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Salahaddin University-Erbil

Approximation of System of Initial Value Problem

Research Project

Submitted to the Department of Mathematics in partial fulfillment of the requirements for the degree of BSc. in MATHEMATIC

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Certification of the Supervisor

I certify that this work was prepared under my supervision at the Department of Mathematics / College of Education / Salahaddin University- Erbil in partial fulfillment of the requirements for the degree of Bachelor of philosophy of Science in Mathematics.

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Abstract

In this research we study three important numerical methods in mathematics: Taylor series, Euler method, and Runge-Kutta method. The Taylor series represents functions as an infinite sum, while Euler method and Runge-Kutta method are used to solve ordinary differential equations. These methods have wide-ranging applications in various fields and are essential tools for researchers and professionals. Finally, some examples were given to illustrate three methods.

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CHAPTER ONE

INTRODUCTION

Today, numerical analysis is a vast and diverse field, with applications in almost every area of science, engineering, and technology. Some of the most important areas of research in numerical analysis today include the development of fast algorithms for solving linear and nonlinear systems of equations, the development of efficient methods for solving partial differential equations, and the study of numerical stability and error analysis. Additionally, there is a growing interest in the development of high-performance computing techniques for solving very large-scale numerical problems, as well as in the use of machine learning and other data-driven techniques in numerical analysis. (Kendall, 1978)

To solve an initial value problem, we need to find a particular solution that satisfies both the differential equation and the initial condition. The initial condition provides a starting point for solving the differential equation, and helps to determine the value of the arbitrary constants that appear in the general solution. (Kendall, 1978)

There are several methods that can be used to solve initial value problems, including separation of variables, integrating factors, and using series solutions. In some cases, numerical methods such as Euler's method or the Runge-Kutta method may be used to approximate the solution. (Coralie, et al., 2021) (Md.Amirul, 2015)

Once a particular solution is found, it can be used to make predictions about the behavior of the system described by the differential equation. This can be especially useful in fields such as physics, engineering, and economics, where differential equations are commonly used to model physical and economic systems. (Coralie, et al., 2021) (Md.Amirul, 2015)

Overall, initial value problems are an important part of calculus and differential equations, and have a wide range of applications in science and engineering. (Coralie, et al., 2021) (Md.Amirul, 2015)

In other words, an initial value problem is a differential equation that has a given value for the dependent variable (usually denoted as y) and its derivative at a specific point, which is referred to as the initial condition. The goal is to find a solution to the differential equation that satisfies this initial condition. (Coralie, et al., 2021) (Md.Amirul, 2015)

To solve an initial value problem, one typically applies techniques from differential calculus and integral calculus to obtain an explicit solution for the differential equation. This solution will be expressed in terms of the unknown function, y, and any constants that appear in the equation. The initial condition is then used to determine the values of these constants. (Coralie, et al., 2021) (Md.Amirul, 2015)

For example, consider the initial value problem given by the differential equation:

$$\frac{dy}{dx} = x + 2y$$

with the initial condition $y(0) = 1$. To solve this initial value problem, one can use the method of separation of variables, which involves rewriting the equation as:

$$\frac{dy}{(x + 2y)} = dx$$

Integrating both sides with respect to their respective variables, we obtain:

$$\frac{1}{2}\ln|x + 2y|| = x + C$$

where C is a constant of integration. Solving for y, we have:

$$y = -x \pm \frac{\sqrt{x^2 + 4e^{2c}}}{2y}$$

Using the initial condition $y(0) = 1$, we can determine the value of the constant C:

$$1 = -(0) \pm \frac{\sqrt{0^2 + 4e^{2c}}}{2}$$

$$e^{2c} = 1$$

$$C = \ln\frac{1}{2}$$

Substituting this value of C into the expression for y, we obtain the solution to the initial value problem:

$$y = -x + \frac{\sqrt{x^2 + 1}}{2}$$

This solution satisfies the differential equation $\frac{dy}{dx} = x + 2y$ and the initial condition $y(0) = 1$. (Coralie, et al., 2021) (Md.Amirul, 2015)

CHAPTER TWO

BACKGROUND

Definition 2.1: (Joel, 2021)

Initial value problem, an IVP is deferential equation together with a place for a solution to start. They are often written

$$y' = f(x, y)$$

$$y(a) = b$$

Where (a, b) is the point the solution $y(x)$ must go through. The initial value problem:

Consider the ordinary differential equation

$$\frac{dy}{dt} = f\big(t, y(t)\big), y(t_0) = y_0$$

Where f is a function from $\mathbb{R}^{N+1}$in to $\mathbb{R}^N$ for some $n > 0$ $(if\ N = 1$, then we have a scalar equation; otherwise, a vector equation), t_0 is a given scalar value, often taken to be $t_0 = 0$, and known as the initial point; and y_0 is known vector in $\mathbb{R}^N$, known as the initial value. We want to find the unknown function $y(t)$. In the sense that $y'(t) - f\big(t, y(t)\big) = 0$

For all $t > t_0$, and $y(t_0) = y_0$.

Definition 2.2: (Charles, 2012)

initial condition, the state of a time-dependent dynamical system, for instance, an NWP model, at a given time used to start a forecast of the future state of the system.

Definition 2.3: (George B. Thomas, et al., 2014)

A function f from a set D to a set Y is rule that assigns a unique (single) element $f(x)\epsilon Y$ to each element $x\epsilon D$

Definition 2.4: (George B. Thomas, et al., 2014)

Let c be a real number on the $x - axis$

The function f is continues at c if $\lim_{n\to c} f(x) = f(c)$

The function f is right-continues at c if $\lim_{n\to c^+} f(x) = f(c)$

The function f is left-continues at c if $\lim_{n\to c^-} f(x) = f(c)$

Definition 2.5: (George B. Thomas, et al., 2014)

The derivative of function f at a point x_0. Denoted $f'(x_0)$. Is $f'(x_0) \lim_{h\to 0} \frac{f(x_0+h)-\mathrm{f}(x_0)}{h}$

Provide this limit exists.

Definition 2.6: (George B. Thomas, et al., 2014)

Given a sequence of numbers $\{a_n\}$, an expression of the form

$a_1 + a_2 + a_3 + \cdots + a_n + \cdots$

Is an infinite series. The number $a_n$is the nth terms of the series. The sequence $\{s_n\}$defined by

$$s_1 = a_1$$

$$s_2 = a_1 + a_2$$

$$\vdots$$

$$s_n = a_1 + a_2 + \cdots + a_n = \sum_{k=1}^{n} a_k$$

is the sequence of partial sums of the series, the number s_n , being the nth partial sum. If the sequence of partial sums converges to a limit L, we say that the series converges and that its sum is L. In this case, we also write

$$a_1 + a_2 + \cdots + a_n + \cdots = \sum_{k=1}^{n} a_k$$

If the sequence of partial sums of the series does not converge, we say that the series diverges.

Definition 2.7: (Homles, 2000)

A Boundary value problem is a system of ordinary differential equations with solution and derivative values specified at more than one point. Most commonly, the solution and derivatives are specified at just two points (the boundaries) defining a two-point boundary value problem. $y'' = f(x, y, y'), \quad a \le x \le b \quad y(a) = \alpha$ and $y(b) = \beta$

Taylor series method 2.8: (James, 2013)

Consider the first order differential equation

$\frac{dy}{dx} = f(x, y)$, $y(y_0) = y_0$ … (1)

If $y(x)$ is the exact solution of (1) then $y(x)$ can be expanded into a Tylor's series about point $x = x_0$ as

$y(x) = y_0 + (x - x_0)y'_0 + \frac{(x-x_0)^2}{2!}y''_0 + \frac{(x-x_0)^3}{3!}y'''_0 + \cdots$ (2)

Differentiating (1) w.r.t x we get

$$y'' = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial x}\frac{\partial y}{\partial x} = \frac{\partial f}{\partial x} + f\frac{\partial f}{\partial y} = f_x + f_y f$$

$$y''' = f_{xx} + f_{xy}f + f_{yx} + f_{yy}f^2 + f_x f_y + f_y^2 f$$

and so on.

Putting $x = x_0$ and $y = y_0$ in expressions for y', y'', y''',… and substituting them in Equ (2), we get a power series for $y(x)$ in powers of $(x - x_0)$.

i.e., $y(x) = y_0 + (x - x_0)y'_0 + \frac{(x-x_0)^2}{2!}y''_0 + \frac{(x-x_0)^3}{3!}y'''_0 + \cdots$ (4)

putting $x = x_0 + h$ in (4), we get

$$y_{1=}y(x_0) = y_0 + y_0'h + \frac{1}{2!}y_0''h^2 + \frac{1}{3!}y_0'''h^3 + \frac{1}{4!}y_0^4h^4 + \ldots \quad (5)$$

Here y_0,y_0', y_0'', y_0''', y_0^4, ... can be found by using (1) and successive differentiations (3) at $x = x_0$. The series (5) can be truncated at any stage if h_1 is small.

After obtaining y_1, we can calculate y_1,y_1', y_1'', y_1''', y_1^4, ... from (1) at $x = x_0 + h$. Now, expanding $y(x)$ by Taylor's series about $x = x_1$, we get

$$y_2 = \ y_1 = hy_1 + \frac{h^2}{2!}y_1'' + \frac{h^3}{3!}y_1''' + \frac{h^4}{4!}y_1^4 + \ldots \quad (5)$$

Proceeding on, we get

$$y_{n+1} = \ y_n = hy_n + \frac{h^2}{2!}y_n'' + \frac{h^3}{3!}y_n''' + \frac{h^4}{4!}y_n^4 + \ldots \quad (5)$$

Taylor series Method for Simultaneous first order O.D. 2.9: (James, 2013)

The Simultaneous first order differential equation of the form:

$\frac{dy}{dx} = f_1(x, y, z)$ and $\frac{dz}{dx} = f_2(x, y, z)$

With initial values y$(x_0) = y_0$ and z$(x_0) = z_0$

To solve this system of equations at interval h ,the incrreaments in y and z are obtained by using the formula:

$$y_1 = y_0 + hy_0' + \frac{h^2}{2!}y_0'' + \frac{h^3}{3!}y_0''' + \cdots \quad \text{and}$$

$$z_1 = z_0 + hz_0' + \frac{h^2}{2!}z_0'' + \frac{h^3}{3!}z_0''' + \cdots$$

Euler's method 2.10: (Md.Amirul, 2015)

Euler's method is simplest one-step method. It is basic explicit method for numerical integration of ordinary differential equations. Euler proposed his method for initial value problem (IVP) in 1768. It is first numerical method for solving IVP and serves to illustrate the concepts involved in the advanced methods. It is important to study because the error analysis is easier to understand. The general formula for Euler approximation is

$$y_{n+1}(x) = y_n(x) + hf(x_n + y_n), n = 0,1,2,3, \dots$$

Leonhard Euler (1707-1783). (James, 2013)

Consider the equation $\frac{dy}{dx} = f(x, y)$

Given that $y(x_0) = y_0$. Its curve solution through $p(x_0, y_0)$ is shown dotted in figure. Now we have to find the ordinate of any other Q on this curve.

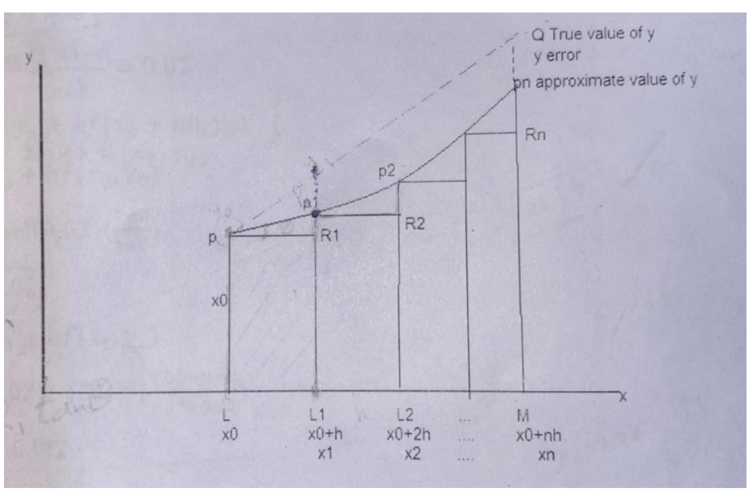

Figure 2.1.3 Curve Solution of $p(x_0, y_0)$

Let us divide LM into n sub-intervals each of width h at L_1, L_2, L_3 …. So that h is quite small. In the interval LL_1, we approximate the curve by tangent at p. if the ordinate through L_1 meets this tangent in $p(x_0 + h, y_1)$, then

$$y_1 = P_1L_1 = LP + R_1P_1 = y_0 + PR_1 \tan\theta = y_0 + h(\frac{dy}{dx})_p = y_0 + hf(x_0, y_0)$$

Let P_1Q_1 be the curve of solution of (1) through p_1 and let its tangent at p_1 meet the ordinate through L_2 in P_2 $(x_0 + 2h, y_0)$. Then

$$y_2 = y_1 + hf(x_0 + h, y_1)$$

Repeating this process n times, we finally reach an approximation MP_n of MQ given by

$$y_n = y_{n-1} + hf(x_0 + (n-1)h, y_{n-1})$$

This is Euler's method of finding an approximate solution of (1).

Geometrically it is an approximation of the curve of $y(x)$ by polygon whose first side is the curve at x_0.

Runge Kutta Method 2.11: (Md.Amirul, 2015)

Runge Kutta Method, this method was devised by two Germain mathematicians, Runge about 1894 and extended by Kutta a few years later. The Runge Kutta Method is most popular because it is quite accurate, stable and easy to program. This method is distinguished by their order in the sense that they agree with Taylor's series solution up to terms of h' where r is the order of the method, it do not demand prior computational of higher derivatives of $y(x)$ as in Tylor's series method. The fourth order Runge Kutta method (RK4) is widely used for solving initial value problem

(IVP) for ordinary differential equation (ODE). The general formula for Runge Kutta approximation is

$$y_{n+1}(x) = y_n(x) + hf(x_n + y_n), n = 0,1,2,3, \dots$$

$$k_1 = hf(x, y), k_2 = hf\left(x + \frac{h}{2}, y + \frac{k_1}{2}\right), k_3 = hf\left(x + \frac{h}{2}, y + \frac{k_2}{2}\right), k_4 = hf(x + h, y + k_3).$$

CHAPTER THREE

EXAMPLES OF TAYLOR SERIES, RUNGE KUTTA METHOD AND EULER METHOD

Example 3.1:

By using Taylor's series find the value of $y\ at\ x = 0.1$ to five places of decimations form

$dy/dx = x^2y - 1 \dots \dots .(1)\ , y(0) = 1$

Solution:

For this example

$h = x - x0\ = 0.1 - 0 = 0.1$

$$y_1 = y_0 + y_0'h_1 + \frac{1}{2!}y_0''h_1^2 + \frac{1}{3!}y_0'''h_1^3 + \frac{1}{4!}y_0^4h_1^4 + \cdots$$

$$y_{i+1} = y_i + 0.1y_i' + 0.05y_i'' + 0.00016y_i''' + 0.0000041y^4 + \cdots$$

The derivatives of equation (1),

$y' = x^2y - 1$

$y'' = x^2y' + 2xy$

$y''' = x^2y'' + 4xy' + 2y$

$y^4 = x^2y''' + 6xy'' + 6y'$

Using the given initial value:

$y_0' = -1$

$y_0'' = 0$

$y_0''' = 2$

$y_0^4 = -6$

Putting these values in the Taylor's series of y_1:

$y_1 = 1 + 0.1(-1) + 0.005(0) + 0.00016(2) + 0.00000(-6) + \cdots$

$y_1 = 0.90033$

Example 3.2:

Solve by Taylor series method of third order equation $\frac{dy}{dx} = \frac{x^3+xy^2}{e^x}$, $y(0) = 1$ for y at $x = 0.1x$, $x = 0.2$ and $x = 0.3$

Solution:

We have $y' = (x^3 + xy^2)e^{-x}$; $y'(0) = 0$

Differentiating successively and substituting $x = 0$, $y = 1$.

$y'' = (x^3 + xy^2)(-e^{-x}) + (3x^2 + x.2.y.y')e^{-x}$

$= (-x^3 - xy^2 + 3x^2 + y^2 + 2xyy')(e^{-x})$;$y''(0) = 1$

$y''' = (-x^3 - xy^2 + 3x^2 + y^2 + 2xyy')(-e^{-x}) + \{-3x^2 - (y^2 + x.2.y.y') +$
$6x + 2yy' + 2[yy' + x(y'^2 + yy'')]\}(e^{-x}) \quad y'''(0) = -2$

Putting these values in the Taylor's series, we have:

$$y(0) = y(0) + xy'(0) + \frac{x^2}{2!}y''(0) + \frac{x^3}{3!}y'''(0) + \cdots$$

$$= 1 + x(0) + \frac{x^2}{2!}(1) + \frac{x^3}{3!}(-2) + \cdots$$

$$= 1 + \frac{x^2}{2} - \frac{x^3}{6} + \cdots$$

Hence

$$y(0.1) = 1 + \frac{(0.1)^2}{2} - \frac{(0.1)^3}{6} + \cdots = 1.005$$

$$y(0.2) = 1 + \frac{(0.2)^2}{2} - \frac{(0.2)^3}{6} + \cdots = 1.017$$

$$y(0.3) = 1 + \frac{(0.3)^2}{2} - \frac{(0.3)^3}{6} + \cdots = 1.036$$

Example 3.3:

Find $y(0.3)$ and $z(0.3)$ given $\frac{dy}{dx} = x + z$, $\frac{dx}{dz} = x - y^2$ and $y(0) = 22$

,$z(0) = 1$, $h = 0.1$

Solution:

$h = 0.1$

$$y(0.1) = y_0 + hy_0' + \frac{h^2}{2!}y_0'' + \frac{h^3}{3!}y_0''' + \cdots$$

$y' = x + z$

$y'' = 1 + z'$

$y''' = 1 - 2y$

$y_0' = 1$

$y_0'' = -3$

$y_0''' = -3$

$$y(0.1) = 2 + (0.1)(1) + \frac{(0.1)^2}{2!}(-3) + \frac{(0.1)^3}{3!}(-3) + \cdots$$

$= 2.08295$

$$z(0.1) = z_0 + hz_0' + \frac{h^2}{2!}z_0'' + \frac{h^3}{3!}z_0''' + \cdots$$

$z' = x - y^2$

$z'' =$ 1-2yy'

$z''' = -2yy''$

$z_0' = -4$

$z_0'' = -3$

$z_0''' = 10$

$$z(0.1) = 1 + (0.1)(-4) + \frac{(0.1)^2}{2!}(-3) + \frac{(0.1)^3}{3!}(10) + \cdots$$

$= 0.58666$

Example 3.4:

Given $\frac{dy}{dx} = x + z$, $\frac{dx}{dz} = \frac{y-x}{y+x}$ with the initial condition $y_0 = 1$, $x_0 = 0$ find y

For $x = 0.1$,by Euler's method.

Solution:

We take n=5

$$h = \frac{x - x_0}{n} = \frac{0.1 - 0}{5} = 0.02$$

$$y_{n+1} = y_n + hf(x_0 + nh, y_n)$$

$$y_1 = y_0 + hf(x_0, y_0)$$

$$= 1 + (0.02)\left(\frac{1 - 0}{1 + 0}\right)$$

$$= 1.02$$

$$y_2 = y_1 + hf(x_1, y_1)$$

$$= 1.02 + (0.02)(\frac{1.02 - 0.02}{1.02 + 0.02})$$

$$= 1.0392$$

$$y_3 = y_2 + hf(x_2, y_2)$$

$$= 1.0392 + (0.02)(\frac{1.0392 - 0.02}{1.0392 + 0.02})$$

$$= 1.0577$$

$$y_4 = y_3 + hf(x_3, y_3)$$

$$= 1.0577 + (0.02)(\frac{1.0577 - 0.02}{1.0577 + 0.02})$$

$$= 1.0738$$

$$y_5 = y_4 + hf(x_4, y_4)$$

$$= 1.0738 + (0.02)(\frac{1.0738 - 0.02}{1.0738 + 0.02})$$

$= 1.0910$

Hence the required approximation value of $y = 1.0910$

Example 3.5:

Use Euler's method with $h = 0.1$ to find approximate values for the solution of the initial value problem:

$$y' + 2y = x^3 e^{-2x}, y(0) = 1 \quad at\ x = 0.1, 0.2, 0.3.$$

Solution: We rewrite equation as

$$y' = -2y + x^3 e^{-2x}, y(0) = 1$$

Which is of the form equation of Euler, with

$$f(x, y) = -2y + x^3 e^{-2x}, x_0 = 0\ and\ y_0 = 1$$

Euler's method yields

$y_1 = y_0 + hf(x_0, y_0)$

$= 1 + (0.1)f(0,1) = 1 + (0.1)(-2) = 0.8$

$y_2 = y_1 + hf(x_1, y_1)$

$= 0.8 + (0.1)f(0.1.0.8) = 0.8 + (0.1)(-2(0.8) + (0.1)^3 e^{-0.2})$

$= 0.64008187$

$y_3 = y_2 + hf(x_2, y_2)$

$= 0.64008187 + +(0.1)(-2(0.64008187) + (0.2)^3 e^{-0.4}$

$= 0.51260175$

Example 3.6:

Find the value of k_1 by Runge-Kutta method of fourth order if $dy/dx = 2x + 3y^2$ and $y(0.1) = 1.1165, h = 0.1$.

Solution:

Given,

$dy/dx = 2x + 3y^2$ and $y(0.1) = 1.1165, h = 0.1$

So, $f(x, y) = 2x + 3y2$

$x_0 = 0.1, y_0 = 1.1165$

By Runge-Kutta method of fourth order, we have

$k_1 = hf(x_0 . y_0)$

$= (0.1)\, f(0.1, 1.1165)$

$= (0.1)[2(0.1) + 3(1.1165)^2]$

$= (0.1)\, [0.2 + 3(1.2465)]$

$= (0.1)(0.2 + 3.7395)$

$= (0.1)(3.9395)$

$= 0.39395$

Example 3.7:

Consider an ordinary differential equation $dy/dx = x^2 + y^2, y(1) = 1.2$ Find $y(1.05)$ using the fourth order Runge- Kutta method.

Solution:

Given,

$dy/dx = x^2 + y^2, y(1) = 1.2$

So, $f(x, y) = x^2 + y^2$

$x_0 = 1$ and $y_0 = 1.2$

Also, $h = 0.05$

Let us calculate the values of k_1, k_2, k_3 and k_4.

$k_1 = hf(x_0, y_0)$

$= (0.05)[{x_0}^2 + {y_0}^2]$

$= (0.05)[(1)^2 + (1.2)^2]$

$= (0.05)(1 + 1.44)$

$= (0.05)(2.44)$

$= 0.122$

$k_2 = hf[x_0 + (1/2)h, y_0 + (1/2)k_1]$

$= (0.05)[f(1 + 0.025, 1.2 + 0.061)]\{since\ h/2 = 0.05/2$
$= 0.025\ and\ k_1/2 = 0.122/2 = 0.061\}$

$= (0.05)[f(1.025, 1.261)]$

$= (0.05)[(1.025)^2 + (1.261)^2]$

$= (0.05)(1.051 + 1.590)$

$= (0.05)(2.641)$

$= 0.1320$

$k_3 = hf[x_0 + (1/2)\, h, y_0 + (1/2)\, k_2]$

$= (0.05)\, [f(1 + 0.025, 1.2 + 0.066)]\ \{since\ h\ /\ 2 = 0.05/2$

$= 0.025\ and\ k_2\ /\ 2 = 0.132/2 = 0.066\}$

$= (0.05)[f(1.025, 1.266)]$

$= (0.05)[(1.025)^2 + (1.266)^2]$

$= (0.05)(1.051 + 1.602)$

$= (0.05)(2.653)$

$= 0.1326$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$= (0.05)\, [f(1 + 0.05, 1.2 + 0.1326\,)]$

$= (0.05)[f(1.05, 1.3326)]$

$= (0.05)[\, (1.05)^2 + (1.3326)^2\,]$

$= (0.05)(1.1025 + 1.7758)$

$= (0.05)\, (2.8783)$

$= 0.1439$

By RK4 method, we have;

$y_1 = y_0 + (1/6)\, (k_1 + 2k_2 + 2k_3 + k_4)$

$y_1 = y(1.05)\, y_0 + (1/6)\, (k_1 + 2k_2 + 2k_3 + k_4)$

By substituting the values of y_0, k_1, k_2, k_3 and k_4, we get,

$y(1.05) = 1.2 + (1/6)\, [0.122 + 2(0.1320) + 2(0.1326) + 0.1439]$

$= 1.2 + (1/6)\ (0.122\ + 0.264\ + 0.2652\ +\ 0.1439)$

$= 1.2 + (1/6)\ (0.7951)$

$= 1.2 + 0.1325$

$= 1.3325$

Example 3.8: Apply Range-Kutta method to find an approximate value of y for $x =\ 0.2$ in steps of 0.1, if$\frac{dy}{dx} = x + y^2$, given that $y\ =\ 1$, where $x = 0$.

Solution: Here we take $h = 0.1$ and carry out the calculations in two steps.

Step 1.$x_0 = 0, y_0 =\ 1, h\ =\ 0.1$

$k_1\ =\ hf\ (x_0\ , y_0)\ =\ 0.1f(0,1)\ =\ 0.1000$

$k_2\ =\ hf\ (x_0\ +\ 1/2h, y_0\ +\ 1/2k_1)\ =\ 0.1f(0.05,1.1)\ =\ 0.1152$

$k_3\ =\ hf\ (x_0 + 1/2h, y_0 +\ 1/2k_2)\ =\ 0.1f(0.05,1.1152)\ =\ 0.1168$

$k_4\ =\ hf\ (x_0 + h, y_0 + k_3)\ = 0.1f(0.1.1,1168)\ =\ 0.1347$

$k\ = 1/6\ (k_1\ +\ 2k_2\ +\ 2k_3\ +\ k_4)$

$= 1/6(0.1000 +\ 0.2304\ + 0.2336\ + 0.1347)\ =\ 0.1165$

Giving $y(0.1) = y_0 + k = 1.1165$

Step II. $x_1\ =\ x_0\ +\ h\ =\ 0.1, y_1 = 1.1165, h\ =\ 0.1$

$k_1\ =\ hf\ (x_1, y_1)\ =\ 0.1f(0.1,1.1165)\ =\ 0.1347$

$k_2\ = hf\ (x_1, 1/2h, y_1 + 1/2k_1) = 0.1f(0.15,1.1838) = 0.1551$

$k_3\ =\ hf\ (x_1 + 1/2h, y_1\ + 1/2k_2)\ =\ 0.1f(0.15,1.194)\ =\ 0.1576$

$k_4 = hf\ (x_1 + h, y_1 + k_3) = 0.1f(0.2,1.1576) = 0.1823$

$k = 1/6(k_1 + 2k_2 + 2k_3 + k_4) = 0.1571$

Hence $y(0.2) = y_1 + k = 1.2736.$

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پوختە

لەم توێژینەوەیەدا هەڵساین بە باسکردنی سێ ڕێگای ژمارەیی لە بابەتی بیرکاری ئەوانیش

Series, Euler Method, Runge Kutta Method Taylor

بۆ شیکارکردنی هاوکێشەکانی جیاکاری ئاسایی. ئەم رێگایانە لە زۆر

بواری جۆراوجۆر کاریان پێ کراوە لەکۆتایدا بەهەندێك نموونە ئەم ڕێگایەمان روونکردیتەوە.

خلاصة

في هذا بحث قمنا بدراسة ثلاثة طرق (ميثود) لحل معادلات سلسلة تايلور و طريقة أويلر و طريقة رونج-كوتا لحل معادلات عادية. هذه أساليب لها تطبيقات واسعة النطاق في مختلف المجالات و هي أدوات أساسية للباحثين و المهنيين.