زانكۆی سەلاحەدین-هەولێر

Salahaddin university – erbil

Application to ordinary differential equation

Research Project

Submitted to the department of (Mathematics) in partial fulfillment of the requirements for the degree of BSc. in (Mathematics)

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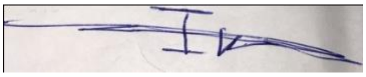
Certain of the supervisors

I certify that this work was prepared under my supervision at that department of mathematics /college of education / salahaddin university – erbil in partial fulfillment of requirements for the degree of bachelor of philosophy of science in mathematics.

Signature : 

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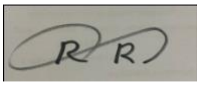
Signature : 

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Abstract

An integral equation is a type of mathematical equation where the unknown function appears under an integral sign. Integral equations arise in various fields of mathematics, physics, engineering and other sciences

In this report we studied the integral equations. Also we classified the integral equation such as volterra fredholm integral equations, linear and non-linear integral equations homogeneous. Secondly we converted differential equation into integral equation. Finally some examples were given to illustrate these methods.

CONTENTS

CHAPTER ONE

INTRODUCTION

Integral equations are equations in which the unknown function appears inside a definite integral. They are closely related to differential equations. Initial value problems and boundary value problems for ordinary and partial differential equations can often be written as integral equation.

And some integral equations can be written as initial or boundary value problems for differential equations. Problems that can be cast in both forms are generally more familiar as differential equations.

Many applications are best modeled with integral equations, but most of these problems require a lengthy derivation. A relatively simple example is the model for population dynamics, with birth and death rates that depend on age. This model was first formulated by A. j. lotka 1922, who is best known for lotka-volterra predator-prey population model. Krivan.v 2007

Integral equations are also important in the theory and numerical analysis of differential equations, this is where the mathematics student is most likely to en counter them. For example, picard's existence and uniqueness theorem for first-order initial value problems (murray F.J and miller K.S 2013) is conveniently proved using integral equations, the proof is constructive and can be used to formulate a method for numerical solution of initial value problems.

Systematic study of integral equations is usually undertaken as part of a course in functional analysis or applied mathematics. This advanced setting is required for a full appreciation of integral equation theory but it makes the subject accessible. By constrast, several of the important results in the theory of integral equations can be demonstrated using nothing more than elementary analysis, and the elaboration of this statement is goal of the present discussion. In fact, all but one of the results presented here will be derived using nothing more than the material presented in a standard advanced calculus course.

Concepts that play an important role in the theory and application of continuous mathematics.

CHAPTER TWO

BACKGROUND

Definition (Integral equation)2.1 Colton, D. and Kress, R., 2013

An equation is called an integral equation in which an unknown function is to be determined appears under one or more integral signs. Naturally in such an equation these occur other terms as well.

For example for $a \leq x \leq b$, $a \leq t \leq b$, the equations

$$g(x) = \int_a^b K(x,t)g(t)dt \qquad (2.1.1)$$

$$g(x) = \int_a^x K(x,t)g(t)dt \qquad (2.1.2)$$

$$g(x) = f(x) + \lambda \int_a^b K(x,t)g(t)dt \qquad (2.1.3)$$

$$g(x) = f(x) + \lambda \int_a^x K(x,t)g(t)dt \qquad (2.1.4)$$

$$g(x) = \lambda \int_a^x K(x,t)[g(t)]^2 dt \qquad (2.1.5)$$

Where the function $g(x)$ is the unknown function while all the other function are known and λ , a and b are constants, are integral equation equation. These equation may be complex-valued functions of the of the real variables x and t.

Definition(Linear and non-linear integral equations) 2.2

An integral equation is equation is said to be a linear if only linear operations are performed in it upon the unknown functions. An integral equation which is not linear is known as non-linear integral equation.

For example, the integral equations(2.1.1)to (2.1.4) are linar while (2.1.3)is not linear linear the most general type of linear integral equation is of the form

$$\alpha(x)\, g(x) = f(x) + \lambda \int_{\Omega} K(x,t)[g(t)]dt \qquad (2.2.1)$$

Where the upper limit may be either variable $x$or fixed. The functions f, α and K are known function, while g is to be determined ; λ is a non-zero real or complex parameter. The function $K(x,t)$ is known as the kernel of the integral equation .the integration extends over the domain Ω of the auxiliary variable t.

The integral equations, which are linear, involved the linear operator

L{ }=$\int_{\Omega} K(x,t)$ { } dt

Having the kernel $K(x,t)$. It satisfies the linearty condition

L {c1g1(t)+c2g2(t)}=c1L{g1(t)}+c2L{g2(t)}

L{g(t)}= $\int_{\Omega} K(x,t)$g(t) dt and c1,c2 are constants.

Linear integral equations are classified into two basic types.

Definition (Voltera Integral Equation) 2.3 Miller,R.K,1975

An integral equation is called a volterra integral equation if the upper limit of integration a variable *e.g.*

$$\alpha(x)\, g(x) = f(x) + \lambda \int_{a}^{x} K(x,t) g(t) dt \qquad (2.3.1)$$

Here the constant, $f(x), \alpha(x)$ and $K(x,t)$ are known function while g(x) is unknown function, non-zero real or complex parameter. Equation (2.3.1) is a volterra integral equation of ird kind.

(a) When $\alpha = 0$, the unknown function g appears only under integral sign and nowhere else in the equation (2.3.1), we get

$$g(x) = f(x) + \lambda \int_{a}^{x} K(x,t) g(t) dt = 0 \qquad (2.3.2\ )$$

(2.3.2) is called the volterra integral equation of first kind.

(b) When $\alpha = 1$, the equation (2.3.1) involves the unknown function g , both inside as well as outside the integral sign, then

$$g(x) = f(x) + \lambda \int_a^x K(x,t)g(t)dt$$

(2.3.3)

Is called the volterra's integral equation of second kind.

(c) When $\alpha = 1, f(x)$=0, the equation (2.3.1)reduces to

$$g(x) = \lambda \int_a^x K(x,t)g(t)dt \qquad (2.3.4)$$

Is called the homogeneous voltera's integral equation of second kind.

Definition (fredholm integral equation) 2.4 Weisstein, E.W., 2002

An integral equation is said to be a fredholm integral equation if the upper limit of integration is fixed , say b, e.g.,

$$\alpha(x)\, g(x) = f(x) + \lambda \int_a^b K(x,t)g(t)dt \qquad (2.4.1)$$

Where a and b are both constants, $\alpha(x), g(x)$ and $K(x,t)$ are known functions while g(x) is unknown function and λ is non-zero real or complex parameter. Equation(2.4.1)is known as fredholm integral equation of third kind.

(a) When $\alpha = 0$,equation(2.4.1)contains unknown function g only under the integral sign, then

$$f(x) = \lambda \int_a^b K(x,t)g(t)dt = 0 \qquad (2.4.2)$$

Is called fredhoolm integral equation of first kind.

(b) When $\alpha = 1$, equation (2.4.1)involves the unknown function g both inside as well as outside the integral sign, then

$$g(x) = f(x) + \Lambda \int_a^b K(x,t)g(t)dt$$

Is known as fredholm integral equation of second kind.

(c) When $\alpha = 1$,f(x)=0 equation (2.4.1) reduces to

$$g(x) = \Lambda \int_a^b K(x,t)g(t)dt$$

Is known as the homogenous fredholm integral equation of second kind.

Example 2.5: Show that the function g(x)=1 is asolution of the fredholm integral equation $g(x) + \int_0^1 x(e^{xt} - 1)g(t)dt = e^x - x$

Solution. Substuting the function g(x)=1 in the L.H.S. of the given equation, we have

L.H.S.=$1 + \int_0^1 x(e^{xt} - 1)g(t)dt$ =$1 + x\left[\frac{e^{xt}}{x} - t\right]_0^1$

$= 1 + e^x - x - 1 = e^x - x$= R.H.S.

Hence g(x)=1 is a solution of the given integral equation

Example 2.6 : define integral equations of fredholm and volterra types. That the Show function g(x)=xe^x Is a solution of the volterra integral equation

g(x)=$\sin x + 2\int_0^x \cos(x-t)$ g(t) dt

Solution: substituting function g(x)=xe^xin the R.S.H. of the given integral equation , we have

R.H.S. =$\sin x$+2$\int_0^x \cos(x-t)$te^tdt

=$\sin x$+2$\left[\cos x \int_0^x te^t \cos t \text{ dt} + \sin x \int_0^x te^t \sin t \text{ dt}\right]$

= $sinx$ +2$cosx\left[\left\{t\frac{e^t}{2}(cost + sint)\right\}_0^x - \frac{1}{2}\int_0^x e^t(\,cost + sint\,)dt\right] +$

$2\, sinx \left[\left\{ t\frac{e^t}{2}(sint - cost) \right\}_0^x - \frac{1}{2}\int_0^x e^t (sint - cost) dt \right]$

$$\left\langle \begin{array}{l} since \int e^{ax} \cos bx \, dx = \dfrac{e^{ax}}{a^2 + b^2} [a \cos bx + b \sin bx] \\ \int e^{ax} \sin bx \, dx = \dfrac{e^{ax}}{a^2 + b^2} [a \sin \mathrm{bx} - b \cos bx \,] \end{array} \right.$$

$=\sin x + 2 \Big[\cos x \left\{\frac{1}{2}\, xe^x (\cos x + \sin x)\right\} + \sin x \left\{\frac{1}{2} xe^x (\sin x -\right.$

$\left.\cos x)\right\} - \frac{1}{2}\cos \mathrm{x} - \left\{\frac{1}{2} e^t (\cos t + \sin t + \sin t - \cos t\right\}_0^x -$

$\frac{1}{2}\sin x \left\{\frac{1}{2} e^t (\sin t - \cos t - \cos t - \sin t\right\}_0^x \Big]$

$=\sin x + xe^x\, (cos^2 x + sin^2 x) - \cos x\, (e^x\, \sin x) + \sin x (e^x \cos x -$
$1) = xe^x$

=L.H.S

Hence g(x)=xe^x is a solution of the given integral equation

Example verify whether the function $g(x) = 1 - \frac{2 \sin x}{[1-(\pi/2)]}$ is the solution of the integral equation

$$g(x) - \int_0^\pi \cos(x + t) gt(t) d = 1$$

Solution. We have

$\int_0^\pi \cos(x + t) g(t) dt = \int_0^\pi \cos(x + t) \left[1 - \frac{2 \sin x}{[1-(\pi/2)]}\right] dt$

$=\int_0^\pi \cos(x + t) - \frac{2}{1-(\frac{\pi}{2})}\int_0^\pi \cos(x + t) \sin t \, dt$

$=- \sin x - \sin x - \frac{2}{\left[1-\frac{\pi}{2}\right]}\int_0^\pi \cos x \cos t \sin t \; - \sin x \sin t \sin t \; dt$

$= - \sin x - \sin x - \frac{2}{\left[1-\frac{\pi}{2}\right]}\int_0^\pi \cos x * \frac{\sin 2t}{2} - \left[\sin t \;\; \left(\frac{\cos(t-x)}{2}\right) + \right.$

$\left.\frac{\cos\,(x+t)}{2}\right] dt$

$= -2 \sin x + \frac{2}{\left[1-\frac{\pi}{2}\right]}\int_0^\pi \left[\frac{1}{2}\left(\frac{\sin(x+2t)}{2} - \frac{\sin(x-2t)}{2}\right) - \frac{1}{2}\left(\frac{\sin x}{2} - \frac{\sin(x-2t)}{2} -\right.\right.$

$\frac{\sin(x+2t)}{2} + \frac{\sin x}{2}\Big)\Big] dt$

$= -2\sin x + \frac{2}{\left[1-\frac{\pi}{2}\right]}\int_0^{\pi}\Big[\frac{\sin(x+2t)}{2} - \frac{\sin(x-2t)}{2} - \frac{2\sin x}{2} + \frac{\sin(x-2t)}{2} +$
$\frac{\sin(x+2t)}{2}\Big] dt$

$= -2\sin x + \frac{2}{\left[1-\frac{\pi}{2}\right]}\int_0^{\pi}[\sin(x+2t) - \sin x\,]dt$

$-2\sin x + \frac{2}{\left[1-\frac{\pi}{2}\right]}\Big[-\frac{\cos(x+2t)}{2} - t\,sin\,x\Big]_0^{\pi}$

$=-2\sin x + \frac{1}{1-(\frac{\pi}{2})}\Big[\frac{\cos x}{2} - \frac{\cos x}{2} + \pi\sin x\Big]$

$= -2\sin x + \frac{1}{1-(\frac{\pi}{2})}\sin x$

$= \frac{4(\pi-1)}{2-\pi}\sin x$

Therefore $1+\int_0^{\pi}\cos(x+t)\,g(t)dt \neq g(x)$

Thus g(x) is not a solution of given integral equation.

Example .verify that the given function $g(x) = \frac{x}{(1+x^2)^{5/2}}$ is a solution of the integral equation

$$g(x) = \frac{3x+2x^3}{3(1+x^2)^2} - \int_0^x \frac{3x+2x^3-t}{(1+x^2)^2}g(t)dt$$

Solution. Substuting the $g(x) = \frac{x}{(1+x^2)^{5/2}}$ in the R.H.S. of given equation, we have

R.H.S.

$=\frac{3x+2x^3}{3(1+x^2)^2} - \int_0^x \frac{3x+2x^3-t}{(1+x^2)^2}.\frac{t}{(1+t^2)^{5/2}}\,dt$

$=\frac{3x+2x^3}{3(1+x^2)^2} - \int_0^x \frac{3x+2x^3}{(1+x^2)^2}.\frac{t}{(1+t^2)^{5/2}}\,dt + \int_0^x \frac{1}{(1+x^2)^2}\cdot\frac{t^2}{(1+t^2)^{5/2}}dt$

$$=\frac{3x+2x^3}{3(1+x^2)^2}+\frac{3x+2x^3}{3(1+x^2)^2}\cdot\left[\frac{1}{(1+t^2)^{3/2}}\right]_0^x+\frac{1}{3(1+x^2)^2}\cdot\left[\frac{t^3}{(1+t^2)^{3/2}}\right]_0^x$$

$$\left[\!\left[since\ \int_0^x \frac{t^2dt}{(1+t^2)^{5/2}}=\frac{tan^2\theta\ sec^2\theta}{sec^5\theta}d\theta, t=tan\,\theta, dt=sec^2\theta\,d\theta=\right.\right.$$
$$\left.\left.\int sin^2\theta\, cos\,\theta d\theta = sin^3\theta = \frac{t^3}{(1+t^2)^{3/2}}\right]\!\right]$$

$$=\frac{3x+2x^3}{3(1+x^2)^2}+\frac{3x+2x^3}{3(1+x^2)^2}\left[\frac{x^3}{(1+x^2)^{3/2}}-1\right]+\frac{1}{3(1+x^2)^2}\cdot\frac{x^3}{(1+x^2)^{3/2}}$$

$$=\frac{1}{3(1+x^2)^{3/2}}\left[\frac{3x+2x^3}{(1+x^2)^2}+\frac{x^3}{(1+x^2)^2}\right]=\frac{x}{(1+x^2)^{5/2}}=\text{L.H.S.}$$

Thus $g(x)=\frac{x}{(1+x^2)^{5/2}}$ is a solution of the given integral equation.

CHAPTER TWO

APPLICATION TO ORDINARY DIFFERENTIAL EQUATION

Definition(Initial value problem)3.1 (nedialkov,N.S., Jackson, K.R and Corliss, G.F., 1999)

When an ordinary differential equation is to be solved under conditions involving dependent variable and its derivative at same value of independent variable , then the problem under consideration is said to be an initial value problem.

Relationship between a linear differential equation and a volterra integral equation .method of converting an initial value problem to a volterra integral equation 3.2

Consider the differential equation of order n as

$$\frac{d^n y}{dx^n} + a_1(x)\frac{d^{n-1}y}{dx^{n-1}} + a_2(x)\frac{d^{n-2}y}{dx^{n-2}} + \dots\dots\dots. + a_n(x)y = f(x) \quad (3.2.1)$$

With the initial conditions

$$y(a) = c_0, y'(a) = c_1, \dots\dots, y^{n-1}(a) = c_{n-1} \quad (3.2.2)$$

Where the functions $a_n(x)$, ……, $a_n(x)$ and $f(x)$ are defined and continuous in $a < x < b$. in order to reduce the initial value problem(3.2.1) – (3.2.2) to the voltera integral equation. We introduce an unknown function $g(x)$. thus we take

$$\frac{d^n y}{dx^n} = g(x) \quad (3.2.3)$$

Integrating both sides of equation (3.2.3) with respect to 'x' from a to x using the initial condition (3.2.2) , we find that

$$\frac{d^{n-1}y}{dx^{n-1}} = \int_a^x g(t)dt + c_{n-1} \qquad (3.2.4)$$

Integrating again equation (3.2.4) with respect to 'x' from a to x and using (3.2.2) we have

$$\frac{d^{n-2}y}{dx^{n-2}} = \int_a^x g(t)dt^2 + (x-a)c_{n-1} + c_{n-2} \qquad (3.2.5)$$

or

$$\frac{d^{n-2}y}{dx^{n-2}} = \int_a^x (x-t)g(t)dt + (x-a)c_{n-1} + c_{n-2} \qquad (3.2.6)$$

Integrating equation (3.2.5) again with to "x" from a to x and using (3.2.2) we have

$$\frac{d^{n-3}y}{dx^{n-3}} = \int_a^x g(t)dt^3 + \frac{(x-a)^2}{2i}c_{n-1} + \frac{(x-a)^1}{1i}c_{n-2} + c_{n-3}$$

or

$$\frac{d^{n-3}y}{dx^{n-3}} = \int_a^x \frac{(x-t)^2}{2^2 i} g(t)dt + \frac{(x-a)^2}{2i}c_{n-1} + \frac{(x-a)^1}{1i}c_{n-2} + c_{n-3} \qquad (3.2.7)$$

And so on finally we arrive at

$$\frac{dy}{dx} = \frac{(x-t)^{n-2}}{(n-2)i} g(t)dt + \frac{(x-a)^{n-2}}{(n-2)i}c_{n-1} + \frac{(x-a)^{n-3}}{(n-3)i}c_{n-2} + \cdots + (x-a)c_2 + c_1 \qquad (3.2.8)$$

$$y = \frac{(x-t)^{n-1}}{(n-1)i} g(t)dt + \frac{(x-a)^{n-1}}{(n-1)i}c_{n-1} + \frac{(x-a)^{n-2}}{(n-2)i}c_{n-2} + \cdots + (x-a)c_1 + c_0 \qquad (3.2.9)$$

Multiplying(3.2.4),(3.2.5),……,(3.2.8)and(3.2.9)by
1,9 $(x), \dots, a_{n-1}(x)$ and$a^n(x)$ respectively and adding , we get

$$\frac{d^n y}{dx^n} + a_1(x)\frac{d^{n-1}y}{dx^{n-1}} + \cdots \dots \dots + a_n(x)y = g(x) + \Big[c_{n-1}\, a_1(x) + \{(x-a)c_{n-1} + c_{n-1}\}a_2(x) + \Big\{\frac{(x-a)^{n-1}}{(n-1)i}c_{n-1} + \cdots + (x-a)c_1 + c_0\Big\}a_n(x) + \int_a^x a_1(x) + (x-t)a_2(x) + \frac{(x-t)^2}{2i}a_3(x) + \cdots +$$

$\frac{(x-t)^{n-1}}{(n-1)i} a_n(x)\Big] g(t)dt$

$$f(x) = g(x) + h(x) - \int_a^x K(x,t)g(t)dt \qquad (3.2.10)$$

Where we have used (3.1) and assumed the following :

$$h(x) = c_{n-1}\, a_1(x) + \cdots + c_{n-1}\, \frac{(x-a)^{n-1}}{(n-1)i} a_n(x) \qquad (3.2.11)$$

and

$$K(x,t) = -\left[a_1(x) + (x-t)a_2(x) + \cdots + \frac{(x-t)^{n-1}}{(n-1)i} a_n(x)\right]$$

$$= \sum_{k=1}^{n} a_k(x) \frac{(x-t)^{k-1}}{(k-1)i} \qquad (3.2.12)$$

Again let $f(x) - h(x) = \emptyset(x)$ (3.2.13)

Using (3.1.13) , (3.1.10) reduced to

$$g(x) = \emptyset(x) + \int_a^x K(x,t)g(t)dt \qquad (3.2.14)$$

Which required volterra integral equation of second kind .

Particular case:

Consider the linear differential equation of second kind

$$\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)y = f(x) \qquad (3.2.15)$$

With initial conditions $y(0) = c_0$ and $y'(0) = c_1$ (3.2.16)

Taking $\frac{d^2y}{dx^2} = g(x)$ (3.2.17)

Integrating both sides of equation (3.2.17)with respect to 'x' from 0 to x and using initial condition (3.2.16) we find that

$$\frac{dy}{dx} = \int_0^x g(t)\, dt + c_1 \qquad (3.2.18)$$

And

$$y = \int_0^x (x-t)\, g(t)\, dt + c_1(x) + c_0 \tag{3.2.19}$$

Using equation (3.2.17) , (3.2.18) and (3.2.19) in (3.2.15) , we have

$g(x) + a_1(x)\left[\int_0^x g(t)\, dt + c_1\right] + a_2(x)\left[\int_0^x (x-t)\, g(t)\, dt + c_1(x) + c_0\right] = f(x)$

Or

$g(x) + \int_0^x [a_1(x) + a_2(x) + (x-t)]g(t)dt = f(x) - c_1a_1(x) - c_1xa_2(x) - c_0a_2(x)$

Or

$$g(x) = \emptyset(x) + \int_0^x K(x,t)g(t)dt$$

Where $K(x,t) = -[a_1(x) + a_2(x) + (x-t)]$

$\emptyset(x) = f(x) - c_1a_1(x) - c_1xa_2(x) - c_0a_2(x)$

Whichb representsthe volterra's integral equation of the second kind.

Illustrative examples

Example: 3.3 from an integral equation corresponding to the differential equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ with the initial condition $y(0) = 1, y'(0) = 0$

Solution. Consider $\frac{d^2y}{dx^2} = g(x)$ (3.3.1)

Integrating both side with respect to 'x' from 0 to x and using y'(0)=0 , we have $\frac{dy}{dx} = \int_0^x g(t)dt$ (3.3.2)

Integrating both sides with respect to 'x' from 0 to x using (3.3.1) and y(0)=1 we get

$$y = 1 + \int_0^x (x-t)g(t)dt \tag{3.3.3}$$

From the relations (3.3.1)-(3.3.3), the given defferential equation reduces to

$g(x) + x\int_0^x g(t)dt + 1 + \int_0^x (x-t)g(t)dt = 0$

Or $g(x) + 1 + \int_0^x (x + x - t)g(t)dt$

Or $g(x) = -1 - \int_0^x (2x-t)g(t)dt = 0$

Which is the required voltera integral equation of second kind.

Example 3.4 : from an integral equation corresponding to differential equation $\frac{d^2y}{dx^2} + y = \cos x\,; y(0) = 0\,, y'(0) = 1$.

Solution: consider $\frac{d^2y}{dx^2} = g(x)$ (3.4.1)

Integrating (3.4.1) with respect to 'x' from 0 to x and using initial conditions $y(0) = 0\,, y'(0) = 1$, we get

$\frac{dy}{dx} = 1 + \int_0^x g(t)dt$ and $y = x + \int_0^x (x-t)g(t)\,dt$

Substuting the values $\frac{d^2y}{dx^2}$ and y in the given integral equation , we get

$g(x) + x + \int_0^x (x-t)g(t)\,dt = \cos x$

$g(x) = -x + \cos x - \int_0^x (x-t)g(t)\,dt$

Which represents a volterra's integral equation of second kind.

Example 3.5 : convert the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 4\sin x$ with initial conditions $y(0) = 1, y'(0) = -2$ into volterra's integral equation of second kind.

Solution : consider $\frac{d^2y}{dx^2} = g(x)$ (3.5.1)

Integrating with respect to 'x' from 0 to x , and using the initial conditions given with the problem , we find that that

$\frac{dy}{dx} = -2 + \int_0^x g(t)\,dt$

Or

$y = 1 - 2x + \int_0^x (x-t)\,g(t)\,dt$

Substuting, the values of $\frac{d^2y}{dx^2}$, $\frac{dy}{dx}$ and y in the given differential equation, we find that

$$g(x) - 3\left[-2 + \int_0^x g(t)dt\right] + 2\left[1 - 2x + \int_0^x (x-t)g(t)dt\right] = 4\sin x$$

or

$$g(x) = 4\sin x + 4x - 8 + \int_0^x (3 - 2x + 2t)g(t)dt$$

Which represents the volterra's integral equation of second kind.

Example 3.6: from an integral equation corresponding to the differential equation $\frac{d^2y}{dx^2} - \sin x\ \frac{dy}{dx} + e^x y = x$, with initial condition $y(0) = 1, y'(0) = -1$

Solution : suppose that $\frac{d^2y}{dx^2} = g(x)$ (3.6.1)

Integrating (3.6.1) with respect to 'x' , from 0 to x , and using $y'(0) = -1$,

We find that $\frac{dy}{dx} = -1 + \int_0^x g(t)\ dt$ (3.6.2)

Integrating again (2.6.2) with to 'x' from 0 to x and $y(0) = 1$

We have

$$y = 1 - x + \int_0^x (x-t)g(t)\ dt \qquad (2.6.3)$$

Substuting, the values of $\frac{d^2y}{dx^2}$, $\frac{dy}{dx}$ and y in the given differential equation, we have

$$g(x) - \sin x\left[-1 + \int_0^x g(t)dt\right] + e^x\left[1 - x + \int_0^x (x-t)\ g(t)dt\right] = x$$

Or$g(x) = x - \sin x - e^x(1 - x) + \int_0^x \sin x\ \ g(t)\ dt -$

$\int_0^x e^x (x-t) g(t) dt$

Or

$g(x) = x - \sin x - e^x (1-x) + \int_0^x \{\sin x - e^x (x-t)\}\ g(t) dt$

Which is required volterra integral equation of second kind.

Example 3.7: reduce the initial value problem to volterra integral equation of second kind. $\frac{d^3y}{dx^3} - 2xy = 0, y(0) = \frac{1}{2}\ , y'(0) = y''(0) = 1.$

Solution. Consider $\frac{d^3y}{dx^3} = g(x)$ (3.7.1)

Integrating with respect to 'x' from 0 to x, and using the initial conditions given with the problem , we have

$\frac{d^2y}{dx^3} = \int_0^x g(t) dt + 1$

$\frac{dy}{dx} = \int_0^x (x-t) g(t) dt + x + 1$

$y = \int_0^x \frac{(x-t)^2}{2!}\ g(t) dt + \frac{x^2}{2} + x + \frac{1}{2}$

Substuting the values of $\frac{d^2y}{dx^3}$ and y in the given differential equation , we find that

$g(x) - 2x \left[\int_0^x \frac{(x-t)^2}{2!} g(t) dt + \frac{x^2}{2} + x + \frac{1}{2}\right] = 0$

Or $g(x) = x(x+1)^2 + \int_0^x \frac{(x-t)^2}{2!}\ g(t) dt$

Which represents the volterra's integral equation of second kind .

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ثوختة

هاوكێشه یەكگرتووەكان جۆرێكه له هاوكێشەی بیركاری كه فەنكشنی نەناسراو له ژێر نیشانەیەكی یەكگرتوودا دەردەكەوێت. هاوكێشه یەكگرتووەكان له بواره جیاوازەكانی بیركاری و فیزیا و ئەندازیاری و زانستەكانی تردا سەرهەڵدەدەن

لەم ڕاپۆرتەدا ئێمه لێكۆڵینەوەمان له هاوكێشه یەكگرتووەكان كرد. هەروەها ئێمه هاوكێشه یەكگرتووەكانمان پۆلێن كرد وەک هاوكێشه یەكگرتووەكانی ڤۆلتێرا فرێدۆڵم، هاوكێشه یەكگرتووەكانی هێڵی و ناهێڵی یەكسان. دووەم: هاوكێشه جیاوازەكانمان گۆڕی بۆ هاوكێشه یه ككرتووه كان. له كۆتاییدا هەندێک نموونه بۆ وەسفكردنی ئەم شێوازانه خرانەڕوو.

خلاصة

المعادلة المتكاملة هي نوع من المعادلات الرياضية حيث تظهر الدالة غير المعروفة تحت

علامة متكاملة. تنشأ المعادلات التكاملية في مختلف مجالات الرياضيات والفيزياء والهندسة والعلوم الأخرى

في هذا التقرير درسنا المعادلات التكاملية. كما قمنا بتصنيف المعادلة التكاملية مثل معادلات فولتيرا فريدهولم المتكاملة والمعادلات التكاملية الخطية وغير الخطية المتجانسة. ثانيًا قمنا بتحويل المعادلة التفاضلية إلى معادلة تفاضلية. أخيرًا تم إعطاء بعض الأمثلة لتوضيح هذه الأساليب.