زانكۆی سەلاحەدین-هەولێر

Salahaddin University-Erbil

Integral equations and their relationship to differential equations

Research project:

Submitted to the department of Mathematics in partial fulfillment of the requirements for the degree of BSc.in MATHEMATICS

Prepared By:

Zaynaba Sabir Abdullah

Supervised By:

Dr. Pakhshan M. Hassan

Assist.pro. Dr. Ivan subhi latif

April-2023

Certification of the Supervisors

I certify that this work was prepared under my supervision at the Department of Mathematics/College of Education/Salahaddin University-Erbil in partial fulfillment of the requirements for the degree of Bachelor of philosophy of Science in Mathematics.

Signature:

Supervisor**: Dr. Ivan Subhi Latif**

Scientific grade: **Assist. Professor**

Signature:

Supervisor: **Dr. Pakhshan M Hassan**

Scientific grade: **Doctor**

Date: / / 2023

In view of the available recommendations, I forward this work for debate by the examining committee.

Signature:

Name: **Dr. Rashad Rashid Haji**

Scientific grade: **Assist. Professor**

Chairman of the Mathematics Department

Date: / / 2023

Acknowledgment

Primarily, I would like to thanks my god for helping me to complete this research with success.

Then I would like to express special of my supervisor Dr. Ivan Subhi Latif and Dr. Pakhshan M. Hasan Who are valuable to guidance has been the once helped me to completing my research.

Words can only inadequately express my gratitude to my supervisor for patiently helping me to think clearly and consistently by discussing every point of this dissertation with me.

I would like to thank my family, friend and library staff whose support has helped me to conceive this research

Abstract

Integral and differential equations have a fundamental importance in the functional analysis and the practice problems, and many domains of scientific research. However, the resolution of differential equations with constant coefficients is easy, but the resolution of these equations with variable coefficients is practically difficult or impossible in more part of the cases. This work presents an analytical method which it transforms a differential equation with initial conditions to a Volterra equations of second kind, efficient methods for approximate numerical solution of these equations.

Table of content

CHAPTER ONE

INTRODUCTION

An integral equation is defined as equations in which the unknown function $u(x)$ to be determined appear under the integral sign. The subject of integral equations is one of the most useful mathematical tools in both pure and applied mathematics. It has enormous applications in many physical problems. Many initial and boundary value problems associated with ordinary differential equation (ODE) and partial differential equation (PDE) can be transformed into problems of solving some approximate integral equations. (M.Rahman, 2007)

The development of science has led to the formation of many physical laws, which, when restated in mathematical form, often appear as differential equations. (M.Rahman, 2007)

Engineering problems can be mathematically described by differential equations, and thus differential equations play very important roles in the solution of practical problems. For example, Newton's law, stating that the rate of change of the momentum of a particle is equal to the force acting on it, can be translated into mathematical language as a differential equation. Similarly, problems arising in electric circuits, chemical kinetics, and transfer of heat in a medium can all be represented mathematically as differential equations.

A typical form of an integral equation in $u(x)$ is of the form

$$u(x) = f(x) + \lambda \int_{\alpha(x)}^{\beta(x)} K(x,t)u(t)dt \qquad (1.1)$$

where $K(x,t)$ is called the kernel of the integral equation (1.1), and $\alpha(x)$ and $\beta(x)$ are the limits of integration. It can be easily observed that the unknown function $u(x)$ appears under the integral sign. It is to be noted here that both the kernel $K(x,t)$ and the function $f(x)$ in equation (1.1) are given functions; and λ is a constant parameter. The prime objective of this text is to determine the unknown function $u(x)$ that will satisfy equation (1.1) using a number of solution techniques. We shall devote considerable efforts in exploring these methods to find solutions of the unknown function. Integral equation (1888-1903) J. Fourier (1768-1830) is the initiator of the theory of integral equations. (M.Rahman, 2007)

A term integral equation first suggested by (Du Bois-Reymond in 1888). Du Bois-Reymond define an integral equation is understood an equation in which the unknown function occurs under one or more signs of definite integration. Late eighteenth and early ninetieth century Laplace, Fourier, Poission, Liouville and Able studies some special type of integral equation. The pioneering systematic investigations goes back to late nineteenth and early twentieth century work of volterra, Fredholm and Hilbert. In (1887), Volterra published a series of famous papers in which he singled out the notion of a functional and pioneered in the development of a theory of functional s in theory of linear integral equation of special type. Fredholm presented the fundamentals of the Fredholm integral equation theory in a paper published in (1903) in the Acta Mathematical. (Mostefa & G., 2016)

A differential equation is an equation in which one or more variables, one or more functions of these variables and also the derivatives of these functions with respect to these variables occur the order of a differential equation is equal to the order of the highest occurring derivative. We write a generic second-order equation for an unknown state

$y = y(t)$ In the form

$$y'' + A(t)\, y' + B(t)y = f(t)$$

Where A, B and f are known functions.

Further, most differential equations cannot be solved by performing a sequence of integrations, involving only elementary functions: polynomials, rational functions, trigonometric functions, exponentials, logarithms, and A, B are constants. For we find the relationship between integral and differential equations, we will need the following lemma which will allow us to replace a double integral by a single one. Differential equations have been a major branch of pure and applied mathematics since their inauguration in the mid7th century. While their history has been well studied, it remains a vital field of on-going investigation, with the emergence of new connections with other parts of mathematics, fertile interplay with applied subjects, interesting reformulation of basic problems and theory in various periods, new vistas in the 20th century, and so on. In this meeting we considered some of the principal parts of this story, from the launch with (Newton and Leibniz up to around 1950). `Differential equations' began with Leibniz, the Bernoulli brothers and others from the (1680s), not long after Newton's `fluxional equations' in the (1670s). Applications were made largely to geometry and mechanics; isoperimetrical problems were exercises in optimization. (Mostefa & G., 2016)

CHAPTER TWO

BAGRAOUND

Definition (Equation) 2.1: (Jean, 2016)

Equations are mathematical statements containing two algebraic expressions on both sides of an 'equal to (=)' sign. It shows the relationship of equality between the expression written on the left side with the expression written on the right side. In every equation in math, we have, L.H.S = R.H.S (left hand side = right hand side). Equations can be solved to find the value of an unknown variable representing an unknown quantity. If there is no 'equal to' symbol in the statement, it means it is not an equation. It will be considered as an expression. You will learn the difference between equation and expression in the later section of this article**.**

Definition (Differential Equation) 2.2: (Mohammed, 2015)

A differential equation is an equation which contains one or more terms and the derivatives of one variable (i.e., dependent variable) with respect to the other variable (i.e., independent variable) Here "x" is an independent variable and "y" is a dependent variable.

For example,

A differential equation contains derivatives which are either partial derivatives or ordinary derivatives. The derivative represents a rate of change, and the differential equation describes a relationship between the quantity that is continuously varying with respect to the change in another quantity. There are a lot of differential equations formulas to find the solution of the derivatives.

Definition (Integral Equation) 2.3: (Dr.Ramesh, 1995)

An integral equation is said to be a linear if only linear operations are performed in it upon the unknown functions. An integral equation which is not linear is known as non-linear integral equation. For example, the integral equations (1.2.1) to (1.2.4) are linear while (1.2.3) is not linear. The most general type of linear integral equation is of the form

$$\alpha(x)\ g(x) = f(x)\ + \lambda \int_{\Omega} K(x,t)\,[g(t)]dt \qquad \ldots(2.3.1)$$

where the upper limit may be either variable x or fixed. The functions f,$\propto$ and k are known functions, while g is to be determined; λ is a non-zero real or complex parameter. The function $k(x,t)$ is known as the kernel of the integral equation. The integration extends over the domain Ω of the auxillary variable t .

The integral equations, which are linear, involved the linear operator

$$L\{\ \} = \int_{\Omega} K(x,t)\,\{\ \}dt$$

Having the kernel $K(x,t)$.It satisfies the linearty condition

$$L\{c_1 g_1(t) + c_2 g_2(t)\} = c_1 L\{g_1(t)\} + c_2 L\{g_2(t)\}$$

$$L\{g(t)\} = \int_{\Omega} K(x,t) g(t) dt\ \ and\ c_1, c_2\ are\ constants.$$

2.1 Classification of Linear Integral Equations (Dr.Lamyaa, 2012)

The most frequently used linear integral equations fall under two main classes namely Fredholm and Volterra integral equations. However, in this text we will distinguish four more related types of linear integral equations in addition to the two main classes. In the following is the list of the Fredholm and Volterra integral equations, and the four more related types:

1. Fredholm Integral Equations

2. Volterra Integral Equations

3. Integro-differential Equations

4. Singular Integral Equations

5. Volterra-Fredholm Integral Equations

6. Volterra-FredholmIntegro-differential Equations

Volterra integral equations 2.1.1:

The most standard form of Volterra linear integral equations is of the form

$$\Phi(x)u(x) = f(x) + \lambda \int_a^x k(x,t)u(t)dt$$

Where the limits of integration are function of x and the unknown function $u(x)$ appears linearly under the integral sign.

If the function $\Phi(x) = 1$, then this equation is known as the Volterra integral equation of the second kind:

$u(x) = f(x) + \lambda \int_a^x k(x,t)u(t)dt$

If the function $\Phi(x) = 0$, then the equation is known as the Volterra integral

Equation of the first kind:

$f(x) + \lambda \int_a^x k(x,t)u(t)dt = 0$

If the function $f(x) = 0$, the equation is called a homogeneous Volterra integral equation, otherwise it is called nonhomogeneous Volterra integral equation.

Leibniz Rule of differentiating under the integral sign 2.1.2 (Dr.Ramesh, 1995)

If $F(x,t)$ and $\frac{\partial F(x,t)}{\partial x}$ are continuous function of x and t in the domain

$\alpha \le x \le \beta, t_0 \le t \le t_1$, then

$$\frac{d}{dx}\int_{a(x)}^{b(x)} f(x,t)$$

$$= \int_{a(x)}^{b(x)} \frac{\partial F(x,t)}{\partial x} dt + \frac{db(x)}{dx} F\big(x,,b(x)\big) - \frac{da(x)}{dx} F(x,a(x))$$

Provided the limit of integration $a(x)$ and $b(x)$ are defined function having continuous derivatives for $\alpha \le x \le \beta$. This rule may be used to convert integral equations to equivalent ordinary differential equations. In particular, we have

(i) **For Volterra integral equation**:

$$\frac{d}{dx}\left[\int_a^x K(x,t)u(t)dt\right] = \int_a^x \frac{\partial K}{\partial x} u(t)dt + K(x,x)u(x)$$

(ii) **For fredholm integral equation**:

$$\frac{d}{dx}\left[\int_a^b K(x,t)u(t)dt\right] = \int_a^b \frac{\partial K}{\partial x}u(t)dt$$

Here $u(t)$ is independent of x and hence on taking partial derivatives with respect to $x, u(t)$is treated as constant.

CHAPTER THREE

TRANSFORMATION OF THE VOLTERRA

3.1 Transformation a Differential Equations to integral equations of Volterra kind

First method 3.1.1

Example 1: Convert the differential equation

$$Y''(t) + 2tY'(t) + Y(t) = 0 \quad ; \quad Y(0) = I \quad , \quad Y'(0) = 0$$

In to an integral equation.

Solution: Let

$$Y"(t) = V(t) \qquad \dots (1)$$

$$Y'(t) = \int_0^t V(u)\,du \qquad \dots (2)$$

$$Y(t) = \int_0^t (t-u)\,V(u)du + 1 \qquad \dots (3)$$

Substituting the equation (1,2,3) in to the differential equation, it follows that

$$V(t) + 2t\int_0^t v(u)du + \int_0^t (t-u)v(u)ddu + 1 = 0$$

$$V(t) + \int_0^t (t-u)v(u)du + 1 = 0$$

Example 2: Convert the differential equation

$Y^{(4)}(t) - 4Y'''(t) + 6Y''(t) - 4Y'(t) = 3cos2t \quad ; \quad Y(0) = -1\,, Y'(0) = 4$
$, Y''(0) = 0\,, Y'''(0) = 2$

In to an integral equation.

Solution:

Let

$Y^{(4)}(t) = v(t)$ ... (1)

$$Y'''(t) - Y'''(0) = \int_0^t V(u)\, du$$

$Y'''(t) = \int_0^t (t-u)\, V(u) du + 2$ ... (2)

$$Y''(t) - Y''(0) = \int_0^t (t-u)\, V(u) du + 2t$$

$Y''(t) = \int_0^t (t-u) V(u)\, du + 2t$ ... (3)

$$Y'(t) - Y'(0) = \int_0^t \frac{(t-u)^2}{2!}\, V(u) du + t^2$$

$Y'(t) = \int_0^t \frac{(t-u)^2}{2!}\, V(u) du + t^2 + 4$ ... (4)

$$Y(t) - Y(0) = \int_0^t \frac{(t-u)^3}{3!}\, V(u) du + \frac{t^3}{3} + 4t$$

$Y(t) = \int_0^t \frac{(t-u)^2}{2!}\, V(u) du + \frac{t^3}{3} + 4t - 1$ ...(5)

Substituting the equation (1, 2, 3, 4, and 5) in to the differential equation, it follows that

$$V(t) - 4\int_0^t V(u)\, du - 8\int_0^t (t-u) V(u) du + 12t - 4\int_0^t \frac{(t-u)^2}{2!} V(u) du + 4t^2 - 16\int_0^t \frac{(t-u)^3}{3!}\, V(u) du + \frac{t^3}{3} + 4t - 1 = 3cos2t$$

Example 3: Convert the differential equation

$$Y'''(t) + 8Y(t) = 3\, sint + cos2t \quad ; \quad Y(0) = 0 \quad , \quad Y\,(0) = -1 \;, Y''(0) = 2$$

In to an integral equation.

Solution: Let

$$Y'''(t) = v(t) \qquad \dots (1)$$

$$Y''(t) = \int_0^t V(u)\, du \ + 2$$

$$Y'(t) = \int_0^t (t-u)\, V(u) du + 2t - 1 \qquad \dots (2)$$

$$Y(t) = \int_0^t \frac{(t-u)^2}{2!}\ V(u) du + t^2 - t$$

Substituting the equation (1,2,3,4) in to the differential equation, it follows that

$$V(t) + 8\int_0^t \frac{(t-u)^2}{2!}\ V(u) du + 8t^2 - 8t = \ 3\ sint \ + \ cos2t$$

$$V(t) = 3sin\ t \ + \ cos\ 2t \ - 8t^2 \ + 8t - 4\int_0^t (t-u)^2\ V(u) du$$

Second method 3.1.2

Example 1: Convert the differential equation

$$Y''(t) + 2Y'(t) - 8Y(t) = 5t^2 - 3t \quad ; \quad Y(0) = -2\ , Y\ (0) = 3$$

In to an integral equation.

Solution: Integrating both sides of the given differential equation, we have

$$\int_0^t \{\mathrm{Y''(u)} + 2\mathrm{Y'(u)} - 8\mathrm{Y(u)}\} du = \int_0^t (5t^2 - 3t) du$$

$$Y'(t) - Y'(0) + 2Y(t) - 2Y(0) - 8\int_0^t Y(u) du = \frac{5}{3}t^3\ \frac{3}{2}t^2$$

$$This\ becomes, using \quad Y(0) = -2 \quad , \qquad Y'(0) = 3$$

$$Y'(t) - 3 + 2Y(t) + 2Y(t) + 4 - 8\int_0^t Y(u) du = \frac{5}{3}t^3\ \frac{3}{2}t^2$$

$$Y'(t) - 2Y(t) - 8\int_0^t Y(u)du = \frac{5}{3}t^3 \ \frac{3}{2}t^2 - 1$$

Integrating again from 0 to t as before, we get

$$Y(t) - Y(0) + 2\int_0^t Y(u)du - 8\int_0^t (t-u)Y(u)du = \frac{5}{12}t^4 - \frac{3}{6}t^3 - t$$

$$Y(t) + \int_0^t \{2 - 8(t-u)\}Y(u)du \ = \frac{5}{12}t^4 - \frac{1}{2}t^3 - t - 2$$

Example 2: Convert the differential equation

$$Y''(t) - 3Y'(t) + 2Y(t) = 4\,sint \quad ; \quad Y(0) = 1\,, Y\,(0) = -2$$

In to an integral equation.

Solution: Integrating both sides of the given differential equation, we have

$$\int_0^t \{Y''(u) - 3Y'(u) + 2Y(u)\}du = \int_0^t 4\sin u\, du$$

$$Y'(t) - Y'(0) - 3Y(t) + 3Y(0) + 2\int_0^t Y(u)du = 4 - 4\,cost$$

$$This\ becomes, using \quad Y(0) = 1 \quad , \qquad Y'(0) = -2$$

$$Y(t) - Y(0) - 3\int_0^t Y(u) + 2\int_0^t (t-u)Y(u)du = -t - 4sint$$

$$Y(t) + \int_0^t \{2(t-u) - 3\}Y(u)du = 1 - t - 4sint.$$

Example 3: Convert the differential equation

$$Y'''(t) - 8Y(t) = 3\ sint\ + cos2t \quad ; \quad Y(0) = 0\ , Y\ (0) = -1\ ,$$
$$Y''(0) = 2$$

In to an integral equation.

Solution: Integrating both sides of the given differential equation, we have

$$\int_0^t \{Y'''(u) + 8Y(u)\}du = \int_0^t \{3\sin u + cosu\}du$$

$$Y''(t) - Y''(0) + 8\int_0^t Y(u)du = -3cost + 3cos0 + 2sint - 2sin0$$

$$This\ becomes, using\ \ Y(0) = 0\ \ , \quad Y'(0) = -1\ \ , \quad Y''(0) = 2$$

$$Y''(t) + 8\int_0^t Y(u)du = 2\ sint - 3\ cost + 5$$

Integrating again from 0 to t as before, we get

$$Y'(t) - Y'(0) + 8\int_0^t (t-u)Y(u)du = -2cost + 2cos0 - 3sint + 3sin + 5t$$

$$Y'(t) + 8\int_0^t (t-u)Y(u)du = -2\ cost - 3\ sint + 5t + 1$$

Integrating again from 0 to t as before, we get

$$Y(t) - Y(0) + 8\int_0^t \frac{(t-u)^2}{2!}\ Y(u)du = -2\,sin\,t + 3\,cos\,t - 3 + \frac{5}{2}t^2 + t$$

$$Y(t) + 4\int_0^t (t-u)^2\ Y(u)du = 3\,cos\,t - 2\,sin\,t - 3 + \frac{5}{2}t^2 + t$$

3.2 Transformation an integral equation of Volterra to ordinary differential equations

First method 3.2.1

Example 1: convert the integral equation

$$y(t) = 3t - 4 - 2sint + \int_0^t \{(t-u)^2 - 3(t-u) + 2\} Y(u)\, du$$

In to a differential equation.

Solution:

$$y(t) = 3t - 4 - 2sint + \int_0^t \{(t-u)^2 - 3(t-u) + 2\} Y(u)\, du \qquad \ldots(1)$$

Differenting both sides of the given integral equation

$$y'(t) = 3 - 2\cos t + \int_0^t 2(\mathrm{t}-\mathrm{u})\mathrm{Y}(\mathrm{u})\mathrm{du} - 3\int_0^t Y(u)\ du + 2Y(t) \qquad \ldots(2)$$

$$y''(t) = 2\sin t + 2\int_0^t \mathrm{Y}(\mathrm{u})\mathrm{du} - 3Y(t) + 2Y'(t) \qquad \ldots(3)$$

$$y'''(t) = 2\cos t + 2\mathrm{Y}(\mathrm{t}) - 3Y'(t) + 2Y''(t)$$

$$y'''(t) - 2\mathrm{Y}(\mathrm{t}) + 3Y'(t) - 2Y''(t) = 2\cos t$$

$$The\ initial\ conditions \quad Let \quad t = 0$$

$$In\ equation(1) \quad we\ get \quad Y(0) = -4$$

$$In\ equation(2) \quad we\ get \quad Y'(0) = -7$$

$$In\ equation(3) \quad we\ get \quad Y''(0) = -2$$

Example 2: convert the integral equation

$$Y(t) = 5\cos t + \int_0^t (\mathrm{t}-\mathrm{u})\, Y(u)\, du$$

In to a differential equation.

Solution:

$Y(t) = 5\cos t + \int_0^t (\mathrm{t} - \mathrm{u})\, Y(u)\, du \qquad \ldots (1)$

Differenting both sides of the given integral equation

$Y'(t) = -5\sin t + \int_0^t Y(u)\, \mathrm{du} \qquad \ldots (2)$

$Y''(t) = 5\cos t + Y(t)$

$Y'(t) - Y(t) = -5\cos t$

The initial conditions let $t = 0$

In equation (1) we get $Y(0) = 5$

In equation (2) we get $Y'(0) = 0$

Example 3: Convert the integral equation

$Y(t) = t^2 - 3t + 4 - 3\int_0^t (t - u)^2\, Y(u)\, du$

In to a differential equation.

Solution:

$Y(t) = t^2 - 3t + 4 - 3\int_0^t (t - u)^2\, Y(u)\, du \qquad \ldots (1)$

Differenting both sides of the given integral equation

$Y'(t) = 2t - 3 - 3\int_0^t 2(\mathrm{t} - \mathrm{u})\mathrm{Y}(\mathrm{u})\mathrm{du} \qquad \ldots (2)$

$Y''(t) = 2 - 6\int_0^t Y(u)\, \mathrm{du} \qquad \ldots (3)$

$Y'''(t) = -6Y(t)$

$Y'''(t) + 6Y(t) = 0$

The initial conditions let $t = 0$

In equation (1) we get $Y(0) = 4$

Second method 3.2.2

Example 1: convert the integral equation

$$F(x) = \int_{\sin x}^{\cos x} \sqrt{1 + x^3}\ dt$$

In to a differential equation

Solution:

$h(x) = \cos(x) \quad , \quad g(x) = \sin(x)$

$f(x,t) = \sqrt{1 + t^3}$

$f(x, h(x)) = \sqrt{1 + \cos t^3}$

$f(x, g(x)) = \sqrt{1 + \sin x^3} \qquad \frac{dh(x)}{dx} = -\sin(x) \quad , \quad \frac{dg(x)}{dx} = \cos(x)$

$$\int_{\sin x}^{\cos x} \frac{\partial}{\partial x}\left(\sqrt{1 + t^3}\right) dt = 0$$

Therefore

$F'(x) = \sqrt{1 + \cos x^3} * (-\sin x) - \sqrt{1 + \sin x^3} * (\cos x)$ **… (By Leibniz Integral Rule)**

Example 2: convert the integral equation

$$F(x) = \int_{x}^{x^2} (x - t) \cos t \ \ dt$$

In to a differential equation.

Solution:

$h(x) = x^2$, $g(x) = x$

$f(x,t) = (x - t)\, cos(t)$

$f(x, h(x)) = (x - x^2)\, cos\,(x^2)$

$f(x, g(x)) = (x - x)\, cos(x) = 0$, $\frac{dh(x)}{dx} = 2x$, $\frac{dg(x)}{dx} = 1$

$$\int_x^{x^2} \frac{\partial}{\partial x}(x\cos(t) - t\cos(\,t))dt = \int_x^{x^2} \cos(t)\, dt$$

$= \sin(\,x^2) - \sin(\,x)$

Therefore

$F'(x) = 2x^2\, cos\, x^2 \; - 2x^3\, cos\, x^2 + sinx^2 - sin\, x \ldots$ **(By Leibniz Integral Rule)**

Reference

Dr.Lamyaa, H., 2012. Numerical Solution Of Intrgral Equation. *Al-Mustansiriyah Journal Of Science,* 23(2), pp. 1-6.

Dr.Ramesh, K., 1995. Integral Equatin. *Directorate of Distance Education Guru Jambheshwar Universty*, 17(7), pp. 1-290.

Jean, P., 2016. *Galois' Theory of Algebraic Equations.* 2nd ed. New York: World Scientific.

M.Rahman, 2007. *Integral Equatins and their Applications.* Billerica: WIT Press.

Mohammed, K., 2015. *A Friendly Introduction Diffrential Equations.* San Bernardino: Create Space.

Mostefa, N. & G., S., 2016. Integral Equations and their Relationship to Differential Equations with Initial Conditions. *General Letters in Mathematics,* 1(1), pp. 23-31.

پوختە

هاوکێشە تەواوکاری و داتاشراوەکان گرنگییەکی بنەڕەتییان هەیە لە شیکاری کارایی و کێشە پراکتیکییەکان،و چەندین جۆری توێژینەوەی زانستی.

بەڵام شیکارکردنی هاوکێشەی داتاشراو کە هاوکۆڵکەی نەگۆڕیان تیایە ئاسانن، بەڵام شیکارکردنی ئەم هاوکێشانە بە هاوکۆڵکەی گۆڕاو بە شێوەیەکی کرداریی قورسە یان مەحاڵە لە زۆربەی حاڵەتەکان.

ئەم کارە شێوازێکی شیکاری دەخاتە ڕوو کە هاوکێشە داتاشراوەکان بە مەرجە سەرەتاییەکانەوە دەگۆڕێت بۆ هاوکێشەکانی ڤۆلتێرا لە جۆری دووەم، ڕێگەی کارا بۆ شیکاری نیومێریکی بە نزیکەی بۆ ئەم جۆرە هاوکێشانە.

الخلاصة

المعادلات التكاملية والتفاضلية لها أهمية أساسية في التحليل الوظيفي و المشاكل العملية والعديد من مجالات البحث العلمي.

ومع ذلك ، فإن حل المعادلات التفاضلية ذات المعاملات الثابتة سهل ، ولكن حل هذه المعادلات ذات المعاملات المتغيرة أمر صعب أو مستحيل عمليا في جزء أكبر من الحالات.

يقدم هذا العمل طريقة تحليلية تقوم بتحويل المعادلات التفاضلية بشروط أولية إلى معادلات فولتيرا من النوع الثاني ، طرق فعالة للحل العددي التقريبي لهذه المعادلات.