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Comparative Study of Interpolation Formula

Research Project
Submitted to the Department of Mathematic in Partial Fulfillment of the Requirements for the Degree of BSc. In Mathematic

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Certification of the supervisors

I certify that this work was prepared under my supervision at the Department of Mathematics/College of Education/Salahaddin University-Erbil in partial fulfillment of the requirements for the degree of Bachelor of philosophy of Science in Mathematics.

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In view of the available recommendations, I forward this work for debate by the examining committee.

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Eman Abdulwahid Mousa

2024

Dedication To

- *My father and mother*

- *My dear supervisor*

- *My brothers and sisters*

- *All who want to read it*

Eman Abdulwahid Mousa

2024

Contents

Abstract

When taking care of issues utilizing numerical strategies it is generally important to set up a model and to record conditions communicating the limitations and actual laws that apply. These conditions should now be tackled and a decision introduces itself. One route is to continue utilizing regular strategies for math, getting an answer as a recipe, or set of formulae. Another strategy is to communicate the conditions so that they might be addressed computationally, i.e. by utilizing strategies for mathematical examination or Numerical analysis. In this paper we are aimed to discuss interpolation, various methods to solve central difference interpolation, their generalizations. Applications of interpolation are also discussed and one can easily understand the concepts of the paper.

Chapter one

1. Introduction

At times, determining the value of a dependent variable for a given independent variable poses a challenge when their explicit relationship is unknown. For instance, consider estimating India's population for the year 2000, where no precise mathematical expression exists for such predictions. However, employing interpolation allows for approximate estimation of the population for any given year. Numerous scholars have explored various interpolation techniques, including central difference interpolation formulas.

Akima (1970) introduced a novel mathematical method involving fitting a smooth curve to a set of numbers in a plane, focusing on local steepness and polynomial representation. Other researchers like Atkinson (1989), Carl and Boor (1980) discussed Newton's divided difference formula for unevenly spaced points, while Burden and Faires (2001) and Suli and Mayers (2003) delved into the Lagrange formula for interpolation. Abdulla et al. (2004) proposed an efficient formula for central difference interpolation, demonstrating its effectiveness compared to existing methods.

Further studies explored various aspects of interpolation, including finite difference methods, radial point interpolation, and void-filling interpolation. Muthumalai (2008) investigated new iterative interpolation methods, while Singh and Bhadauria (2009) proposed finite difference formulae for unequal sub-intervals. Interpolation techniques have found applications in diverse fields such as lidar technology for vegetation assessment.

More recent research has focused on refining interpolation methods, comparing different central difference formulas and their applications. Scholars like Roseline et al. (2019) have developed and analyzed central difference interpolation formulas, aiming to provide clarity and utility for students and researchers alike. These studies contribute to a better understanding of interpolation techniques and their practical implications, offering valuable insights for further exploration in academic settings.

Chapter Two

2- Some basic definitions:

In this section, some basic terms regarding interpolation have been discussed:

2.1 - Interpolation: (Muthumalai, , 2014)"Interpolation is the art of reading between the lines of the table", says Thiele.

It also refers to the addition or filling up of middle terms of the series. Hence, Interpolation is thetechnique of approximating the function value for any undetermined independent variable, whereas Extrapolation is the technique of computing the function value outside the stated range.

2.2 - Premise of interpolation:

For interpolation to work, certain premises must be fulfilled:

1. During the period under examination, there are no significant changes in the values.

2. Values should increase and fall consistently. For example, if the given data is on the number of fatalities in various years in a particular town, and some of the observations are for years when the town was affected by war or epidemic, interpolation approaches are ineffective.

3. In the methodology of finite differences, when a given set of values is expressed in polynomial form, it is observed that if values of the function are given in a clear and verified manner then one can easily compute 'Y' in accordance with 'X'.

And if the value of the function is not known, then one will introduce some simpler function. For e.g. $\emptyset(x)$ such that both the functions have the same set of values. This process is what someone called Interpolation.

Then after that, there comes a major term that has its importance in interpolation i.e. Finite differences.

Finite difference function (:[singh A. , 2009)The math of finite differences gives out the changes that occur in the value of the dependent variable by the finite changes in the independent variable.

So, there are have three types of differences in the math of finite differences:

Let us consider a function x =g(y) and tabulate it for the uniformly spaced values

$y=y_0,y_0+h,y_0+2h\dots y_0+nh$

given. $x=x_0,x_1,x_2\dots x_n$

To find the values of f(y) and f'(y), for some middle values of x, the given three differences are helpful:

Forward difference: The given set of values such as

$x_0-x_1,x_2-x_1,x_3-x_2\dots x_n-x_{n-1}$

When expressed by Δx_0, Δx_1, $\Delta x_2,\dots,\Delta x_{n-1}$

respectively are known as first forward differences and the symbolΔ is known as the forward difference operator.

Backward difference: The given set of values such as

$x_1 - x_0$, $x_2 - x_1$, $x_3 - x_2$, … … … … … . $x_n - x_{n-1}$

when expressed by ∇x_1, ∇x_2, $\nabla x_3 \dots \nabla x_n$,respectively, are known as first backward differences and the symbol $\nabla$is known as the backward difference operator.

Central difference: The relation that defines the central difference operator 'd' is given as :

$$x_1 - x_0 = \delta X_{\frac{1}{2}},\ x_2 - x_1 = \delta X_{\frac{3}{2}},\ \dots,\ x_n - x_{n-1} = \delta X_{n-\frac{1}{2}}$$

3- Study of different central difference Interpolation formulas (kumar, 2018)

It is easy to calculate the value of y corresponding to any value of x if the value of the function g(x) is known. On the other hand, if one is unfamiliar with the value of g(x), then it is quite difficult to find the solution of g(x) by using a tabulated collection of numbers (x_i, y_i).

In such circumstances, simply substitute a normal function ∅(x) for g(x) which shows the same values as that of g(x) at the given tabulated collection of numbers. As a result, one can find any other value from ∅(x) which is called as an interpolating or smoothing function

The methodology of finite differences helps in understanding the concept of Interpolation. By using forward and backward interpolation differences of a function the two fundamental interpolation formulae can be obtained. These formulas are frequently used in engineering and scientific research. Before discussing central difference interpolation formulas let's take a small look at the forward and backward interpolation formulas.

Newton's forward interpolation formula:

Newton's forward interpolation formula: let us consider a function x = g(y), take the values x_0 ,x_1 ,x_2 ,x_3 ,x_4 ....x_n in accordance with the values y_0 ,y_1 ,y_2 ,y_3 ....y_n of y. Let these values of y to be stated in a spaced manner such that y_j =y_0 +jh(j =0

2........). assuming x(y) to be an n^{th} degree polynomial in y such that x (y_0) = x_0 , $x(y_n$) = x_1 , ... $x(y_n$) = x_n , we can write,

$$x_p = x_0 + p\Delta x + \frac{p(p-1)}{2!}\Delta^2 x_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 x_0 + \ldots\ldots\ldots\ldots\ldots..(1)$$

This method is beneficial for determining values of x at the beginning of a tabulated set of values and extrapolating values of x a bit backward (i.e. to the left) of $x0$.

One more result is there: The linear interpolation is given by the first two terms of this formula, while the parabolic interpolation is given by the first three terms, and so on.

And

$$p_n(x) = f(x_0) + \frac{\Delta f(x_0)}{h}(x - x_0) + \frac{1}{2!}\frac{\Delta^2 f(x_0)}{h^2}(x - x_0)(x - x_1) + \ldots$$
$$+ \frac{1}{n!}\frac{\Delta^n f(x_0)}{h^n}(x - x_0)(x - x_1)\ldots(x - x_{n-1}).$$

Form (5.25) let $s = \dfrac{x - x_0}{h}$,

then, we have $x - x_i = x - x_0 - (x_i - x_0) = sh - ih = (s - i)h$, $\quad i=0,1,\ \ldots,\ n.$

With this, the Newton forward formula (5.25) becomes

$$p_n(x) = f(x_0) + s\Delta f(x_0) + \frac{s(s-1)}{2!}\Delta^2 f(x_0) + \ldots + \frac{s(s-1)(s-2)\ldots(s-n+1)}{n!}\Delta^n f(x_0),$$

and the corresponding error (5.8) becomes

$$E_n(x) = f(x) - p_n(x) = \frac{s(s-1)(s-2)\ldots(s-n)}{(n+1)!}h^{n+1} f^{(n+1)}(c), \text{ where } a<c<b.$$

Newton Backward Difference Interpolation Formula

let us consider a function x= g(y), take the values

x_0, x_1, x_2 ... ...in accordance with the valuesy_0, y_0 + h,

y_0 + 2h.......... of y. Consider

the problem of determining the values of g(y) for y = $y + ph$, where p may be any real

number. Then we have:

$$x_p = x_n + p\nabla x_n + \frac{p(p+1)}{2!}\nabla^2 x_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 x_n + \cdots \dots \dots \dots (2)$$

It is termed Newton's backward interpolation formula as the given equations consist of xn and the backward differences of x_n.

This method is beneficial for determining values of x at the end of the tabulated set of values and for extrapolating values of x a little ahead (to the right) of x_n.

Following similar steps as in subsection 5.4.1, with $s = \frac{x - x_n}{h}$.

We can obtain the following Newton backward difference interpolation formula:

$$p_n(x) = f(x_n) + s\nabla f(x_n) + \frac{s(s+1)}{2!}\nabla^2 f(x_n) + \dots + \frac{s(s+1)(s+2)\dots(s+n-1)}{n!}\nabla^n f(x_n),$$

and the corresponding error:

$$E_n(x) = f(x) - p_n(x) = \frac{s(s+1)(s+2)\dots(s+n)}{(n+1)!} h^{n+1} f^{(n+1)}(c). \text{ where } a<c<b.$$

Central difference interpolation: (Uddin, 2019)

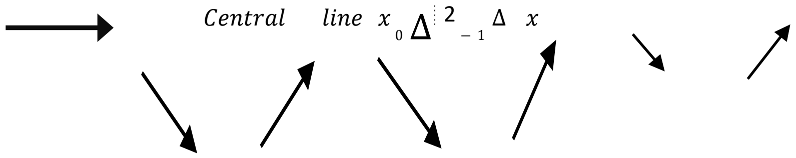
So, in Newton's backward and forward difference interpolation formula, it is learned that they can be used for interpolation around the beginning and end of a table of data. Now let’s introduce the central difference interpolation formula that is best applicable for determining the values near the middle of the table. In the central difference interpolation, there are four methods, that are required to formulate the central difference:

Gauss Forward interpolation formula:

If y takes the values $y_0 - 2h$, $y_0 - h$, y_0, $y_0 + h$, $y_0 + 2h$ and corresponding values of x = g(y) are x_{-2}, x_{-1}, x_0, x_1, x_2, then the formula for Gauss forward interpolation is:

$$x_p = x_0 + p\Delta x_0 + \frac{p(p-1)}{2!}\Delta^2 x_{-1} + \frac{(p+1)p(p-1)}{3!}\Delta^3 x_{-1} + \frac{(p+1)p(p-1)(p-2)}{4!}\Delta^4 x_{x-2} + \cdots \ldots \ldots \ldots \ldots \quad (3)$$

As seen below, this formula uses odd differences slightly below the central line,and even differences are taken on the central line:



$\Delta x_0 \Delta^3 x_{-1} \Delta^5 x_{-2}$

This method is beneficial to estimate values of x for p (0<p<1), calculated in a forward direction from the origin.

When ‘p’ is between 0 and $\frac{1}{2}$, this formula is applied.

Gauss backward interpolation:

Gauss backward interpolation formula is derived by modifying Newton’s forward interpolation formula. It's beneficial for interpolating x values for a negative ‘p’ value that's between -1 and 0 or in terms of central difference notation the values lie between -1/2 and 0.

The gauss backward formula is written as:

$$x_p = x_0 + p\Delta x_{-1} + \frac{p(p+1)}{2!}\Delta^2 x_{-1} + \frac{(p+1)p(p-1)}{3!}\Delta^3 x_{-2} + \frac{(p+2)(p+1)p(p-1)}{4!}\Delta^4 x_{-2} \ldots \ldots \ldots \ldots \quad (4)$$

As illustrated below, this approach has odd differences above the central line , and even differences are taken on the central line:

$$\Delta x_{-1} \Delta^3 x_{-2} \; \Delta^5 x_{-3}$$

Central line: $x_0\,\Delta^2\ x_{-1}\ \Delta^4 x_{-2}$

This is the rule formulation of gauss backward interpolation.

The Gauss forward and backward interpolation formulas aren't very useful. These, on the other hand, might be considered as the first phase in attaining the crucial formula for the next sections:

Stirling's central difference interpolation formula:

After taking the arithmetic mean of Gauss forward and backward interpolation we will obtain Stirling's Interpolation formula. This formula is used for an odd number of equally spaced values.

So, aftertaking the mean of Gauss forward and backward interpolation we get Stirling's formula as

$$x = x_0 + p\left[\frac{\Delta x_{0+}\Delta x_{-1}}{2}\right] + \frac{p^2}{2!}\Delta^2 x_{-1} \pm 1 + \frac{p(p^2-1)}{3!}\left[\frac{\Delta x_{-1+}\Delta x_{-2}}{2}\right] + \frac{p^2(p^2-1)}{4!}\Delta^4 x_{-2+\cdots} \text{..............}(5)$$

The means of odd differences just above and below the central line and even differences onthe central line are used in this formula, as illustrated below:

$\Delta x_{-1}\Delta^3 x_{-2}$

Central line: $x_0\Delta^2 x_{-1}\Delta^4\ x_{-2}$

$\Delta x_0\Delta^3 x_{-1}$

By applying this format one can make up Stirling's formula an easy way to remind the terms in the formula.

Stirling's interpolation formula gives the best approximate result when 'p' lies between $\frac{-1}{4}$and$\frac{1}{4}$.

Bessel's Interpolation Formula

To derive BIF, since $\Delta^i y_0 = \delta^i y_{\frac{i}{2}}$ for i=1, 2, … from (5.28), we get:

$$p_n(x) = y_0 + s\delta y_{\frac{1}{2}} + \frac{s(s-1)}{2!}\delta^2 y_1 + \frac{s(s-1)(s-2)}{3!}\delta^3 y_{\frac{3}{2}} + ... + \frac{s(s-1)(s-2)...(s-n+1)}{n!}\delta^n y_{\frac{n}{2}}.$$

Using average operator, it yields:

$$y_0 = \frac{1}{2}y_0 + \frac{1}{2}y_0 + \frac{1}{2}y_1 - \frac{1}{2}y_1 = \frac{1}{2}(y_1 + y_0) - \frac{1}{2}(y_1 - y_0) = \mu y_{\frac{1}{2}} - \frac{1}{2}\delta y_{\frac{1}{2}}.$$

$$\delta^2 y_1 = \frac{1}{2}\delta^2 y_1 + \frac{1}{2}\delta^2 y_1 + \frac{1}{2}\delta^2 y_0 - \frac{1}{2}\delta^2 y_0$$

$$= \frac{1}{2}(\delta^2 y_1 + \delta^2 y_0) + \frac{1}{2}(\delta^2 y_1 - \delta^2 y_0) = \mu\delta^2 y_{\frac{1}{2}} + \frac{1}{2}\delta^3 y_{\frac{1}{2}}.$$

$$\delta^3 y_{\frac{3}{2}} = \delta^3 y_{\frac{3}{2}} + \delta^3 y_{\frac{1}{2}} - \delta^3 y_{\frac{1}{2}} = \delta^3 y_{\frac{1}{2}} + (\delta^3 y_{\frac{3}{2}} - \delta^3 y_{\frac{1}{2}}) = \delta^3 y_{\frac{1}{2}} + \delta^4 y_1$$

$$= \delta^3 y_{\frac{1}{2}} + \frac{1}{2}\delta^4 y_1 + \frac{1}{2}\delta^4 y_1 + \frac{1}{2}\delta^4 y_0 - \frac{1}{2}\delta^4 y_0$$

$$= \delta^3 y_{\frac{1}{2}} + \frac{1}{2}(\delta^4 y_1 + \delta^4 y_0) + \frac{1}{2}(\delta^4 y_1 - \delta^4 y_0)$$

$$= \delta^3 y_{\frac{1}{2}} + \mu\delta^4 y_{\frac{1}{2}} + \frac{1}{2}\delta^5 y_{\frac{1}{2}}.$$

and so on.

Substitute these values in (5.37) and add the coefficients, we get:

$$p_n(x) = \mu y_{\frac{1}{2}} + (s-0.5)\delta y_{\frac{1}{2}} + \frac{s(s-1)}{2!}\mu\delta^2 y_{\frac{1}{2}} + \frac{s(s-1)(s-0.5)}{3!}\delta^3 y_{\frac{1}{2}} + \frac{s(s^2-1)(s-2)}{4!}\mu\delta^4 y_{\frac{1}{2}} + \cdots .$$

The formula (5.38) is known as **Bessel's interpolation formula**.

1- Newton's forward interpolation formula :

$$xp = x_0 + p\Delta x_0 + \frac{p(p-1)}{2!}\Delta^2 x_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 x_0 + \cdots$$

2- Newton's forward interpolation formula :

$$xp = x_n + p\nabla x_n + \frac{p(p+1)}{2!}\nabla^2 x_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 x_n + \cdots$$

3- Gauss forward interpolation formula :

$$xp = x_0 + p\Delta x_0 + \frac{p(p-1)}{2!}\Delta^2 x_{-1} + \frac{p(p+1)(p-1)}{3!}\Delta^3 x_{-1} + \frac{p(p+1)(p-1)(p-2)}{4!}\Delta^4 x_{-1} + \cdots$$

4- Gauss backward interpolation :

$$xp = x_0 + p\Delta x_0 + \frac{p(p-1)}{2!}\Delta^2 x_{-1} + \frac{p(p+1)(p-1)}{3!}\Delta^3 x_{-1} + \frac{p(p+1)(p-1)(p-2)}{4!}\Delta^4 x_{-1} + \cdots$$

5- Stirling's central difference interpolation formula :

$$xp = x_0 + p\left[\frac{\Delta x_0 + \Delta x_{-1}}{2}\right] + \frac{p^2}{2!}\Delta^2 x_{-1} + \frac{p(p^2-1)}{3!}\left[\frac{\Delta x_{-1} + \Delta x_{-2}}{2}\right] + \frac{p^2(p^2-1)}{4!}\Delta^4 x_{-2} + \cdots$$

6- Bessel's central difference interpolation formula :

$$xp = x_0 + p\Delta x_0 + \frac{p(p-1)}{2!}\left[\frac{\Delta^2 x_0 + \Delta^2 x_{-1}}{2}\right] + \frac{\left(p-\frac{1}{2}\right)p(p-1)}{3!}\Delta^3 x_{-1}$$

Chapter Three

3. Numerical discussion (1Ndu, 2019)

3.1. Consider a function x = $4y^2 + y + 1$, and value of x for equidistant spaced values of y are**:**

X	1	3	5	7	9	11	13
Y	6	40	106	204	334	496	690

Solution: let us solve this problem with all the four central difference interpolation formulae:

Here let us find this for y=6, for that, we have y_0

y0=7 and h=2 and p=

y	x	Δx	Δ^2x	Δ^3x
1	6			
		34		
3	40		32	
		66		0
5	106		32	
		98		0
7	204		32	
		130		0
9	334		32	
		162		0
11	496		32	
		194		
13	690			

1. **Gauss forward interpolation formula**:

$(6) = 204 + (-0.5)(130) + \frac{(-0.5)(-0.5-1)}{2}(32)$......... [using(3)]

$= 204 - 65 + (-0.5)(-1.5)(16)$

$= 204 - 65 + 12 = 151$

Hence we get x(6) = 151 by gauss forward interpolation formula.

using (3)]

2. Gauss backward interpolation formula:

$(6) = 204 + (-0.5)(98) + \frac{(-0.5)(-0.5+1)}{2}(3)$ ... ... [using (4)]

$= 204 - 49 + (-0.5)(0.5)(16)$

$= 204 - 49 - 4$

$=151$

3. Stirling's formula:

$(6) = 204 + (-0.5)\left[\frac{98+130}{2}\right] + \frac{(-0.5)^2}{2!}(32)$............. [using (5)]

$= 204 + (-0.5)(114) + 0.25(16)$

$= 204 - 57 + 4$

$=151$

4. Bessel's formula:

$(6) = 204 + (-0.5)(130) + \frac{(-0.5)(-0.5-1)}{2}\left[\frac{32+32}{2}\right]$ ... ... [using(6)]

$=204- 65+(0.375)(32)$

$=151$

Hence, by all the methods we get the same result as x (6) =151

3.2. The population of a town in a year is: (Das, 2016)

Year	1931	1941	1951	1961	1971
Population(in thousand)	15	20	27	39	52

Sol: let us solve this problem with all the four central difference interpolation formulae: Here let us find this for the year=1946, for that, we have $x0$ = 1951 and h=2 and

p=$\frac{x-x_0}{h}$=$\frac{1946-1951}{2}$=-0.5

Year	Population(in thousand)	Δ_x	$\Delta^2{}_x$	$\Delta^3{}_x$	$\Delta^4{}_x$
1931	15				
		5			
1941	20		2		
		7		3	
1951	27		5		-7
		12		-4	
1961	39		1		
		13			
1971	52				

1-Gauss forward interpolation:

$(1946) = 27 + (-0.5)12 + \frac{(-0.5)(-1.5)}{2}(5) \ldots \ldots \ldots \ldots . .$ [using(3)]

=27-6+1.875

=22.875 thousand

2- Gauss backward interpolation:

$(1946) = 27 + (-0.5)7 + \frac{(-0.5)(-0.5+1)}{2}(5) \ldots \ldots \ldots . .$ [using(4)]

= 27-3.5-0.625

= 22.875

=22875 thousand

3. Stirling's formula:

(1946)=27+(−0.5)[$\frac{7+12}{2}$]+ $\frac{(0.5)^2}{2}$ (5)............[using(5)]

= 27-4.75+0.625

= 22.875

= 22875 thousand

4. Bessel's formula:

(1946) = 27 + (−0.5)12 + $\frac{(-0.5)(-0.5-1)}{2}[\frac{5+1}{2}]$ ... ... [using(6)]

= 27-6+1.875

=22.875

=22875 thousand

Hence, by all the methods we get the same result as x (1946) =22875 thousand

5.Alternatives for interpolation formulas:

So far, we have come to know about the several interpolation formulae likeNewton'sforward, Newton's backward, Gauss's backward and Gauss's forward, Stirling's formula, Bessel's formula for calculating $xp$for equally spaced tabulated values.Now we will do the comparative study of all of them that which formula or method is best applicable at which stage:The central difference formula has smaller coefficients that converge faster than Newton's formula. The coefficients in Stirling's formula fall more rapidly than those in Bessel's formula after a few terms, while the coefficients in Bessel's formula decline more rapidly than those in Newton's formula after a few terms.As a result,wherever possible, we will prefer the central difference formula instead of Newton's formula.The correct interpolation method, on the other hand, is determined by the position of the interpolated values in the given data. The following rules will assist you in comprehending all of the strategies.:

1. For calculating the tabulated set of values at the starting of the table, use Newton’s forward interpolation formula.

2. For calculatingvalue nearby the end of the table,prefer Newton’s backward interpolation formula.

3. For calculating value close to the center of the table, prefer either Stirling’s formula or Bessel’s formula.

• If someone wants to interpolate a value of 'p' that is between$\frac{-1}{4}$ $and\frac{1}{4}$, prefer

Stirling’s formula.

• Use Bessel’s formula, if someone wants to interpolate for a value of 'p' between

$\frac{1}{4}$ and $\frac{3}{4}$.

• By reminding all these rules one can easily make the right choice that in which problem which method is best applicable.

6. Applications of Interpolation in Real life:

As we know, the method of finding values at unknown points using known values or sample points is known as interpolation. So, various practical uses of interpolation as follows:

1. The concept of interpolation can be used to estimate unknown parameters for any geographic point data, such as rainfall,elevation,noise levels, chemical concentrations, and so on.

2. Using interpolation techniques to zoom digital images: Image processing for low-resolution digital images is a very challenging problem nowadays. It is because of errors that occur in quantization and sampling. Zooming in on such images is extremely difficult. As a result, when zooming in low-resolution images, we use interpolation functions.

3. Interpolation is often used outside the domain of mathematics to scale images and transform the sample rate of digital signals.

4. Interpolation is helpful whenever you have to scale things up or down regularly.

For example- We may know how much food costs for a 10person event, 50 person event, or 100 person event, but we need a precise estimate of how much catering will cost for 25 people or 75 people. That's what interpolation allows us to achieve.

These are some of the examples of interpolation in real life as well as in other disciplines.

7. Observation of the Study: Based on the observations and readings, it is concluded from this paper that Interpolation is mostly used to assist users, who may be scientists, photographers, engineers, or mathematicians, in determining what information may exist outside of their collected information. The comparative study of all the methods helps us to understand that how one can solve various problems by using the best suitable interpolation method. Or how to make the right choice that which formula gives the result of the given problem more accurately.

EX 1: Determine the best method among N.F.I.F , N.B.I.F and (contral.I.f for estimating (0.1) on the following table , whare f(x)= $\cos^2\left(\frac{\pi}{2}x\right)$

Sol 1:

X_m0.1

x	y	Δ	Δ^2	Δ^3	Δ^4
-2	1	-1			
-1	0	1	2	-4	
0	1	-1	-2	4	8
1	0	1	2		
2	1				

Note that , since $\Delta yi=\nabla y_{i+1}$ $=\delta y_{i+\frac{1}{2}}$

Then for above interpolations , we have the some table

X_m=0.1 , x_0=0 , h=1 ⇒ m=$\frac{0\cdot 1-0}{1}$=0.1

I. Forward : f(0.1) ≅ 1+(0.1)(-1) + $\frac{0.1(0\cdot 1-1)}{2!}$(2)=0.81

II. Backward: f(0.1) ≅ 1+(0.1)(1) + $\frac{0.1(0\cdot 1+1)}{2!}$(2)=1.21

III. Central : f(0.1) ≅ $\frac{1}{2}$ + (0.1-0.5)(-1)+$\frac{0.1(0.1+1)}{2!}$(0)+$\frac{0.1(0.1-1)(0.1-0.5)}{3!}$ (4)=0.9194

Since f(0.1) = $\cos^2\left(\frac{\pi}{2}(0\cdot 1)\right)$ ≅0.975528 , then

E_1= 0.975528 -0.81= 0.,165528 (Forward)

E_2= 0.975528 – 1.21 = 0.234472 (Backward)

E_3=0.975528 – 0.9194 = 0.056128 (Central)

Thus, the central method is the best to interpolate f(0.1)

EX 2: Determine the best method among N.F.I.F , N.B.I.F and (Contral.i.f for estimating F(4.5) on the following table , whare f(x)= $3X^2+2X+1$

Sol 2 :

X	Y	Δ	Δ^2
2	17	17	
3	34	23	6
4	57	29	6
5	86	35	6
6	121	41	6
7	162	47	6
8	209		

4.5 →

$X_M=4.5$, $X_O=5$, $h=1 \Rightarrow m=\frac{4.5-6}{1}=-0.5$

I. Forward: $F(4.5) \cong 86+(-0.5)(35)+(-0.5-1)3=70.75$

II. Backward: $F(4.5) \cong 86+(-0.5)(29)+(-0.5-1)3=70.75$

III. Central: $F(4.5) \cong \left(\frac{86-121}{2}\right)+(-0.5-0.5)+\frac{(-0\cdot 5)(-0.5-1)}{2!}\left(\frac{6+6}{3}\right)=70.75$

since $f(4.5) = 3(4.5)^2 + 2(4.5)+1=1241$

$E_1=E_2=E_3 = 1241-70.75= 1170.25.$

Thus , All above methods ore the seme to interpolate f(4.5)

EX3: Determine the best method among N.F.I.F , N.B.I.F and (Bessel.i.f for estimating F(6), on the following data , whare f(x)= $4X^2+X+1$

SOL3:

$X_m=6$ →

		Δ	Δ^2	Δ^3
	6			
	4	0		
	4	2		
	6	4		
	0			

$X_M=6$, $X_O=7$, h=2 ⇒ m=$\frac{6-7}{2}$=-0.5

I. Forward : f(6) ≅ 204+(-0.5)(130)+$\frac{(-0.5)(-0.5+1)}{2}$(32)=151

II. Backward: f(6) ≅204+(-0.5)(98)+ $\frac{(-0.5)(-0.5+1)}{2}$(32)=151

III. Central : f(6) ≅204+(-0.5)(130)+ $\frac{(-0.5)(-0.5+1)}{2}\left(\frac{32+32}{2}\right)$ =151

Since F(6)=4(6^2)+(6)+1=0

$E_1=E_2=E_{3=}$151-151=0

Thus , All above methods ore the seme to estimate F(6)

And F(6)= 151 is an exact solution.

EX4: Determine which one of these methods among newton, backwand forward and central is the best for estimating f(3.1) where $f(x)=x^2$.on [-3,3]

Sol4:

X	Y	Δ	Δ^2	Δ^3
-3	9	-5		
-2	4	-3	2	0
-1	1	-1	2	0
0	0	1	2	0
1	1	3	2	0
2	4	5	2	
3	9			

$X_m=3.1$

$X_M=3.1$

$X_O=3$, h=1 $\Rightarrow$ m=$\frac{3.1-3}{1}$=0.1

I. Forward is failing here, because we can't determine Y_0.

II. Backward: $f(3.1) \cong 9+(0.1)(5) + \frac{(0.1)(0\cdot 1+1)}{2!}(2)=9.61$

III. Central is failing here, because we can't determine Y_1 to calculate $\mu y_{\frac{1}{2}}$.

So thet , backward is the best method for (3.1)

EX 5: Determine the best method among N.F.I.F , N.B.I.F and (Contral.i.f for interpolating F(1.45), whure F(x)= [x] is 0 floor function.

Sol 5:

X_m=1.45

x	Y	Δ	Δ^2
-2.5	-3	1	
-1.5	-2	1	0
-0.5	-1	1	0
0.5	0	1	0
1.5	1	1	0
2.5	2		

X_M=1.45, X_O=1.5 , h=1 $\Rightarrow$ m=$\frac{1.45-1.5}{1}$=-0.05

I. Forward : f(1.45) $\cong$ 1+(-0.05)(-1) = 0.95

II. Backward: f(1.45) $\cong$ 1+(-0.05)(1) =0.95

III. Central : f(1.45) $\cong$ $(\frac{1+2}{2})$ + (-0.05-0.5)(1)=0.95

Since f(1.45)= [1.45]=1 , them

$E_1=E_2=E_3$=1-0.95=0.05

Thus , All the methods ore the seme to interpolate f(1.45).

Conclusion

In conclusion, the field of interpolation offers a rich tapestry of methods and approaches that researchers have explored over the years. From classical techniques like Lagrange's and Newton's interpolation to innovative variations such as the central difference interpolation proposed by Mohiuddin et al. (2019), the spectrum of tools available for estimating values between data points is diverse and continually evolving. Studies by various scholars have delved into the nuances of different interpolation methods, shedding light on their strengths, limitations, and applications in diverse domains.

The comparative analyses presented in these research efforts have highlighted the efficiency, accuracy, and versatility of newer interpolation formulas in contrast to traditional approaches. Singh and Bhandari (2009) introduced finite difference formulae for unequal sub-intervals, while Batter et al. (2009) explored interpolation's utility in lidar technology applications. Garner and Go (2013), Muthumala and Uthra (2014), Srivastav et al. (2015), Das and Chakrabarty (2016) have contributed unique perspectives on interpolation methods tailored to specific real-world challenges.The ongoing pursuit of refining interpolation techniques is evident in the research endeavors documented in the introduction. Researchers have strived to enhance accuracy, efficiency, and adaptability in interpolation methodologies to cater to the complexities of modern computational and analytical tasks. The comparative evaluations undertaken have revealed promising advancements and novel insights that pave the way for a deeper understanding and utilization of interpolation in diverse scientific and practical contexts.

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پوختە

کە سنووردارکردن و یاسا ڕاستەقینەکانی کارپێکراو دەگەیەنن. ئێستا پێویستە مامەڵە لەگەڵ ئەم مەرجانەدا بکرێت و بڕیارێک خۆی بناسێنێت. یەکێک لە ڕێگاکان ئەوەیە کە بەردەوام بیت لە بەکارهێنانی ستراتیژییە ئاساییەکان بۆ بیرکاری، وەرگرتنی وەڵام وەک ڕێچکەیەک، یان کۆمەڵێک فۆرمول. ستراتیژییەکی دیکە بریتییە لە گەیاندنی مەرجەکان بۆ ئەوەی بە شێوەیەکی حیساباتی چارەسەر بکرێن، واتە بە بەکارهێنانی ستراتیژییەکان بۆ تاقیکردنەوەی بیرکاری یان شیکاری ژمارەیی. لەم توێژینەوەیەدا ئامانجمان باسکردنی ئینتەرپۆلاسیۆنە، شێوازە جۆراوجۆرەکان بۆ چارەسەرکردنی ئینتەرپۆلاسیۆنی جیاوازی ناوەندی، گشتاندنەکانیان. هەروەها باس لە بەکارهێنانی ئینتەرپۆلاسیۆن دەکرێت و مرۆڤ دەتوانێت بە ئاسانی لە چەمکەکانی توێژینەوەکە تێبگات.