bmatrix}$$

Characteristic equation is

$$\left|\lambda[I]-[A]\right|=0$$

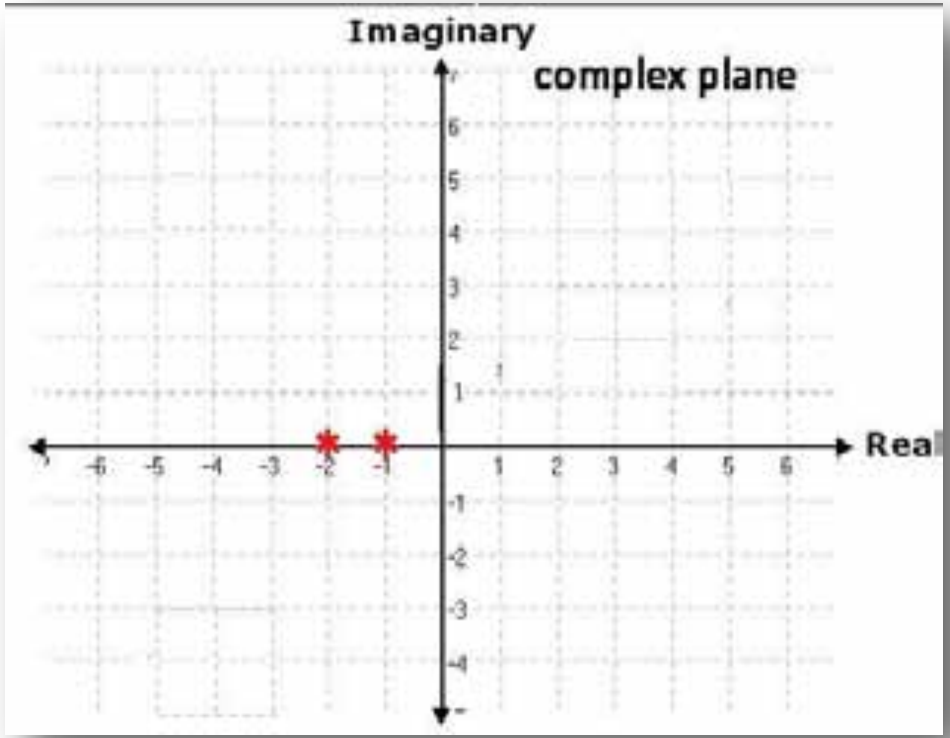
$$\left|\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}-\begin{bmatrix}-3 & 2\\ -1 & 0\end{bmatrix}\right|=0$$

$$\begin{vmatrix}\lambda+3 & -2\\ 1 & \lambda\end{vmatrix}=0$$

$$\lambda^2+3\lambda+2=0$$

$$(\lambda+2)(\lambda+1)=0$$

The eigenvalues are $\lambda_1=-2$ and $\lambda_2=-1$



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Example: Consider a 2-dimensional system with the following system and input matrices.

$$[A]=\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \quad [B]=\begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ and } [C]=\begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$(s[I]-[A])=\begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}$$

$$(s[I]-[A])^{-1}=\frac{\operatorname{adj}(s[I]-[A])}{|s[I]-[A]|}=\frac{1}{s^2+3s+2}\begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix}$$

The transfer function of the system is

$$[G(s)]=[C](s[I]-[A])^{-1}[B]$$

$$[G(s)]=\frac{\begin{bmatrix} 1 & 0 \end{bmatrix}}{s^2+3s+2}\begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}=\frac{s+3}{(s+1)(s+2)}$$

The denominator of the transfer function is

$$|s[I]-[A]|=s^2+3s+2=(s+1)(s+2)$$

The roots of the characteristic equation, i.e., the eigenvalues are -1 and -2 and therefore, the system is stable.

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Solution of state space equation

a. Homogenous state response

The state-variable response of a system described by

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t)$$

with zero input, and an arbitrary set of initial conditions $x(0)$ is the solution of the set of n homogeneous first-order differential equations.

To derive the homogeneous response $x_h(t)$, we begin by considering the response of a first-order (scalar) system with state equation

$$x\ (t) = ax(t)$$

With initial condition $x(0)$. In this case the homogeneous response $x_h(t)$ has an exponential form defined by the system time constant $\tau = -1/a$, or

$$x_h(t) = e^{at} x(0).$$

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The exponential term e^{at} may be expanded as power series to give,

$$x_h(t) = \left(1 + at + \frac{a^2t^2}{2!} + \frac{a^3t^3}{3!} + \ldots + \frac{a^kt^k}{k!} + \ldots\right) x(0),$$

The above solution is true for higher order systems, say n^{th} order system to be,

$$\mathbf{x}_h(t) = \left(\mathbf{I} + \mathbf{A}t + \frac{\mathbf{A}^2t^2}{2!} + \frac{\mathbf{A}^3t^3}{3!} + \ldots + \frac{\mathbf{A}^kt^k}{k!} + \ldots\right) \mathbf{x}(0)$$

The system homogeneous response $x_h(t)$ may therefore be written in terms of the matrix exponential

$$e^{\mathbf{A}t} = \mathbf{I} + \mathbf{A}t + \frac{\mathbf{A}^2 t^2}{2!} + \frac{\mathbf{A}^3 t^3}{3!} + \ldots + \frac{\mathbf{A}^k t^k}{k!} + \ldots$$

The solution is often written as,

$$\mathbf{x}_h(t) = e^{\mathbf{A}t}\mathbf{x}(0)$$

$$\mathbf{x}_h(t) = \Phi(t)\mathbf{x}(0)$$

$\Phi(t)$ is called state transition matrix.

Solution: The system matrix is

$$\mathbf{A} = \begin{bmatrix} -2 & 0 \\ 1 & -1 \end{bmatrix}.$$

From Eq. (9) the matrix exponential (and the state transition matrix) is

$$\begin{aligned}
\mathbf{\Phi}(t) &= e^{\mathbf{A}t} \\
&= \left(\mathbf{I} + \mathbf{A}t + \frac{\mathbf{A}^2 t^2}{2!} + \frac{\mathbf{A}^3 t^3}{3!} + \ldots + \frac{\mathbf{A}^k t^k}{k!} + \ldots \right) \\
&= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 1 & -1 \end{bmatrix} t + \begin{bmatrix} 4 & 0 \\ -3 & 1 \end{bmatrix} \frac{t^2}{2!} \\
&\quad + \begin{bmatrix} -8 & 0 \\ 7 & -1 \end{bmatrix} \frac{t^3}{3!} + \ldots
\end{aligned}$$

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$$= \begin{bmatrix} 1 - 2t + \frac{4t^2}{2!} - \frac{8t^3}{3!} + \dots & 0 \\ 0 + t - \frac{3t^2}{2!} + \frac{7t^3}{3!} + \dots & 1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + \dots \end{bmatrix}$$

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and the homogeneous response to initial conditions $x_1(0)$ and $x_2(0)$ is

$$\mathbf{x}_h(t) = \mathbf{\Phi}(t)\mathbf{x}(0)$$

or

$$\begin{aligned} x_1(t) &= x_1(0)e^{-2t} \\ x_2(t) &= x_1(0)\left(e^{-t} - e^{-2t}\right) + x_2(0)e^{-t}. \end{aligned}$$

With the given initial conditions the response is

$$\begin{aligned} x_1(t) &= 2e^{-2t} \\ x_2(t) &= 2\left(e^{-t} - e^{-2t}\right) + 3e^{-t} \\ &= 5e^{-t} - 2e^{-2t}. \end{aligned}$$

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The forced state response

Matrix differentiation and integration are defined to be element by element operations, therefore if the state equations are rearranged, and all terms pre-multiplied by the square matrix e^{-At}:

$$e^{-\mathbf{A}t}\dot{\mathbf{x}}(t) - e^{-\mathbf{A}t}\mathbf{A}\mathbf{x}(t) = \frac{d}{dt}\left(e^{-\mathbf{A}t}\mathbf{x}(t)\right) = e^{-\mathbf{A}t}\mathbf{B}\mathbf{u}(t).$$

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Integration of the equation gives,

$$\int_0^t \frac{d}{d\tau}\left(e^{-\mathbf{A}\tau}\mathbf{x}(\tau)\right)d\tau = e^{-\mathbf{A}t}\mathbf{x}(t) - e^{-\mathbf{A}0}\mathbf{x}(0) = \int_0^t e^{-\mathbf{A}\tau}\mathbf{B}\mathbf{u}(\tau)d\tau$$

Because $e^{A0}=I$ and $[e^{-At}]^{-1} = e^{At}$, the complete state vector response may be written in tow similar forms:

$$\mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}(0) + e^{\mathbf{A}t}\int_0^t e^{-\mathbf{A}\tau}\mathbf{B}\mathbf{u}(\tau)d\tau$$

$$\mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}(0) + \int_0^t e^{\mathbf{A}(t-\tau)}\mathbf{B}\mathbf{u}(\tau)d\tau.$$

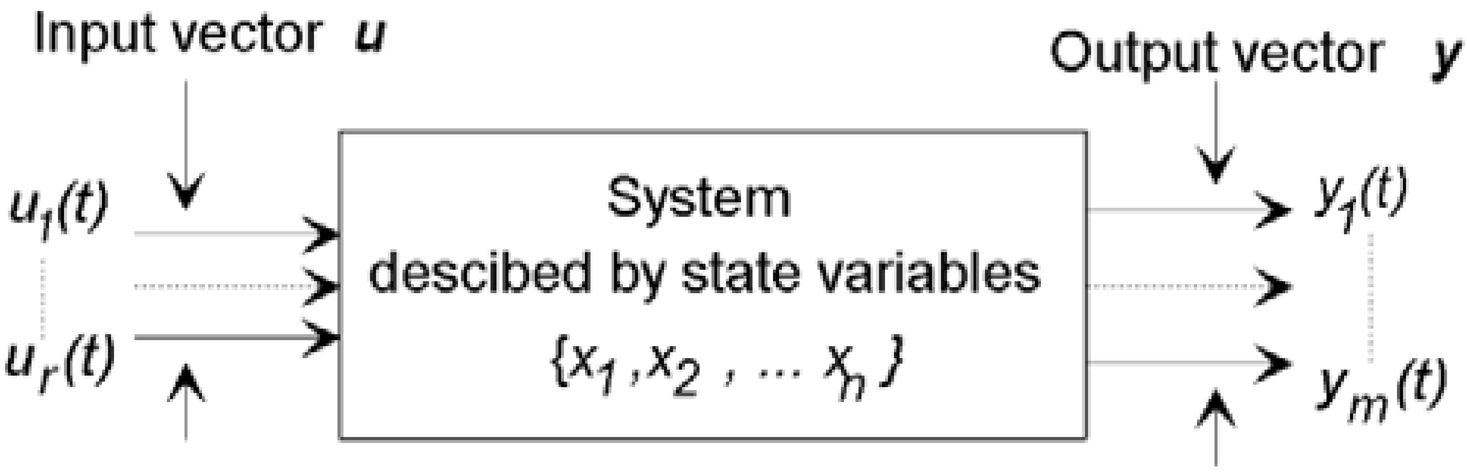
Example:

Find the response of the two state variables of the system

$$\begin{aligned} \dot{x}_1 &= -2x_1 + u \\ \dot{x}_2 &= x_1 - x_2. \end{aligned}$$

to a constant input $u(t) = 5$ for $t > 0$, if $x_1(0) = 0$, and $x_2 = 0$.

Example: MIMO System



Solution: A set of ODE

$$
\begin{aligned}
\dot{x}_1 &= a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n + b_{11}u_1 + \dots + b_{1r}u_r \\
\dot{x}_2 &= a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n + b_{21}u_1 + \dots + b_{2r}u_r \\
\vdots & \quad \vdots \\
\dot{x}_n &= a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n + b_{n1}u_1 + \dots + b_{nr}u_r
\end{aligned}
$$

$$
\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & & & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} b_{11} & \dots & b_{1r} \\ b_{21} & & b_{2r} \\ \vdots & & \vdots \\ b_{n1} & \dots & b_{nr} \end{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ u_r \end{bmatrix}
$$

$$
\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}
$$

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Output equation

$$\begin{array}{ccccccccccccccc} y_1 & = & c_{11}x_1 & + & c_{12}x_2 & + & \dots & + & c_{1n}x_n & + & d_{11}u_1 & + & \dots & + & d_{1r}u_r \\ y_2 & = & c_{21}x_1 & + & c_{22}x_2 & + & \dots & + & c_{2n}x_n & + & d_{21}u_1 & + & \dots & + & d_{2r}u_r \\ \vdots & & \vdots & & & & & & & & & & & & \\ y_m & = & c_{m1}x_1 & + & c_{m2}x_2 & + & \dots & + & c_{mn}x_n & + & d_{m1}u_1 & + & \dots & + & d_{mr}u_r \end{array}$$

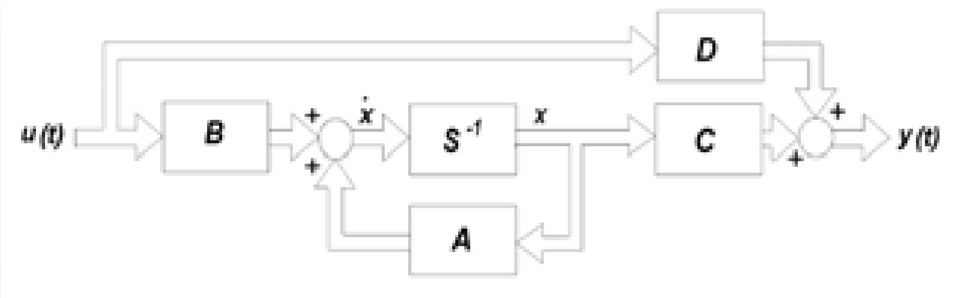
$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1n} \\ c_{21} & c_{22} & \dots & c_{2n} \\ \vdots & & & \vdots \\ c_{m1} & c_{m2} & \dots & c_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} d_{11} & \dots & d_{1r} \\ d_{21} & & d_{2r} \\ \vdots & & \vdots \\ d_{m1} & \dots & d_{mr} \end{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ u_r \end{bmatrix}$$

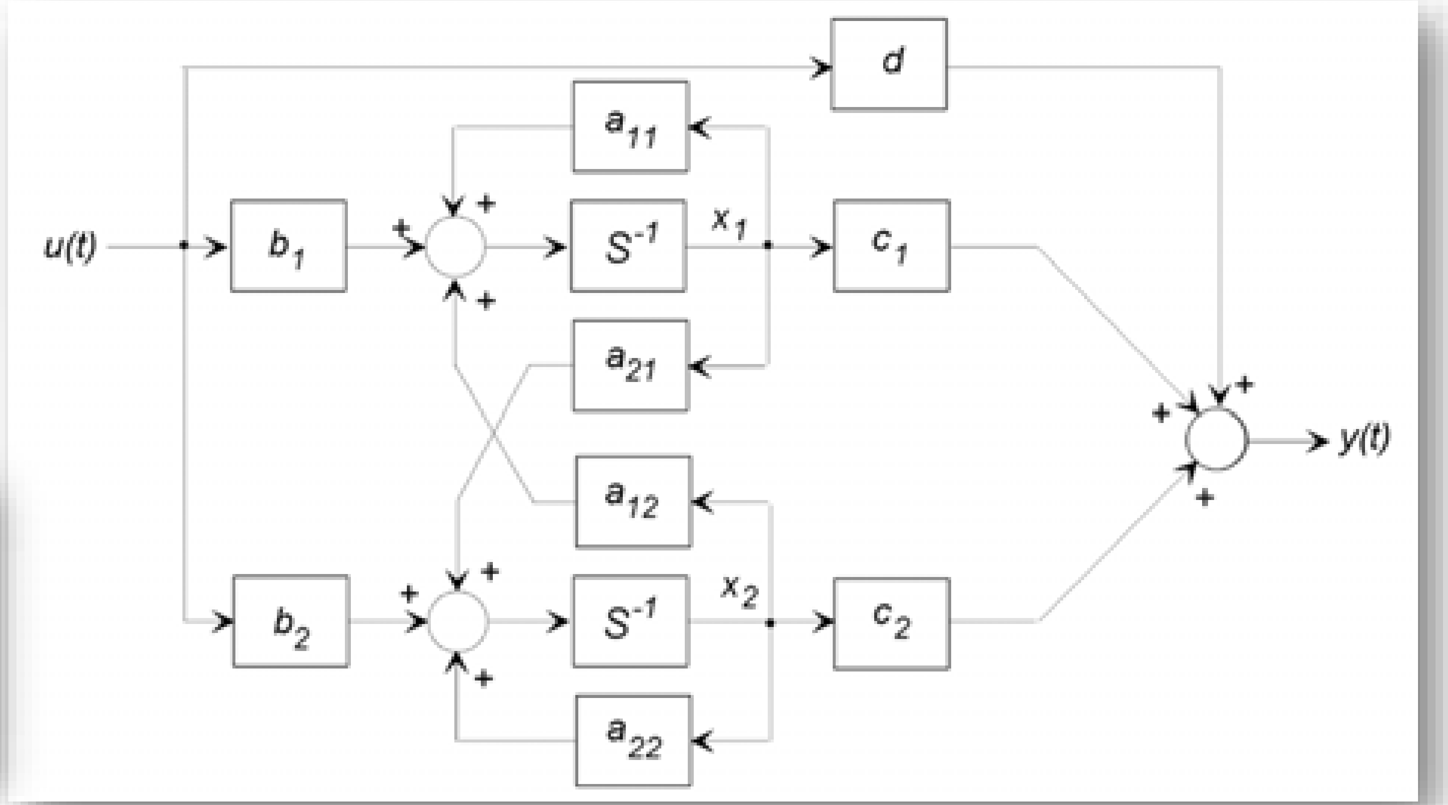
$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}$$

Example: Draw a block diagram for the following SISO system

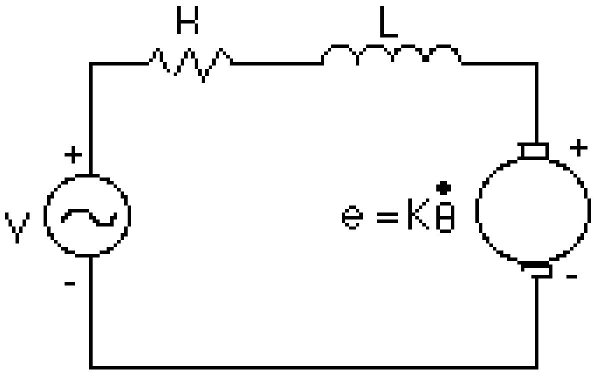
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u(t)$$

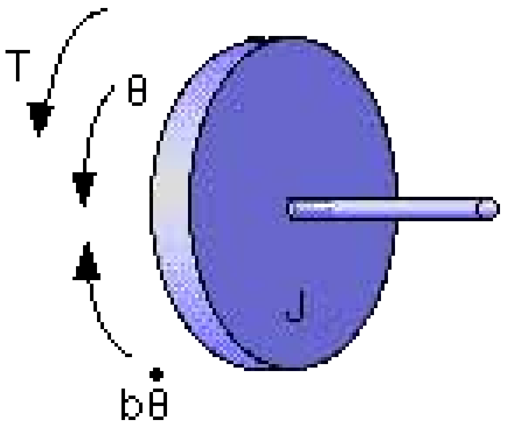
$$y(t) = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + du(t).$$





HW. DC Motor





Datasheet

* moment of inertia of the rotor (J) = 3.2284E-6 kg.m^2/s^2
* damping ratio of the mechanical system (b) = 3.5077E-6 Nms
* electromotive force constant (K=Ke=Kt) = 0.0274 Nm/Amp
* electric resistance (R) = 4 ohm
* electric inductance (L) = 2.75E-6 H
* input (V): Source Voltage
* output (theta): position of shaft
* The rotor and shaft are assumed to be rigid

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Solution

$$T \propto i$$

$$T = K_t \mathrm{i}$$

$$e \propto \dot{\theta}$$

$$e = b\,\dot{\theta}$$

$$J\,\ddot{\theta} + b\,\dot{\theta} = K_t \mathrm{i}$$

$$L\,\frac{di}{dt} + \mathrm{R\,I} = \mathrm{v} - \mathrm{k}\,\dot{\theta}$$

$$\frac{d}{dt}\begin{bmatrix}\theta \\ \dot{\theta} \\ i\end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{b}{J} & \frac{K}{J} \\ 0 & -\frac{K}{L} & -\frac{R}{L}\end{bmatrix}\begin{bmatrix}\theta \\ \dot{\theta} \\ i\end{bmatrix} + \begin{bmatrix}0 \\ 0 \\ \frac{1}{L}\end{bmatrix} V$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\begin{bmatrix}\theta \\ \dot{\theta} \\ i\end{bmatrix}$$

```
J=3.2284E-6;
b=3.5077E-6;
K=0.0274;
R=4;
L=2.75E-6;

A=[0 1 0
   0 -b/J K/J
   0 -K/L -R/L];
B=[0 ; 0 ; 1/L];
C=[1 0 0];
D=[0];
```

```
[y,x,t]=step(A,B,C,D);
plot(t/tscale,y)
ylabel('Amplitude')
xlabel('Time (sec)')
```

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<u>Reading list</u>

Jan Machowski, et. al. (2020). Power System Dynamics, Stability and Control, John Weily and Sons.

Gibbard, M.J., and Pourbeik P., Vowles D. J., (2015). Small Signal Stabilit , Control and Dynamic Performance of Power Systems. Adelaide University Press.

Kwatny, H. G. and Miller K.M. (2016). Power System Dynamics and Control, Berkhauser Press.

Thank you