زانكۆی سەلاحەدین - هەولێر
Salahaddin University-Erbil

Golden Ratio

Research Project Submitted to the department of (Mathematic) in partial fulfillment of the requirements for the degree of BSc. in (Mathematic)

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April -2022

Certification of the Supervisors

I certify that this work was prepared under my supervision at the Department of Mathematics / College of Education / Salahaddin University-Erbil in partial fulfillment of the requirements for the degree of Bachelor of philosophy of Science in Mathematics.

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Date: 10 / 4 / 2022

In view of the available recommendations, I forward this work for debate by the examining committee.

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Acknowledgement

To begin with, I would like to thank "**Allah**" for the virtues of this blessing for implanting the soul of endurance and faith in me for completing this study. I would like to thank my supervisor **Dr. Ivan S. Latif** for suggesting the project and for his guidance, support and encouragement during the course of the work. I am indebted to the Mathematic Department for giving the chance of having this course of study. I would like to thank my dear friends Amin.

Abstract

In this research, we talk about the golden ratio in general what is the golden ratio and the definition of golden ratio and the amount of golden ratio in a formula and Calculating the amount of golden ratio and then we talk about the history of Fibonacci and we analyzed the example of flowchart and we put it in this way program and analyzed the sample with the program

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INTRODUCTION

The golden ratio, also known as the golden number, golden proportion, or the divine proportion, is **a ratio between two numbers that equals approximately 1.618**. Usually written as the Greek letter phi, it is strongly associated with the Fibonacci sequence, a series of numbers wherein each number is added to the last. We assume that we have two values a and b which specify an interval, may be a very crude

one, in which the true minimum point lies, With such methods we literally search for the minimum of f(x) in some range a< x <b in which we suspect the minimum lies, by evaluating the function at chosen points in the interval. This strategy may be the only one available. For instance the cost of running a chemical process may depend on the operating temperature and hence find the cost at these temperatures and hope in this way to locate the minimum of the cost function, and the temperature at which to run the process for least cost. And that with in this interval that function is unimodal, i.e. has one minimum at x**, Thus our function has a form similar to that show in Fig.1. For such a function, if we know its value at three points

CHAPTER ONE

1.1. What is the Golden Ratio?

The golden ratio, which is also referred to as the golden mean, divine proportion, or golden section, exists between two quantities if their ratio is equal to the ratio of their sum to the larger quantity between the two. With reference to this definition, if we divide a line into two parts, the parts will be in the golden ratio if:

The ratio of the length of the longer part, say "a" to the length of the shorter part, say "b" is equal to the ratio of their sum " (a + b)" to the longer length.

Refer to the following diagram for a better understanding of the above concept:

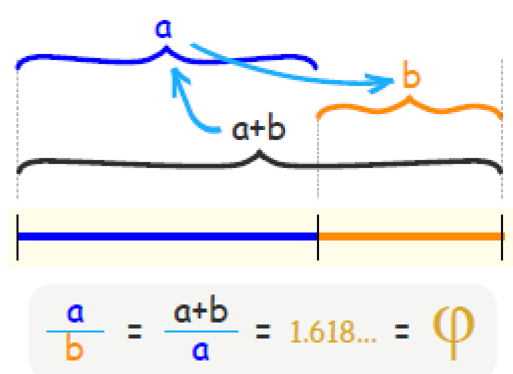


Figure 1 Golden ratio length

It is denoted using the Greek letter ϕ, pronounced as "phi". The approximate value of ϕ is equal to 1.61803398875... It finds application in geometry, art, architecture, and other areas. Thus, the following equation establishes the relationship for the calculation of golden ratio: $\phi = a/b = (a + b)/a = 1.61803398875...$ where a and b are the dimensions of two quantities and a is the larger among the two.

1.2. Definition of Golden Ratio [2 3 4]

When a line is divided into two parts, the long part that is divided by the short part is equal to the whole length divided by the long part is defined as the golden ratio. Mentioned below are the golden ratio in architecture and art examples.

There are many applications of the golden ratio in the field of architecture. Many architectural wonders like the Great Mosque of Kairouan have been built to reflect the golden ratio in their structure. Artists like Leonardo Da Vinci, Raphael, Sandro Botticelli, and Georges Seurat used this as an attribute in their artworks.

1.3. Formula of Golden Ratio [3 4]

The Golden ratio formula can be used to calculate the value of the golden ratio. The golden ratio equation is derived to find the general formula to calculate golden ratio.

We saw above that the Golden Ratio has this property:

$$\frac{a}{b} = \frac{a+b}{a}$$

We can split the right-hand fraction like this:

$$\frac{a}{b} = \frac{a}{a} + \frac{b}{a}$$

$\frac{a}{b}$ is the Golden Ratio φ , $\frac{a}{a} = 1$ and $\frac{b}{a} = \frac{1}{\varphi}$, which gets us:

$$\Phi = \frac{1}{\varphi}$$

So the Golden Ratio can be defined in terms of itself. Let us test it using just a few digits of accuracy:

$$\varphi = 1 + \frac{1}{1.618}$$

= 1 + 0.61805…

= 1.61805…

With more digits we would be more accurate.

1.4. Golden Ratio Equation [3 4]

From the definition of the golden ratio,

$$\frac{a}{b} = \frac{a+b}{a} = \phi$$

From this equation, we get two equations:

$\frac{a}{b} = \phi \rightarrow (1)$

$\frac{a+b}{a} = \phi \rightarrow (2)$

From equation (1),

$\frac{a}{b} = \phi$

$\Rightarrow$ a = b

Substitute this in equation (2),

$(b\,\phi + b)/b\,\phi = \phi$

$B\,(\phi + 1)/b\,\phi = \phi$

$(\phi + 1)/\phi = \phi$

$1 + 1/\phi = \phi$

$1 + 1/\phi = \phi$

CHAPTER TWO

2.1. Golden ratio Calculating [1 4]

The value of the golden ratio can be calculated using different methods. Let us start with a basic one.

Hit and trial method

We will guess an arbitrary value of the constant, then follow these steps to calculate a closer value in each iteration.

Calculate the multiplicative inverse of the value you guessed, i.e., 1/value. This value will be our first term.

Calculate another term by adding 1 to the multiplicative inverse of that value.

Both the terms obtained in the above steps should be equal. If not, we will repeat the process till we get an approximately equal value for both terms.

For the second iteration, we will use the assumed value equal to the term 2 obtained in step 2, and so on.

Example 1

Since $\phi = 1 + 1/\phi$, it must be greater than 1. Let us start with value 1.5 as our first guess.

Term 1 = Multiplicative inverse of 1.5 = 1/1.5 = 0.6666...

Term 2 = Multiplicative inverse of 1.5 + 1 = 0.6666.. + 1 = 1.6666...

Since both the terms are not equal, we will repeat this process again using the assumed value equal to **term 2**.

The following table gives the data of calculations for all the assumed values until we get the desired equal terms:

Iteration	**Assumed value**	**Term 1 (1/value)**	**Term 2 (1/value + 1)**
1.	**1.5**	**11.511.5 = 0.6666..**	**0.6666.. + 1 = 1.6666..**
2.	**1.6666…**	**11.666..11.666.. = 0.6**	**0.6 + 1 = 1.6**
3.	**1.6**	**11.611.6 = 0.625**	**0.625 + 1 = 1.625**
4.	**1.625**	**11.62511.625 = 0.61538..**	**0.61538.. + 1 = 1.61538..**
5.	**1.61538…**	**..**	**.. and so on**

Table 1: gives the data of calculations for example 1

2.2. What is Golden Rectangle? [2 4]

In geometry, a golden rectangle is defined as a rectangle whose side lengths are in the golden ratio. The golden rectangle exhibits a very special form of self-similarity. All rectangles that are created by adding or removing a square are golden rectangles as well.

2.3. Constructing a Golden Rectangle [4]

We can construct a golden rectangle using the following steps:

Step 1: First, we will draw a square of 1 unit. On one of its sides, draw a point midway. Now, we will draw a line from this point to a corner of the other side.

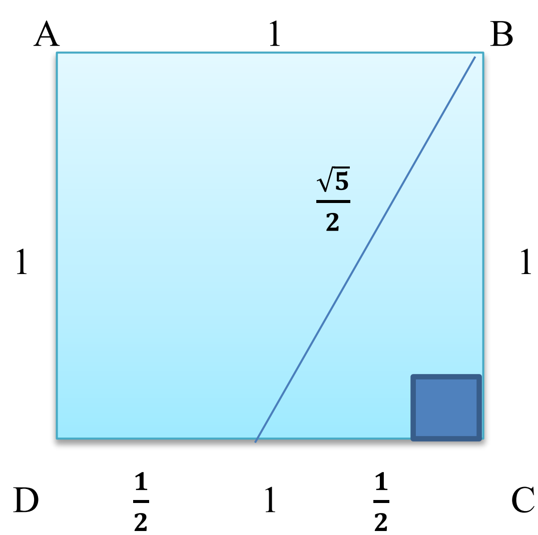


Figure 2: Construction of a golden rectangle

Step 2: Using this line as a radius and the point drawn midway as the center, draw an arc running along the square's side. The length of this arc can be calculated using Pythagoras Theorem: $\sqrt{(1/2)^2 + (1)^2} = \sqrt{5}/2$ units.

Step 3: Use the intersection of this arc and the square's side to draw a rectangle as shown in the figure below:

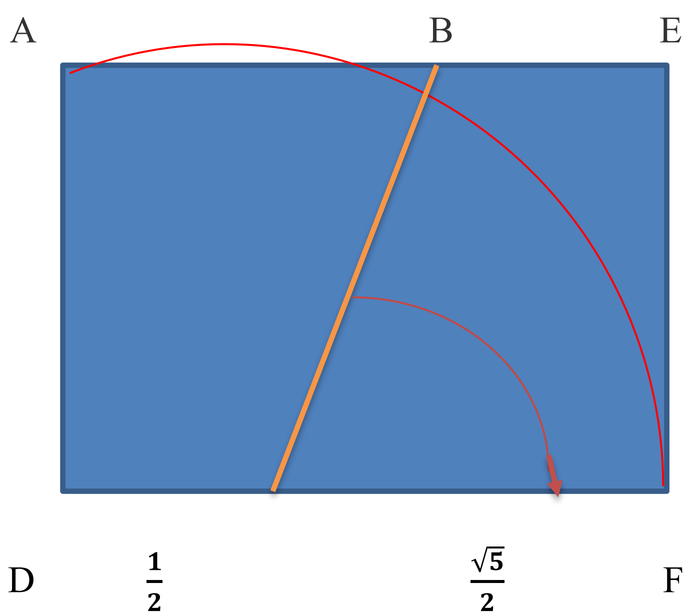


Figure 3: the intersection of this arc

This is a golden rectangle because its dimensions are in the golden ratio. i.e., $\phi = (\sqrt{5}/2 + 1/2)/1 = 1.61803$

CHAPTER THREE

3.1. History of Fibonacci [5 6]

Leonardo Bigollo' Pisano (Fibonacci) (ca 1170-1245) Italy [2, 3], Leonardo (known today as Fibonacci) introduced the decimal system and other new methods of arithmetic to Europe, and relayed the mathematics of the Hindus, Persians, and Arabs. Others had translated Islamic mathematics, e.g. the works of al-Khowârizmi, into Latin, but Leonardo was the influential teacher. He also re- introduced older Greek ideas like Mersenne numbers and Diophantine equations. Leonardo's writings cover a very broad range including new theorems of geometry, methods to construct and convert Egyptian fractions (which were still in wide use),

irrational numbers, the Chinese Remainder Theorem, theorems about Pythagorean triplets, and the series 1, 1, 2, 3, 5, 8, 13, ... which is now linked with the name Fibonacci. In addition to his great historic importance and fame (he was a favorite of Emperor Frederick II) [2], Leonardo Fibonacci' is called "the greatest number theorist between Diophantus and Fermat" and "the most talented mathematician of the Middle Ages."

Leonardo is most famous for his book Liber Abaci, but his Liber Quadrat rum provides the best demonstration of his skill. He defined congruums and proved theorems about them, including a theorem establishing the conditions for three square numbers to be in consecutive arithmetic series; this has been called the finest work in number theory prior to Fermat (although a similar statement was made about one of Bhaskara's theorems). Although often overlooked, this work includes a proof of the n= 4 case of Fermat's Last Theorem. (Leonardo's proof of FLT4 is widely ignored or considered incomplete.)

Leonardo provided Europe with the decimal system, algebra and the 'lattice method of multiplication, all far superior to the methods then in use. He introduced notation like 3/5; his clever extension of this for quantities like 5 yards, 2 feet, and 3 inches is more efficient than today's notation. It seems hard to believe but before the decimal system, mathematicians had no notation for zero. Referring to this system, Gauss was later to exclaim "To what heights would

science now be raised if Archimedes had made that discovery!"

Some histories describe him as bringing Islamic mathematics to Europe, but in Fibonacci's own preface to Liber Abaci, he specifically credits the Hindus:

.as a consequence of marvelous instruction in the art, to the nine digits of the Hindus, the knowledge of the art very much appealed to me before all others, and for it I realized that all its aspects were studied in Egypt, Syria, Greece, Sicily, and Provence, with their varying methods;

... But all this even, and the algorism, as well as the art of Pythagoras, I considered as almost a mistake in respect to the method of the Hindus. Therefore, embracing more stringently that method of the Hindus, and taking stricter pains in its study.

3.2. some concept definition for Fibonacci number [7 10]

Definition: **3.2.1.** : The open interval contains neither of the endpoints. If a and b are real numbers,

then the open interval of numbers between a and b is written as (a,b) and

(a,b) ={ x ∈R; a<x< b}.

Definition: 3.2.2.: The closed interval contains both endpoints. If a and b are real numbers, then

the closed interval is written as [a,b] and

[a,b] ={ x ER ; a≤ x ≤b }.

Definition: 3.2.3. : all the numbers between two given numbers is called Interval.

Definition: 3.2.4.: Let I be an interval. A subinterval of I is a subset of I that is also an interval is S

called subinterval.

Definition: 3.2.5.: A ratio shows the relative sizes of two or more values.

Definition: 3.2.6.: A relation F between two non empty set A and B such that for all a∈A there exist only one element b∈ B such that b= f(a) denoted by F:A⊂ R→ B ⊂R.

Definition: 3.2.7.: The "range" is the set of all possible values of a function for the values of the

variable.

Definition: 3.2.8.: The function f(x) is unimodal on I = [a, b], if there exists a unique number

such that

f(x) is decreasing on [a, p]

And

f(x) is increasing on [p , b].

Definition: 3.2.9.: A function f (x) is "increasing" at a point x_0 if and only if there exists some

interval I containing x_0 such that f(x_0) $>$ f(x) for all x in I to the left of x_0 and f(x_0) $<$f(x) for

all X in I to the right of x_0.

Definition: 3.2.10.: A function f(x) is "decreasing" at a point x_0 if and only if there exists some

interval I containing Xo such that f(x_0) $<$f(x) for all x in I to the left of x_0 and f(x_0) $>$ f(x) for

all x in I to the right of x_0.

3.3. Fibonacci search [8 9 13]

Suppose that we want to locate the minimum as accurately as possible, i.e. with the shortest

possible interval of uncertainty, but can only afford n function evaluations. How should we choose the n values at which we evaluate the function? In the first place it would seem clear that we should not make the decision for all the points , we should let the function values we obtain

from the early experiments determine the position of subsequent point. In effect as we obtain function values, we obtain information about the function and the position of its minimum. We

use this information to guide us in our search.

Thus suppose, as in Fig.1, that we have an interval of uncertainty (x_1 , x_3) and have a function

value f(x_2) within this interval. If we could carry out just one further experiment at the point x_4,

where should we place x4 So as to obtain the smallest possible interval of uncertainty?

Suppose x_2-x_1=L and x_3-x_2= R with L> R (as in Fig. 1) and these will be fixed if x_1, x_2 and x3 are known. If xais placed in (x_1, x_2) then

(i) If f(x_4) < f(x_2) the new uncertainty interval will be (x_1, 2) of length

x_2-x_1 $=$ L

(ii) If f(x_4) > f(x_2) the new uncertainty interval will be

(x_a, x_3) of length

x_3-x_4 .

Since we do not know which of these outcomes will occur we choose x_4 So as to minimize the larger of x_3-x_4 and x_2-x_1. We achieve this by making x_3-x_4 and x_2-x_1 equal, i.e., by placing x_4

symmetrically in the interval with respect to x_2, the point already in the interval. Any other

position for x_4 could result in an interval longer than L. Placing x_4 in this position means that we are not gambling on getting a particular outcome.

If we then found that we were allowed one more evaluation we should apply the same strategy

to

(i) The interval (x_4, x_2) in which we already have the value at x_4

(ii) The interval (x_4, x_3) in which we already have the value at x_2

Thus the strategy is clear once we have started. We place the next point with in the interval being searched symmetrically with respect to the point already there. Paradoxically, to see how should start we have to consider how we will finish.

At the n^{th}evaluation we place the nth point symmetrically with respect to the (n - 1^{th}) point. The position of this latter point is in principle under our control. In order to get the greatest interval reduction at this stage it should bisect the penultimate interval. Then x_n would coincide

With x_{n-1} . We appear to have a problem here since no new information is being obtained. In practice $x_{n-1}$and xn are separated just sufficiently to enable us to decide which half, left or right,

is the final uncertainty interval. They are placed at a distance $\boldsymbol{\varepsilon}$ /2 either side of the middle of L_{n-1}; $\boldsymbol{\varepsilon}$ may be at our choice or it may be the minimum separation of two point that is possible . (Our plant engineer could only control temperature to the nearest degree perhaps, then $\boldsymbol{\varepsilon}$ = 1.)

The interval of uncertainty will be of length Ln and

L_{n-1}= = 2 $L_{n-\varepsilon}$ (See Fig. 4, bottom layer)

At the preceding stage x_{n-1} and x_{n-2}must be symmetrically placed with in L_{n-2} and distant L_{n-1}

from the ends of that interval. Thus

L_{n-2}= L_{n-1}+ L_n (See Fig. 4, middle layer)

[1] As drawn it is evidently x_{n-2}which remains as the included point at the penultimate stage.

Similarly at the previous stage

L_{n-3}= L_{n-2}+ L_{n-1} (See Fig. 4, top layer)

In general

L_{j-1}=L_j+ L_{j-1} for 1<j <n

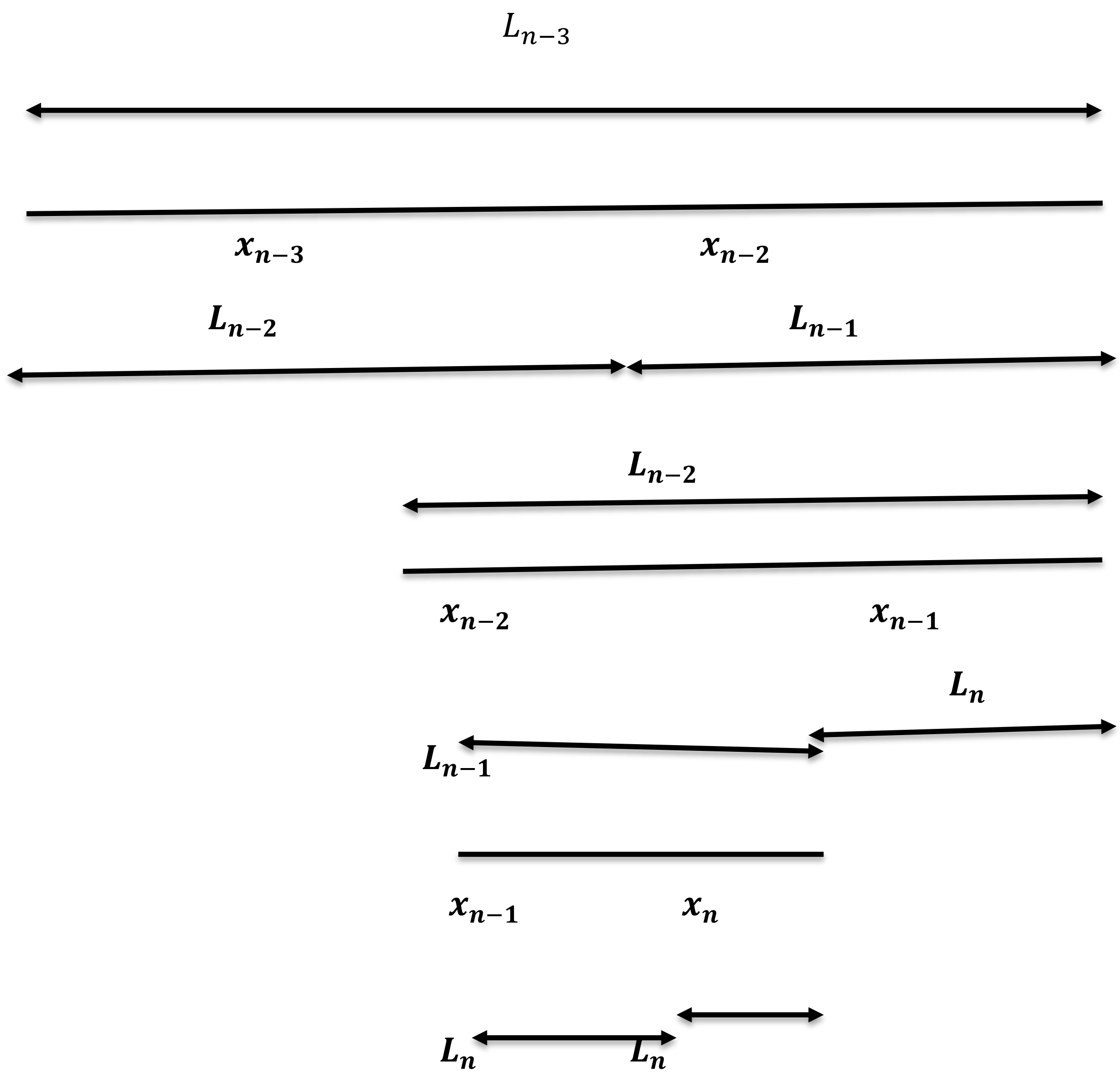


Figure 4: Fibonacci interval

Thus

$L_{n-1}=2L_n-\varepsilon$

$L_{n-2}=L_{n-1}+L_n=3L_n-\varepsilon$

$L_{n-3}=L_{n-2}+L_{n-1}=5L_n-2\varepsilon$

$L_{n-4}=L_{n-3}+L_{n-2}=8\,L_n-3\,\varepsilon$ *etc.*

If we define the Fibonacci sequence of number by $\boldsymbol{f_0}$=1 , $\boldsymbol{f_1}$=1 , $\boldsymbol{f_k}$=$\boldsymbol{f_{k-1}}$+ $\boldsymbol{f_{k-2}}$ for k= 2,3,… then

$\boldsymbol{L_{n-j}}$=$\boldsymbol{f_{j+1}}$ $\boldsymbol{L_n}$-$\boldsymbol{f_{j-1}\varepsilon}$ j=1 , 2 , … , n-1

If the original interval (a,b) is of length

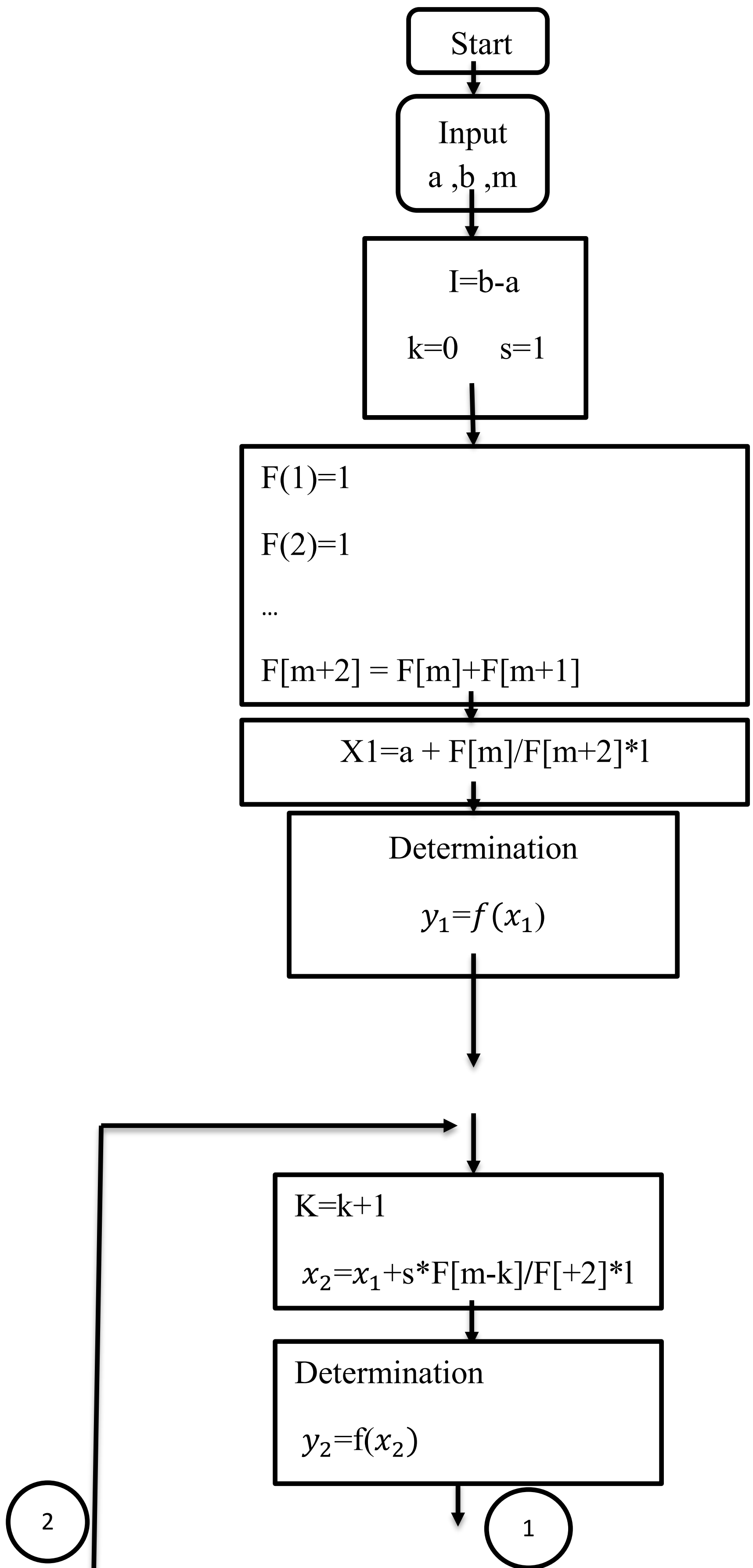
$\boldsymbol{L_1}$=b-a

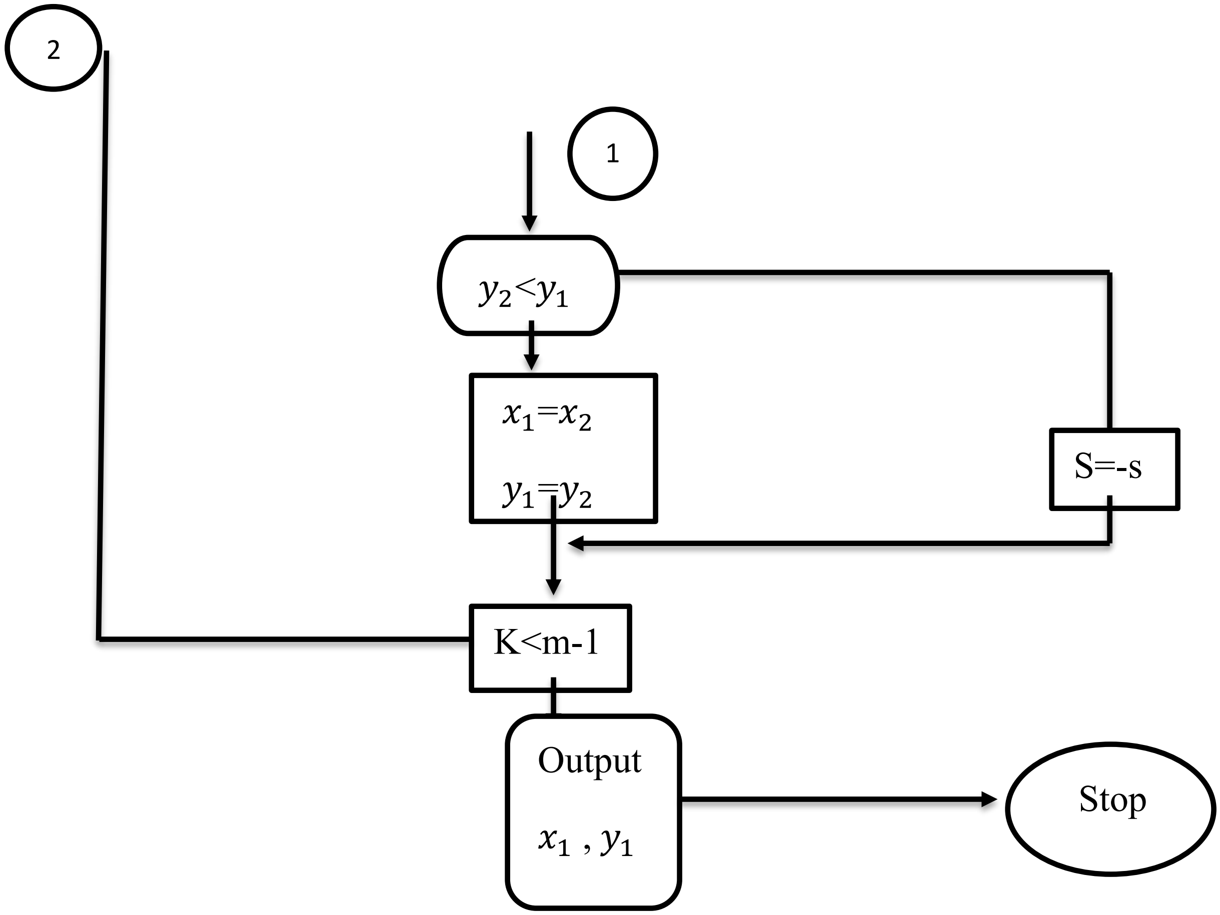
$\boldsymbol{L_1}$=$\boldsymbol{f_n L_n}$- $\boldsymbol{\varepsilon f_{n-2}}$

$\boldsymbol{L_n}$=$\boldsymbol{\frac{L_1}{f_n}} + \boldsymbol{\varepsilon}\ \boldsymbol{\frac{f_{n-2}}{f_n}}$

Thus with n function evaluation we reduce the original uncertainty interval to a function $\boldsymbol{\frac{1}{f_n}}$ of its value (neglecting $\boldsymbol{\varepsilon}$), and this is the best that can be done

3.4. Using Flowchart to solve Fibonacci Direction Search [11 15]





Flowchart 1 Fibonacci direction search

3.5. Using C++ Algorithm to Solve Direction Search (Fibonacci Search) [12 14]

```
#include <iostream.h>

#include<canio.h>

#include<math.h>

Float fa(float x);

main() {

Int const m=20;

Float I ,e ,q ,x[m],y[m],i ,c[m],d[m],a[m],b[m],f[m];

Int j ,k ,n;

cin>>a[0]>>b[0];

q=b[0];

i=b[0]-a[0];

f[0]=0;

f[1]=1;

cout<<"c       d        f(c)       f(d)"<<endl;

for (j=0; j<=m; j++)

f [j+2] = f [j] +f [j+1];

c[0]=a[0]+(1-(f[m-1]/f[m]))*i;

d[0]=a[0]+(f[m-1]/f[m])*I;

x[0]=fa(c[0]);

y[0]=fa(d[0]);

cout<<"     "<<d[0]<"     "<<x[0]<<"       "<<y[0]<<endl;

for (k=1; k<m-1; k++)
```

```
{ b[0]=q;
If(fa(c[k-1])>(fa(d[k-1])))
{
a[k]=c[k-1];
b[k]=b[k-1];
}
else
{ b[k]=d[k-1];
a[k]=a[k-1]; }
i=b[k]-a[k];
c[k]=a[k]+(1-(f[m-1-k]/f[m-k]))*i;
d[k]=a[k]+(f[m-1-k]/f[m-k])*i;
x[k]=fa(c[k]);
y[k]=fa(d[k]);
cout<<a[k]<<"    "<<b[k]<<"    "<<c[k]<<"    "<<d[k]<<"    "<<x[k]<<"
"<<y[k]<<endl;
}
getch( );
}
float fa(float x)
{
float y;
y=pow (x,2) – sin(x);
return (y); }
```

Example : find the minimum of the unimodal function f(x)=x^2-sin(x) on the interval [0,1] using the Fibonacci search method. Use 20 function evaluation . [12 14 16]

Solution

A	C	d	b	f(a)	f(c)	f(d)	f(b)
0.0000000	0.3819660	0.6180340	1.0000000	0.0000000	-0.2268475	-0.1974679	0.1585290
0.0000000	0.2360680	0.3819660	0.6180340	0.0000000	-0.1781534	-0.2268475	-0.1974679
0.2360680	0.3819660	0.4721359	0.6180340	-0.1781534	-0.2268475	-0.2318772	-0.1974679
0.3819660	0.4721359	0.5278641	0.6180340	-0.2268475	-0.2318772	-0.2250488	-0.1974679
0.3819660	0.4376941	0.4721359	0.5278641	-0.2268475	-0.2322759	-0.2318772	-0.2250488
0.3819660	0.4164078	0.4376941	0.4721359	-0.2268475	-0.2310824	-0.2322759	-0.2318772
0.4164078	0.4376941	0.4508496	0.4721359	-0.2310824	-0.2322759	-0.2324650	-0.2318772
0.4376941	0.4508496	0.4589804	0.4721359	-0.2322759	-0.2324650	-0.2323713	-0.2318772
0.4376941	0.4458250	0.4508496	0.4589804	-0.2322759	-0.2324425	-0.2324650	-0.2323713
0.4458250	0.4508496	0.4539558	0.4589804	-0.2324425	-0.2324650	-0.2324482	-0.2323713
0.4458250	0.4489311	0.4508496	0.4539558	-0.2324425	-0.2324637	-0.2324650	-0.2324482
0.4489311	0.4508496	0.4520373	0.4539558	-0.2324637	-0.2324650	-0.2324614	-0.2324482
0.4489311	0.4501188	0.4508496	0.4520373	-0.2324637	-0.2324656	-0.2324650	-0.2324614
0.4489311	0.4496620	0.4501188	0.4508496	-0.2324637	-0.2324652	-0.2324656	-0.2324650
0.4496620	0.4501188	0.4503928	0.4508496	-0.2324652	-0.2324656	-0.2324655	-0.2324650
0.4496620	0.4499360	0.4501188	0.4503928	-0.2324652	-0.2324655	-0.2324656	-0.2324655
0.4499360	0.4501188	0.4502101	0.4503928	-0.2324655	-0.2324656	-0.2324656	-0.2324655
0.4501188	0.4502101	0.4503015	0.4503928	-0.2324656	-0.2324656	-0.2324656	-0.2324655
0.4501188	0.4502101	0.4502101	0.4503015	-0.2324656	-0.2324656	-0.2324656	-0.2324656

Table 2 using the Fibonacci search method

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پوختە

لەم توێژینەوەیەدا باسی ڕێژەی زێڕین دەکەین بە گشتی ڕێژەی زێڕین چیە وە پێناسەی ڕێژەی زێڕین وە ڕێژەی زێڕین بە شێوەی هاوکێشە وە ژماردنی ڕێژەی زێڕین وە پاشان باس لە مێژوی فیبۆناچی دەکەین وە نمونەی فلۆچارتمان شیکار کردوە وە بە پڕۆگرامی ئەم ڕێگەیەمان داناوەو نمونەمان بە پڕۆگرام شیکار کردوە