Robotic Systems

Chapter II
Configuration Space: Degree of Freedom and Topology

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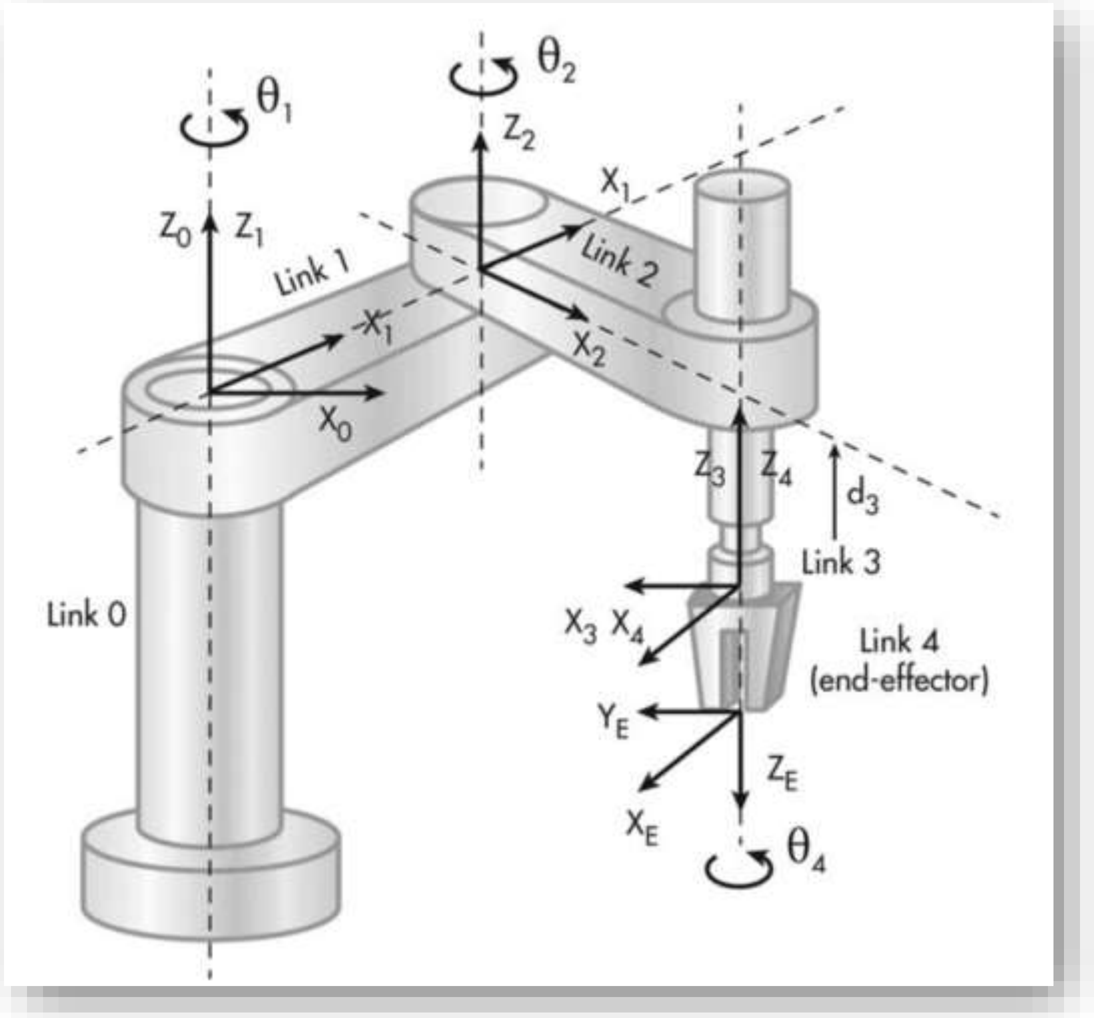
Configuration Space

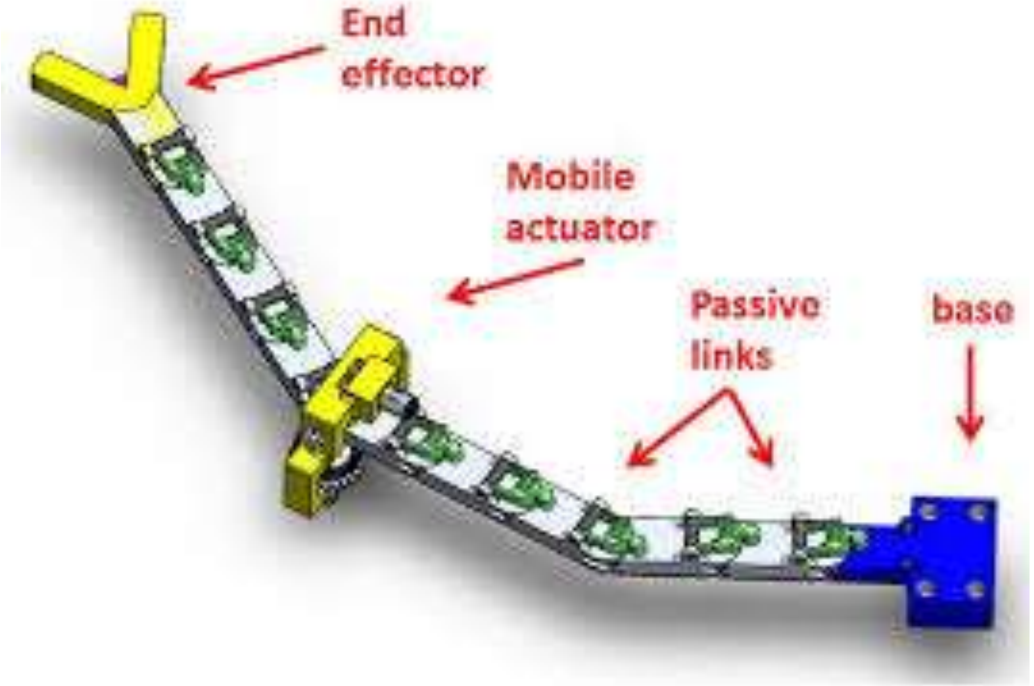
The most fundamental question you can ask about a robot is, "Where is it?"

The answer to this question is the robot's configuration, which is a specification of the positions of all the points of the robot.

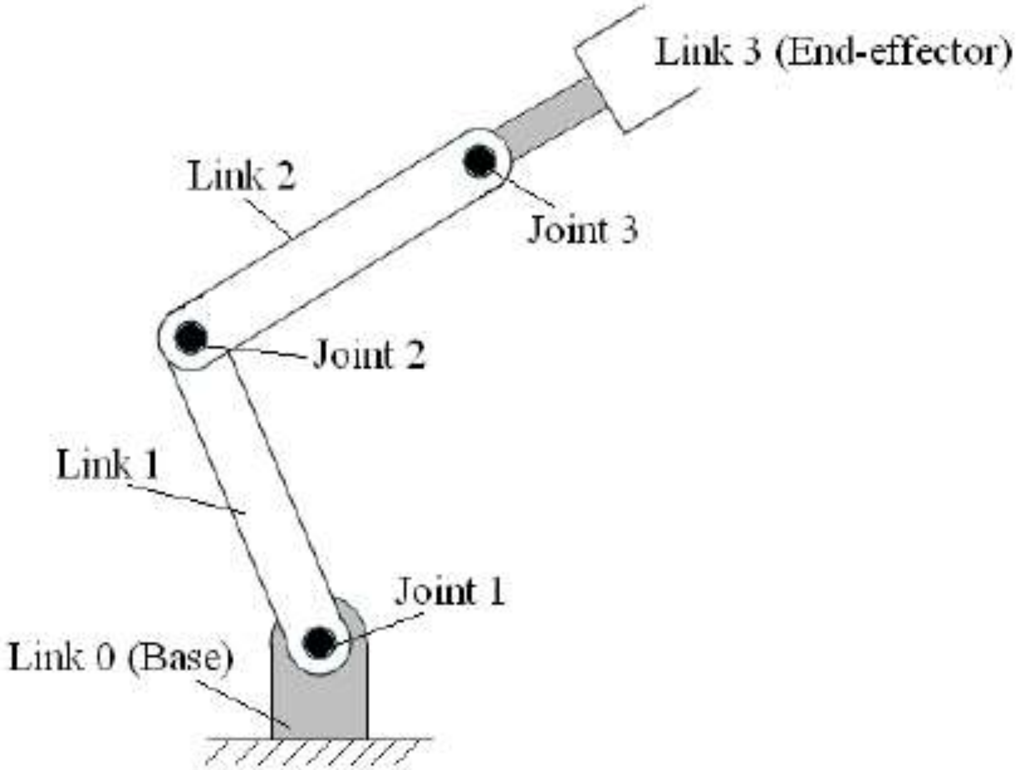


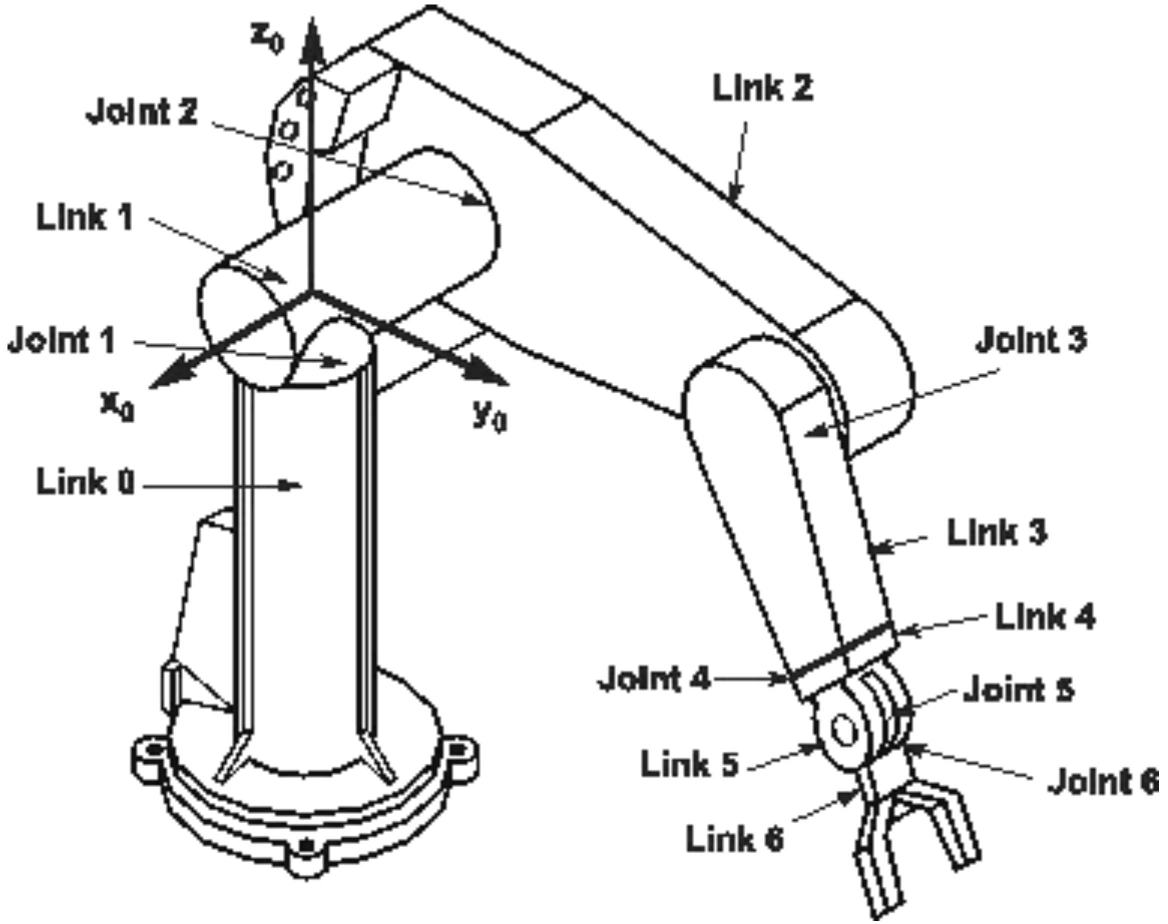
Links and Joints

- A Robot is mechanically constructed by connecting a set of bodies, called links, to each other using various types of joints.
- Actuators such as electric motors deliver forces or torque that cause the robots link to move.
- An end effector such as gripper or hand for grasping and manipulating objects , is attached to a specific link.

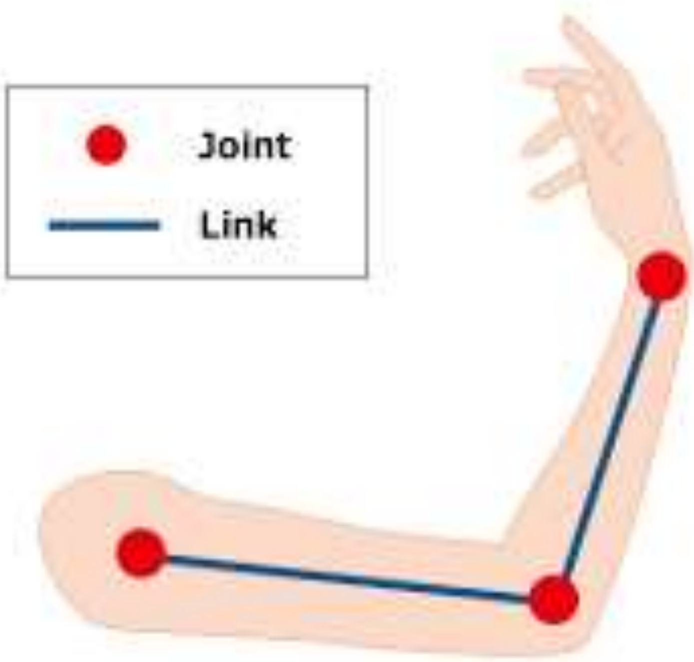


Example: Links and Joints

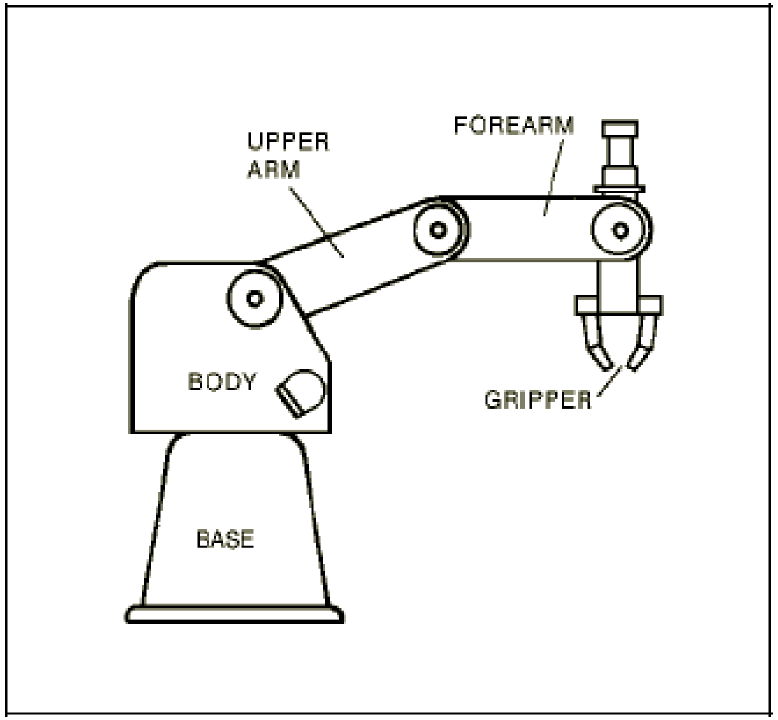


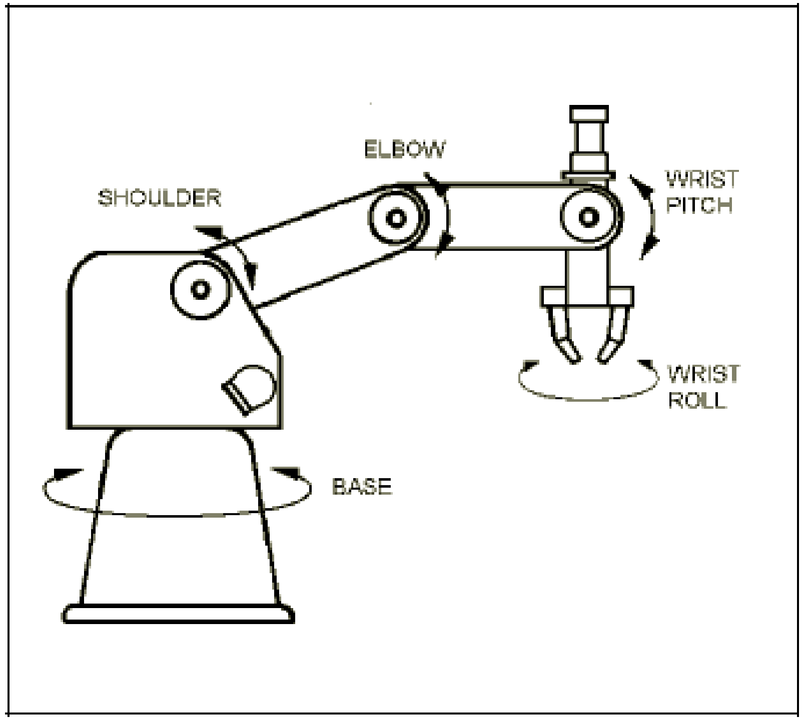


E

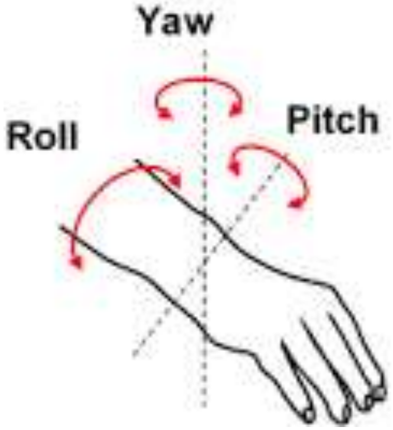

Joint
Link

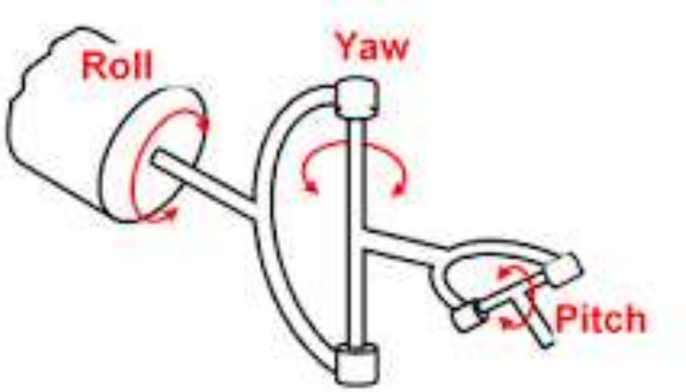
o

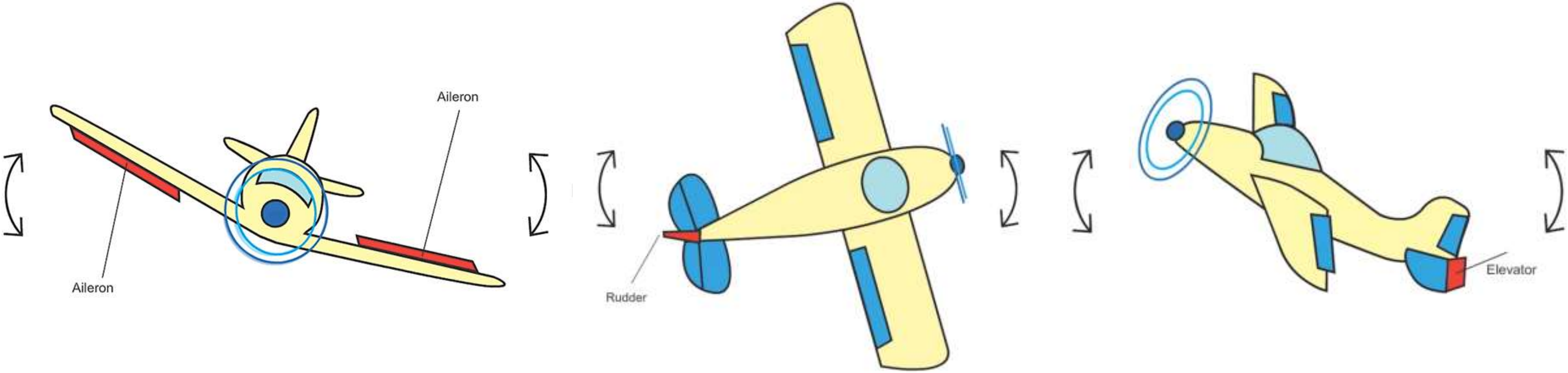

UPPER
ARM
FOREARM
BODY
GRIPPER
BASE


ELBOW
SHOULDER
WRIST
PITCH
WRIST
ROLL
BASE

Basic Motion: Roll, Yaw, Pitch







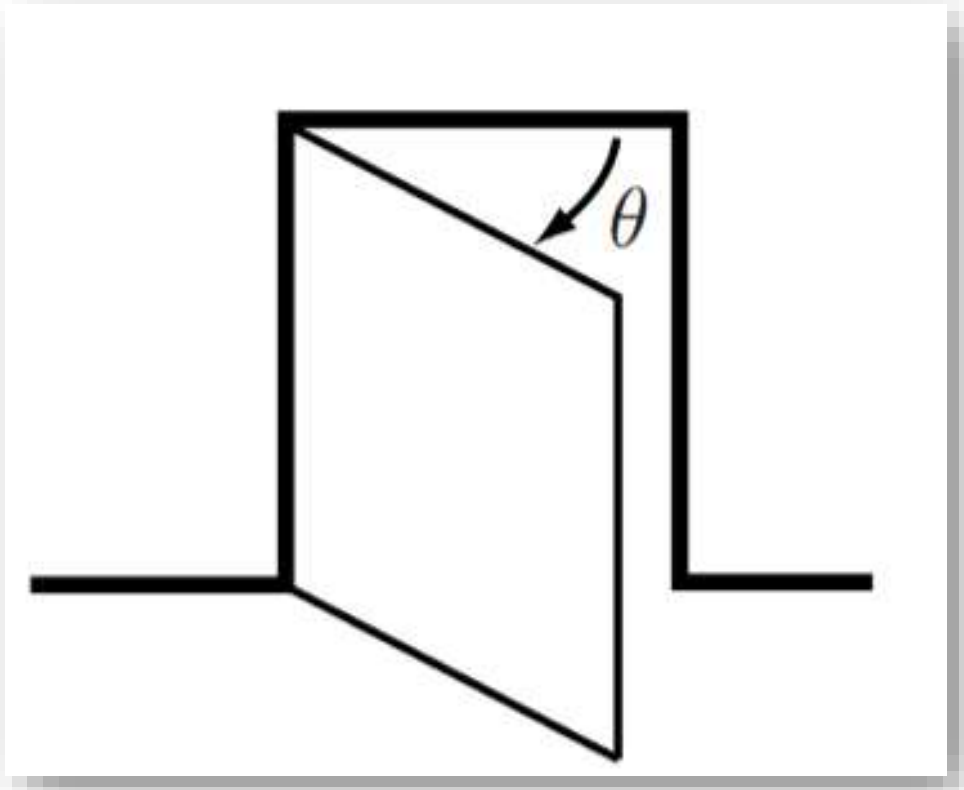
Example of Motion:

A door is represented by a single number (Θ) about its hinge,

or

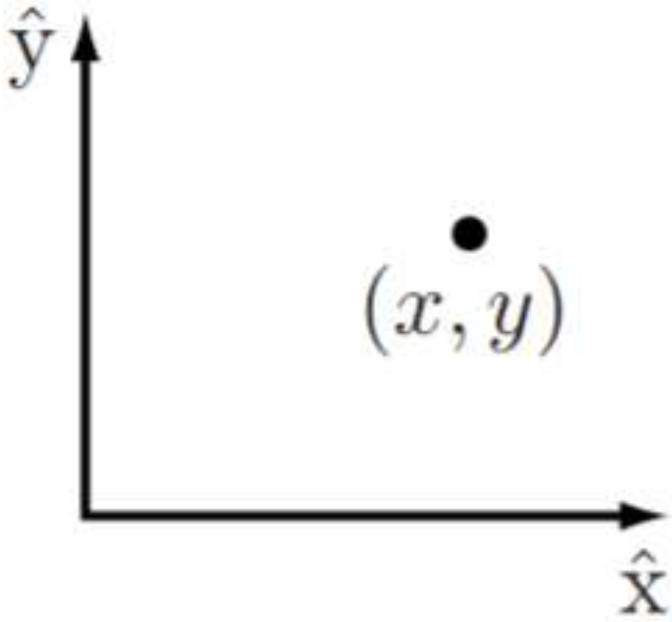
Only one variable is needed to find the position of the door,

So, it's degree of freedom is 1.



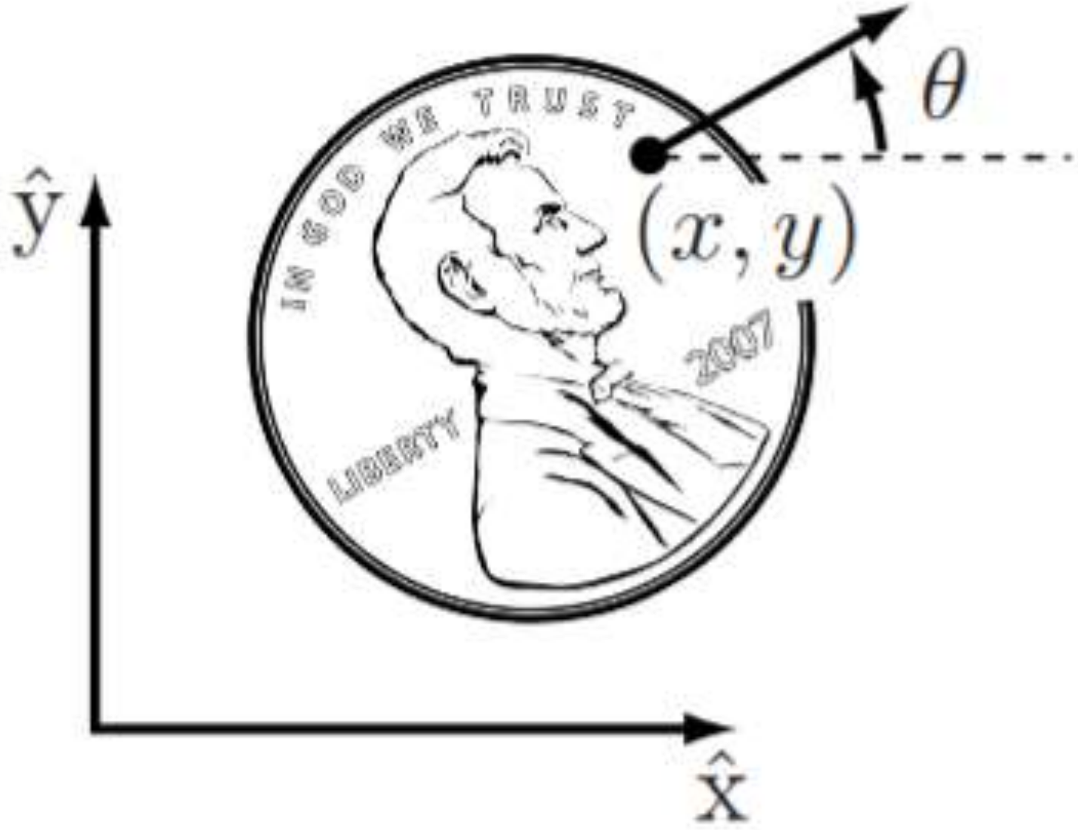
Example: a point (planar surface)

The configuration of a point on a plane can be described by two coordinates, (x,y).



Example: Coin (planar surface)

The configuration of a coin lying heads up on a flat table can be described by three coordinates: two coordinates (x,y)that specify the location of a particular point on the coin, and one coordinate(θ) that specifies the coin's orientation.



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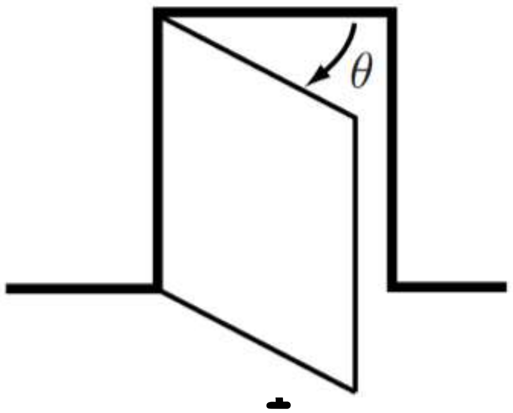
REMARK

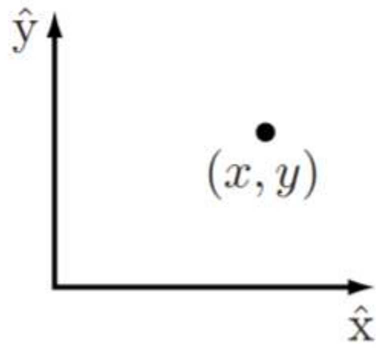
In the previous examples , coordinates all take values over a continuous range of real numbers.

Definition:

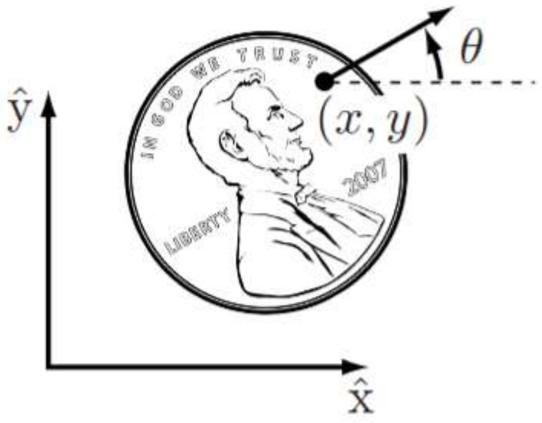
The number of degrees of freedom **(DoF)** of a robot is the smallest number of real-valued coordinates needed to represent its configuration.

Examples: Degree of Freedom (DoF)





DoF=2

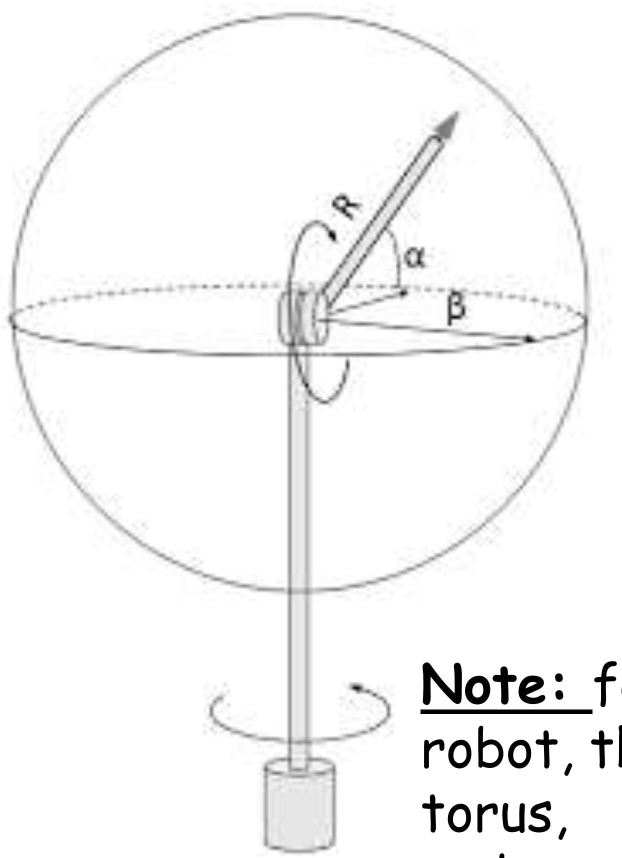


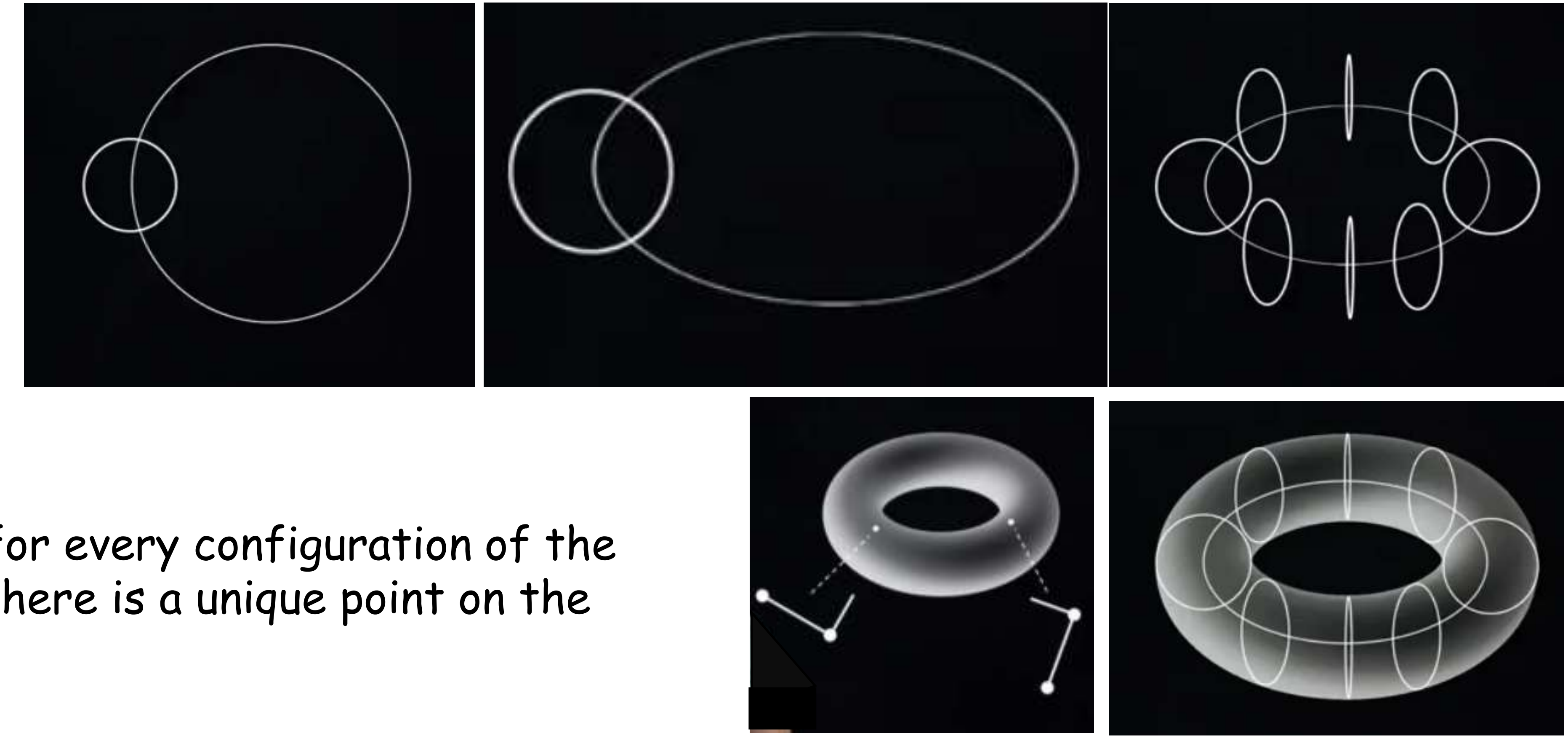
DoF=3

Definition:

The configuration of a robot is a complete specification of the position of every point of the robot. The minimum number n of real-valued coordinates needed to represent the configuration is the number of degrees of freedom(dof) of the robot. Then n-dimensional space containing all possible configurations of the robot is called the configuration space(C-space). The configuration of a robot is represented by a point in its C-space.

Example (C-Space) : Two joint arm





Note: for every configuration of the robot, there is a unique point on the torus,
and
for every point on the torus, there is a unique configuration of the robot.

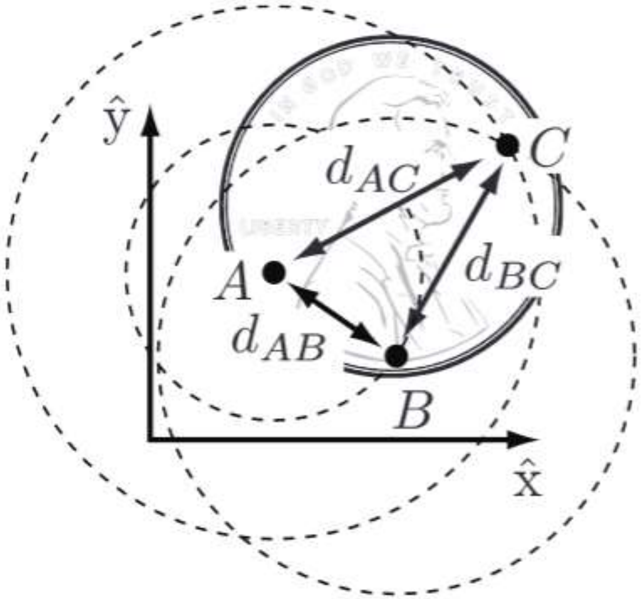
Example: A coin is lying on a table

Chose three points A, B,C

The positions of these points in the plane are written (x_A,y_A), (x_B,y_B),(x_C,y_C)

REMARK

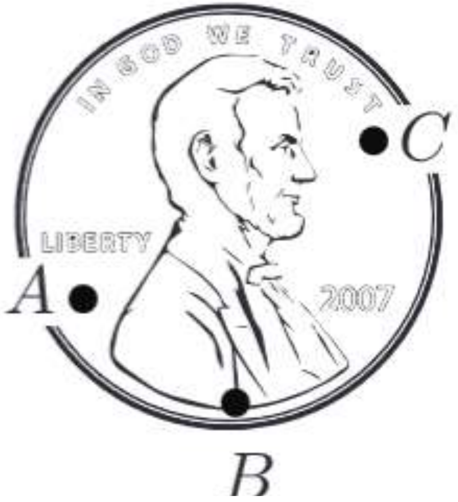
If the points could be placed independently anywhere in the plane, the coin would have six degrees of freedom – two for each of the three points.



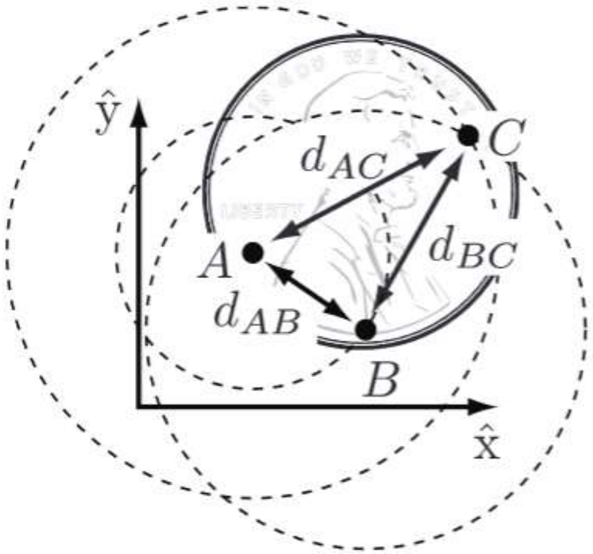
Ex. A coin is lying on a table, cont.

According to the definition of **rigid body**, the distances between points A and B, is always constant regardless of where the coin is. The same is true between A and C, B and C.

The following equality constraints on the coordinates (x_A,y_A), (x_B,y_B),(x_C,y_C) must always be satisfied

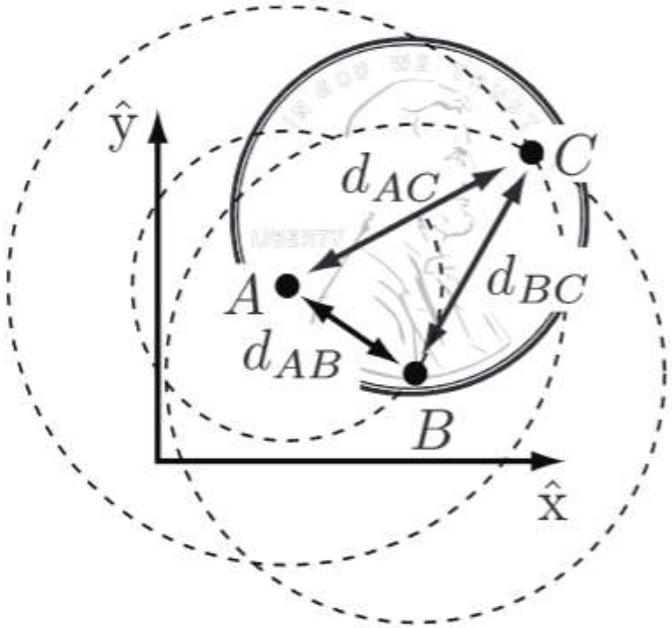


$$d(A,B)=\sqrt{(x_A-x_B)^2+(y_A-y_B)^2}=d_{AB},$$
$$d(B,C)=\sqrt{(x_B-x_C)^2+(y_B-y_C)^2}=d_{BC},$$
$$d(A,C)=\sqrt{(x_A-x_C)^2+(y_A-y_C)^2}=d_{AC}.$$



Ex. A coin is lying on a table, cont.

Chose a position of A, any where, there are two degree of freedom
The constraint d (A,B)=d_{AB} restricts the choice of (x_B,y_B) to those points of the circle of radius d_{AB} centered at A.

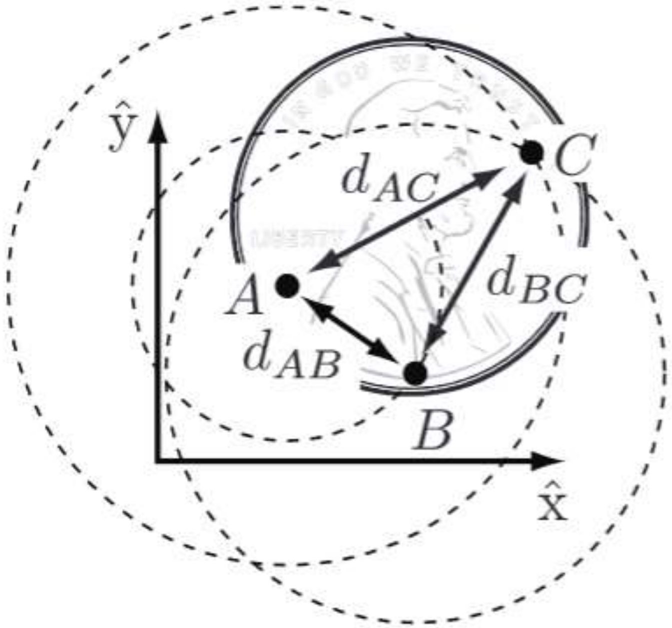


A point on the circle can be specified by a single parameter, e.g, the angle specifying the location of B on the circle centered at A. Let's call this angle φ_{AB} and define it to be the angle that the vector AB makes with the x-axis.

Ex. A coin is lying on a table, cont.

Once we have chosen the location of point B, the location of C is fixed. Hence, coin has three degree of freedom in the plane which can be specified by (x_A,y_A,φ_{AB}).

Therefore, Dof=3



REMARK

We have been applying the following general rule for determining the number of degrees of freedom of a system:

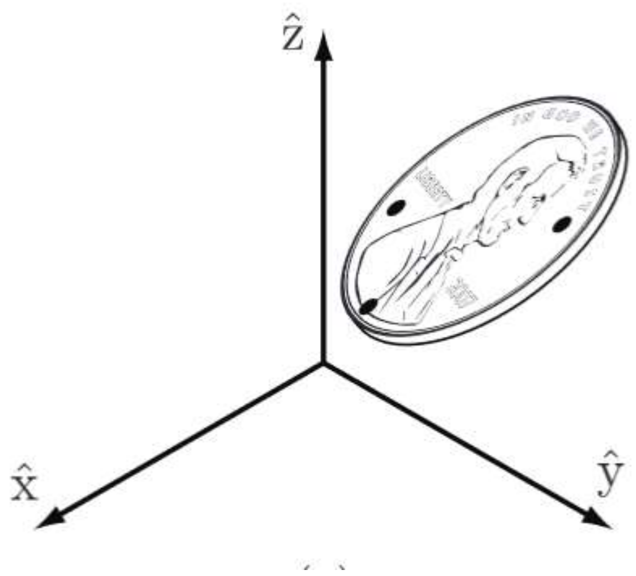
degrees of freedom = (sum of freedoms of the points)−(number of independent constraints)

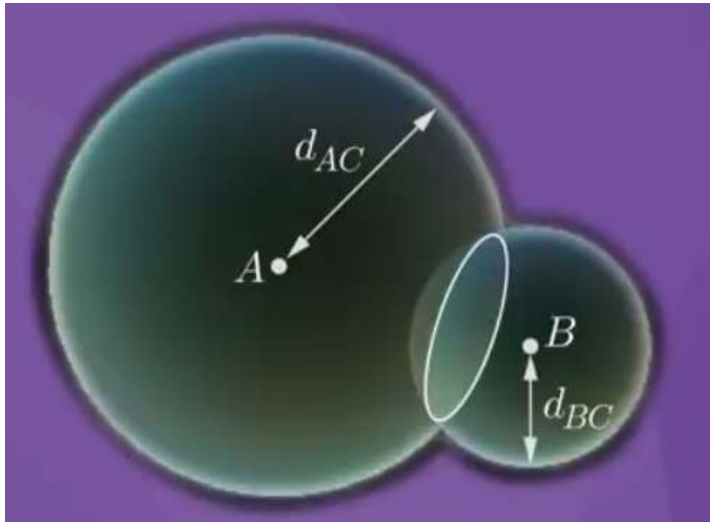
or, in terms of variables

degrees of freedom = (number of variables)− (number of independent equations)

Ex. A coin in a sphere

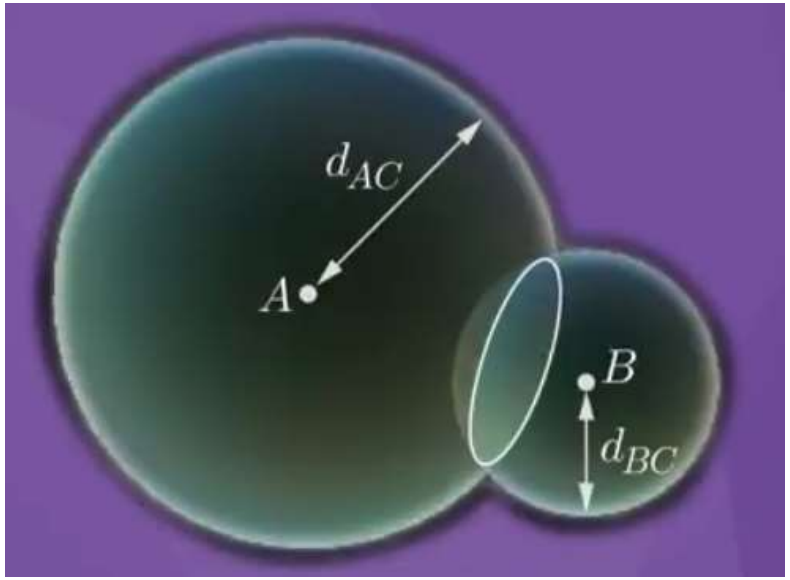
The points A,B, and C are given by (x,y,z)
Point A can be placed freely (3dof)The location of B is subject to the constraint d(A,B)=d_{AB}. Means it must lie on the sphere of radius dAB centered at A. Thus we have 3-1=2 freedom to specify, which can be expressed as the latitude and longitude for the point on the sphere. The point C must lie at the intersection of spheres centered at A and B of radius dAC and dBC. In the general case, the intersection of two spheres is a circle, hence the location of the point C can be described by an angle. Point C therefore adds 3-2=1 freedom.





The idea of Constraints

Thus a rigid body in space has six total degrees of freedom, three of which are linear, or x-y-z, and three of which are angles, the roll, the pitch, and the yaw.



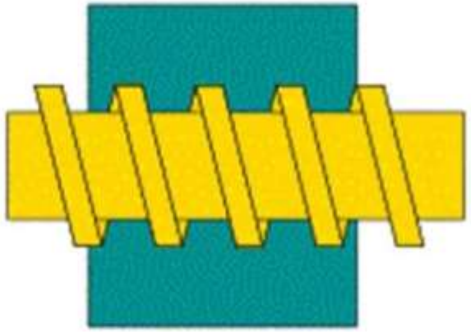
Point	Coordination	Independent constraints	Real freedoms
A	3	0	3
B	3	1	2
C	3	2	1
D, etc.	3	3	0
total			6

REMARK

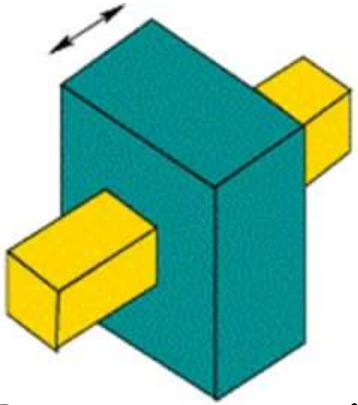
- The dimension of the C-space, or the number of degrees of freedom, equals the sum of the freedoms of the points minus the number of independent constraints acting on those points.

- Since our robots are made of rigid bodies, we can express the number of degrees of freedom more simply as the sum of the freedoms of the bodies minus the number of independent constraints acting on the bodies.

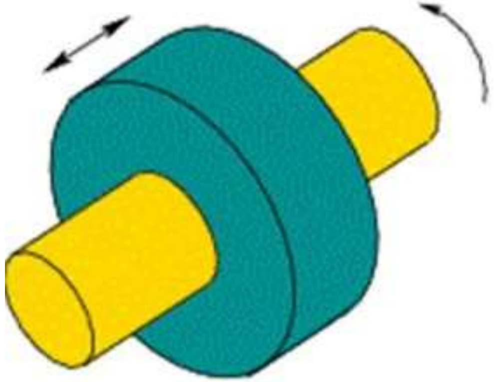
Robot Joints



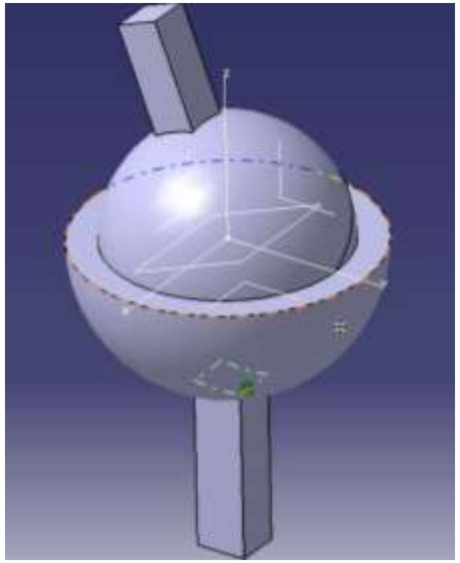
Helical (H)



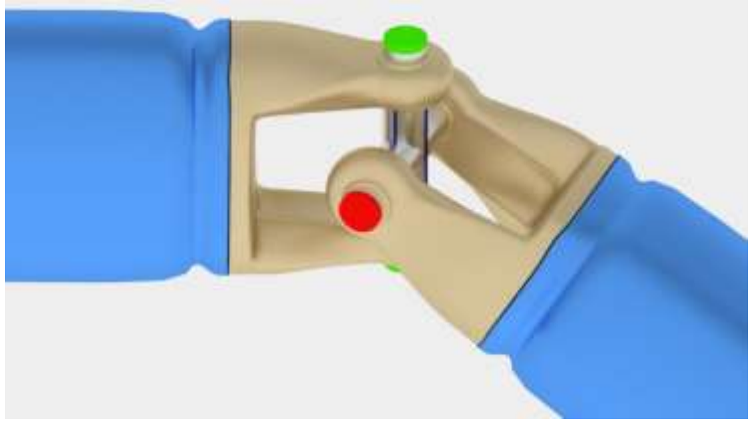
Prismatic (P)



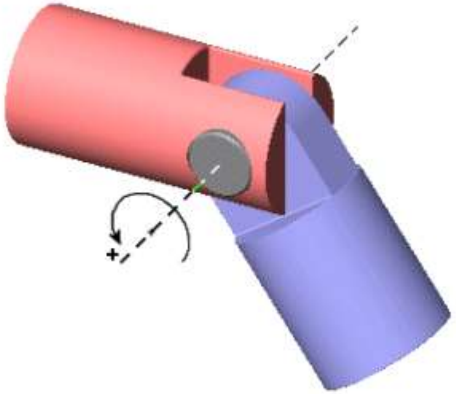
Cylindrical (C)



Spherical (S)



Universal (U)

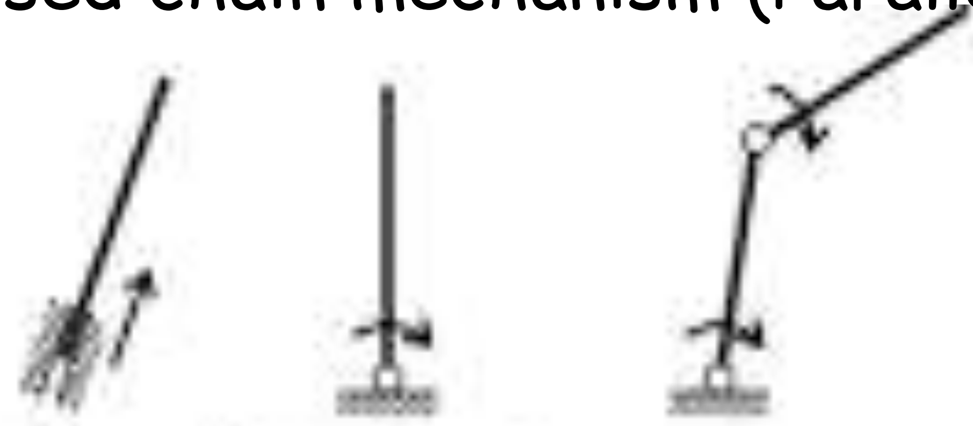


Revolute (R)

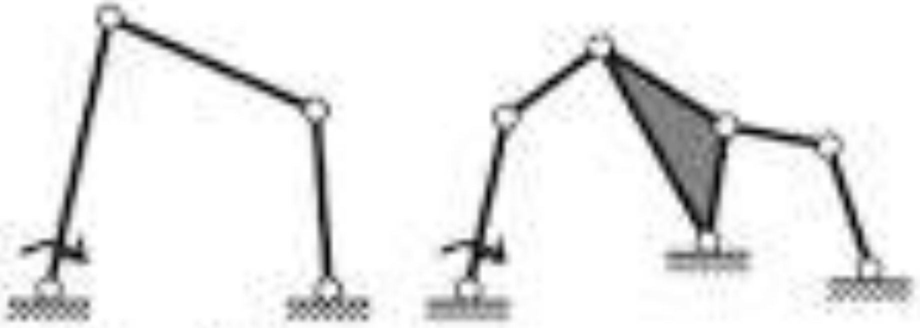
DoF and constraints of joints

Joint type	dof f	Constraints c between two planar rigid bodies	Constraints c between two spatial rigid bodies
Revolute (R)	1	2	5
Prismatic (P)	1	2	5
Helical (H)	1	N/A	5
Cylindrical (C)	2	N/A	4
Universal (U)	2	N/A	4
Spherical (S)	3	N/A	3

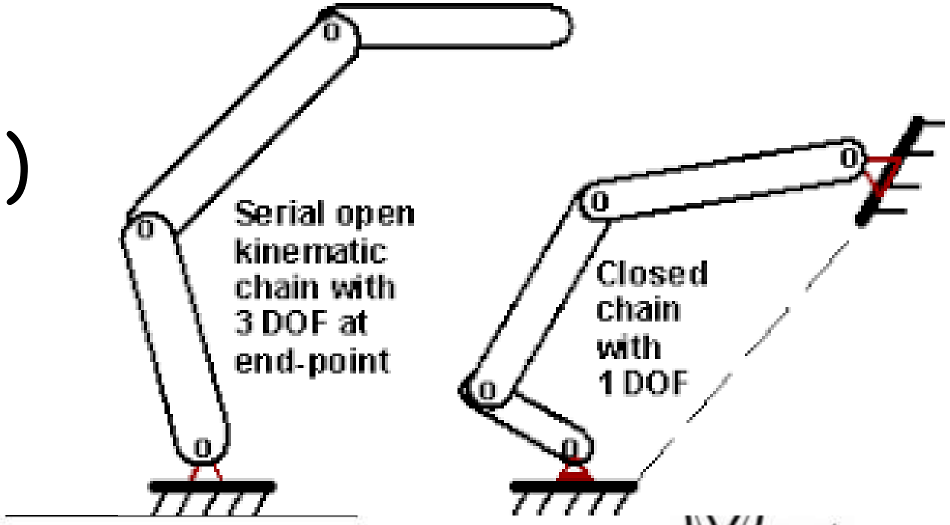
Open chain mechanism (Serial mechanism)
Closed chain mechanism (Parallel mechanism)

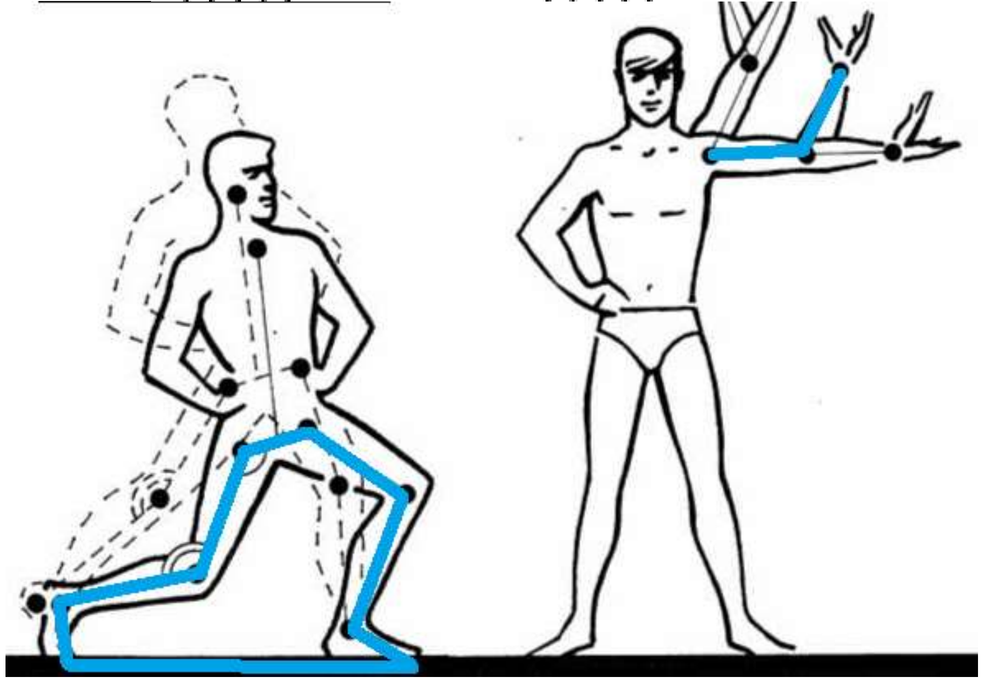


a) Open-chain mechanisms



b) Closed-chain mechanisms





DoF ; Grubler's Formula

$$\text{dof} = \underbrace{m(N-1)}_{\text{rigid body freedoms}} - \underbrace{\sum_{i=1}^{J} c_i}_{\text{joint constraints}}$$

$$= m(N-1) - \sum_{i=1}^{J}(m - f_i)$$

$$= m(N-1-J) + \sum_{i=1}^{J} f_i.$$

Where

N: Number of links (ground is regarded as a link)

J: Number of joints

m: number dof freedom of a rigid body (m=3 for planar and m=6 for spatial bodies)

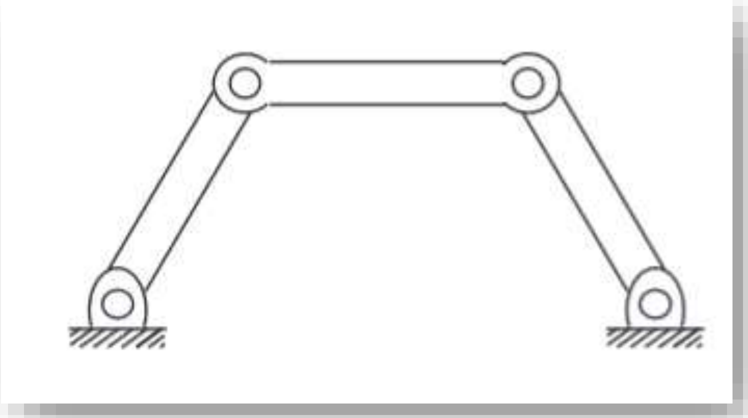
f_i: number of freedom provided by joint i.

c_i: number of constraints provided by joint i.

$f_i+c_i=m$

Application of Grubler Formula

Example: Four-bar linkage



Four links (one of them ground), N=4

Single closed loop connected by four revolute joints. J=4

Since all the links are confined to move in the same plane, we have m= 3

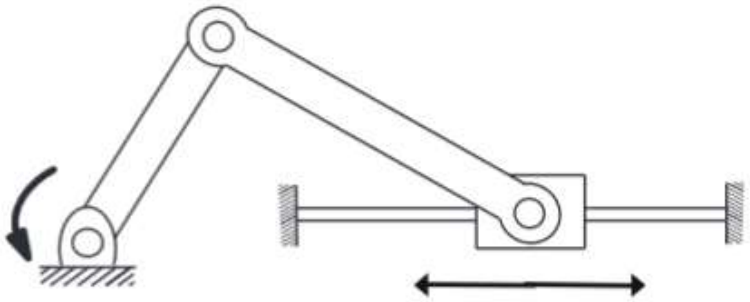
$f_i=1,\ i=1,\ldots,4$

$$\text{dof} = m(N-1-J) + \sum_{i=1}^{J} f_i.$$

Dof=1

Application of Grubler Formula

Example: The slider-crank closed-chain mechanism



1. Four links (one of them ground), N=4
Single closed loop connected by three revolute joints and one prismatic . J=4 , F_i=1, i=1,…,4

$$\text{dof} = m(N-1-J) + \sum_{i=1}^{J} f_i.$$

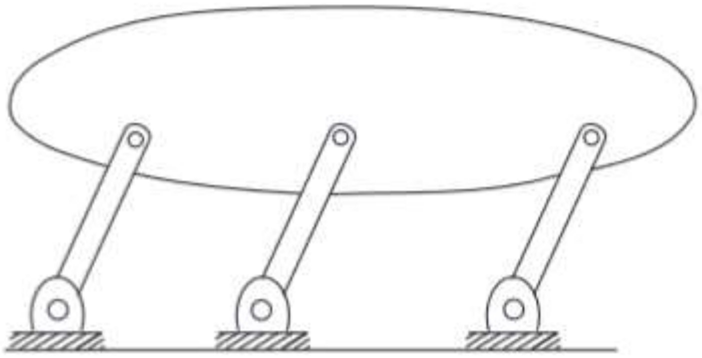
2. Two revolute joint (f_i=1) and one PR joint (f_i=2) and three links N=3.

In both cases dof=1.

Note: each joint connects precisely two bodies

Application of Grubler Formula

Example: A parallelogram linkage



N=5, J=6R, (fi=1)

$$\text{dof} = m(N-1-J) + \sum_{i=1}^{J} f_i.$$

$$3(5-1-6)+6=0.$$

Dof=0

A mechanism with zero degrees of freedom is by definition a rigid structure. It is clear from examining the figure, though, that the mechanism can in fact move with one degree of freedom. The constraints provided by the joints are not independent, as required by Grubler's formula.

Application of Grubler Formula

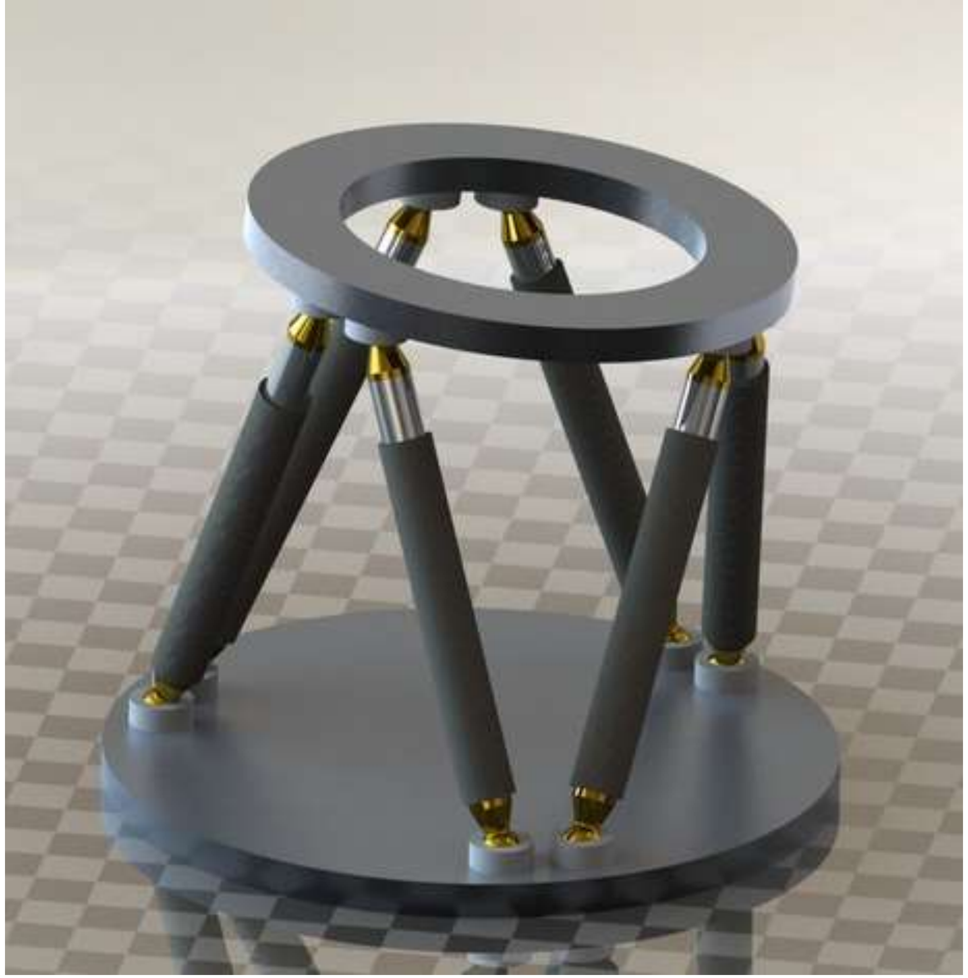
Example:

Stewart Platform

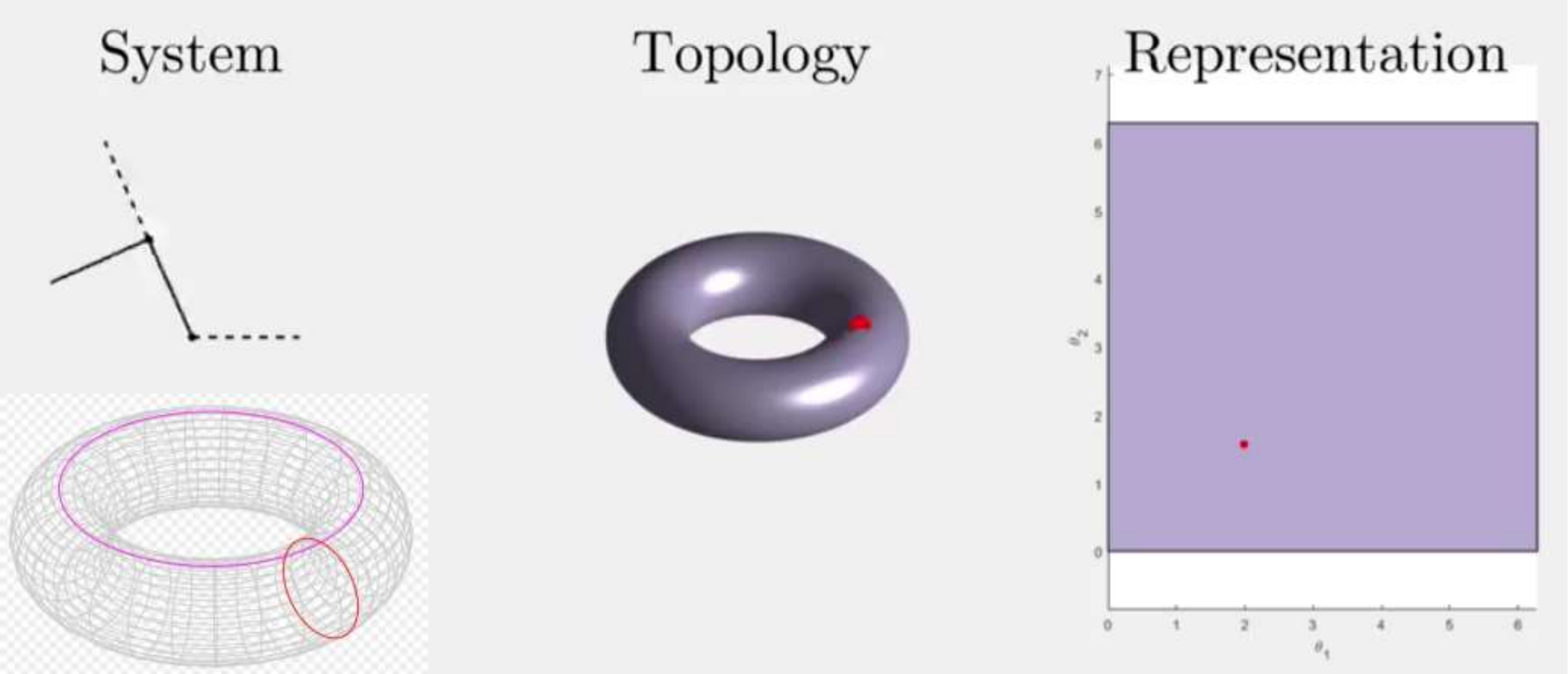
N=12+2, J=6 S, 6 U, 6 P

$$\text{dof} = m(N - 1 - J) + \sum_{i=1}^{J} f_i.$$

Dof=?

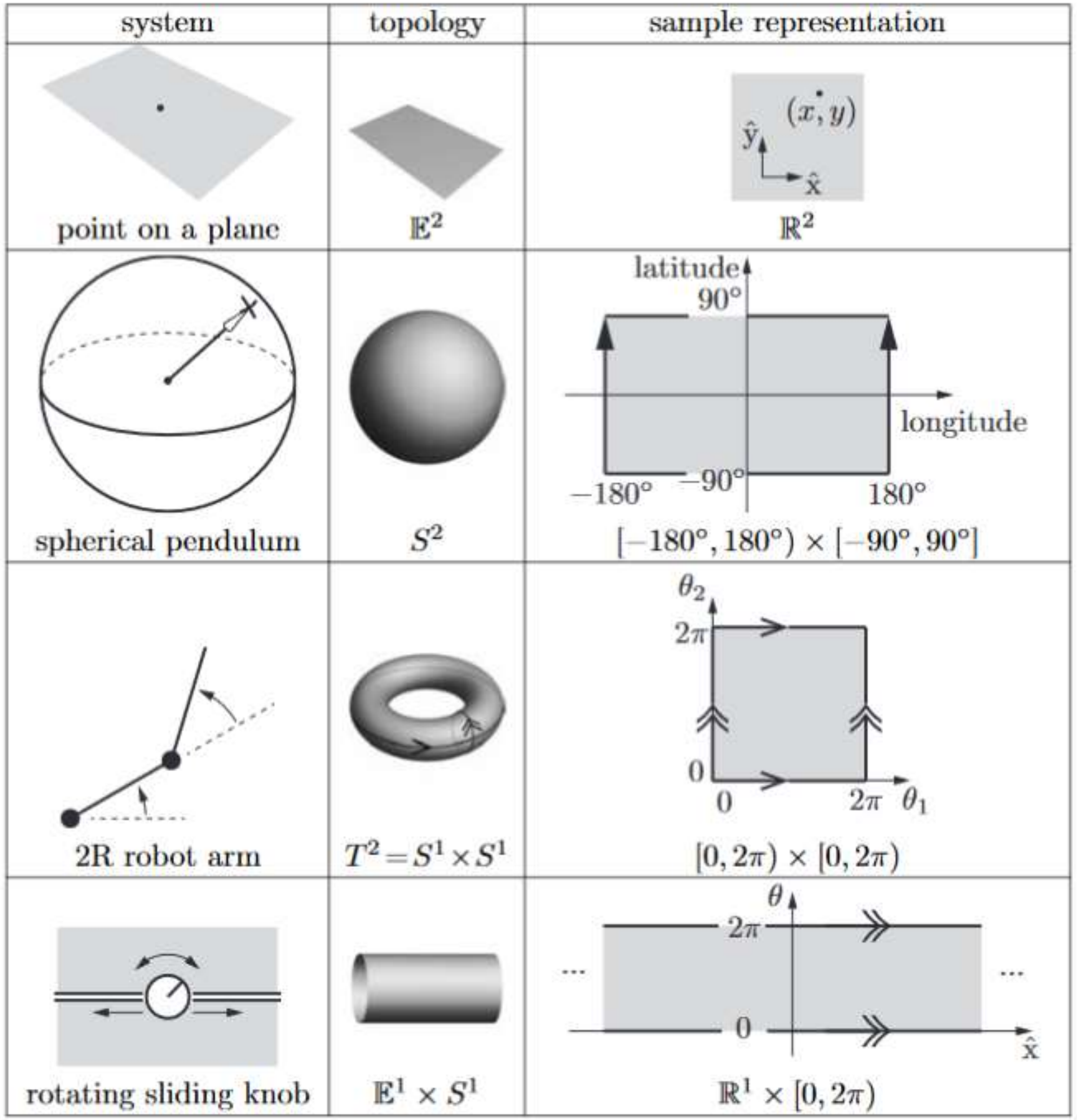


Topology:
Example: 2R robot arm



Topology Representation Notation

- A circle is written mathematically as S^1((a one dimensional sphere space).
- A line is written mathematically as E^1 (or R^1) (a one dimensional Euclidian space).



REMARK

- The topology of a space is a fundamental property of the space itself and is independent of how we choose coordinates to represent points in the space .

- For example, to represent a point on a circle, we could refer to the point by the angle θ from the center of the circle to the point, relative to a chosen zero angle. Or, we could choose a reference frame with its origin at the center of the circle and represent the point by the **two coordinates (x,y)** subject to the Constraint $x^2+y^2= 1$. No matter what our choice of coordinates is, the space itself does not change.

Examples, Topology Representation Notation:

Space Representation

- ✓ The C-space of a rigid body in the plane can be written as R^2*S^1 since the configuration can be represented as the concatenation of the coordinates (x , y) representing R^2 and an angle Θ representing S^1.

- ✓ The C-space of a 2R robot arm can be written $S^1 \times S^1 = T^2$, where T^n is then n-dimensional surface of a torus in an (n+ 1)-dimensional space. Note that $S^1 \times S^1 \times \cdots \times S^1$ (n copies of S1) is equal to T^n, not S^n; for example, a sphere S^2 is not topologically equivalent to a torus T^2.

REMARK

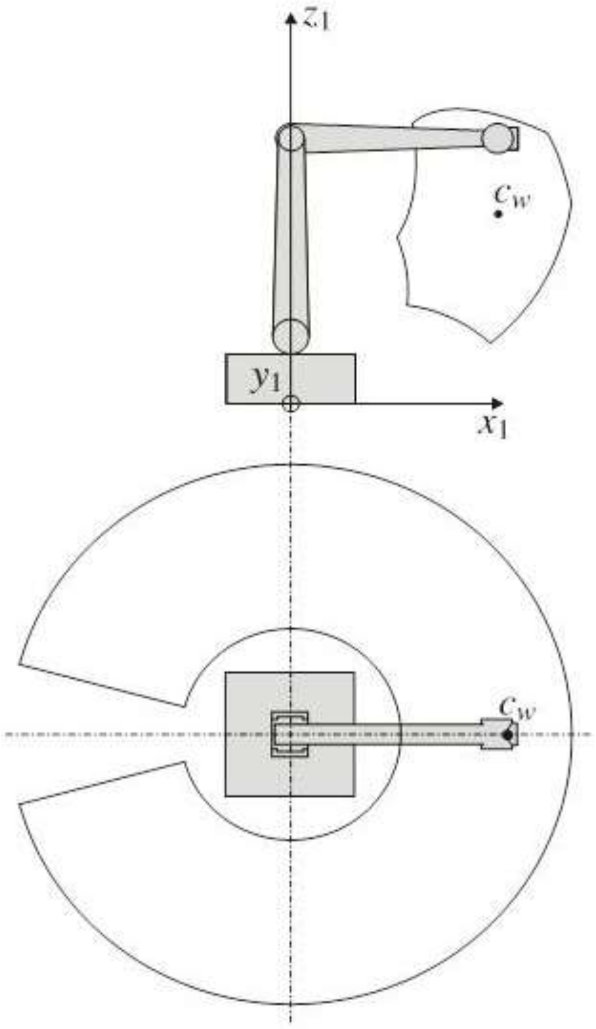
- ✓ The latitude-longitude representation of a sphere is unsatisfactory near the poles. Taking small steps result in large change in coordinates. So, the north and south poles are singularities.

- ✓ The solution is to embed a two-dimensional unit sphere into a three dimensional Euclidean space **(x,y,z).** The constraint **$x^2+y^2+z^2=1$** reduces the dof to 2. This method is called **implicit representation**, while the first one, using n-coordinates to represent n-dimensional spaces is called **explicit parametrization**.

The work space

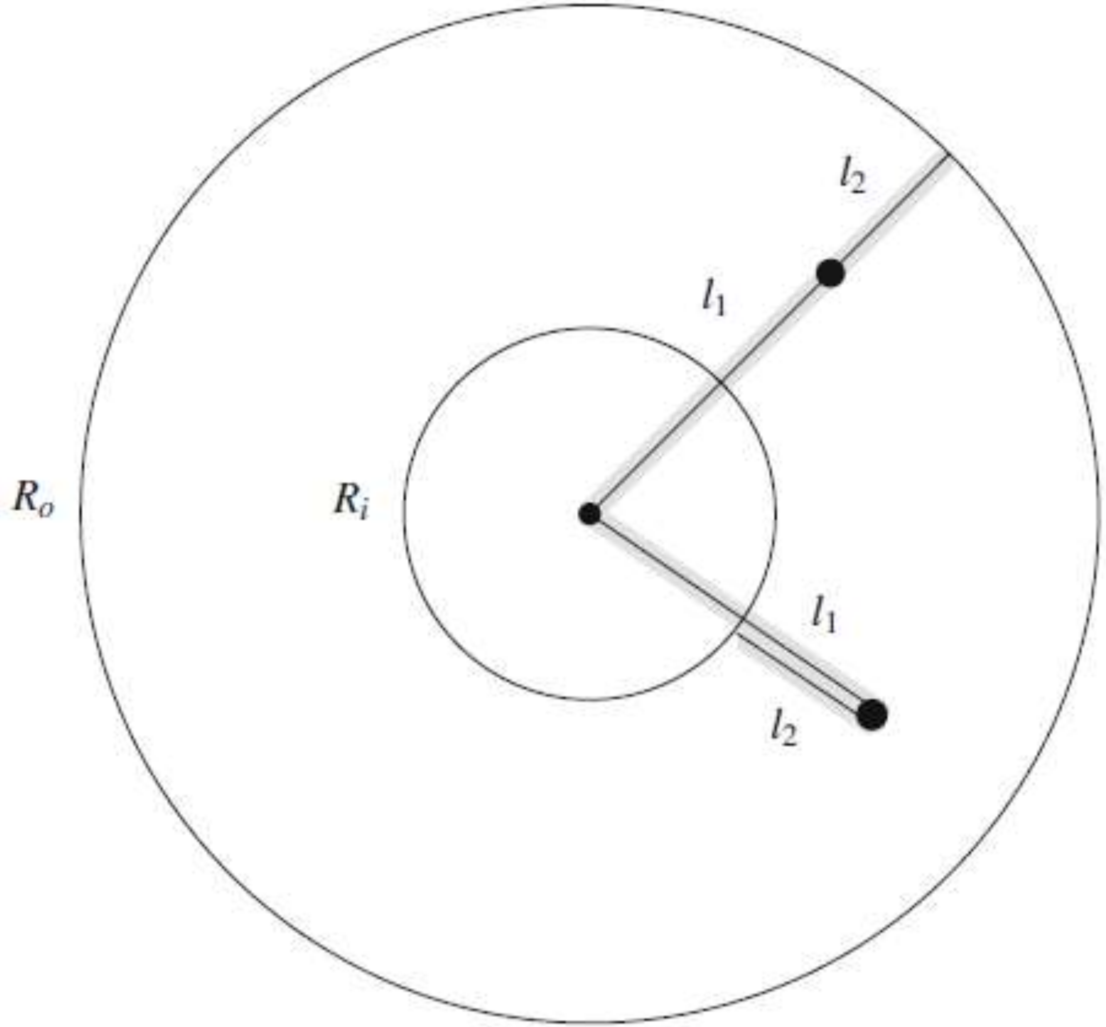
- The robot workspace (sometimes known as **reachable space**) is all places that the end effector (gripper) can reach.

- The workspace is dependent on the DOF angle/translation limitations, the arm link lengths, the angle at which something must be picked up at, etc. The workspace is highly dependent on the robot configuration. .



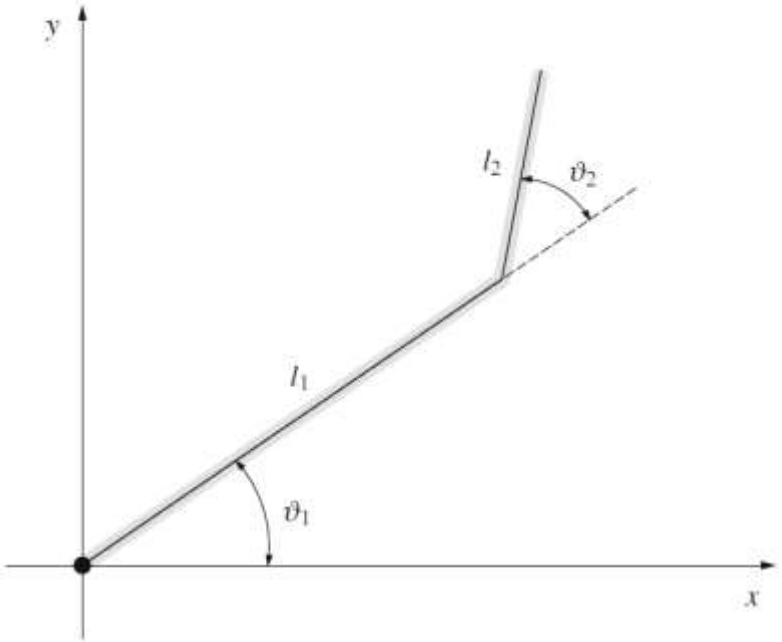
Example

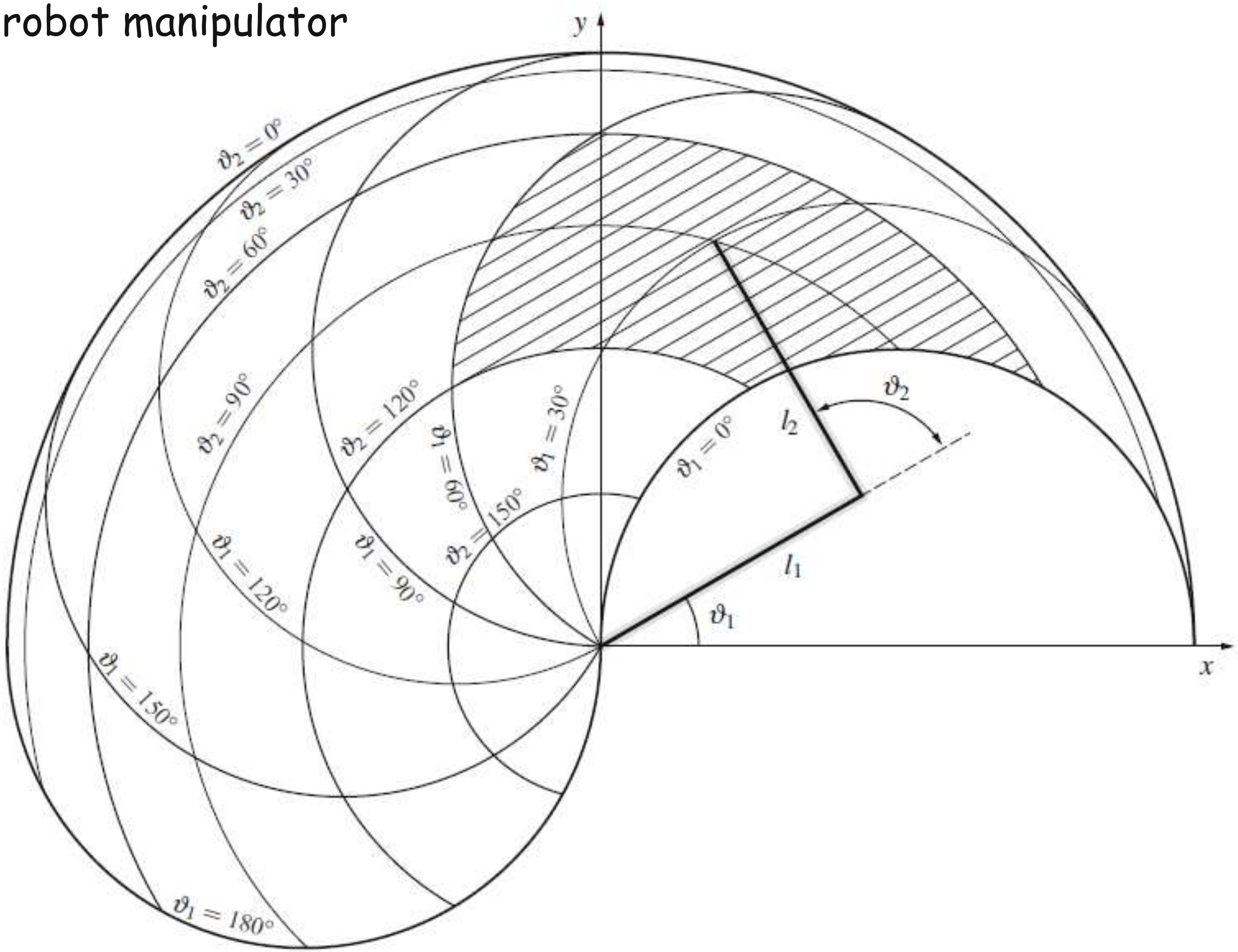
Working area of two-segment manipulator with the second segment shorter.



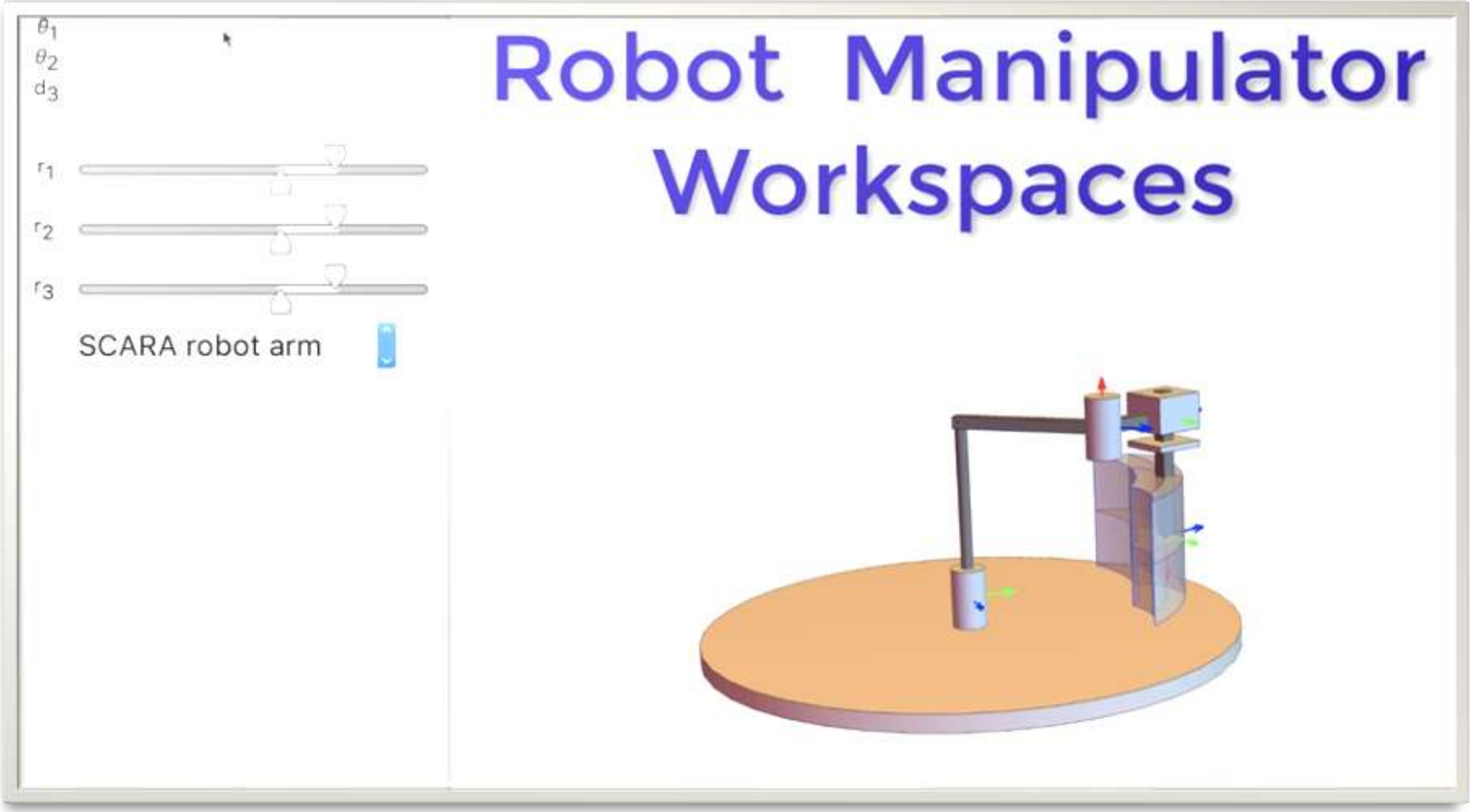
Example

Workspace of a planar two-segment robot manipulator ($l1 = l2$, $0^\circ \leq \vartheta 1 \leq 180^\circ$, $0^\circ \leq \vartheta 2 \leq 180^\circ$)





Task Space and Work Space



Reading list

Kevin M. Lynch and Frank C. Park, 2017, Modern Robotics, 1st Edition, Cambridge University Press, chapter 2.

Link:

https://robotacademy.net.au/

Questions?