زانكۆی سەلاحەدین - هەولێر
Salahaddin University-Erbil

Optimization in Graph Theory

Research Project

Submitted to the Department of Mathematics in partial fulfillment of the requirements for the degree of BSc. in MATHEMATIC

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Certification of the Supervisor

I certify that this research was prepared under my supervision at the Department of Mathematics / College of Education / Salahaddin University-Erbil in partial fulfillment of the requirements for the degree of Bachelor of philosophy of science

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Date: 6/ 4 / 2023

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Abstract

Traffic congestion has become a significant problem in many cities worldwide, including Erbil. The city's rapid population growth, coupled with an increase in the number of vehicles, has led to congestion on the roads, causing delays, increased travel times, and environmental pollution. This study proposes a solution to the traffic congestion in Erbil by using the Chinese Postman Problem (CPP) and suggesting the use of trains.

The CPP is a mathematical problem that seeks to find the shortest possible route for a person who must traverse each street at least once before returning to the starting point. This problem can be applied to traffic management by optimizing the traffic flow in a city's road network, reducing congestion and travel time. By using the CPP in Erbil, we can identify the most efficient routes for vehicles to take, reducing traffic congestion.

Moreover, introducing a train system in Erbil can further alleviate traffic congestion. Trains are faster, safer, and more efficient than cars, and they can carry a large number of passengers. By offering an alternative mode of transportation, we can reduce the number of cars on the road, and consequently, the traffic congestion.

In conclusion, traffic congestion in Erbil is a serious problem that requires a multifaceted solution. By utilizing the CPP and introducing a train system, we can reduce traffic congestion, improve travel time, and promote a greener and more sustainable city.

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CHAPTER ONE

INTRODUCTION

1.1 History of graph theory

The study of graphs, also known as graph theory, is an important part of many disciplines, including mathematics, engineering, physical, social, biological, and computer science, linguistics, and many others. The history of graph theory can be traced back to 1735, when Leonhard Euler, a Swiss mathematician, solved the Königsberg bridge problem.

A graph is a collection of points known as nodes or vertices that are linked together by a network of lines known as edges. Because graph theory is considered a branch of applied mathematics, it is no surprise that theory has been independently discovered numerous times. (agarwal, et al., 2019)

Who invented graph theory:

In 1736, Euler (1707-1782) published a paper in which he solved Königsberg bridge problem, which gave birth of graph theory. Because graph theory is thought to have begun in 1736 with the publication of Euler's solution to the Königsberg bridge problem, Euler became known as the "Father of Graph Theory." (agarwal, et al., 2019)

Nothing more was done in this field for the next 100 years.

Then, in 1847, G. R. Kirchhoff (1824-1887) developed the tree theory in electrical networks for their applications. Kirchhoff's research into electric networks led to the development of the fundamental concepts and theorems relating to trees in graphs.

10 years later, a. Cayley (1821-1895) thought about trees that arose from the counting of organic chemical isomers and discovered trees while trying to count the isomers of saturated hydrocarbons CnH2n+2.

Two other milestone in graph theory were established around the time of Kirchhoff and Cayley.

1. One milestone is the four-color conjecture, and
2. The other milestone is puzzle invented by Hamilton. (agarwal, et al., 2019)

Four-color conjecture:

According to the four-color conjecture, four-color are sufficient for coloring any atlas (a map on a plane) so that countries with common borders have different colors.

In 1840, the four-color problem is considered to have been first presented by A. F. Möbius (1790-1868) in his lectures.

About 10 years later, A. De Morgen (1806-1871) discussed this problem in London.

In 1879, Cayley published this problem in the "first volume of the proceedings of the Royal Geographic Society", and it quickly became well known. Following this, the famous four-color conjecture gained fame and has remained popular ever since. (agarwal, et al., 2019)

Puzzle invented by Hamilton:

In 1859, Sir E. R. Hamilton (1805-1865) invented a puzzle approach to graph and sold it to a game manufacturer in Dublin for guineas.

The puzzle made up of a wooden regular dodecahedron (a polyhedron, with 12 faces all of which are regular pentagons, and 20 corners and 30 edges. 3 edges meet at each corner). The name of 20 important cities were marked on the corners.

The goal of the puzzle was to find a path along the dodecahedron's edges that passed through each of the 20 cities exactly once.

However, the solution to this particular problem is simple, no one could yet find a necessary and sufficient condition for the existence of such a path (known as a Hamiltonian circuit) in any graph. (agarwal, et al., 2019)

After this fruitful period, there was a half-century of relative inactivity.

Then, in 1920s, a resurgence of interest in graphs began. D. König was one of the pioneers during this time period. He organized the work of other mathematicians as well as his own and published the first book on the subject in 1936.

The last 30 years have seen intense activity in graph theory – both pure and applied. A significant amount of research work has been done and continues to be done in this area. Thousands of papers and more than dozen books have been published in the last decade. (agarwal, et al., 2019)

1.2 Use of the graph theory

1.2.1 Transportation Networks:

In mathematics, the term "graph" can have two different meaning, depending on the context. The first meaning is related to the graphical representation of mathematical function or a relation, which shows the relationship between the input and output values of the function or the relation.

However, the second meaning of the term "graph" is the one most commonly used in graph theory, which is a mathematical structure consisting of a set of vertices(also called nodes or points) and a set of edges (also called links or arcs) connecting them. This type of graph is used in a wide range of applications including transportation networks, social networks, computer networks, and many others.

In transportation networks, graph theory is a powerful tool for solving problems such as finding the shortest route between two points, determining the optimal location of facilities such as airports or train station, and analyzing traffic flow patterns. By representing the transportation network as a graph, mathematicians and transportation planners can use graph theory to develop models that help them understand and optimize transportation systems. (Mrs.M.durgadevi & Ms.Ch, 2018)

1.2.2 Intelligent Transportation System:

Sensing technologies play a crucial role in the operation intelligent transportation systems (ITS). These technologies are used to collect real-time data on traffic conditions and transmit information to the ITS control center for analysis and decision-making.

For example, in-vehicle sensors can detect changes in speed, acceleration, and braking, which can be used to predict potential accidents or traffic congestion. Roadside sensors can detect the presence of vehicle and pedestrians and can provide real-time data on traffic flow and volume. Additionally, video vehicle detection technologies can capture images of vehicles passing through a designated area and transmit this information to the control center for analysis.

The use of sensing technologies in ITS can lead to significant benefits, such as improved traffic flow, reduced congestion, and increased safety for drivers and pedestrians. Real time traffic data can be used to adjust traffic signal timings and optimize traffic flow, which can reduce travel times and improve fuel efficiency. Additionally, sensing technologies can be used to detect and respond to accidents or other incidents quickly, reducing the risk of further accidents and improving emergency response times.

Overall, the integration of sensing technologies in ITS can help to address many of the challenges associated with modern transportation systems, such as traffic congestion, safety concerns, and environmental impacts. As technology continues to advance, we can expect to see further innovations in sensing technologies and their applications in intelligent transportation systems. (Mrs.M.durgadevi & Ms.Ch, 2018)

CHAPTER TWO

Background

Definitions 2.1: (Bondy, 2006) Graph, a graph is basically a collection of dots, with some pairs of dots being connected by lines. The dots are called vertices, and the lines are called edges.

Definitions 2.2: (Bondy, 2006) Vertex, a vertex is a 'dot' in a graph. The plural of vertex is 'vertices', 'this graph has five vertices. Other synonyms for vertex are node and point. Here are the vertices of a graph.

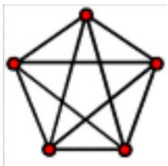


Figure 1 graph showing vertices

Definitions 2.3: (Bondy, 2006) Edge, an edge connects two vertices in a graph. We call those vertices endpoints of the edge. Other synonyms for edge are arc,

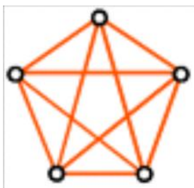


Figure 2 graph showing Edges

link and line. here is the edge of a graph.

Definitions 2.4: (Bondy, 2006) Adjacent, two vertices are adjacent if they are connected by an edge. We often call these two vertices' neighbors. Two adjacent vertices:

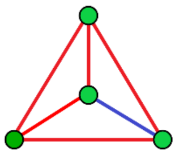


Figure 3 graph showing adjacent vertices

Definitions 2.5: (Bondy, 2006) Bridge, an edge in a graph whose removal (leaving the vertices) results in a disconnected graph.

Definitions 2.6: (Bondy, 2006) Bipartite Graph, a graph is bipartite if the vertices can be partitioned into two sets, X and Y, so that the only edges of the graph are between the vertices in X and the vertices in Y.

Definitions 2.7: (Bondy, 2006) Size, the size of the graph is the number of edges it has.

Definitions 2.8: (Bondy, 2006) Degree, the degree of a vertex is the size of its neighborhood. The degree of a graph is maximum degree of all of its vertices.

Definitions 2.9: (Bondy, 2006) Pendent Vertex, a vertex of degree 1. Also known as a leaf.

Definitions 2.10: (Bondy, 2006) Order, the order of a graph is the number of vertices it has.

Definitions 2.11: (Bondy, 2006) Walk, a walk is a series of vertices and edges.

Definitions 2.12: (Bondy, 2006) Closed Walk, is a walk from a vertex back to itself.

Definitions 2.13: (naduvath, 2017) Trial, a trial is a walk that does not pass the same edge twice. A trail might visit the same vertex twice, but only if its comes and goes from a different edge each time.

Definitions 2.14: (naduvath, 2017) Tour, a tour is a trial that begins and ends on the same vertex.

Definitions 2.15: (Bondy, 2006)Cycle, cycle is a closed walk with no repeated vertices (expect that the first and last vertices are the same).

Definitions 2.16: (Bondy, 2006) Path, a path is a walk where no repeated vertices. A **u-v** path is a path beginning at u and ending at v.

Definitions 2.17: (stephanie, 2019) **(**U-V) Walk, a (u-v) walk would be a walk beginning at (u) and ending at (v).

Definitions 2.18: (stephanie, 2019) Directed Graph, a directed graph is a graph where the edges have direction, that is, they are ordered pairs of vertices.

Definitions 2.19: (stephanie, 2019)Loop, a loop is an edge or arc that joins a vertex to itself.

Definitions 2.20: (stephanie, 2019) Multigraph, a multigraph is a graph without loops, but which may have multiple edges.

Definitions 2.21: (stephanie, 2019) Null Graph, a null graph is a graph with no edges. It may have one or more vertices.

Definitions 2.22: (stephanie, 2019) Simple Graph, a simple graph that doesn't have any loops or multiple edges.no multiple edges means that no two edges have the same endpoints.

Definitions 2.23: (stephanie, 2019) Trivial Graph, a trivial graph is a graph with only one vertex.

Definitions 2.24: (stephanie, 2019) Undirected Graph, an undirected graph is a graph where none of the edges have direction, the pairs of vertices that make up each edge are unordered.

Definitions 2.25: (Bondy, 2006) Subgraph, a subgraph of graph is some smaller portion of that graph. Here is an example of a subgraph:

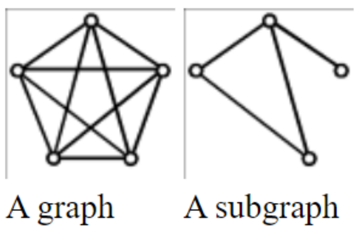


Figure 4 Graph, showing all vertices and edges

Subgraph, showing a subset of the vertices and edges from

Definitions 2.26: (Bondy, 2006) Spanning Subgraph, a subgraph of the graph G which contains of all the vertices of G.

Definitions 2.27: (Bondy, 2006) Tree, a connected graph containing no circuits.

Definitions 2.28: (Bondy, 2006)Spanning Tree, a spanning subgraph of a graph which is also tree.

Definitions 2.29: (Bondy, 2006)Circuit, in a graph, a circuit is a simple, closed walk.

Definitions 2.30: (Bondy, 2006)Connected, a connected graph is one in which every pair of vertices are joined by a chain.

Definitions 2.31: (Mohtashim, 2019) Euler Graph, a connected graph G is called an Euler graph, if there is a closed trial which includes every edge of the graph G.

Definitions 2.32: (Mohtashim, 2019) Euler Path, a Euler path is a path that uses every edge of a graph exactly once. a Euler path starts and ends at different vertices.

Definitions 2.33: (Mohtashim, 2019) Euler Circuit, a Euler circuit is a circuit that uses every edge of a graph exactly once. A Euler circuit always starts and ends at the same vertex a connected graph G is a Euler graph if and only if all vertices of G are of even degree, and connected graph G is Eulerian if and only if its edge set can be decomposed into cycle.

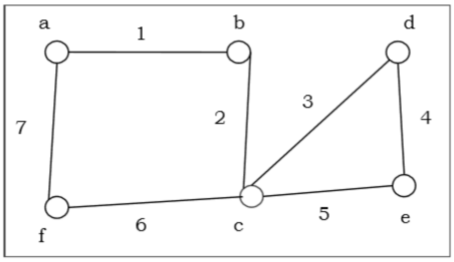


Figure 5 graph showing a Euler circuit

The above graph is a Euler graph as a 1 b 2 c 3 d 4 e 5 c 6 f 7 g covers all edges of the graph.

Theorem 2.1: (Sinha, 2015) The edge connectivity of a graph G cannot exceed the degree of the vertex with the smallest degree in G.

Proof:

The vertex v_i be the vertex with the smallest degree in G. Let $d(v_i)$ be the degree of $v_{i.}$ Vertex v_i can be separated from G by removing the $d(v_i)$ edges incident on vertex $v_{i.}$ hence the theorem.

Theorem 2.2: (Sinha, 2015) The vertex connectivity of any graph G can never exceed the edge connectivity of G.

Proof:

Let a denote the edge connectivity of G. Therefore, Ǝ a cut set S in G with α edges. Let S partition the vertices of G into subsets V_1 and V_2. By removing almost α vertices V_1 or V_2 which the edges in S are incident, we can affect the removal of S (together with all other edges incident on these vertices) from G. Hence the theorem.

Theorem 2.3: (Sinha, 2015) The maximum vertex connectivity one can achieve with a graph G of n vertices and e edges ($e \geq n-1$) is the integral part of the number 2e/n. [2e/n]

Proof:

Every edge in G contributes two degrees. The total (2e degree) is divided among n-vertices. Therefore, there must be at least one vertex in G whose degree is equal to or less than the number 2e/n. The vertex connectivity of G cannot exceed this number.

To show that this value can actually be achieved, one can first construct an n-vertex regular graph of degree equal to the integral part of the number [2e/n] and then add the remaining e-(2/n). [integral part of the number 2e/n] edges arbitrary.

Thus, we can summarize as follows:

$$Vertex\ connectivity\ \leq\ Edge\ connectivity\ \leq\ 2e/n$$

and

maximum vertex connectivity possible = [integral part of the number 2e/n]. thus, for a graph with 8 vertices and 16 edges, we can achieve a vertex connectivity (and therefore edge connectivity) s high as four$(= 2\frac{16}{8})$

The traffic sensors can be placed on each edge in a cut-set of G determined by its edge connectivity as well as on each vertex of G determined by its vertex connectivity. These sensors will provide complete traffic information for the control system. Thus, optimal locations for the traffic sensors can be obtained by using edge connectivity and vertex connectivity of the compatibility graph G.

Theorem 2.4: (Mrs.M.durgadevi & Ms.Ch, 2018) **One way street problem: Robin's theorem,** the problem of orienting every edge in a graph in such a way that it remains possible to travel between any two vertices is known as Robin's Theorem, named after mathematician guy Robin who first proved it in 1969.

To solve this problem, we can use the concept of strong connectivity in a directed graph. A directed graph is strongly connected if there is a directed path between every pair of vertices. In other words, we can get from any vertex to any other vertex by following the direction of the edges.

To apply Robin's theorem to the problem of making every street in a city one-way, we first represent the streets as edges of a graph and the street corners as vertices. We then start with the initial undirected graph, where each edge represents a two-way street.

To make every street one-way, we need to assign a direction to each edge in such a way that the resulting directed graph is strongly connected. This means that we need to ensure that for any two vertices in the graph, there is a directed path between them.

One way to achieve this is to choose a spanning tree of the initial graph and orient all edges in the tree to point away from the root of the tree. This ensures that there is a directed path from the root to any other vertex in the graph. We then orient the remaining edges arbitrarily, as long as they do not create cycles in the graph.

By using this method, we can ensure that it is still possible to get from any place to any other place in the city, even after making every street one-way. This can help to alleviate traffic congestion and reduce air pollution in urban areas.

Chinese postman's problem (CPP): (Mrs.M.durgadevi & Ms.Ch, 2018) in 1962, a Chinese mathematician called Kuan mei-ko was interested in postman delivering mail. The (CPP) is a classic problem in graph theory that seeks to find the shortest possible route that visits every edge of an undirected graph at least once. In the context of the problem you described, this could be interpreted as finding the shortest route for a postman to deliver mail to a number of streets.

There are several approaches to solving the CPP, but one common method is to add additional edges to the graph to make it Eucerin (i.e., having an even number of edges at every vertex). Once the graph is Eulerian, the postman can traverse every edge exactly twice (once in each direction) and return to the starting point, thus ensuring that all streets are visited.

To add the necessary edges, we can use the following algorithm:

1. Identify all vertices in the graph with odd degree (i.e., an odd number of edges incident to the vertex).
2. For each pair of odd-degree vertices, odd an edge between them with a weight equal to the shortest distance between them.
3. Find the minimum-weight perfect matching on the set of new edges (i.e., the set of edges connecting the odd-degree vertices).
4. Add the matching edges to the original graph to create a new, Eulerian graph.

5. Find a Eulerian circuit on the new graph using any standard algorithm (e.g., Hierholzer's algorithm).
6. Traverse the Eulerian circuit, taking each edge exactly twice (once in each direction), to visit every edge in the original graph.

By following these steps, the postman can be sure to cover very street in the shortest possible distance.

CHAPTER THREE

Introduction About Erbil

Erbil, also known as Hawler in Kurdish, is the capital city of the Kurdistan Region in Iraq. With a population of approximately 1.3 million in 2009, it was the fourth largest city in Iraq after Baghdad, Basra, and Mosul. However, since then, the city's population has grown significantly, and it is estimated to be around 2 million people as of 2021. Erbil is an important cultural and economic center in the region, with a rich history dating back thousands of years. This is the map of Erbil. (gunter, 2023)

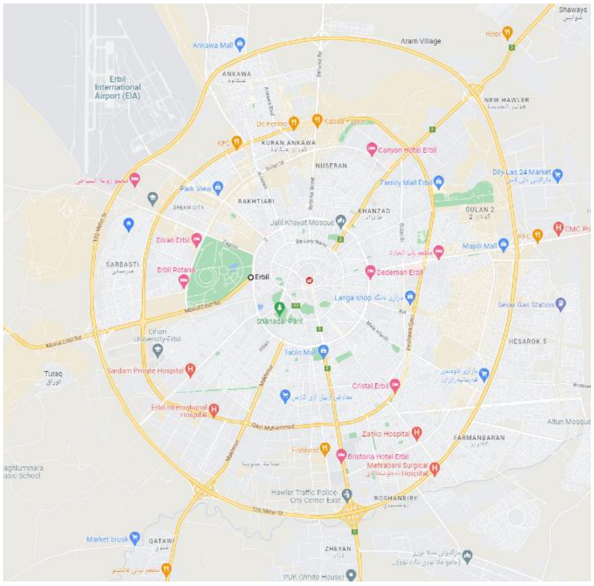


Figure 6 map of Erbil city

Like many growing cities, Erbil has been facing traffic congestion issues due to a rapidly increasing population, the expansion of the city, and an increase in the number of cars on the roads. This has led to longer commuting times, increased air pollution, and reduced road safety.

To address this problem, the government of Erbil has implemented several measures, including building new roads and highways, improving public transportation, and introducing carpooling schemes. Additionally, the city has been working to improve its urban planning, such as developing pedestrian-friendly zones and bike lanes.

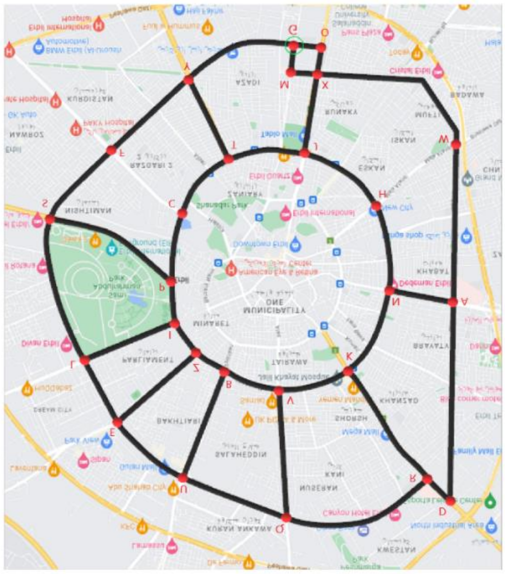
Despite these efforts, traffic congestion remains a significant issue in Erbil, and the city continues to work towards finding sustainable solutions to address the problem.

The Chinese postman problem is a mathematical problem that seeks to find the shortest possible route that visits every edge of a given undirected graph at least once. It is often used in transportation and logistics to optimize delivery routes and reduce travel time.

While the Chinese postman problem could potentially be used to address traffic congestion in Erbil by optimizing the routes of public transportation and delivery vehicles, it may not necessarily solve the problem of traffic congestion caused by private vehicles.

Private vehicles are often a major contributor to traffic congestion in cities, and reducing the number of cars on the roads can be challenging. Some potential solutions could include promoting the use of public transportation, encouraging carpooling and ridesharing, and improving infrastructure to support non-motorized modes of transportation like cycling and walking.

Ultimately, a combination of solutions will likely be necessary to address traffic congestion in Erbil and other growing cities. Following example: In the graph 1 where each intersection is a vertex and each street is an edge with a length equal to the length of the street. The delivery guy needs to find the shortest path that visits every vertex (intersection) at least once and returns to the starting vertex (G).

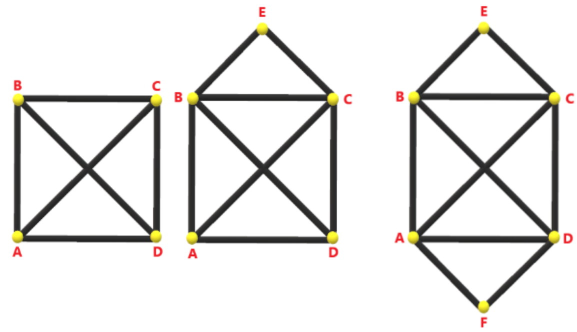


graph 1

Figure 7 Erbil Street 40-meter and street 60-meter map

In the graph 1, The 40-meter street appears as an outside circle that connects vertices G, O, M, X, Y, F, S, L, E, U, Q, R, D, A and W while 60-meter street appears as an inside circle connects vertices T, J, H, N, K, V, B, Z, I, P and C. the starting vertex is G.

To find the shortest path that visits every vertex at least once and returns to G, we can use the Chinese postman algorithm, we will return to solving this actual problem later, but initially we will look at drawing various graphs. The Chinese postman is traversable graphs given below.



Graph 1 graph 2 graph 3

Figure 8 Graph 1 has four vertices with an odd number of edges connected to them. Graph 2 has two vertices with an odd number of edges connected to them, and three vertices with an even number of edges connected to them. Graph 3 has six vertices with an even number of edges connected to them.

we find:

- It is impossible to draw graph 1 without either taking the pen off paper or re-tracing an edge.
- We can draw graph 2, but only by starting at either A or D, in each case the path will end at the other vertex of D or A.
- Graph 3 can be drawn regardless of starting position and you will always return to the start vertex.

In order to establish the differences, we must consider the order of the vertices for each graph. The following when the order of all the vertices is even the graph is Travers able. When there are two odd vertices, we can draw the graph but the start and end vertices are different. When there are four odd vertices the graph can't be drawn without repeating an edge.

vertex	order
A	3
B	3
C	3
D	3

Graph 1

vertex	order
A	3
B	4
C	4
D	3
E	2

Graph 2

vertex	order
A	4
B	4
C	4
D	4
E	2
F	2

Graph 3

We can use the following algorithm:

1. Identify all vertices in the graph with odd degree (i.e., an odd number of edges incident to the vertex).
2. For each pair of odd-degree vertices, odd an edge between them with a weight equal to the shortest distance between them.
3. Find the minimum-weight perfect matching on the set of new edges (i.e., the set of edges connecting the odd-degree vertices).

4. Add the matching edges to the original graph to create a new, Eulerian graph.
5. Find a Eulerian circuit on the new graph using any standard algorithm (e.g., Hierholzer's algorithm).
6. Traverse the Eulerian circuit, taking each edge exactly twice (once in each direction), to visit every edge in the original graph.

To find the shortest path in (figure 2.10), while each node represents a street intersecting, and each edge represents a street. the numbers on the edges represent the distance between two intersections for example, the edge between G and Y has a distance of 1.6 kilo meter.

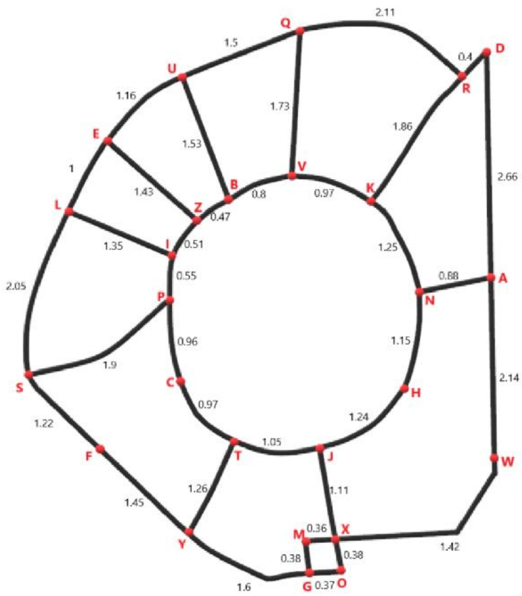


Figure 9 graph showing street map of Erbil

First, you need to identify the odd-degree vertices in the graph, which are G, Y, S, P, L, I, E, Z, U, B, Q, V, K, R, A, N, J and T, then we need to find a minimum-weight matching of these vertices.

Since there are more than two odd-degree vertices, of each degree, there is more than one way to matching them, we can compute the weights of the edges connecting the odd-degree vertices:

$(G,Y) = 1.6$,$(S,L) = 2.05$,$(E,U) = 1.16$,$(Q,R) = 2.11$,$(K,V) = 0.97$,

$(P,I) = 0.55$,$(Z,B) = 0.47$,$(A,N) = 0.88$,$(T,J) = 1.05$

Using an algorithm to find a minimum-weight. Perfect matching, we can select the following edges:

(G, Y), (S, L), (E, U), (Q, R), (K, V), (P, I), (Z, B), (A, N), (T, J)

The total weight of this matching is:

$1.6 + 2.05 + 1.16 + 2.11 + 0.97 + 0.55 + 0.47 + 0.88 + 1.05 = 10.84$

In the matching that we have chosen for the odd-degree vertices in the graph, the total weight of the nine edges connecting these vertices is 10.84 kilometers. This represents the minimum amount of additional distance that the delivery guy would need to travel in order to visit all edges in the graph at least once, and return to the starting point.

To calculate the total weight of the graph, we need to sum up the weights of all the edges in the graph. From the given graph, we can see that the weights of the edges are as follows:

$(G, Y) = 1.6$, $(Y, F) = 1.45$, $(F, S) = 1.22$, $(S, L) = 2.05$, $(L, E) = 1$,

$(E, U) = 1.16$, $(U, Q) = 1.5$, $(Q, R) = 2.11$, $(R, D) = 0.4$,$(D, A) = 2.66$,

$(A, W) = 2.14$, $(W, X) = 1.42$,$(X, M) = 0.36$,$(X, O) = 0.38$,$(M, G) = 0.38$,

$(O, G) = 0.37$,$(X, J) = 1.11$,$(Y, T) = 1.26$, $(S, P) = 1.9$,$(L, I) = 1.35$,

$(E, Z) = 1.43$,$(U, B) = 1.53$,$(Q, V) = 1.73$,$(R, K) = 1.86$,$(A, N) = 0.88$,

$(J, T) = 1.05$,$(T, C) = 0.97$,$(C, P) = 0.96$, $(P, I) = 0.55$,$(I, Z) = 0.51$,

$(Z, B) = 0.47$,$(B, V) = 0.8$,$(V, K) = 0.97$,$(K, N) = 1.25$,$(N, H) = 1.15$,

$(H, J) = 1.24$

Adding up all these weights, we get:

$$1.6 + 1.45 + 1.22 + 2.05 + 1 + 1.16 + 1.5 + 2.11 + 0.4 + 2.66 + 2.14 + 1.42 + 0.36 + 0.38 + 0.38 + 0.37 + 1.11 + 1.26 + 1.9 + 1.35 + 1.43 + 1.53 + 1.73 + 1.86 + 0.88 + 1.05 + 0.97 + 0.96 + 0.55 + 0.51 + 0.47 + 0.8 + 0.97 + 1.25 + 1.15 + 1.24 = 43.17$$

Therefore, the total weight of the graph is 43.17 kilometers.

After adding the edges from, the minimum-weight matching, the resulting graph is:

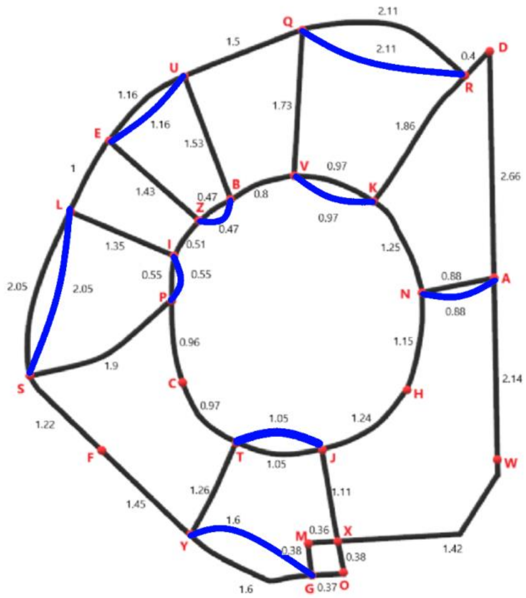


Figure 10 graph showing street map of Erbil after adding edges to odd vertices

the graph now has no odd-degree vertices, which means that it has an Eulerian circuit. The delivery guy can simply traverse the circuit to visit every edge in the graph exactly once and return to G.

one possible Eulerian circuit is:

$$G \to M \to X \to O \to G \to Y \to T \to J \to X \to W \to A \to N \to H \to J \to T \to C \to P \to I \to L \to S \to P \to I \to Z \to B \to U \to E \to Z \to B \to V \to K \to R \to Q \to V \to K \to N \to A \to D \to R \to Q \to U \to E \to L \to S \to F \to Y \to G$$

The total distance traveled is the sum of the lengths of the edges in the circuit:

$$
\begin{aligned}
Distance \; = & \; GM + MX + XO + OG + GY + YT + TJ + JX + XW + WA + AN \\
& + NH + HJ + JT + TC + CP + PI + IL + LS + SP + PI + IZ \\
& + ZB + BU + UE + EZ + ZB + VK + KR + RQ + QV + VK \\
& + KN + NA + AD + DR + RQ + QU + UE + EL + LS + SF + FY \\
& + YG
\end{aligned}
$$

$$
\begin{aligned}
Distance \; = & \; 0.38 + 0.36 + 0.38 + 0.37 + 1.6 + 1.26 + 1.05 + 1.11 + 1.42 \\
& + 2.14 + 0.88 + 1.15 + 1.24 + 1.05 + 0.97 + 0.96 + 0.55 \\
& + 1.35 + 2.05 + 1.9 + 0.55 + 0.51 + 0.47 + 1.53 + 1.16 + 1.43 \\
& + 0.47 + 0.97 + 1.86 + 2.11 + 1.73 + 0.97 + 1.25 + 0.88 \\
& + 2.66 + 0.4 + 2.11 + 1.5 + 1.16 + 1 + 2.05 + 1.22 + 1.45 \\
& + 1.6 = 43.17 + 10.48 = 53.62
\end{aligned}
$$

Therefore, the shortest path that visits every vertex at least once and returns to G is 53.62 kilometers.

train system can potentially help alleviate traffic congestion in certain areas, as it provides an alternative mode of transportation that can reduce the number of vehicles on the road. However, it is important to consider several factors before implementing such a system.

Firstly, the cost of building and maintaining a train system can be substantial, and the government would need to carefully consider the financial implications of such a project. Additionally, the train system would need to be convenient and accessible to users, with stations located in areas where there is a high demand for transportation.

Another factor to consider is the existing infrastructure in the area. The train system would require a dedicated track, and the government would need to ensure there is sufficient space and resources to accommodate the new system without disrupting existing traffic flows or infrastructure.

Furthermore, the train system would need to be integrated with other modes of transportation, such as buses, taxis, and private vehicles, to ensure seamless and efficient travel for commuters. The government would need to consider how the train system would fit into the overall transport network and how it would impact the existing transport options available to the public.

Conclusion

The Chinese Postman Problem is a classic problem in graph theory that involves finding the shortest closed path that visits every edge in a given graph. The problem has real-world applications in areas such as transportation, logistics, and network design.

In order to solve the Chinese Postman Problem, we first need to identify the odd-degree vertices in the graph. We then find a minimum-weight matching of these vertices, which will add a set of even-length paths to the graph, effectively transforming it into a Eulerian graph. We can then use an algorithm to find a Eulerian circuit in the modified graph, which will correspond to the shortest closed path that visits every edge in the original graph.

The solution to the problem can be computed using various algorithms, such as the Hierholzer's algorithm or the Edmonds' algorithm. It is also worth noting that some graphs may not have a solution, as they may have an odd number of odd-degree vertices, which cannot be paired up in a minimum-weight matching.

Overall, the Chinese Postman Problem is an interesting and important problem in graph theory, and its solution has practical applications in many areas of science and engineering.

The weight of a graph is an important metric that can be used to evaluate the efficiency and cost of different paths or routes. In the context of the Chinese Postman Problem, the weight of the graph represents the total distance that the delivery guy would need to travel in order to visit all edges in the graph at least once, and return to the starting point.

In general, finding the total weight of a graph can help us to better understand the structure and characteristics of the graph, and to make informed decisions about how to navigate or optimize paths within the graph.

In conclusion, introducing a train system can be a viable solution to reduce traffic congestion in certain areas. However, it requires careful planning and consideration of various factors such as cost, accessibility, infrastructure, and integration with existing transportation systems.

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پوختە

قەرەباڵغی هاتوچۆ بۆتە کێشەیەکی بەرچاو لە زۆرێک لە شارەکانی جیهان، لەوانەش هەولێر. زیادبوونی خێرای دانیشتوانی شارەکە، لەگەڵ زیادبوونی ژمارەی ئۆتۆمبێلەکان، بووەتە هۆی قەرەباڵغی لەسەر ڕێگاکان و بووەتە هۆی دواکەوتن و زیادبوونی کاتی گەشت و پیسبوونی ژینگە. ئەم توێژینەوەیە پێشنیاری چارەسەرێک دەکات بۆ قەرەباڵغی هاتوچۆ لە هەولێر بە بەکارهێنانی کێشەی پۆستەچی چینی و پێشنیاری بەکارهێنانی شەمەندەفەر.

کێشەی پۆستەچی چینی کێشەیەکی بیرکارییە کە هەوڵدەدات کورتترین ڕێگا بدۆزێتەوە بۆ کەسێک کە دەبێت لانیکەم جارێک لە هەر شەقامێکدا تێپەڕێت پێش ئەوەی بگەڕێتەوە بۆ خاڵی دەستپێک. ئەم کێشەیە دەتوانرێت بۆ بەڕێوەبردنی هاتوچۆ بەکاربهێنرێت بە باشکردنی ڕۆیشتنی هاتوچۆ لە تۆری ڕێگاوبانەکانی شارێکدا، کەمکردنەوەی قەرەباڵغی و کاتی گەشتکردن. بە بەکارهێنانی کێشەی پۆستەچی چینی لە هەولێر دەتوانین کارامەترین ڕێگاکان دەستنیشان بکەین بۆ ئەوەی ئۆتۆمبێلەکان بیگرنە بەر، ئەمەش قەرەباڵغی هاتوچۆ کەم دەکاتەوە.

جگە لەوەش هێنانەکایەی سیستەمی شەمەندەفەر لە هەولێر دەتوانێت زیاتر قەرەباڵغی هاتوچۆ کەم بکاتەوە. شەمەندەفەرەکان خێراتر و سەلامەتتر و کاراترن لە ئۆتۆمبێلەکان و دەتوانن ژمارەیەکی زۆر سەرنشین هەڵبگرن. بە پێشکەشکردنی شێوازێکی گواستنەوەی جێگرەوە، دەتوانین ژمارەی ئۆتۆمبێلەکان لەسەر ڕێگاکە کەم بکەینەوە و لە ئەنجامدا قەرەباڵغی هاتوچۆ کەم بکەینەوە.

لە کۆتاییدا دەڵێین قەرەباڵغی هاتوچۆ لە هەولێر کێشەیەکی جدییە و پێویستی بە چارەسەری فرەلایەنە هەیە. بە بەکارهێنانی کێشەی پۆستەچی چینی و ناساندنی سیستەمی شەمەندەفەر، دەتوانین قەرەباڵغی هاتوچۆ کەم بکەینەوە، کاتی گەشتکردن باشتر بکەین، و شارێکی سەوزتر و بەردەوامتر بەرەوپێش ببەین.

ملخص

أصبح الازدحام المروري مشكلة كبيرة في العديد من المدن حول العالم ، بما في ذلك أربيل. أدى النمو السكاني السريع للمدينة ، إلى جانب زيادة عدد المركبات ، إلى الازدحام على الطرق ، مما تسبب في حدوث تأخير ، وزيادة أوقات السفر ، والتلوث البيئي. تقترح هذه الدراسة حلاً للازدحام المروري في أربيل باستخدام مشكلة ساعي البريد الصيني واقتراح استخدام القطارات.

مشكلة ساعي البريد الصيني هي مشكلة رياضية تسعى إلى إيجاد أقصر طريق ممكن للشخص الذي يجب أن يجتاز كل شارع مرة واحدة على الأقل قبل العودة إلى نقطة البداية. يمكن تطبيق هذه المشكلة على إدارة حركة المرور عن طريق تحسين تدفق حركة المرور في شبكة طرق المدينة ، وتقليل الازدحام ووقت السفر. باستخدام مشكلة ساعي البريد الصيني في أربيل، يمكننا تحديد أكثر الطرق كفاءة للمركبات، مما يقلل الازدحام المروري.

علاوة على ذلك ، فإن إدخال نظام القطار في أربيل يمكن أن يخفف من الازدحام المروري. القطارات أسرع وأكثر أمانًا وفعالية من السيارات ، ويمكنها أن تحمل عددًا كبيرًا من الركاب. من خلال تقديم وسيلة نقل بديلة ، يمكننا تقليل عدد السيارات على الطريق ، وبالتالي تقليل الازدحام المروري.

في الختام ، يعتبر الازدحام المروري في أربيل مشكلة خطيرة تتطلب حلاً متعدد الأوجه. من خلال استخدام مشكلة ساعي البريد الصيني وإدخال نظام القطار ، يمكننا تقليل الازدحام المروري وتحسين وقت السفر وتعزيز مدينة أكثر خضرة واستدامة.