زانکۆی سەلاحەدین - هەولێر

Salahaddin University-Erbil

Cubic spline Interpolation

a special case for Spline interpolation that is used very often to avoid the problem of Runge's phenomenon.

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Certification of the supervisors

I certify that this work was prepared under my supervision at the Department of

Mathematics/College of Education/Salahaddin University-Erbil in partial

fulfillment of the requirements for the degree of Bachelor of philosophy of Science

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Date: /4/ 2023

In view of the available recommendations, I forward this work for debate by the

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Date: /4/ 2023

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Primarily, I would like to thanks my god for helping me to complete this research with success.

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Abstract

Spline interpolation is a vital tool in numerical analysis, offering a flexible approach to approximating functions between data points.

This research project explores various spline techniques, their theoretical foundations,

practical implementations, and applications across diverse fields.

By synthesizing theoretical insights with practical considerations, this study aims to provide valuable guidance for researchers and practitioners leveraging spline interpolation methods.

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Introduction

Spline interpolation serves as a cornerstone in the realm of numerical analysis and computational mathematics, offering a powerful method for approximating functions between discrete data points. With applications spanning diverse fields such as
engineering, computer graphics, finance, and scientific computing, understanding the principles and applications of spline interpolation is crucial for researchers and practitioners alike.

In this research project, we embark on a comprehensive exploration of spline interpolation techniques, aiming to elucidate their theoretical underpinnings, computational aspects, and practical implications. Spline interpolation involves

Chapter one

Definitions

Definition of numerical 1.1:

Numerical refers to anything related to number or numerical value . it involves the use pf numbers , calculation . or measurements in various contexts such as mathematics statics or data analysis.

Definition of function 1.2 :

In mathematics a function is a related between a set of inputs (domain) where each input is related to exactly one output .

Definition of spline function 1.3 :

A spline function is piecemise defined polynomial function use in mathematical and computational applications particularly in the representation and interpolation of curves or surface .

Definition of polynomial 1.4 :

A polynomial is a mathematical expression consisting of variables raised to nonnegative integer exponents, combined through addition subtraction and multiplication .

Definition of interval 1.5 :

An interval in mathematics is a set of real numbers that includes all the numbers between any two given numbers within the set it can be expressed in various forms such as open intervals , closed intervals or half-open intervals Interpolation by spline function

Definition 1.6 : a function S is called spline function of degree K if

1- The domain of s is an interval [a , b]

2- $s\ \acute{s}, \acute{s}, \acute{\acute{s}} \ldots s^{(k-1)}$ are all continuous function on [a, b]

3- The are points x_i partitioning where $a = x_0 < x_1 < \cdots < x_n = b$ and such that S is a polynomial of degree at most K an each subinterval $[\ , x_{i+1}]$

Definition 1.7 : a spline function is a function of polynomial pieces joined together with certain smooth conditions .

We are forced to write.

$$S(x) = \begin{cases} s_o(x) & if\ x_0 \le x \le x_1 \\ s_1(x) & if\ x_1 \le x \le x_2 \\ \vdots & \\ s_{n-1}(x) & if\ x_{n-1} \le x \le x_n \end{cases}$$

Or can write

$$F(x) = \begin{cases} s_0(x) & if\ \ x \in [\,x_0, x_1] \\ s_1(x) & if\ \ x \in [\,x_1, x_2] \\ \vdots & \\ s_{n-1}(x) & if\ \ x \in [\,x_{n-1}, x_n] \end{cases}$$

Note : the function S(x) that we wish to construct consists of (n-1) polynomial pieces the interpolation conditions are $s(x_i) = y_i$, $1 \leq i \leq n$

The continuity condition are imposed only at the interior knots $x_2, x_3, \ldots x_{n-1}$ these condition are written as . $\lim_{x \to x_i -} s^{(i)}(x) = \lim_{x \to x_j +} s^{(i)}(x)$, j=1,2 … , n-1 . i=0,1, … , k-1

learning objective : after successful completion of this lessen you should be able to .

1) Develop linear spline interpolation to given data points
2) Estimate unknown data points from the linear spline interpolation .

First – degree spline [linear spline interpolation]

A polygonal function is a spline of degree one which consists linear polynomials joined together to achieve continuity The points $x_0, x_1, \ldots x_n$ are called knots .

Definition 1.8: a function S is called a spline of degree one (first degree spline)

1- The domain of S is an interval [a,b]

2- S is continuous on [a,b]

There is a partitioning of the interval $a = x_0 < x_1 < \cdots < x_n = b$ such that

S is a linear polynomial on each subinterval [, x_{i+1}]

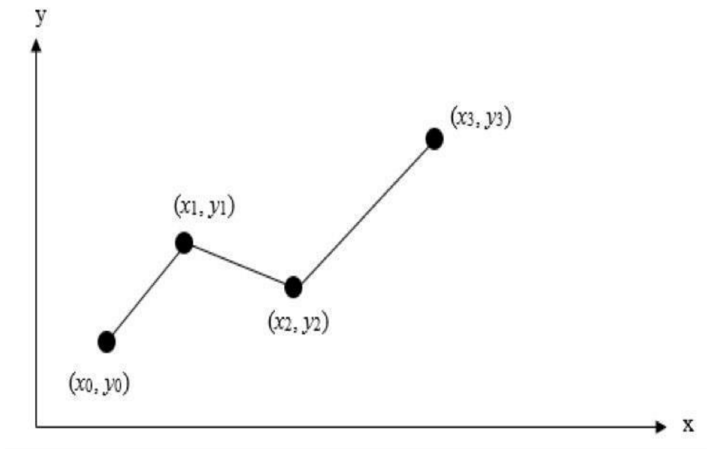
Where each piece of S(x) is a linear polynomial for first degree spline .

$$\boldsymbol{s_i(x) = a_i(x) + b_i}$$

A function such as S(x) is called piecewise linear

$$\boldsymbol{s_i(x) = m_i(x - x_i) + y_i}$$

Where $m_i = \frac{y_{i+1} - y_i}{x_{i+1} - x_i}$ for i= 0 , 1, … , n-1 m is a slopes between x_{i-1} *and* x_i



S(x) is give above data by

$$s_0(x) = m_0(x - x_0) + y_0 \quad , \quad x_0 \leq x \leq x_1$$
$$s_1(x) = m_1(x - x_1) + y_1 \quad , \quad x_1 \leq x \leq x_2$$

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$$s_{n-1}(x) = m_{n-1}(x - x_{n-1}) + y_{n-1} \quad , \quad x_{n-1} \leq x \leq x_n$$

Example 1.1 :

x_i	y_i
0	4
2	-2
5	19
6	59

$$S(x) = \begin{cases} s_0(x) \ \ if \quad x \in [0,2] \\ s_1(x) \ \ if \quad x \in [2,5] \\ s_2(x) \ \ if \quad x \in [5,6] \end{cases}$$

1) $s_0(x) = m_0(x - x_0) + y_0$

$m_0 = \frac{-2-4}{2-0} = \frac{-6}{2} = -3$

$s_0(x) = -3(x-0) + 4$

$s_0(x) = -3x + 4$

2) $s_1(x) = m_1(x - x_1) + y_1$

$m_1 = \frac{19-(-2)}{5-2} = \frac{21}{3} = 7$

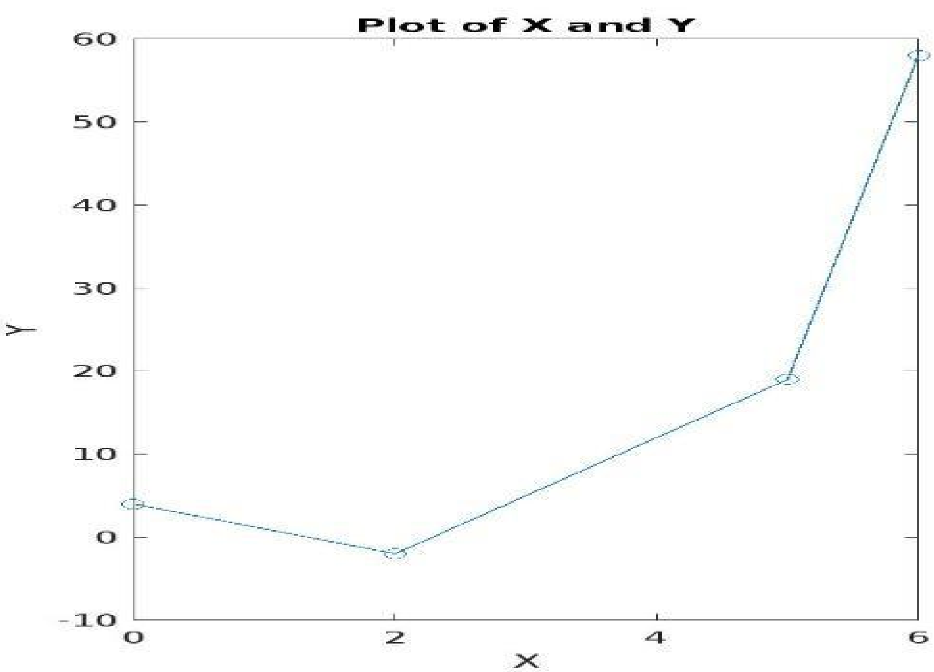
$s_1(x) = 7(x-2) + (-2)$

$s_1(x) = 7x - 16$

3) $s_2(x) = m_2(x - x_2) + y_2$

$m_2 = \frac{58-19}{8-5} = 39$

$s_2(x) = 39x - 176$



Chapter two

Spline of degree two (quadratic spline)

- Spline of degree 2

A function Q is called spline of degree 2 if

- The domain of Q is an interval [a , b]
- Q and $\acute{Q}$ are continuous function on [a , b]
- There are points x_i(called knots) such that $a = x_0 < x_1 < \cdots < x_n = b$ And Q is a polynomial of degree at most 2 each subinterval $[x_i, x_{i+1}]$
- A quadratic spline is a continuously differentiable piecewise quadratic function .

$Q(x_i) = y_i \quad , \quad i = 0,1, \dots , n \quad , since\ Q(x)$ is continuous we can put

$m_i = \acute{s}\ (x_i)$ and $m_{i+1} = \acute{s}(x_{i+1})$

From lagrange interpolation of degree one we get :

$$\acute{Q}\ (x) = \frac{x-x_{i+1}}{x_i+x_{i+1}} m_i + \frac{x-x_i}{x_{i+1}-x_i}\ m_{i+1}$$

Integration both sides first degree spline we get :

$$Q(x) = \frac{(x - x_{i+1})^2}{2(x_{i+1} - x_i)}\ m_i + \frac{(x - x_i)^2}{2(x_{i+1} - x_i)}\ m_{i+1} + c$$

Where c is the constant of integration to find c use the interpolation condition.

$Q(x_i) = y_i$ we obtain $c = y_i - \frac{(x_i-2x_{i+1})}{2} m$

Substituting the value of in the above equation we get :

$$Q(x) = \frac{m_{i+i+1}-mx_i}{2(x_{i+1}- x_i)}\ (x - x_i)^2 + (x - x_i)m_i + y_i$$

Where $Q(x_i) = y_i \quad , \quad \acute{Q}\ (x_i) = m_i \qquad i = 0,1, \dots , n - 1$

And $\acute{Q}\ (x_{i+1}) = m_{i+1}$

These three conditions defined he function $Q(x)$ uniquely on $[x_i, x_{i+1}]$ As given in the equation

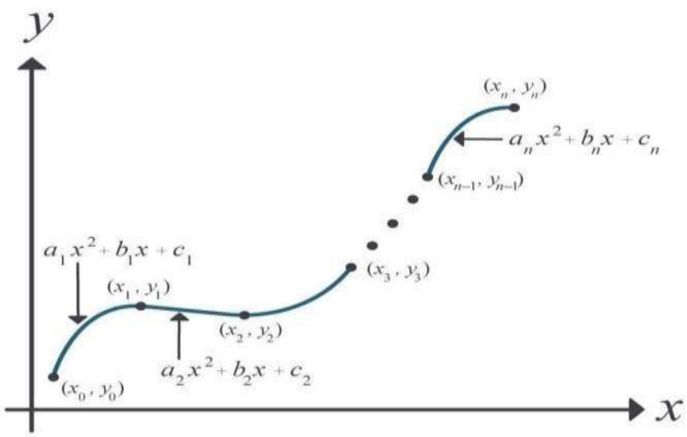
$m_{i+1} = -m_i + 2\ (\frac{y_{i+1}-y_i}{x_{i+1} - x_i})\ ,\ i = 0,1, \dots , n - 1$ Where m_0 is arbitrary

Note :

$$\lim Q(x) = \quad \lim Q(x)$$

$x \to xi-$ $\qquad$ $x \to xi+$

$$\lim Q(x) = \quad \lim Q(x)$$



$x \to xi-$ $\qquad$ $x \to xi+$

Example 2.1:find a quadratic splinenterpolation for these data

x	-1	1	3	3.5
y	1	2	0	5

Solution : Let

$$Q(x) = \begin{cases} Q_0(x) & \text{if } x \in [-1,1] \\ Q_1(x) & \text{if } x \in [-1,3] \\ Q_2(x) & \text{if } x \in [3,3.5] \end{cases}$$

Where $(x) = \frac{m_{i+1} - m_i}{2(x_{i+1} - x_i)} \quad (x - x_i)^2 + (x - x_i)m_i + y_i \quad for \ \ i = 0,1,2$

To find $m_0, m_1, m_2, and\ m_3$

Let $m_0 = 0$

$$m_1 = m_0 + 2\left(\frac{y1 - y0}{x_1 - x_0}\right) = 0 + 2\left(\frac{2-1}{1-(-1)}\right) = 1$$

$$m_2 = m_1 + 2\left(\frac{y2 - y1}{x_2 - x_1}\right) = -1 + 2\left(\frac{0-2}{3-1}\right) = 3$$

$$m_3 = m_2 + 2\left(\frac{y3 - y2}{x_3 - x_2}\right) = -(-3) + 2\left(\frac{5-0}{3.5-3}\right) = 23$$

$$Q_0(x) = \left(\frac{m1 - m0}{2(x_1 - x_0)}\right)(x - x_0)^2 + (x - x_0)m_0 + y_0$$

$$Q_0(x) = \left(\frac{1-0}{2(1-(-1)}\right) \quad (x - (-1))^2 + (x - (-1))(0) + 0 = \frac{1}{4} \ (x + 1)^2$$

$$Q_1(x) = \frac{m2 - m1}{2(x_2 - x_1)} \quad (x - x_1)^2 + (x - x_1)m_1 + y_1$$

$$Q_1(x) = \frac{-3-1}{2(3-1)} \quad (x - 1)^2 + (x - 1)(1) + 2 = -(x - 1)^2 + (x - 1) + 2$$

$$Q_2(x) = \frac{m3 - m2}{2(x_3 - x_2)} \ (x - x_2)^2 + (x - x_2)\ m_2 + y_2$$

$$Q_2(x) = \frac{23-(-3)}{2(3.5-3)}(x - 3)^2 + (x - 3)(-3) + 0 = 26(x - 3)^2 - 3(x - 3)$$

$$Q(x) = \begin{cases} \frac{1}{4}(x + 1)^2 & \text{if } x \in [-1,1] \\ -(x - 1)^2 + (x - 1) + 2 & \text{if } x \in [1,3] \\ 26(x - 3)^2 - 3(x - 3) & \text{if } x \in [1,3] \end{cases}$$

Chapter 3

Spline of degree three (cubic spline)

Definition 3.1 : a function C is called a spline of degree three if

1. The domain of C is an interval [a,b]
2. $C, C' \, and \, C''$ are continuous on [a,b]
3. There are points x_i the knots of C such that

   $a = x_0 < x_1 < \cdots < x_n = b$ and such that C is a polynomial of degree at most 3 on each subinterval $[x_i, x_{i+1}]$

   We next turn to interpolation a table of given values a cubic spline whose knots coincide with the x value in the table

x	x_0	x_1	…	x_{n-1}	x_n
y	y_0	y_1	…	y_{n-1}	y_n

the function C(x) the we are constructing consists of (n-1) cubic polynomial pieces :

$$Q(x) = \begin{cases} c_{0(x)} & if \;\; x \in [x_0, x_1] \\ C(x)=_{1(x)} & if \;\; x \in [x_1, x_2] \\ c_{n-1(x)} & if \;\; x \in [x_{n-1}, x_n] \end{cases}$$

The interpolation condition

$$\boldsymbol{(x_i) = y_i \quad , \quad 0 \le i \le n}$$

The continuity conditions are imposed only at the interior knots

$x_1, x_2, \ldots x_{n-1}$

$$\lim_{x - xi^-} C^{(k)}(\mathrm{xi}) = \lim_{x - xi^+} C^{(k)}(\mathrm{xi}) \qquad i = 0,1,2, \ldots, n-1, \quad k = 0,1,2$$

The continuity condition are needed in order to use all the degree of freedom available A cubic spline function s called a natural cubic spline if $(x_0'') = C(x_n'') = 0$

Algorithm for natural cubic spline :

To derive a natural cubic spline function since $\acute{}(x)$ is continuous

are the number $M_i = \acute{C}(x_i)$ $0 \le i \le n$ we do not now the values of M_1, M_2, ... M_{n-1} , but of course $M_0 = M_n = 0$

If M_i were known , we construct a linear polynomial taking the values M_i and M_{i+1} at the end point thus :

$$\acute{C}(x) = \frac{M_i}{h_i}(x - x_i) + \frac{M_i}{h_i}(x_{i+1} - x)\ h_i$$

Where $h_i = x_{i+1} - x_i$ integrate this twice gives the function

$$\mathrm{C}(x) = \frac{M_{i+1}}{6h_i}(\mathrm{x} - \mathrm{xi})^3 + \frac{M_i}{6h_i}(\mathrm{x}_{i+1} - \mathrm{x})^3 + cx + d$$

Where c and d are constants of integration by adjusting the integration constants .

We obtain a form $C_i(x)$:

$$\mathrm{C}(x) = \frac{M_{i+1}}{6h_i}(\mathrm{x} - \mathrm{xi})^3 + \frac{M_i}{6h_i}(\mathrm{x}_{i+1} - \mathrm{x})^3 + c(x - x_i) + D_i(x_{x+1} - x)$$

Where Ci and Di are constants of integration .

the interpolation condition $C_i(x_i) = y_i$ *and* $C_i(x_i) = y_{i+1}$ *can be* imposed to the appropriate values Ci an Di we get :

$$C(x) = \frac{M_{i+1}}{6h_i}(\mathrm{x} - \mathrm{xi})^3 + \frac{M_i}{6h_i}(\mathrm{x}_{i+1} - \mathrm{x})^3 + \left(\frac{y_{i+1}}{h_i} - \frac{h_i}{6}m_{i+1}\right)(\mathrm{x} - \mathrm{xi}) + \left(\frac{y_i}{h_i} - \frac{h_i}{6}\right)(\mathrm{x}_{i+1} - \mathrm{x})$$

To find Mi , i=1,2, … ,n-1 , we have form the above equation

$$\acute{C}(x) == \frac{M_{i+1}}{2h_i}(\mathrm{x} - \mathrm{xi})^2 + \frac{M_i}{2h_i}(\mathrm{x}_{i+1} - \mathrm{x})^2 + \left(\frac{y_{i+1}}{h_i} - \frac{h_i}{6}m_{i+1}\right) + \left(\frac{y_i}{h_i} - \frac{h_i}{6}\right)(\mathrm{x}_{i+1} - \mathrm{x})$$

This gives : $\acute{C}_i(x) = \frac{h_i}{6}(M_{i+1}) - \frac{h_i}{3}M_i + b_i$

Where $b_i = \frac{1}{h_i}(y_{i+1} - y_i)$

and $h_i = x_{i+1} - x_i$
analogously we have

$$C'_{i-1}(x_i) = \frac{h_{i-1}}{6}(M_{i-1}) - \frac{h_{i-1}}{3} M_i + b_{i-1}$$

Where $b_{i-1} = \frac{1}{h_{i-1}}(y_i - y_{i-1})$

When these are set as equals we get after rearrangement

$h_{i-1}M_{i-1} + 2(h_{i-1} - h_i) + h_i M_{i+1} = 6(b_i - b_{i-1}) \quad for\ 1 \le i \le n - 1$ By letting $u_i = 2(h_{i-1} + h_i)$ and $v_i = 6(b_i - b_{i-1})$ We obtain a tridiagonal system of equation :

$$M_0 = 0$$

$$h_{i-1}M_{i-1} + u_i M_i + h_i M_{i+1} = v_i \qquad 1 \le i \le n - 1$$

$$M_n = 0$$

To be solved for the $M_i S$ the simplicity of the first and last equations is a result of the natural cubic spline conditions

$C'(x_0) = S'(x_n) = 0$

Example 3.1:

Derive the equation of the natural cubic interpolation spline for the following data

x	1	2	3
y	2	6	2

Solution : we know that

$$C(x) = \begin{cases} C_0(x) & if \quad x \in [1,2] \\ C_1(x) & if \quad x \in [2,3] \end{cases}$$

$$C_0(x) = \frac{M_1}{6h_0}(x - x_0)^3 + \frac{M_0}{6h_0}(x_1 - x)^3 + \left(\frac{y_1}{h_0} - \frac{h_0}{6}M_1\right)(x - x_0) + \left(\frac{y_0}{h_0} - \frac{h_0}{6}M_0\right)(x_1 - x)$$

$$C_1(x) = \frac{M_2}{6h_1}(x - x_1)^3 + \frac{M_1}{6h_1}(x_2 - x)^3 + \left(\frac{y_2}{h_1} - \frac{h_1}{6}M_2\right)(x - x_1) + \left(\frac{y_1}{h_1} - \frac{h_1}{6}M_1\right)(x_2 - x)$$

$h_0 = x_1 - x_0 = 2 - 1 = 1$

$b_0 = \frac{y1 - y0}{h_0} = \frac{0 - 2}{1}\ 1 = -2$

$u = 2(h + h_1) = 2(1 + 1) = 4$

$h_1 = x_2 - x_1 = 3 - 2 = 1$

$b_1 = \frac{y2 - y1}{6h_1} = \frac{1 - 0}{1} = 1$

$v = 6(b_1 + b) = 6(1 - (-2)) = 18$

Thus $M_0 = 0 \quad h_0M_0 + u_1M_1 + h_1M_1 = v_1$, $\quad M_2 = 0$

Since $M_0 = M_2 = 0$

Then $uM_1 = v$

$4M_1 = 18 \quad M_1 = \frac{18}{4} = \frac{9}{2}$

$$C(x) = \begin{cases} \frac{-3}{8}(x-1)^3 + (x+1) & if \quad x \in [1,2] \\ \frac{-3}{8}(2-x)^3 + 2x + \frac{11}{2} & if \quad x \in [2,3] \end{cases}$$

Is cubic spline function

Chapter 4

Problem: the statistics and number of students in the 12 grade in the center khabat district

Who use these materials daily

variable					
x_0	coffee	y_0	127	1-2
x_1	Milk	y_1	154	1-2
x_2	Tea	y_2	376	1-5
x_3	juice	y_3	219	1-3
x_4	gum	y_4	148	1-2

Cubic spline formula is

$$f(x) = \frac{(x_i - x)^3}{6h} M_{i-1} + \frac{(x - x_{i-1})^3}{6h} M_i + \frac{((x_i - x))}{h}\left(y_{i-1} - \frac{h^2}{6} M_{i-1}\right) + \frac{((x - x_{i-1}))}{h}\left(y_i - \frac{h^2}{6} \quad M_i\right) \rightarrow (1)$$

We have, $M_{i-1} + 4M_i + M_{i+1} = \frac{6}{h^2}(y_{i-1} - 2y_i + y_{i+1}) \qquad \rightarrow \ (2)$

xHere $h = 1, n = 4$ and $M_0 = 0, M_4 = 0$

Substitute $i = 1$ in equation (2)

$$M_0 + 4M_1 + M_2 = \frac{6}{h^2}(y_0 - 2y_1 + y_2)$$

$$\Longrightarrow 0 + 4M_1 + M_2 = \frac{6}{1} * \ (127 - 2 * 154 + 376)$$

$$\Longrightarrow 4M_1 + M_2 = 1170$$

Substitute $i = 2$ in equation (2)

$$M_1 + 4M_2 + M_3 = \frac{6}{h^2}(y_1 - 2y_2 + y_3)$$

$$\Longrightarrow M_1 + 4M_2 + M_3 = \frac{6}{1} * (154 - 2 * 376 + 219)$$

$$\Longrightarrow M_1 + 4M_2 + M_3 =- 2274$$

Substitute $i = 3$ in equation (2)

$$M_2 + 4M_3 + M_4 = \frac{6}{h^2}(y_2 - 2y_3 + y_4)$$

$\Longrightarrow M_2 + 4M_3 + 0 = \frac{6}{1} * (376 - 2 * 219 + 148)$

$\Longrightarrow M_2 + 4M_3 = 516$

Substitute $i = 1$ in equation (1), we get cubic spline in 1st interval $[x_0, x_1] = [1,2]$

$$f_{1(x)} = \frac{(x_1-x)^3}{6h} M_0 + \frac{(x-x_0)^3}{6h} M_1 + \frac{(x_1-x)}{h}\left(y_0 - \frac{h^2}{6} M_0\right) + \frac{(x-x_0)}{h}\left(y_1 - \frac{h^2}{6} M_1\right)$$

$$f_{1(x)} = \frac{(2-x)^3}{6} * 0 + \frac{(x-1)^3}{6} * 485.0357 + \frac{(2-x)}{1}\left(127 - \frac{1}{6} * 0\right) + \frac{(x-1)}{1}\left(154 - \frac{1}{6} * 485.0357\right)$$

$$f_{1(x)} = 80.8393x^3 - 242.5178x^2 + 188.6786x + 100 \quad for\ 1 \le x \le 2$$

Substitute $i = 2$ in equation (1), we get cubic spline in 2nd interval $[x_1, x_2] = [2,3]$

$$f_{2(x)} = \frac{(x_2-x)^3}{6h} M_1 + \frac{(x-x_1)^3}{6h} M_2 + \frac{(x_2-x)}{h}\left(y_1 - \frac{h^2}{6} M_1\right) + \frac{(x-x_1)}{h}\left(y_2 - \frac{h^2}{6} M_2\right)$$

$$f_{2(x)} = \frac{(3-x)^3}{6} * 485.0357 + \frac{(x-2)^3}{6} * -770.1429 + \frac{(3-x)}{1}\left(154 - \frac{1}{6} * 485.0357\right) + \frac{(x-2)}{1}\left(376 - \frac{1}{6} * -770.1429\right)$$

$$f_{2(x)} = -209.1964x^3 + 1497.6965x^2 - 3291.75x + 2420.2857 \quad for\ 2 \le x \le 3$$

Substitute $i = 3$ in equation (1), we get cubic spline in 3rd interval $[x_2, x_3] = [3,4]$

$$f_{3(x)} = \frac{(x_3-x)^3}{6h} M_2 + \frac{(x-x_2)^3}{6h} M_3 + \frac{(x_3-x)}{h}\left(y_2 - \frac{h^2}{6} M_2\right) + \frac{(x-x_2)}{h}\left(y_3 - \frac{h^2}{6} M_3\right)$$

$$f_{3(x)} = \frac{(4-x)^3}{6} * -770.1429 + \frac{(x-3)^3}{6} * 321.5357 + \frac{(4-x)}{1}\left(376 - \frac{1}{6} * -770.1429\right) + \frac{(x-3)}{1}\left(219 - \frac{1}{6} * 321.5357\right)$$

$$f_{3(x)} = 181.9464x^3 - 2022.5894x^2 + 7269.1074x - 8140.5718 \quad for\ 3 \le x \le 4$$

Substitute $i = 4$ in equation (1), we get cubic spline in 4th interval $[x_3, x_4] = [4,5]$

$$f_4(x) = \frac{(x_4-x)^3}{6h} M_3 + \frac{(x-x_3)^3}{6h} M_4 + \frac{(x_4-x)}{h}\left(y_3 - \frac{h^2}{6} M_3\right) + \frac{(x-x_3)}{h}\left(y_4 - \frac{h^2}{6} M_4\right)$$

$$f_{4(x)} = \frac{(5-x)^3}{6} * 321.5357 + \frac{(x-4)^3}{6} * 0 + \frac{(5-x)}{1}\left(219 - \frac{1}{6} * 321.5357\right) + \frac{(x-4)}{1}\left(148 - \frac{1}{6} * 0\right)$$

$$f_{4(x)} = -53.5893x^3 + 803.8392x^2 - 4036.607x + 6933.714 \quad for\ 4 \leq x \leq 5$$

For $y(1),\ 1 \in [1,2]$, so

substitute $x = 1$ in $f_1(x)$, we get

$$f_1(1) = 127$$

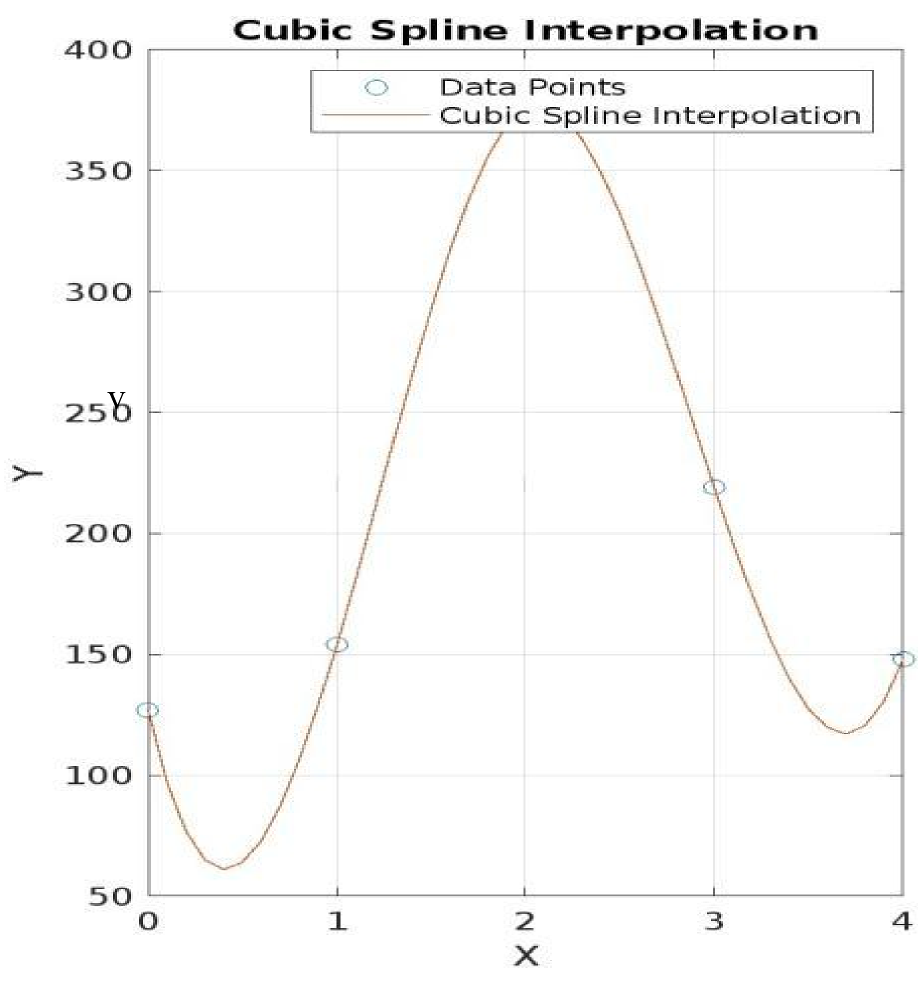
For $y'(1),\ 1 \in [1,2]$, so find $f'_1(x)$

$$f'_1(x) = 242.5178x^2 - 485.0357x + 188.6786$$

Now

substitute $x = 1$ in $f'_1(x)$, we get

$$f'_1(1) = -53.8393$$



Problem: the statistics and number of students in the 12 grade in the center khabat district

Who use these materials daily

variable				
x_0	Tiger	y_0	288	1-4
x_1	Pepsi	y_1	352	1-3
x_2	Cigarettes	y_2	101	5-15
x_3	Hookah	y_3	121	1-2
x_4	Vape	y_4	77	5-10

Cubic spline formula is

$f(x) = \frac{(x_i - x)^3}{6h} M_{i-1} + \frac{(x - x_{i-1})^3}{6h} M_i + \frac{((x_i - x))}{h} \left(y_{i-1} - \frac{h^2}{6} M_{i-1}\right) + \frac{((x - x_{i-1}))}{h} \left(y_i - \frac{h^2}{6} M_i\right) \rightarrow (1)$

We have, $M_{i-1} + 4M_i + M_{i+1} = \frac{6}{h^2}(y_{i-1} - 2y_i + y_{i+1}) \qquad \rightarrow \quad (2)$

Here $h = 1, n = 4$ and $M_0 = 0, M_4 = 0$

Substitute $i = 1$ in equation (2)

$M_0 + 4M_1 + M_2 = \frac{6}{h^2}(y_0 - 2y_1 + y_2)$

$\Longrightarrow 0 + 4M_1 + M_2 = \frac{6}{1} * (288 - 2 * 352 + 101)$

$\Longrightarrow 4M_1 + M_2 =- 1890$

Substitute $i = 2$ in equation (2)

$M_1 + 4M_2 + M_3 = \frac{6}{h^2}(y_1 - 2y_2 + y_3)$

$\Longrightarrow M_1 + 4M_2 + M_3 = \frac{6}{1} * (352 - 2 * 101 + 121)$

$\Longrightarrow M_1 + 4M_2 + M_3 = 1626$

Substitute $i = 3$ in equation (2)

$M_2 + 4M_3 + M_4 = \frac{6}{h^2}(y_2 - 2y_3 + y_4)$

$=> M_2 + 4M_3 + 0 = \frac{6}{1} * (101 - 2 * 121 + 77)$

$=> M_2 + 4M_3 = -384$

Substitute $i = 1$ in equation (1), we get cubic spline in 1st interval $[x_0, x_1] = [1,2]$

$$f_{1(x)} = \frac{(x_1-x)^3}{6h} M_0 + \frac{(x-x_0)^3}{6h} M_1 + \frac{(x_1-x)}{h}\left(y_0 - \frac{h^2}{6} M_0\right) + \frac{(x-x_0)}{h}\left(y_1 - \frac{h^2}{6} M_1\right)$$

$$f_{1(x)} = \frac{(2-x)^3}{6} * 0 + \frac{(x-1)^3}{6} * -629.25 + \frac{(2-x)}{1}\left(288 - \frac{1}{6} * 0\right) + \frac{(x-1)}{1}\left(352 - \frac{1}{6} * -629.25\right)$$

$$f_1(x) = -104.875x^3 + 314.625x^2 - 145.75x + 224 \; for \; 1 \le x \le 2$$

Substitute $i = 2$ in equation (1), we get cubic spline in 2nd interval $[x_1, x_2] = [2,3]$

$$f_{2(x)} = \frac{(x_2-x)^3}{6h} M_1 + \frac{(x-x_1)^3}{6h} M_2 + \frac{(x_2-x)}{h}\left(y_1 - \frac{h^2}{6} M_1\right) + \frac{(x-x_1)}{h}\left(y_2 - \frac{h^2}{6} M_2\right)$$

$$f_2(x) = \frac{(3-x)^3}{6} * -629.25 + \frac{(x-2)^3}{6} * 627 + \frac{(3-x)}{1}\left(352 - \frac{1}{6} * -629.25\right) + \frac{(x-2)}{1}\left(101 - \frac{1}{6} * 627\right)$$

$$f_2(x) = 209.375x^3 - 1570.875x^2 + 3625.25x - 2290 \text{ , } for \; 2 \le x \le 3$$

Substitute $i = 3$ in equation (1), we get cubic spline in 3rd interval $[x_2, x_3] = [3,4]$

$$f_{3(x)} = \frac{(x_3-x)^3}{6h} M_2 + \frac{(x-x_2)^3}{6h} M_3 + \frac{(x_3-x)}{h}\left(y_2 - \frac{h^2}{6} M_2\right) + \frac{(x-x_2)}{h}\left(y_3 - \frac{h^2}{6} M_3\right)$$

$$f_3(x) = \frac{(4-x)^3}{6} * 627 + \frac{(x-3)^3}{6} * -252.75 + \frac{(4-x)}{1}\left(101 - \frac{1}{6} * 627\right) + \frac{(x-3)}{1}\left(121 - \frac{1}{6} * -252.75\right)$$

$$f_3(x) = -146.625x^3 + 1633.125x^2 - 5986.75x + 7322 \text{ , } for \; 3 \le x \le 4$$

Substitute $i = 4$ in equation (1), we get cubic spline in 4th interval $[x_3, x_4] = [4,5]$

$$f_4(x) = \frac{(x_4-x)^3}{6h} M_3 + \frac{(x-x_3)^3}{6h} M_4 + \frac{(x_4-x)}{h}\left(y_3 - \frac{h^2}{6} M_3\right) + \frac{(x-x_3)}{h}\left(y_4 - \frac{h^2}{6} M_4\right)$$

$$f_4(x) = \frac{(5-x)^3}{6} * -252.75 + \frac{(x-4)^3}{6} * 0 + \frac{(5-x)}{1}\left(121 - \frac{1}{6} * -252.75\right) + \frac{(x-4)}{1}\left(77 - \frac{1}{6} * 0\right)$$

$$f_4(x) = 42.125x^3 - 631.875x^2 + 3073.25x - 4758 \; for \; 4 \le x \le 5$$

For $y(1)$, $1 \in [1,2]$, so

substitute $x = 1$ in $f_1(x)$, we get

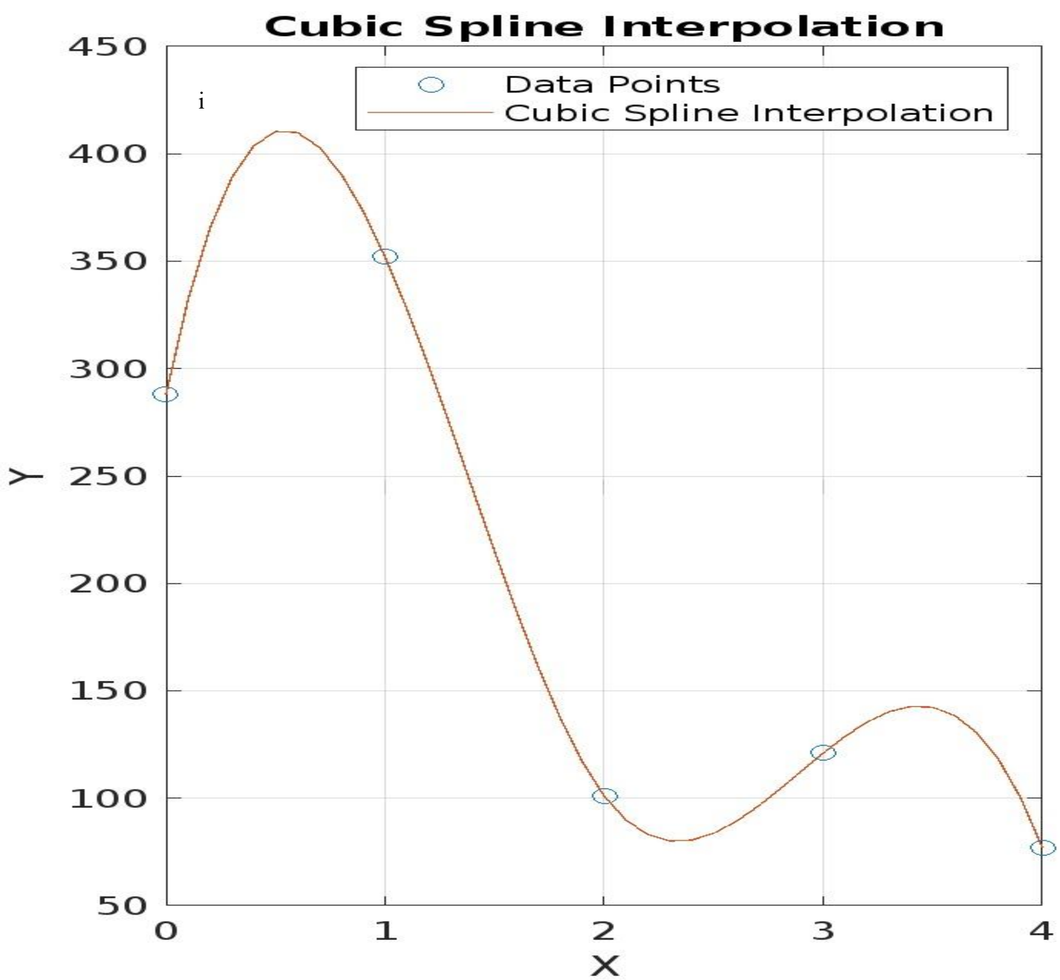
$f_1(1) = 288$

For $y'(1)$, $1 \in [1,2]$, so find $f'_1(x)$

$f'_1 x = -314.625x^2 + 629.25x - 145.75$

Now substitute $x = 1$ in $f'_1(x)$, we get

$f'_1(1) = 168.875$



References[ii]

Al-Ameri, J. M., 2011. Cubic Spline Interpolation for Data Infections of COVID-19. *Al-Qadisiyah Journal of Pure Science,* 26(2411-3514), pp. 23-32.

B. d Bojanov; H. A. Hakopian;A. A. Sahakian, 1993. *Spline Functions and Multivariate Interpolations.* New York : Springer Dordrecht.

C. S., 2023. *Applications of Cubic Spline Interpolation On Functionsin Normed Spaces,* California: s.n.

Elena, 2015. *Linear and Cubic Spline Interpolation.* [Online]
Available at: https://codefying.com/2015/06/07/linear-and-cubic-spline-interpolation/
[Accessed 18 12 2022].

Larry, S. S., 2007. *Spline Functions;basic Theory.* London: Cambridge University press.

McKinely S, L. J., 1998. *Cubic spline interpolation..* s.l.:College Redwoods.

R.Champision, C.T.Lenard & T.M.Mills, 1996. *"An introduction to abstract splines." Matematical Scientist.* s.l.:s.n.

T. L. & K. M., 2011. *Splinecmethods Draft.* s.l.:s.n.

Zyarah, A. A. A. H. I., 2013. *B-Spline Interpolation Method.* Kufa-iraq: s.n.

پوختە:

Spline interpolation

بەرهەمهێنانی فرە ژمارەی ئامرازێکی گرنگە لە شیکاری ژمارەییدا، پێشکەشکردنی نەرم و نیان ڕێگەچارەیەک بۆ نزیککردنەوەی ئەرکەکان لە نێوان خاڵەکانی داتا.

ئەم توێژینەوەیە پڕۆژەکە بەدواداچوون بۆ تەکنیکە جۆراوجۆرەکانی سپلاین دەکات، بنەما تیۆرییەکانیان، جێبەجێکردنی پراکتیکی، و بەکارهێنان لە سەرانسەری بوارە جیاوازەکاندا.

تا کۆکردنەوەی تێڕوانینە تیۆریەکان بە ڕەچاوکردنی پراکتیکی، ئەم توێژینەوەیە ئامانجیەتی بۆ پێشکەشکردنی ڕێنمایی بەنرخ بۆ توێژەران و پزیشکان کە سوود وەردەگرن شێوازەکانی ئینتەرپۆلاسیۆنی سپلاین.