زانکۆی سەلاحەدین-هەولێر

Salahaddin University-Erbil

An optimal spline method for solving numerical problem

Research project:

Submitted to the department of Mathematics in partial fulfillment of the requirements for the degree of BSc. in MATHEMATICS

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April-2023

Certification of the supervisors

I certify that this work was prepared under my supervision at the Department of Mathematics/College of Education/Salahaddin University-Erbil in partial fulfillment of the requirements for the degree of Bachelor of philosophy of Science in Mathematics.

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Scientific grade: Assist. Professor

Date: /4/ 2023

In view of the available recommendations, I forward this work for debate by the examining committee.

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Scientific grade: Assist. Professor

Chairman of the Mathematics Department

Date: /4/ 2023

Acknowledgment

Primarily, I would like to thanks my god for helping me to complete this research with success.

Then I would like to express special of my supervisor Dr. Ivan S. Latif

Whose valuable to guidance has been the once helped me to completing my research.

Words can only inadequately express my gratitude to my supervisor for patiently helping me to think clearly and consistently by discussing every point of this dissertation with me.

Also, I would like to thank my family, friend and library staff whose support has helped me to conceive this research

Abstract

In this paper, we present the spline function of degree one transformation for the sine and cosine functions of degree one. We first introduce the concept of spline functions and its application. We then discuss the properties of the sine and cosine functions and how they can be transformed using spline functions of degree one. Finally, we provide examples and graphical representations to illustrate the effectiveness and applicability of these transformations.

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CHAPTER ONE

INTRODUCTION

1.1 Introduction:

The study of splines has been developed, mostly by French mathematicians. It is well known that interpolating cubic splines can be derived as the solutions of certain variational problems. In its simple form, the mathematical spline is continuous and has both a continuous first derivative and a continuous second derivative. Normally, for many important applications, this mathematical model of the draftsman's spline is highly realistic. (R.Champision:C.T.Lenard:T.M.Mills 1996) (J.H.Ahlbery:E.N.Nilson, The Theorey of Spline and Their Application 1997)

Let's begin with the simplest method linear interpolation. The idea is that we are given a set of numerical points and function values at these points. The task is to use the given set and approximate the function's value at some different points. That is, given x_i where $i = 0, \ldots, n - 1$ our task is to estimate $f(x)$ for $x_0 \geq x \leq x_n$. Of course we may require to go outside of the range of our set of points, which would require extrapolation (or projection outside the known function values).Almost all interpolation techniques are based around the concept of function approximation. Mathematically, the backbone is simple:

$$f(x) = y_i + \frac{y_{i+1} - y_i}{x_{i+1} - x_i}(x - x_i)$$

The above expression tells us that the value of function we are approximating at point x will be around y_i and that $x_i \geq x \leq x_{i+1}$. We can rewrite this expression as follows:

$$s_k(x) = a_k + b_k(x - x_k)$$

Where we have substituted y_i for a_k and $\frac{y_{i+1}-y_i}{x_{i+1}-x_i}$ for b_k. Our linear interpolation is now taking a form of linear regression around a_k.Linear interpolation is the most basic type of interpolations. It works remarkably well for smooth functions with sufficient number of points. However, because it is such a basic method, interpolating more complex. (Elena 2015)

Spline interpolation consist of piecewise polynomial interpolation. Thus, a spline function is a piecewise polynomial whose value and the value of some of its first derivatives coincide at the interpolation point. A spline is a simple mechanical device for drawing smooth curve, it is a slender flexible bar made of wood or some other elastic material. The spline is placed on the sheet of graph paper and held in place at various points by means of certain heavy objects (called "dogs" or "rots") such as to take the shape of the curve we wish to draw. (C.Brezinski and L.Wuytack 2001)

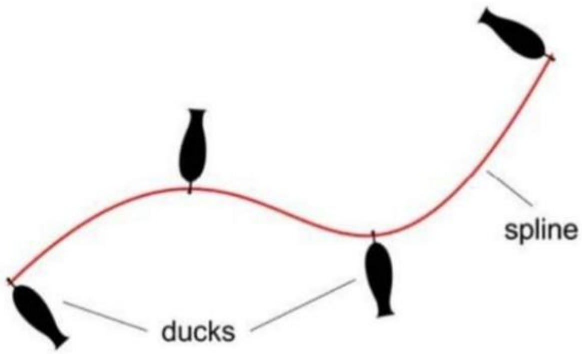

Figure 1.1 curve of spline mechanical

1.2 History of Spline Function:

Spline is the subject had been studied as early as 1906 by George David Birkhoff (1884-1944); its development really starts in 1946 with two papers by Issac Jacob Scheoenbery (1903-1990). (C.Brezinski:L.Waytack 2001)

The history of spline functions is rooted in the work of draftsmen, who often needed to draw a gently turning curve between points on a drawing. This process is called fairing and can be accomplished with a number of ad hoc devices, such as the French curve, made of plastic and presenting a number of curves of different curvature for the draftsman to select. Long strips of wood were also used, being made to pass through the control points by weights laid on the draftsman's table and attached to the strips. The weights were called ducks and the strips of wood were called splines, even as early as 1891. Spline functions have proved to be very useful in numerical analysis and statistics. (kincaid 2013) (R.Champision:C.T.Lenard:T.M.Mills 1996)

CHAPTER TWO

SPLINE FUNCTION

2 Spline function

It seems appropriate to begin a research on spline theory by defining a spline in its simplest and most widely used form, and also to indicate the motivation leading to this definition. For many years, longs, thin strips of wood or some other material have been used much like French curve by draftsmen to fair in a smooth curve between specified points. These strips or spline are anchored in place by attaching lead weights called "ducks" at points along the spline. By varying the points where the ducks are attached to the spline itself and the position of both the spline and the duck relative to the drafting surface, the spline can be made to pass through the specified points provided a sufficient number of ducks are used.
If we regard the draftsmen's spline as a thin beam, then the Bernoulli-Euler law

$M(x) \ = \ EI[1/R(x)]$

Is satisfied. Here $M(x)$ is the bending moment, E is young's modulus's, I is the geometric moment of inertial, and $R(x)$ is the radius of curvature of the elastic, i.e, the curve assumed by the deformed axis of the beam. For small deflection, $R(x)$ is replaced by $1/y''(x)$, where $y(x)$ denotes the elastic. Thus we have

$$y''(x) = (1/EI)M(x).$$

Since the ducks act effectively as simple supports, the variation of $M(x)$ between duck positions is linear.

The mathematical spline is the result of replacing the draftsmen's spline by it's elastic and then approximating the latter by a piecewise cubic(normally a different cubic between each pair of adjacent duck) with certain discontinuities of derivatives permitted at the junction points(the ducks) where two cubics join. (J.H.Ahlbery:E.N.Nilson, The Theorey of Spline and Their Application 1997)

2.1 Mathematical Definition of spline

A spline function is a function consisting of polynomial we are forced to write

$$S(x) = \begin{cases} S_0(x) & x \in [x_0, x_1] \\ S_1(x) & x \in [x_1, x_2] \\ \vdots & \vdots \\ S_{n-1}(x) & x \in [x_{n-1}, x_n] \end{cases}$$

First-degree spline 2.1.1:

A spline function is a function that consists of polynomial piece joined together with certain smoothness conditions. A simple example is the polygonal function (or spline of degree one) (kincaid 2013)

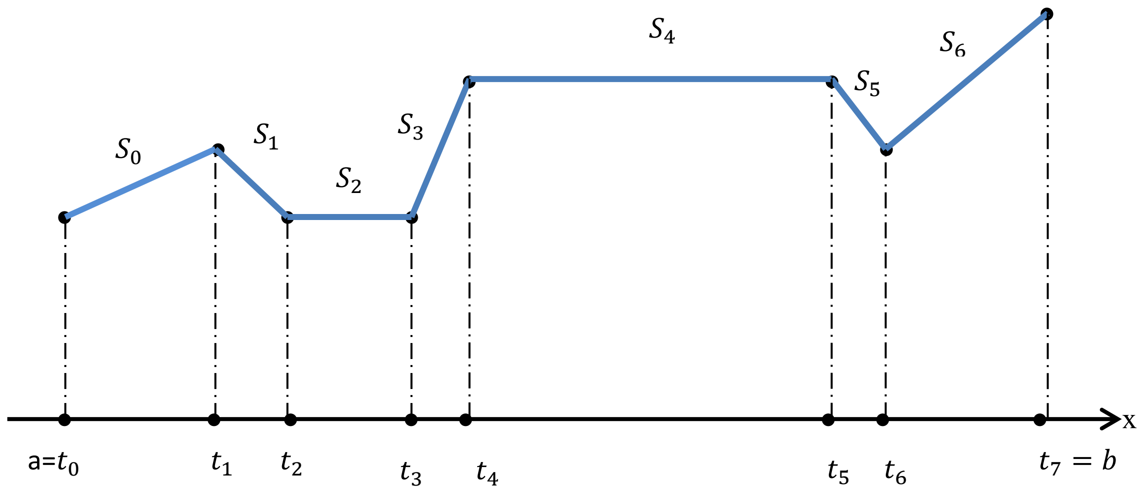

Figure 2.1 first-degree spline function

Whose piece are linear polynomial joined together to achieve continuity, as in figure2.1. The points $t_0, t_1, \dots, t_n$ at which the function changes it's character are termed knots in the theory of spline. Thus, the spline function shown in figure 2.1 has eight knot. Such a function appears somewhat complicated when defined in explicit terms. We are forced to write $\boldsymbol{S(x)}$ **piecewise linear**

$$S(x) = \begin{cases} S_0(x) & x \in [t_0, t_1] \\ S_1(x) & x \in [t_1, t_2] \\ \vdots & \vdots \\ S_{n-1}(x) & x \in [t_{n-1}, t_n] \end{cases} \qquad ----(2.1)$$

Where

$$\boldsymbol{S(x) = a_i x + b_i} \qquad -----(2.2)$$

Because each piece of $\boldsymbol{S}(\boldsymbol{x})$ is piecewise linear. If the knots $t_0, t_1, \dots, t_n$ were given and if the coefficients $a_0, b_0, a_1, b_1, \dots, a_{n-1}, b_{n-1}$ were all known, then the evaluation of $\boldsymbol{S}(\boldsymbol{x})$ at a specific $\boldsymbol{x}$ would proceed by first determining the interval the contains $\boldsymbol{x}$ and then using the appropriate linear function for that interval. If the function $\boldsymbol{S}$ defined by Equation (1) is continuous. We call it a first-degree spline. It is characterized by the following three properties. (kincaid 2013)

Property of first- degree spline2.1.1

A function S is called a spline of degree one if:

1- The domain of S is an interval$[a, b]$.
2- S is continuous on $[a, b]$.
3- There is partitioning of the interval $a = t_0 < t_1 < \cdots < t_n = b$ such that S is a linear polynomial on each subinterval $[t_i, t_{i+1}]$.

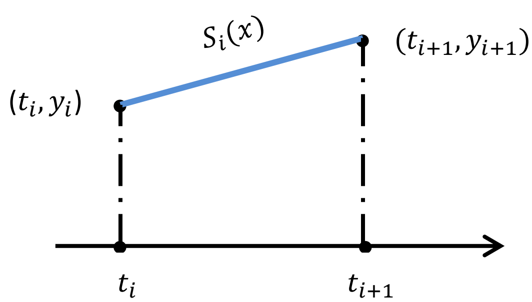

Figure 2.2 First-degree spline: linear $\boldsymbol{S_i}(\boldsymbol{x})$

$$s_i(x) = y_i + m_i(x - t_i) \qquad ----- (2.3)$$

On the interval $[t_i, t_{i+1}]$, where m_i is the SLOP of the line and is therefore given the formula

$$\mathrm{m_i} = \frac{\mathrm{y_{i+1}} - \mathrm{y_i}}{\mathrm{t_{i+1}} - \mathrm{t_i}} \qquad ----(2.4)$$

Substitute equation (2.4) in equation (2.3)

$$S_i(x) = y_i + (x - t_i)[\frac{y_{i+1}-y_i}{t_{i+1}-t_i}] \qquad ---- \ (2.5)$$

End function spline one. (kincaid 2013)

Example 2.1: Determine a spline function of degree one which interpolates the following data:

X	1	2	3	4
Y	0	1	0	1

Solution: since

$$S(x) = \begin{cases} S_0(x) & x \in [1,2] \\ S_1(x) & x \in [2,3] \\ S_2(x) & x \in [3,4] \end{cases}$$

$$S_0(x) = y_0 + (x - x_0)\frac{y_1-y_0}{x_1-x_0}$$

$$S_0(x) = 0 + (x - 1)\frac{1-0}{2-1}$$

$$S_0(x) = (x - 1)$$

$$S_1(x) = y_1 + (x - x_1)\frac{y_2-y_1}{x_2-x_1}$$

$$S_1(x) = 1 + (x-2)\frac{0-1}{3-2}$$

$$S_1(x) = -x + 3$$

$$S_2(x) = y_2 + (x - x_2)\frac{y_3 - y_2}{x_3 - x_2}$$

$$S_2(x) = 0 + (x-3)\frac{1-0}{4-3}$$

$S_2(x)$ =x-3

$$\lim_{x\to 2} S_0(x) = \lim_{x\to 2} S_1(x)\ ;\ x = 2$$

$$\left.\begin{array}{l} S_0(2) = (2-1) = 1 \\ S_1(2) = -2 + 3 = 1 \end{array}\right\} \quad (2,1)$$

$$\lim_{x\to 3} S_1(x) = \lim_{x\to 3} S_2(x)\ ;\ x = 3$$

$$\left.\begin{array}{l} S_1(3) = -3 + 3 = 0 \\ S_2(3) = 3 - 3 = 0 \end{array}\right\} \quad (3,0)$$

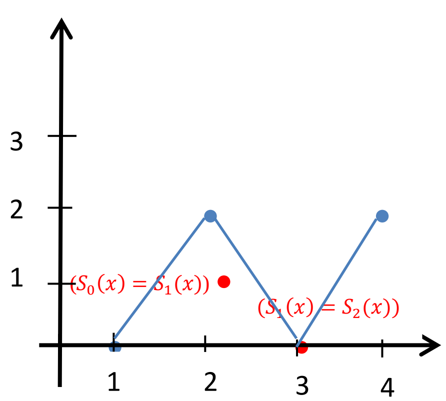

Figure 2.3 First-degree spline function: linear spline $\boldsymbol{S_i(x)}$

CHAPTER THREE

TRANSFORMATION SPLINE FUNCTION OF POLYNOMIAL DEGREE ONE

3.1 Transformation for sine function and cosine function of polynomial degree one

Sine Function 3.1.1: $f(x) = sin(x)$

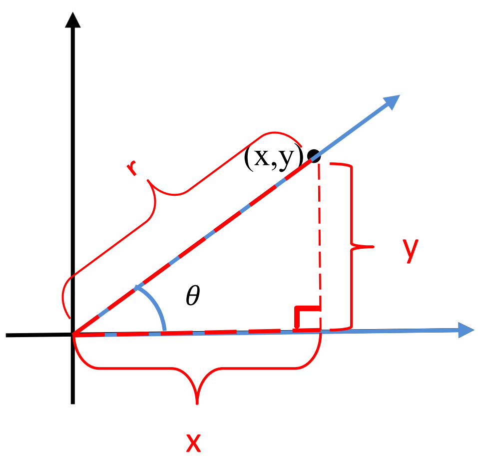

Figure 3.1.1 illustration of sine function using co-ordinate.

NOTATION: Let (x, y) be any point, other than the origin, on the terminal side of an angel θ in standard position. Let r be the distance from the point (x, y) to the origin.

$$Sin\theta = \frac{y}{r}$$

$$r = \sqrt{x^2 + y^2}$$

Property of sin(x) 3.1.2 :

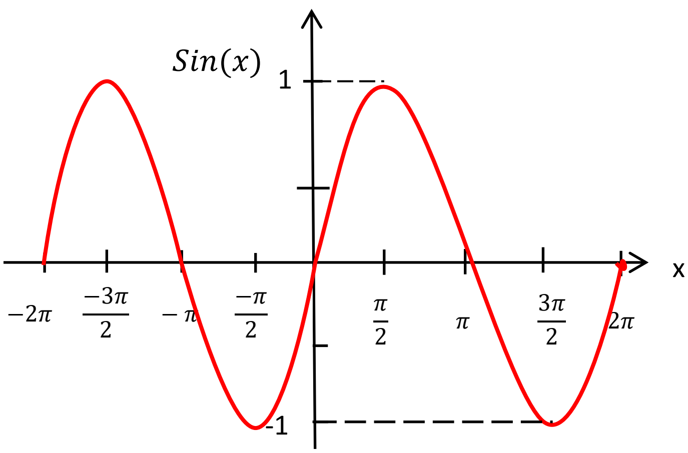

Figure 3.1.2 sine function

1. Domain: $(-\infty,\infty)$ or $-\infty < x < \infty$.
2. Range:$[-1,1]$ $or -1 \leq y \leq 1$.
3. The sine function is a periodic function with fundamental period 2π.
4. The x-intercepts $0, \pm\pi$, $\pm 2\pi, \ldots,$ are integer multiples of, $n\pi$, where n is an integer: $(n\pi, 0)$.
5. The Maximum(1) and Minimum(-1) value of the sine function correspond to x value that are odd integer multiples of $\frac{\pi}{2}, \frac{(2n+1)\pi}{2}$, such as $\pm\frac{\pi}{2}$, $\pm\frac{3\pi}{2}$, $\pm\frac{5\pi}{2}, \ldots$.
6. The sine function is an odd function:-
   - Symmetric about the origin.
   - $Sin(-x) = -sin(x)$. (Young 2012)

Cosine Function 3.1.3: $f(x) = cos(x)$

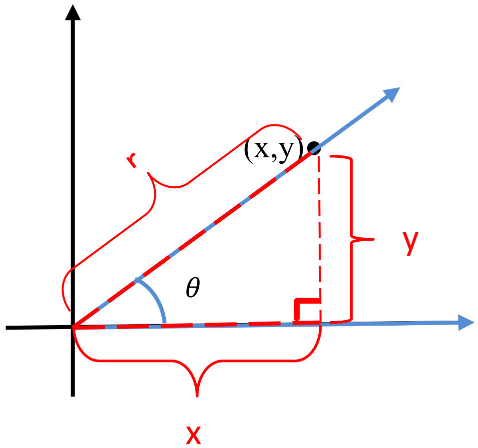

Figure 3.1.3 illustrations of cosine function using co-ordinate.

$$Cos\theta = \frac{x}{r}$$

$$r = \sqrt{x^2 + y^2}$$

Property of cos(x) 3.1.4: (Young 2012)

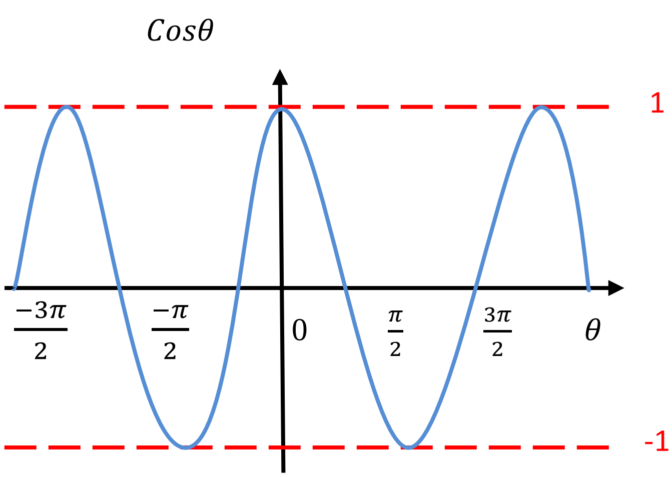

Figure 3.1.4 cosine function

1. Domain:$(-\infty,\infty)$ or $-\infty < x < \infty$.
2. Range: $[-1,1]$ or $-1 \leq y \leq 1$.
3. The cosine function is periodic function with fundamental period 2π.
4. The x-intercepts,$\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$, are odd integer multiples of $\frac{\pi}{2}$, which have the form $\frac{(2n+1)\pi}{2}$,, where n is integer :- $(\frac{(2n+1)\pi}{2}, 0)$.
5. The Maximum (1) and Minimum (-1) values of the cosine function correspond to x-value that are integer multiples of $\pi,, n\pi$ such as $0, \pm\pi, \pm 2\pi, \ldots$.

6. The cosine function is an even function:
   - Symmetric about the y-axis.
   - $Cos(-x) = cos(x)$

Example 3.1: The terminal side of an angel θ in standard position passes through the point(2,5) . (Young 2012)

Solution:

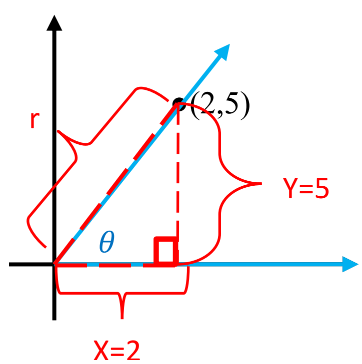

Figure 3.1.5 illustrations using co-ordinate.

$$\boldsymbol{r = \sqrt{2^2 + 5^2}}$$

$$\boldsymbol{r = \sqrt{29}}$$

$$x = 2, y = 5, \; \boldsymbol{r = \sqrt{29}}$$

$$\boldsymbol{sin\theta = \frac{y}{r} = \frac{5}{\sqrt{29}}}$$

$$\boldsymbol{sin\theta = \frac{5}{\sqrt{29}} \cdot \frac{\sqrt{29}}{\sqrt{29}} = \frac{5\sqrt{29}}{29}}$$

$$\boldsymbol{Cos\theta = \frac{x}{r} = \frac{2}{\sqrt{29}}}$$

$$\boldsymbol{Cos\theta = \frac{2}{\sqrt{29}} \cdot \frac{\sqrt{29}}{\sqrt{29}} = \frac{2\sqrt{29}}{29}}$$

3.2 Spline by geometric function

Spline function transformation of sine function 3.2.1:

$$S(x) = \begin{cases} S_0(x) & x \in [x_0, x_1] \\ S_1(x) & x \in [x_1, x_2] \\ \vdots & \vdots \\ S_{n-1}(x) & x \in [x_{n-1}, x_n] \end{cases} \qquad --- (3.2.1)$$

Where

$$S(x) = Sin(\, a_i x + b_i) \qquad --- (3.2.2)$$

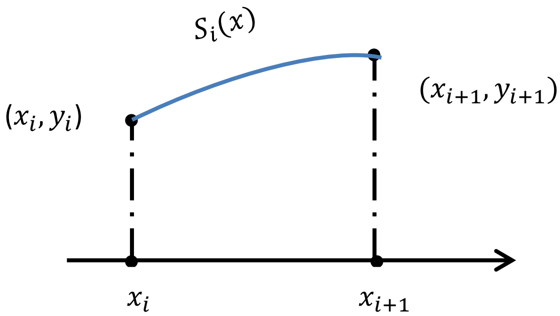

Figure 3.2.1 First-degree spline: sine function $\boldsymbol{S_i(x)}$

$$s_i(x) = Sin(y_i + m_i(x - x_i)) \qquad ------ (3.2.3)$$

On the interval $[x_i, x_{i+1}]$, where m_i is the SLOP of the line and is therefore given the formula

$$\boldsymbol{m_i = \frac{y_{i+1} - y_i}{x_{i+1} - x_i}} \qquad --- (3.2.4)$$

Substitute equation (3.1.4) in equation (3.1.3)

$$S_i(x) = Sin(y_i + (x - x_i)\left[\frac{y_{i+1} - y_i}{x_{i+1} - x_i}\right]) \qquad --- (3.2.5)$$

Example 3.2: Determine a spline function of degree one for transformation for sine function of polygonal degree one which interpolates the following data:

X	1	2	3	4
Y	0	1	0	1

Solution: since

$$S(x) = \begin{cases} S_0(x) & x \in [1,2] \\ S_1(x) & x \in [2,3] \\ S_2(x) & x \in [3,4] \end{cases}$$

$$S_0(x) = Sin(y_0 + (x - x_0)\left(\frac{y_1 - y_0}{x_1 - x_0}\right))$$

$$S_0(x) = Sin(0 + (x - 1)\left(\frac{1-0}{2-1}\right))$$

$$S_0(x) = Sin(x - 1)$$

$$S_1(x) = Sin(y_1 + (x - x_1)(\frac{y_2 - y_1}{x_2 - x_1}))$$

$$S_1(x) = Sin(1 + (x - 2)\left(\frac{0-1}{3-2}\right))$$

$$S_1(x) = Sin(-x + 3)$$

$$S_2(x) = Sin(y_2 + (x - x_2)\left(\frac{y_3 - y_2}{x_3 - x_2}\right))$$

$$S_2(x) = Sin(0 + (x - 3)\left(\frac{1-0}{4-3}\right))$$

$$S_2(x) = Sin(\text{x-3})$$

$$\lim_{x\to 2} S_0(x) = \lim_{x\to 2} S_1(x)\ ;\ x = 2$$

$$\left.\begin{aligned} S_0(2) &= Sin(2 - 1) = Sin1{=}0.0174 \\ S_1(2) &= Sin(-2 + 3) = Sin(1){=}0.0174 \end{aligned}\right\} \quad (2, 0.0174)$$

$$\lim_{x\to 3} S_1(x) = \lim_{x\to 3} S_2(x)\ ;\ x = 3$$

$$\left.\begin{aligned} S_1(3) &= Sin(-3 + 3) = Sin(0) = 0 \\ S_2(3) &= Sin(3 - 3) = Sin(0) = 0 \end{aligned}\right\} \quad (3, 0)$$

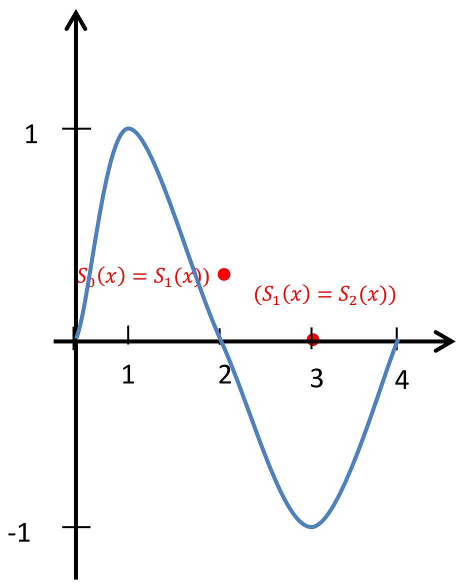

Figure 3.2.2 spline function of degree one for transformation for sine function of polygonal degree one.

Spline transformation of cosine function 3.2.2:

$$S(x) = \begin{cases} S_0(x) & x \in [x_0, x_1] \\ S_1(x) & x \in [x_1, x_2] \\ \vdots & \vdots \\ S_{n-1}(x) & x \in [x_{n-1}, x_n] \end{cases} \qquad \_\_\_\_ \ (3.2.1)$$

Where

$$\boldsymbol{S}(\boldsymbol{x}) = Cos\,(\boldsymbol{a_i x} + \boldsymbol{b_i}) \qquad ----- \ (3.2.2)$$

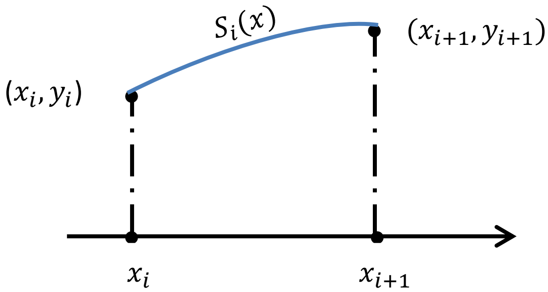

Figure 3.2.3 First-degree spline: cosine function $\boldsymbol{S_i(x)}$

$$s_i(x) = Cos(y_i + m_i(x - x_i)) \qquad \text{--------} \ (3.2.3)$$

On the interval $[x_i, x_{i+1}]$, where m_i is the SLOP of the line and is therefore given the formula

$$\boldsymbol{m_i = \frac{y_{i+1} - y_i}{x_{i+1} - x_i}} \qquad \text{-----} \ (3.2.4)$$

Substitute equation (3.2.4) in equation (3.2.3)

$$S_i(x) = Cos(y_i + (x - x_i)\left[\frac{y_{i+1} - y_i}{x_{i+1} - x_i}\right]) \qquad \text{-----} \ (3.2.5)$$

Example 3.3: Determine a spline function of degree one for transformation for cosine function of polygonal degree one which interpolates the following data:

X	1	2	3	4
Y	0	1	0	1

Solution: since

$$S(x) = \begin{cases} S_0(x) & x \in [1,2] \\ S_1(x) & x \in [2,3] \\ S_2(x) & x \in [3,4] \end{cases}$$

$$S_0(x) = Cos(y_0 + (x - x_0)\left(\frac{y_1 - y_0}{x_1 - x_0}\right))$$

$$S_0(x) = Cos(0 + (x - 1)\left(\frac{1-0}{2-1}\right))$$

$$S_0(x) = Cos(x - 1)$$

$$S_1(x) = Cos(y_1 + (x - x_1)(\frac{y_2 - y_1}{x_2 - x_1}))$$

$$S_1(x) = Cos(1 + (x - 2)\left(\frac{0-1}{3-2}\right))$$

$$S_1(x) = Cos(-x + 3)$$

$$S_2(x) = Cos(y_2 + (x - x_2)\left(\frac{y_3 - y_2}{x_3 - x_2}\right))$$

$$S_2(x) = Cos(0 + (x - 3)\left(\frac{1-0}{4-3}\right))$$

$$S_2(x) = Cos(x - 3)$$

$$\lim_{x\to 2} S_0(x) = \lim_{x\to 2} S_1(x)\ ;\ x = 2$$

$$\left.\begin{aligned} S_0(2) &= Cos(2 - 1) = Cos1 = 0.9998 \\ S_1(2) &= Cos(-2 + 3) = Cos1 = 0.9998 \end{aligned}\right\} (2, 0.9998)$$

$$\lim_{x\to 3} S_1(x) = \lim_{x\to 3} S_2(x)\ ;\ x = 3$$

$$\left.\begin{aligned} S_1(3) &= Cos(-3+3) = Cos0 = 1 \\ S_2(3) &= Cos(3-3) = Cos0 = 1 \end{aligned}\right\} \quad (3,1)$$

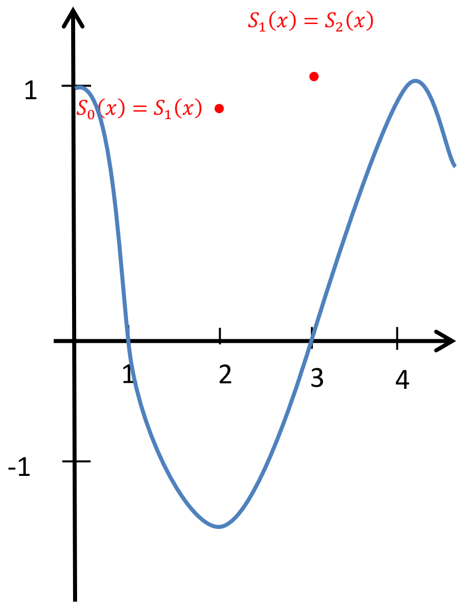

Figure 3.2.4 spline function of degree one for transformation for cosine function of polygonal degree one.

3.3 Matlab of transformation of spline degree one

Spline functions of degree one 3.3.1:

```
clc
n=input('n=');    %n=4
x1=input('x1='); %x1=2.5
x=input('x=');    %x=[1 2 3 4]
y=input('y=');    %y=[1 0 1 0]
m=input('m=');    %m=[1 2;2 3;3 4]
s=zeros(n-1,1);
for i=1:n-1
    if (x1>=m(i,1))&&(x1<=m(i,2))

 s(i,1)=(y(1,i+1)-y(1,i))/(x(1,i+1)-x(1,i))*(x1-
x(1,i))+y(1,i);
        disp(s(i,1))
    end
    plot(x,y,x1,s(i,1))
end
```

Spline function of degree one transformation of Sine function 3.3.2:

```
clc
n=input('n=');    %n=4
x1=input('x1='); %x1=2.5
x=input('x=');    %x=[1 2 3 4]
y=input('y=');    %y=[1 0 1 0]
m=input('m=');    %m=[1 2;2 3;3 4]
s=zeros(n-1,1);
for i=1:n-1
    if (x1>=m(i,1))&&(x1<=m(i,2))

 s(i,1)=sin(y(1,i+1)-y(1,i))/(x(1,i+1)-x(1,i))*(x1-
x(1,i))+y(1,i);
        disp(s(i,1))
```

```
    end
    plot(x,y,x1,s(i,1))
end
```

Spline function of degree one transformation of Cosine function

3.3.3:

```
clc
n=input('n=');    %n=4
x1=input('x1='); %x1=2.5
x=input('x=');    %x=[1 2 3 4]
y=input('y=');    %y=[1 0 1 0]
m=input('m=');    %m=[1 2;2 3;3 4]
s=zeros(n-1,1);
for i=1:n-1
    if (x1>=m(i,1))&&(x1<=m(i,2))

 s(i,1)=cos(y(1,i+1)-y(1,i))/(x(1,i+1)-x(1,i))*(x1-
x(1,i))+y(1,i);
        disp(s(i,1))
    end
    plot(x,y,x1,s(i,1))
end
```

3.4 Algorithm of transformation of spline degree one

Spline functions of degree one 3.4.1:

Step1: Assume that value of interval for x and $f(x)$, and the number of data valued n , and the range for m using formula (2.1)

Step 2: Construct the spline function using formula (2.2)

Step 3: Polite the spline function of degree one using formula (2.5)

Step 4: end.

Spline functions of degree one transformation of Sine function 3.4.2:

Step1: Assume that value of interval for x and $f(x)$,and the number of data valued n, and the range for m using formula (3.3.1)

Step 2: Construct the spline function using formula (3.1.2)

Step 3: Polite the spline function of degree one transformation sine for degree one using formula (3.1.5)

Step 4: end.

Spline functions of degree one transformation of cosine function 3.5.3:

Step1: Assume that value of interval for x and $f(x)$,and the number of data valued n, and the range for m using formula (3.2.1)

Step 2: Construct the spline function using formula (3.2.2)

Step 3: Polite the spline function of degree one transformation cosine for degree one using formula (3.2.5)

Step 4: end.

CHAPTER FOUR

NUMERICAL RESULT

Example 4.1.1: Determine a spline function of degree one which interpolates the following data:

X	1	2	3	4
Y	0	1	0	1

Solution: use linear spline to produce curve for the following data:

X	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
Y	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1

X	2	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3
Y	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0

X	3	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4
Y	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1

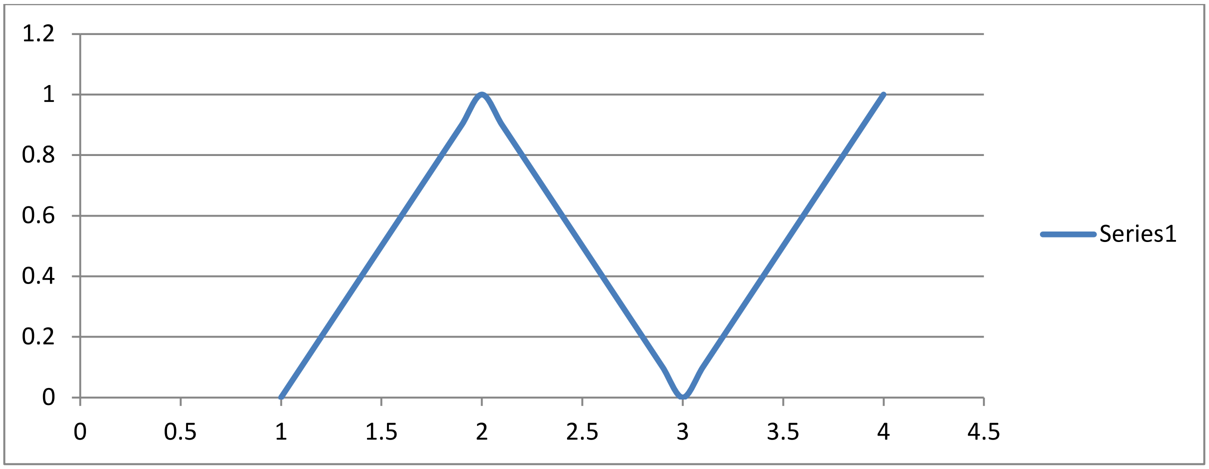

Figure 4.1.1 spline function of degree one

Example 4.1.2: Determine a spline function of degree one for transformation for sine function of polygonal degree one which interpolates the following data:

X	1	2	3	4
Y	0	1	0	1

Solution: use spline function of degree one for transformation for sine function of polygonal degree one to produce curve for the following data:

X	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
y	0	0.099	0.198	0.295	0.389	0.479	0.564	0.644	0.717	0.783	0.841

X	2	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3
y	0.841	0.783	0.717	0.644	0.564	0.479	0.389	0.295	0.198	0.099	0

X	3	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4
y	0	0.099	0.198	0.295	0.389	0.479	0.564	0.644	0.717	0.783	0.841

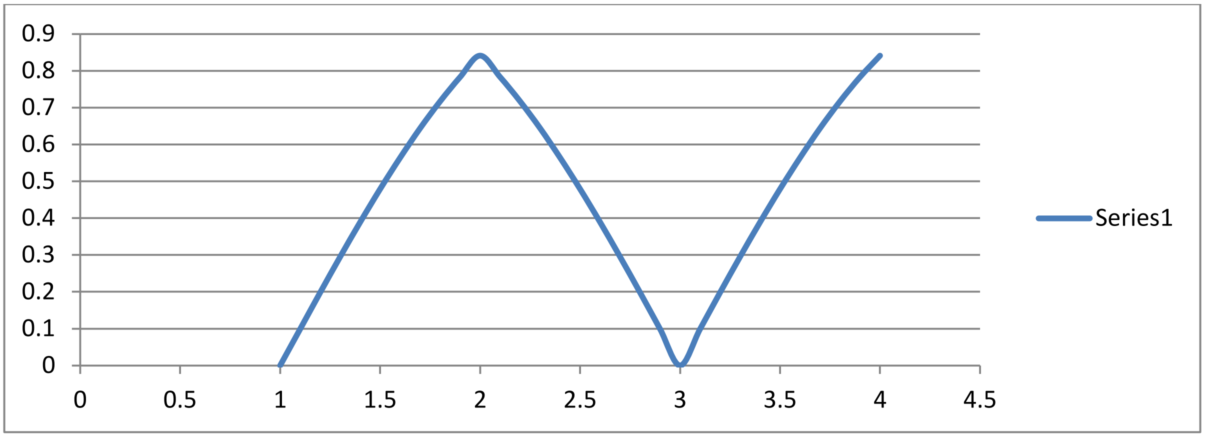

Figure 4.1.2 spline function transformation of sine function

Example 4.1.3: Determine a spline function of degree one for transformation for cosine function of polygonal degree one which interpolates the following data:

X	1	2	3	4
Y	0	1	0	1

Solution: use spline function of degree one for transformation for cosine function of polygonal degree one to produce curve for the following data:

X	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
y	1	0.995	0.980	0.955	0.921	0.877	0.825	0.764	0.696	0.621	0.540

X	2	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3
y	0.540	0.621	0.696	0.764	0.825	0.877	0.921	0.955	0.980	0.995	1

X	3	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4
y	1	0.995	0.980	0.955	0.921	0.877	0.825	0.764	0.696	0.621	0.540

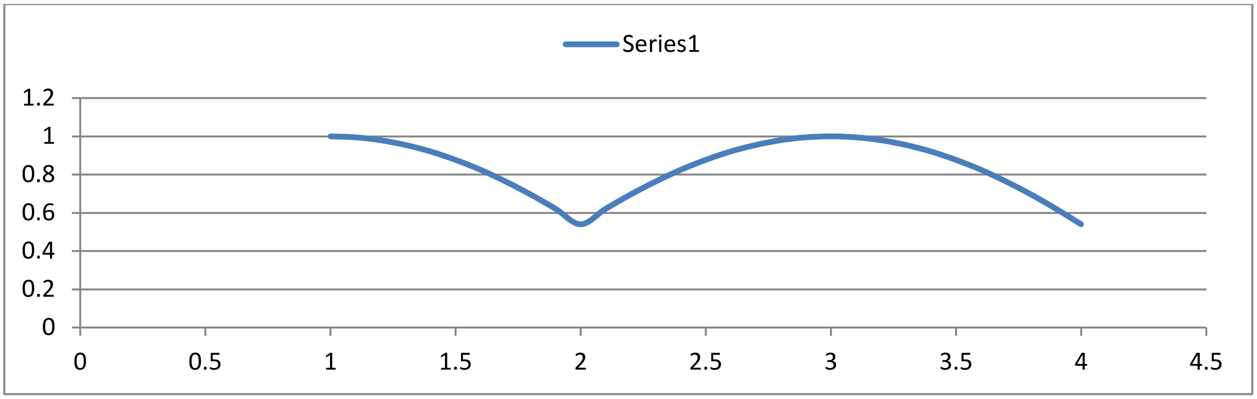

Figure 4.1.3 spline function transformation of cosine function

Example 4.2.1: Determine a spline function of degree one which interpolates the following data:

X	0	1	3
Y	2	4	5

Solution: use linear spline to produce curve for the following data:

X	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
y	2	2.2	2.4	2.6	2.8	3	3.2	3.4	3.6	3.8	4

X	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.1	2.2	2.3	2.4	2.5
y	4	4.05	4.1	4.15	4.2	4.25	4.3	4.35	4.4	4.45	4.5	4.55	4.6	4.65	4.7	4.75

X	2.6	2.7	2.8	2.9	3
y	4.8	4.85	4.9	4.95	5

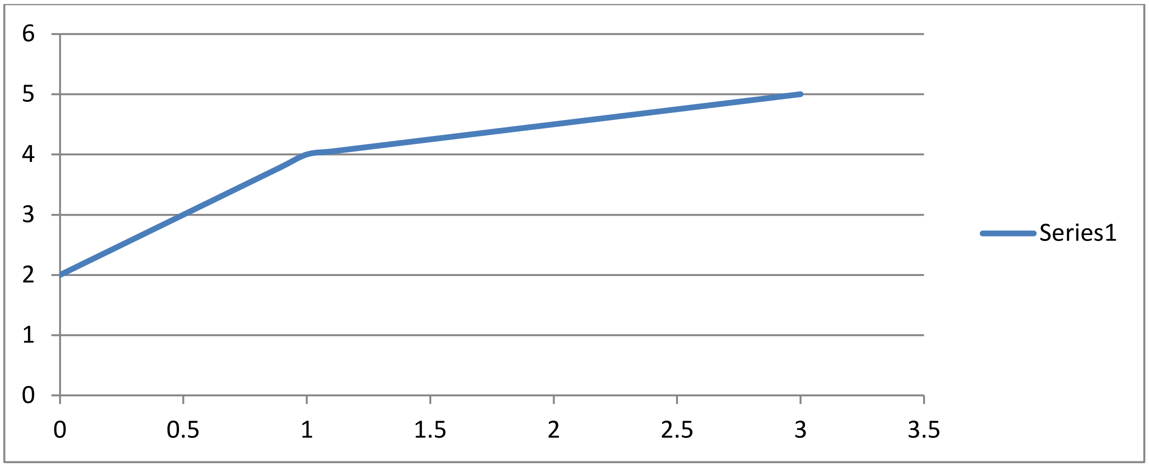

Figure 4.1.4 spline function of degree one by example 4.2.1

Example 4.2.2: Determine a spline function of degree one for transformation for sine function of polygonal degree one which interpolates the following data:

X	0	1	3
Y	2	4	5

Solution: use spline function of degree one for transformation for sine function of polygonal degree one to produce curve for the following data:

X	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Y	0.909	0.808	0.675	0.515	0.334	0.141	-0.058	-0.255	-0.442	-0.611	-0.756

X	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
y	-0.788	-0.818	-0.845	-0.871	-0.894	-0.916	-0.935	-0.951	-0.965	-0.977

X	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3
y	-0.986	-0.993	-0.998	-0.9999	-0.9992	-0.996	-0.990	-0.982	-0.971	-0.958

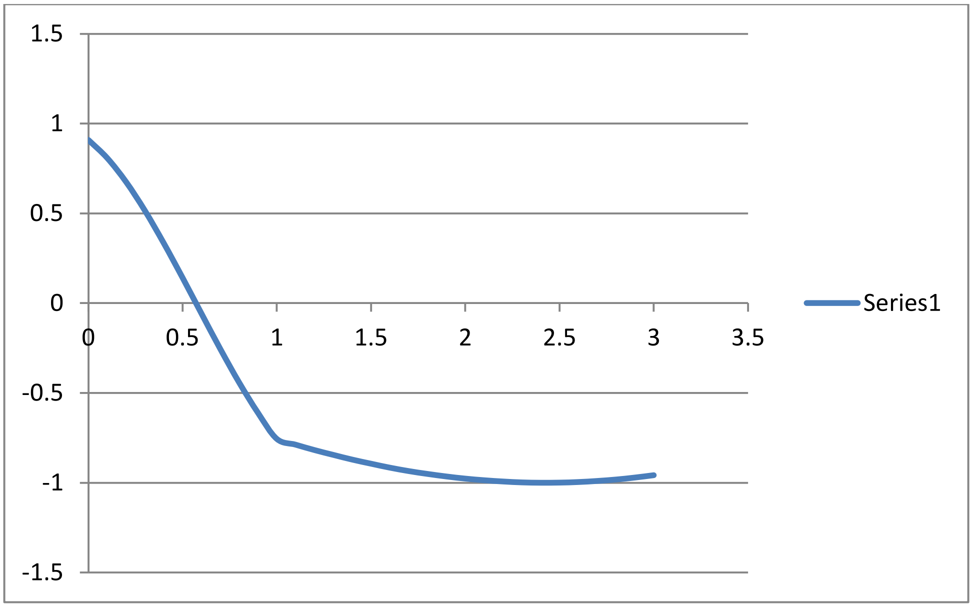

Figure 4.1.5 spline function transformation of sine function by example 4.2.2

Example 4.2.3: Determine a spline function of degree one for transformation for cosine function of polygonal degree one which interpolates the following data:

X	0	1	3
Y	2	4	5

Solution: use spline function of degree one for transformation for cosine function of polygonal degree one to produce curve for the following data:

X	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
y	-0.416	-0.588	-0.737	-0.856	-0.942	-0.989	-0.998	-0.966	-0.896	-0.790	-0.653

X	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
y	-0.615	-0.574	-0.533	-0.490	-0.446	-0.400	-0.354	-0.307	-0.259	-0.210

X	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3
y	-0.161	-0.112	-0.062	-0.012	0.037	0.087	0.137	0.186	0.235	0.283

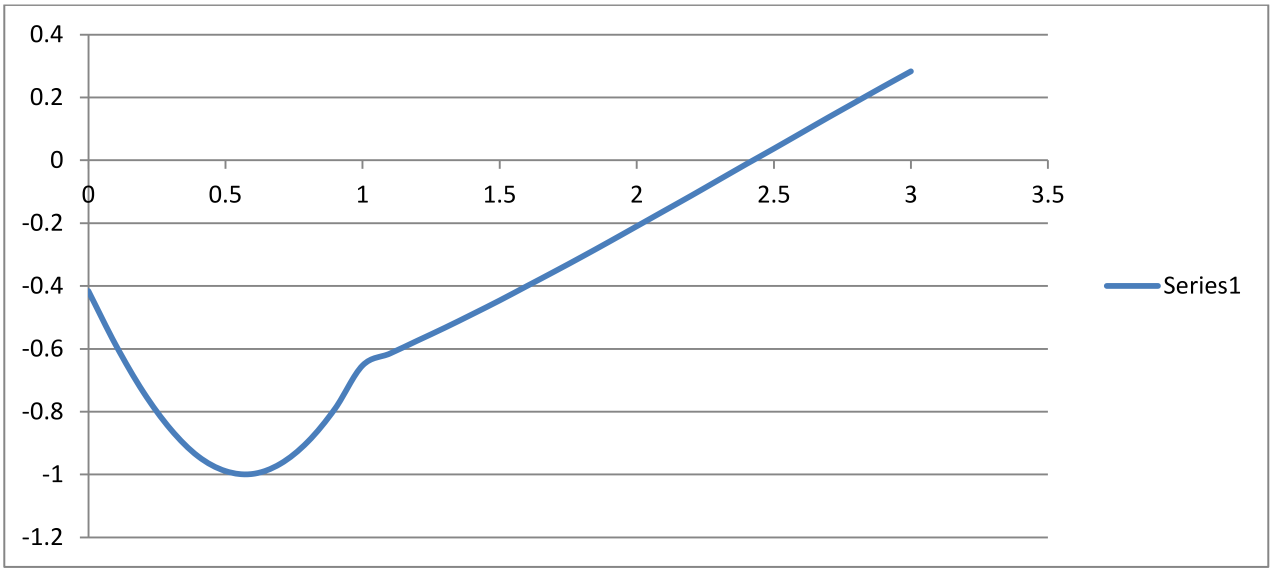

Figure 4.1.6 spline function transformation of cosine function by example 4.2.3

Note: using MatlabR2010a and Microsoft Excel 2010

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پوختە

لەم توێژینەوەیەدا، ئێمە کرداری سپلاینی پلەی یەک گۆڕانکاری پێشکەش دەکەین بۆ ئەرکەکانی ساین و کۆساین لە پلەی یەک. ئێمە سەرەتا چەمکی کرداری سپلاین و جێبەجێکردنی دەخەینە ڕوو. دواتر باس لە تایبەتمەندییەکانی کارەکانی ساین و کۆساین دەکەین و چۆن دەتوانرێت بگۆڕدرێن بە بەکارهێنانی کرداری سپلاینی پلەی یەک. لە کۆتاییدا، ئێمە نموونە و نوێنەرایەتی گرافیکی پێشکەش دەکەین بۆ ڕوونکردنەوەی کاریگەری و جێبەجێکردنی ئەم گۆڕانکارییانە.

ملخص

في هذا البحث ، نقدم وظائف خدد التحويل من الدرجة الأولى لوظائف الجيب وجيب التمام من الدرجة الأولى. نقدم أولاً مفهوم عملية الشريحة وتنفيذها. بعد ذلك ، نناقش خصائص وظائف الجيب وجيب التمام وكيف يمكن تحويلهما باستخدام عمليات خدد من الدرجة الأولى. أخيرًا ، نقدم أمثلة وتمثيلات رسومية لشرح تأثير وتنفيذ هذه التغييرات.