



Marketing Mathematics

Research Project
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Certification of the Supervisor

I certify that this work was prepared under my supervision at the Department of Mathematics / College of Education / Salahaddin University- Erbil in partial fulfillment of the requirements for the degree of Bachelor of philosophy of Science in Mathematics.

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Acknowledgment

First of all, I would like to thanks God for helping me to complete this project with success.

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My thanks appreciations go to Mathematical Department and all my valuable teachers.

Abstract

Marketing mathematics, a fusion of traditional marketing principles and advanced mathematical models, is pivotal in today's data-driven business landscape. This paper delves into the application of mathematical frameworks in marketing, focusing on pricing strategies, cost analysis, discounts, markups, and markdowns. Through quantitative analysis and statistical techniques, marketers can gain deeper insights into consumer behavior, market trends, and campaign performance. The paper explores various mathematical formulas and methodologies employed in marketing decision-making processes, emphasizing the importance of data-driven strategies for business success.

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Introduction

Mathematics Marketing mathematics stands at the nexus of traditional marketing theory and modern data analytics, offering a systematic approach to understanding and optimizing marketing strategies through mathematical frameworks. Beyond its foundational definition, it embodies a dynamic fusion of quantitative analysis and strategic insight that drives innovation and growth in today's fast-paced business landscape amidst the chaos. By harnessing the power of statistical models, machine learning algorithms, and predictive analytics, businesses can unlock hidden patterns within vast datasets, uncovering actionable insights that inform strategic decision-making. from retail and e-commerce to finance and healthcare. Whether it's optimizing pricing strategies, segmenting target markets, or measuring campaign effectiveness, the principles of Marketing Mathematics empower organizations to stay agile, adaptive, and ahead of the competition. moving beyond intuition and anecdotal evidence to embrace a culture of evidence-based decision-making. By embracing the quantitative tools and techniques of Marketing Mathematics, marketers can navigate the complexities of today's marketplace with confidence, precision, and strategic foresight.

Chapter One

1-Definition and basic results

1.1-Cost Price:

is an outlay of money required to produce, acquire, or maintain a product, which includes both physical goods and services.

Costs can come in three forms:

1. A **fixed cost** is a cost that does not change with the level of production or sales (call this "output" for short). In other words, whether the business outputs nothing or outputs 10,000 units, these costs remain the same. Some examples include rent, insurance, property taxes, salaries unrelated to production (such as management), production equipment, office furniture, and much more. Total fixed costs are the sum of all fixed costs that a business incurs.
2. A **variable cost** is a cost that changes with the level of output. In other words, if the business outputs nothing there is no variable cost. However, if the business outputs just one unit (or more) then a cost appears. Some examples include material costs of products, production lab our (hourly or piecework wages), sales commissions, repairs, maintenance, and more. Total variable costs are the sum of all variable costs that a business incurs at a particular level of output.
3. A **blended cost** is a cost that comprises both fixed cost and variable cost components. In other words, a portion of the total cost remains unchanged while another portion depends on the output. For calculation purposes, you must separate a blended cost into its fixed and variable cost components. For examples

A cellphone bill includes a fixed charge for the phone service plus any additional charges for usage, such as long distance, texting, or data. (Olivier 2018, p. 141)

In calculating business costs, fixed costs are commonly calculated on a total basis only since the business incurs these costs regardless of any production. However, variable costs are commonly calculated both on a total and per-unit basis to reveal the overall cost along with the cost associated with any particular unit of output. When these variable costs are assigned on an individual basis it is called a unit variable cost. The calculation of unit variable cost has a further benefit because it allows managers to explore how the total business costs vary at different levels of output.

$$\text{Unit Variable Cost formula : } VC = \frac{TVC}{n} \quad (1.1)$$

where

VC is unit variable cost: The assignment of total variable costs on a per-unit basis.

TVC is Total Variable Cost: The total variable costs in dollars that were incurred at a particular level of output. In the simple average formula, this is represented by the symbol Σx , which stands for the total of all pieces of data.

n is Level of Output: In the simple average formula, n represented the number of pieces of data. For this chapter, the definition is further specified to represent the total number of units produced or sold or the total output that incurred the total variable costs. (Olivier 2018, p. 165)

1.2-Net Income:

The amount of money left over after all costs are deducted from all revenue.

Net income will be split into two parts:

A: Net Income Using a Total Revenue and Total Cost Approach:

Net Income Using a Total Revenue and Cost Approach:

$$NI = n(s) - (TFC + n(VC)) \quad (1.2)$$

Where

NI is Net Income: Net income is the amount of money left over after all costs are deducted from all revenues. If the number is positive, then the business is profitable. If the number is negative, then the business suffers a loss since the costs are exceeding the revenues.

n(S) is Total Revenue: This term in the formula calculates how much money or gross income the sale of the product at a certain output level brings into the organization. **Total revenue** is the entire amount of money received by a company for selling its product, calculated by multiplying the quantity sold by the selling price.

TFC + n(VC) is Total Costs: This term in the formula calculates how much money is spent to generate the revenue. **Total cost** is the sum of all costs for the company, including both the total fixed costs and total variable costs. Two terms make up the costs: Total fixed costs (TFC) are a constant since these costs do not change with the level of output; total variable costs, represented mathematically by $n(VC)$, are the level of output multiplied by the unit variable cost.

n is Level of Output: The number of units produced or sold or the output that incurred all of the variable costs.

S is Unit Selling Price: The unit selling price of the product.

TFC is Total Fixed Costs: The total of all costs that are not affected by the level of output. (Olivier 2018, p. 145)

B: Net Income Using a Total Contribution Margin Approach

Contribution Margin: the amount each unit sold adds to the net income of the business.

The difference between the increase in the total revenue and the increase in the total variable costs is how much the sale of an individual product contributes toward the change in your net income. Formula 1.3 expresses this relationship.

Unit Contribution: $CM = S - VC$ (1.3)

If you have no units sold, your net income is negative and equal to the total fixed costs associated with your business, since there is no offsetting revenue to pay for those costs. With each unit sold, the contribution margin of each product is available to pay off the fixed costs. Formula 1.4 expresses this relationship when calculating net income.

Net Income Using Total Contribution Margin Approach:

$NI = n(CM) - TFC$ (1.4)

where

n(CM) is Total Contribution Margin: This term in the formula calculates how much money is left over to pay the total fixed costs. Use the contribution margin calculated in Formula 1.3 and multiply it by the level of output to determine the total monies remaining after all variable costs are paid. (Olivier 2018, p. 148)

Contribution Rate:

A contribution margin is expressed as a percentage of the selling price.

Your choice between two formulas for calculating a contribution rate depends on whether you have unit information or only aggregate information. If unit information is known, including the unit variable cost and unit selling price, then calculate the contribution rate using unit information as expressed in Formula 1.5.

Contribution Rate if Unit Information Is Known:

$$CR = \frac{CM}{S} \times 100 \text{ (1.5)}$$

If any unit information, including the unit variable cost or unit selling price, is unknown or unavailable, then you cannot apply Formula 1.5. Sometimes only aggregate information is known. When total revenue and total variable costs for any product are known or can at least be calculated, then you must calculate the contribution rate from the aggregate information as expressed in Formula 1.6.

Contribution Rate if Aggregate Information Is Known:

$$CR = \frac{TR - TVC}{TR} \times 100 \text{ (1.6)}$$

where

CR is Contribution Rate: This represents the percentage of the unit selling price that is available to pay for all of the fixed costs of the business.

× 100 is Percent Conversion: The contribution rate is always expressed as a percentage.

Example 1.1: Running a Pizza Delivery Business

In the commercial section of the newspaper, you come across an ad for a pizza delivery business for sale. Upon inquiry, you discover that the owner, who wants to sell the business and then retire, has four salaried employees and owns two delivery vehicles. He invites you to look through his books, where you acquire the following information:

Employees:	
Owner's salary	\$5,000 per month
Employee salaries and premiums	\$2,000 per month each
Delivery Vehicles:	
Car insurance	\$100 per month per vehicle
Fuel	\$1,125 per month per vehicle
Oil changes	vehicle
Vehicle maintenance and repairs	\$225 per month per vehicle \$562.50 per month per vehicle
Business:	
Phone	\$85 per month
Business insurance	\$300 per month
Building lease	\$2,000 per month
Operations:	
Pizza ingredients, materials, and packaging	\$19,125 per month
Selling price of pizza	\$9 per pizza
Average number of pizzas sold	4,500 per month

Assume that this is all of the key information. You need to understand this business and therefore want to determine:

- a. A unit variable cost
- b. A typical monthly net income for the business

c. The contribution margin per pizza

d. The contribution rate

Solution:

Plan:

All calculations require the sorting of costs into fixed and variable components, which will be your first step. You then calculate a unit variable cost (VC) using the historical operations information. Once you obtain this, you can calculate the net income (NI), contribution margin (CM), and contribution rate (CR).

Understand:

What You Already Know

You have unit information on costs, volume, and revenue, as listed above.

How You Will Get There

a. Unit variable cost:

Step 1: Sort the costs into fixed and variable categories. Total the monthly fixed costs.

Step 2: Total the monthly variable costs.

Step 3: Calculate the unit variable cost by applying Formula 1.1.

b. Net income:

Step 1: Calculate total revenue at the level of output.

Step 2: Calculate total costs at the level of output.

Step 3: Calculate monthly net income by applying Formula 1.2.

c. and d. Contribution margin and rate:

Step 1: Calculate the contribution margin by applying Formula 1.3.

Step 2: Calculate the contribution rate by applying Formula 1.5.

Perform:

a. Unit variable cost:

Steps 1 and 2:

Fixed Costs per Month		Variable Costs per Month	
Employee salaries and premiums	$\$2,000 \times 4 = \$8,000$	Fuel vehicles =	$\$1,125 \times 2 = \$2,250$
Car insurance	$\$1,200 \times 2 \text{ cars} \div 12 \text{ months} = \200	Oil changes	$\$225 \times 2 \text{ vehicles} = \450
Phone	$\$85$	Vehicle maintenance and repairs	$\$562.50 \times 2 \text{ vehicles} = \$1,125$
Business Insurance	$\$3,600 \div 12 \text{ months} = \300	Pizza ingredients, materials, and packaging	$\$19,125$
Owner's salary	$\$5,000$	TOTAL VARIABLE COSTS	TVC = \$22,950 per month
Building lease	$\$2,000$		
TOTAL FIXED COSTS	TFC = \$15,585 per month		

$$\text{Step 3: } VC = \frac{\$2,250 + \$450 + \$1,125 + \$19,125}{4,500 \text{ pizzas}} = \$5.10 \text{ per pizza}$$

b. Net income:

$$\text{Step 1: Total Revenue} = 4,500(\$9) = \$40,500$$

$$\text{Step 2: } TFC = \$15,585; TVC = 4,500(\$5.10) = \$22,950$$

$$\begin{aligned} \text{Step 3: } NI &= \$40,500 - (\$15,585 + \$22,950) \\ &= \$40,500 - \$38,535 = \$1,965 \end{aligned}$$

c. and d. Contribution margin and rate:

$$\text{Step 1: } CM = \$9.00 - \$5.10 = \$3.90$$

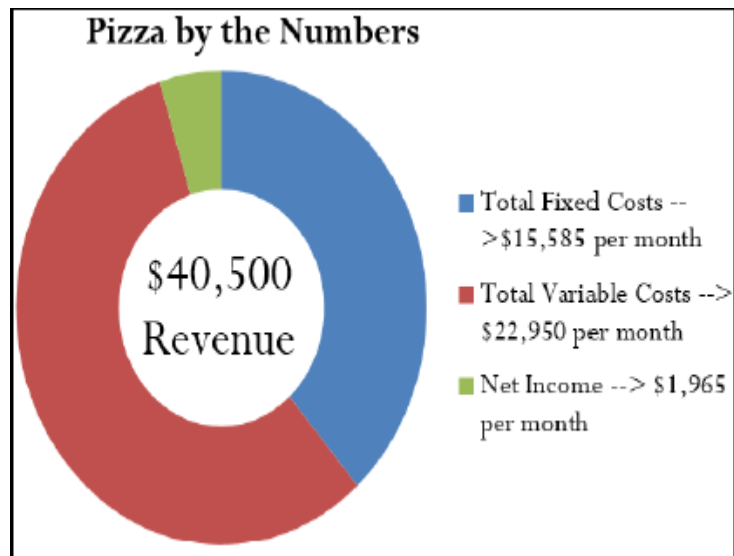
$$\text{Step 2: } CR = \frac{\$3.90}{\$9.00} \times 100 = 43.3\%$$

Calculator Instruction

FC	VC	P	PFT	Q
15585	5.1	9	Answer:1,965	4500

Present:

This is a profitable business. For an average month, the business incurs \$15,585 in fixed costs along with \$22,950 in variable costs, which works out to a unit variable cost of \$5.10 for each of the 4,500 pizzas sold. With total revenue of \$40,500, after removing both fixed and variable costs, net income is \$1,965 per month. Every pizza, after paying for the unit variable costs, has \$3.90, or 43.3%, left over.



Break-Even Analysis:

An analysis of the relationship between costs, revenues, and net income with the sole purpose of determining the point at which total revenue equals total cost.

break-even point:

A quantity that represents the level of output (in units or dollars) at which all costs are paid but no profits are earned, resulting in a net income equal to zero. To determine the break-even point, you can calculate a break-even analysis in two different ways, involving either the number of units sold or the total revenue in dollars.

Method 1: Break-Even Analysis in Units

The objective of this approach is to identify the output level that generates zero net income. To apply this method, you will need to have data on individual units, such as their selling price and variable cost per unit.

$$\text{Unit Break-Even: } n = \frac{TFC}{CM} \text{ (1.7)}$$

where

n is Breakeven Level of Output (Units): This is the level of output in units that produces a net income equal to zero.

Example 1.2: Determining the Break-Even Unit

You sell only gourmet burgers at your restaurant. You run a tight ship and are able to keep your costs fairly low. Here's a peek at your financials. TFC: \$1,200, Variable costs per unit: \$4 and Sales price per unit: \$12, Your fixed costs consist of your monthly rent, utilities, a point of sales system and any, payments on your business loan. Your variable costs per unit are the beef, buns, and toppings used to make your delicious gourmet burgers.

Solution:

Plan:

Calculate the break-even point in units sold, or n at break-even

Understand:

What You Already Know

Step 1: The total fixed costs are known: $TFC = 1200\$$

Step 2: The Variable costs per unit: are $\$4$ and Sales price per unit: $\$12$.

How You Will Get There

Step 3: By formula 1.3, we find the Contribution Margin through Variable costs per unit and Sales price per unit.

Step 4: Apply Formula 1.7.

Perform:

$$\text{Step 2: } CM = 12\$ - 4\$ = 6\$$$

$$\text{Step 3: } n = \frac{1200\$}{6\$} = 150$$

Present: This means that you'll need to sell 150 burgers over the course of the month to break even.

You can then start experimenting with your pricing and other aspects of your business strategy by inputting different figures to this formula. (Olivier 2018, p. 157)

Method 2: Break-Even Analysis in Dollars

When looking at a company's income statement, it's important to note that it doesn't provide information on a per-unit basis. Instead, all the information is aggregated, including total revenue, fixed costs, and variable costs. This means that information on the unit selling price, variable costs per unit, or level of output isn't usually

included. As a result, it's impossible to use Formula 1.7 to calculate the break-even point. Instead, the second method for calculating the break-even point relies only on aggregate information. This means that you can't calculate the break-even point in terms of units, but rather in terms of total revenue.

$$\text{Dollar Break-Even: } TR = \frac{TFC}{CR} \quad (1.8)$$

where

TR is Total Revenue at Break-even: The total amount of dollars that the company must earn as revenue to pay for all of the fixed and variable costs. At this level of revenue, the net income equals zero.

Example 1.3: Determining the Break-Even Dollars

In the annual report to shareholders, Borland Manufacturing reported total gross sales of \$7,200,000, total variable costs of \$4,320,000, and total fixed costs of \$2,500,000. Determine Borland's break-even point in dollars.

Solution:

Plan:

Calculate the dollar break-even point, which is the total revenue (TR) at break-even.

Understand:

What You Already Know

Step 1: The total fixed costs are known: $TFC = \$2,500,000$. Other known information includes the following: $TR = \$7,200,000$, $TVC = \$4,320,000$

How You Will Get There

Step 2: Calculate the contribution rate by applying Formula 1.6.

Step 3: Apply Formula 1.8.

Perform:

$$\text{Step 2: } CR = \frac{\$7,200,000 - \$4,320,000}{\$7,200,000} \times 100 = 40\%$$

$$\text{Step 3: } TR = \frac{\$2,500,000}{40\%} = \frac{\$2,500,000}{0.4} = \$6,250,000$$

Present:

Borland Manufacturing achieved its break-even point at \$6,250,000 in total revenue. At this point, total fixed costs are \$2,500,000 and total variable costs are \$3,750,000, producing a net income of zero.

Chapter two

This chapter delves into the topic of merchandising, which includes determining the cost of a single product and establishing its corresponding retail price. It also covers the effects of decisions regarding markup and markdown.

Outline Topics

2.1- Figuring Out the Cost: Discounts

2.2- Markup: Setting the Regular Price

2.3- Markdown: Setting the Sale Price

2.1- Figuring Out the Cost: Discounts

Discount:

The reduced amount in the selling price of a product.

Cost Price:

The original price of the product is the cost price.

Types of Discounts

You will perform discount calculations more effectively if you understand how and why single-pricing discounts and multiple-pricing discounts occur. Businesses or consumers are offered numerous types of discounts, of which five of the most common are trade, quantity, loyalty, sale, and seasonal.

1. Trade Discounts:

A trade discount is a discount offered to businesses only based on the type of business and its position in the distribution system (e.g., as a retailer, wholesaler, or any other member of the distribution system that resells the product). Consumers are

ineligible for trade discounts. In the discussion of the figure, two trade discounts are offered. The first is a 40% retail trade discount, and the second is a 20% wholesale trade discount. Typically, a business that is higher up in the distribution system receives a combination of these trade discounts. For example, the wholesaler receives both the 40% retail trade discount and the 20% wholesale trade discount from the MSRP. The wholesaler's cost is calculated as an MSRP of \$2.00 less 40% less 20% = \$0.96.

2. Quantity Discounts: A quantity discount (also called a volume discount) is a discount for purchasing larger quantities of a certain product. I'm sure you've walked by a store you probably noticed many shelf tags that indicate quantity discounts, such as “Buy one product for \$2” or “Take two products for \$3.

3. Loyalty Discounts: A loyalty discount is a discount that a seller gives to a purchaser for repeat business. Usually, no time frame is specified; the offer is continually available. For example, in business-to-business circles, sellers typically reward loyal customers by deducting a loyalty discount percentage from the selling price, commonly ranging from 1% to 5%.

4. Sale Discounts: A sale discount is a temporary lowering of the price from a product's regular selling price. Businesses put items on sale for a variety of reasons, such as selling excess stock or attracting shoppers.

5. Seasonal Discounts: A seasonal discount is a discount offered to consumers and businesses for purchasing products out of season.

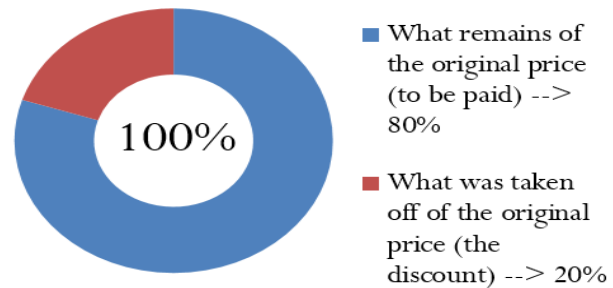
Single Discounts:

Let's start by calculating the cost when only one discount is offered. Later you will learn how to calculate a cost involving multiple discounts.

Figuring out the price after applying a single discount is called a net price calculation. When a business calculates the net price of a product, it is interested in what you still have to pay, not in what has been removed.

Note in Formula 1 below that you take 1 and subtract the discount rate to determine the rate owing. If you are eligible for a 20% discount, then you must pay 80% of the list price, as illustrated in the figure to the right.

20% Discount Interpretation



$$\text{Single Discount: } N = L \times (1 - d) \quad (2.1)$$

where

N is Net Price: The **net price** is the price of the product after the discount is removed from the list price. It is a dollar amount representing what price remains after you have applied the discount.

L is List Price: The **list price** is the normal or regular dollar price of the product before any discounts. It is the **Manufacturer's Suggested Retail Price (MSRP** – a price for a product that has been published or advertised in some way), or any dollar amount before you remove the discount.

d is Discount Rate: The discount rate represents the percentage (in decimal format) of the list price that is deducted.

Formula 2.1 once again applies **Formula *** on rate, portion, and base, where the list price is the base, the $(1 - d)$ is the rate, and the net price represents the portion of the price to be paid. Notice that **Formula 2.1** requires the discount to be in a percentage (decimal) format; sometimes a discount is expressed as a dollar amount, though, such as "Save \$5 today." **Formulas 2.2a** and **2.2b** relate the discount dollar amount to the list price, discount percent, and net price. Choose one formula or the other depending on which variables are known.

Discount Amount: $D\$ = L \times d$ (2.2a)

Discount Amount: $D\$ = L - N$ (2.2b)

where

D\$ is Discount Amount: Determine the discount amount in one of two ways, depending on what information is known. If the list price and discount rate are known, apply **Formula 2.2a**. If the list price and net price are known, apply **Formula 2.2b**. Either of these formulas can be algebraically manipulated to solve for any other unknown variable as well.

NOTE

Formula *: $\text{Portion} = \text{Rate} \times \text{Base}$

Portion is The Part of The Quantity: The portion represents the part of the whole. Compare it against the base to assess the rate.

Rate is The Relationship: The **rate** is the decimal form expressing the relationship between the portion and the base. Convert it to a percentage if needed by applying Formula **. This variable can take on any value, whether positive or negative.

Base is The Entire Quantity: The base is the entire amount or quantity that is of concern. It represents a whole, standard, or benchmark that you assess the portion against.

Formula **: $\% = \text{dec} \times 100$

% is Percentage: This is the decimal expressed as a percentage. It must always be written with the percent (%) symbol immediately following the number.

dec is Decimal Number: This is the decimal number needing to be converted into a percentage.

100 is the Conversion Factor: A percentage is always expressed in hundredths.

Example 2.1: Determining the List Price of a Jacket

Winners pay a net price of \$27.50 for a winter jacket after receiving a retail trade discount of 45%. What was the manufacturer's suggested retail price (MSRP) for the jacket?

Solution:

plan: Calculate the MSRP for the jacket before Winners received the retail trade discount to arrive at the net price. This is called the list price for the product, or L.

Understand: What You Already Know

Step 1: The net price and the discount rate are known: $N = \$27.50$ $d = 0.45$

How You Will Get There

Step 2: List price is the unknown variable; skip this step.

Step 3: Apply Formula 2.1.1, rearranging for L.

Perform:

$$\text{Step 3: } N = L \times (1 - d)$$

$$\$27.50 = L(1 - 0.45)$$

$$L = \$27.50 \div (1 - 0.45)$$

$$L = \$27.50 \div 0.55 = \$50.00$$

Present: The MSRP, or list price, of the winter jacket is \$50.00.

Example 2.2: Determining the Discount Percent and Discount Amount

You are shopping at Mountain Equipment Co-op for a new environmentally friendly water bottle. The price tag reads \$14.75, which is \$10.24 off the regular price. Determine the discount rate applied.

Solution:

Plan: You need to find out how the sale price translates into the discount rate or d.

Understand: What You Already Know

Step 1: The discount amount and net price are known: $D\$ = \10.24 $N = \$14.75$

How You Will Get There

Step 2: Use Formula 2.2b to calculate the list price, rearranging for L.

Step 3: Convert the discount amount into a percentage by applying Formula 2.2a, rearranging for d.

Perform

Step 2: List price: $\$10.24 = L - \14.75 , which results in $L = \$24.99$

Step 3: Discount rate: $d = D\$ \div L$

$$d = \$10.24 \div \$24.99 = 0.409764 \text{ or } 40.9764\%$$

Present: The water bottle today has been reduced in price by the amount of \$10.24. This represents a sale discount of 40.9674%.

Multiple Discounts

You are driving down the street when you see a large sign at Old Navy that says, “Big sale, take an additional 25% off already reduced prices!” In other words, products on sale (the first discount) are being reduced by an additional 25% (the second discount). Because **Formula 2.1** handles only a single discount, you must use an extended formula in this case. Businesses commonly receive more than one discount when they make a purchase. Consider a transaction in which a business receives a 30% trade discount as well as a 10% volume discount. First, you have to understand that this is not a $30\% + 10\% = 40\%$ discount. The second discount is always applied to the net price after the first discount is applied. Therefore, the second discount has a smaller base upon which it is calculated. If there are more than two discounts, you deduct each subsequent discount from continually smaller bases. Formula 2.3 expresses how to calculate the net price when multiple discounts apply.

Multiple Discounts:

$$N = L \times (1 - d_1) \times (1 - d_2) \times \dots \times (1 - d_n) \quad (2.3)$$

where

d1 is First Discount, d2 is Second Discount, dn is nth Discount: When there is more than one discount, you must extend beyond Formula 2.1 by multiplying another discount expression. These discounts are represented by the same d symbol; however, each discount receives a subscript to make its symbol unique. Therefore, the first discount receives the symbol of d_1 , the second discount receives the symbol d_2 , and so on. Recall that the symbol n represents the number of pieces of data (a count), so you can expand or contract this formula to the exact number of discounts being offered.

And It is often difficult to understand exactly how much of a discount is being received when multiple discounts are involved. Often it is convenient to summarize the multiple discount percentages into a single percentage. This makes it easier to calculate the net price and aids in understanding the discount benefit. Simplifying multiple percent discounts into a single percent discount is called finding the single equivalent discount. Whether you apply multiple discounts or just the single equivalent discount, you arrive at the same net price. The conversion of multiple discount percentages into a single equivalent discount percent is illustrated in Formula 2.4

Single Equivalent Discount:

$$d_{equiv} = 1 - (1 - d_1) \times (1 - d_2) \times \dots \times (1 - d_n) \quad (2.4)$$

where

d_{equiv} (or just d) is the **single equivalent discount rate** that is equal to the series of multiple discounts. Recall that taking $(1 - d)$ calculates what you pay. Therefore, if you take 1, which represents the entire amount, and reduce it by what you pay, the rate left over must be what you did not pay. In other words, it is the discount rate.

Example 2.3: Retailer Purchasing Ski-Doos with Multiple Discounts

A retail dealership purchases some Expedition TUV Yeti II Ski-Doos to stock in its stores. Examining the merchandising terms of the manufacturer, Bombardier, the dealership notices that it would be eligible to receive a 35% trade discount, 15% volume discount, and 3% loyalty discount. Because it is June and Ski-Doos are out of season, Bombardier offers a seasonal discount of 12% for purchases made before June 30. If the MSRP for the Ski-Doo is \$12,399.00 and the dealership purchases this item on June 15, what price would it pay?

Solution: plan: You are looking for the net price that the retail dealership will pay for the Ski-Doo or N.

Understand: What You Already Know

Step 1: The retail dealership is eligible for all four discounts (it qualifies for the seasonal discount since it is purchasing before June 30). Therefore,

$L = \$12,399.00$, $d1 = 0.35$, $d2 = 0.15$, $d3 = 0.03$,
and $d4 = 0.12$

How You Will Get There

Step 2: You know the list price, so skip this step.

Step 3: Apply Formula 2.1.3.

Perform: step3:
$$N = L \times (1 - d1) \times (1 - d2) \times \dots \times (1 - dn)$$

$$N = \$12,399 \times (1 - 0.35) \times (1 - 0.15) \times (1 - 0.03) \times (1 - 0.12)$$

$$N = \$12,399 \times 0.65 \times 0.85 \times 0.97 \times 0.88 = \$5,847.54$$

Present: After all four discounts, the retail dealership could purchase the Ski-Doo for \$5,847.54.

Example 2.4: Reducing Multiple Discounts to a Single Equivalent Discount

The retail dealership in Example 2.3, purchases more products subject to the same discounts. It needs to simplify its calculations. Using the information from Example 2.3, what single equivalent discount is equal to the four specified discounts?

Solution: plan: You are looking for a single equivalent discount that is equal to the four discount percentages, or equal (or just d).

Understand: What You Already Know

You know the discount rates:

$d1 = 0.35$, $d2 = 0.15$, $d3 = 0.03$, and $d4 = 0.12$

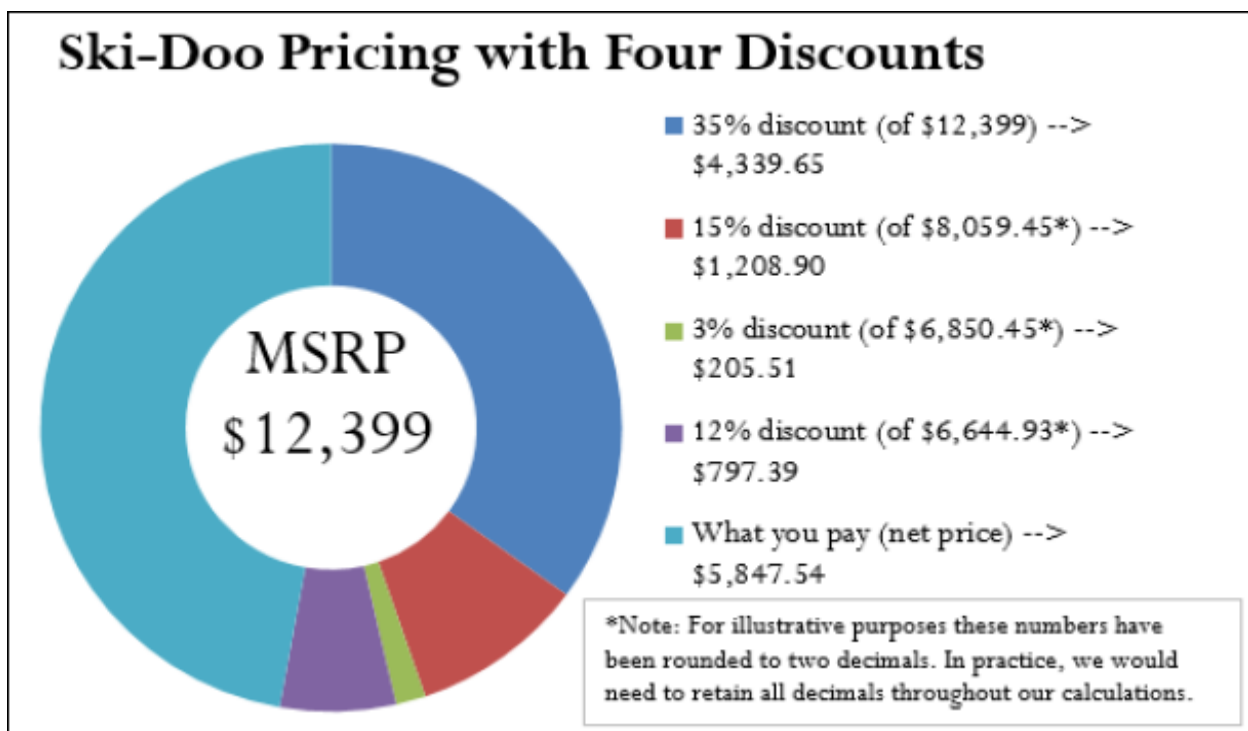
How You Will Get There

Apply Formula 2.4.

$$d_{equiv} = 1 - (1 - d_1) \times (1 - d_2) \times \dots \times (1 - d_n)$$

$$\begin{aligned} d_{equiv} &= 1 - (1 - 0.35) \times (1 - 0.15) \times (1 - 0.03) \times (1 - 0.12) \\ &= 1 - (0.65)(0.85)(0.97)(0.88) = 1 - 0.471614 \\ &= 0.528386 \text{ or } 52.8386\% \end{aligned}$$

Present: The retail dealership can apply a 52.8386% discount to all the products it purchases. (Olivier 2018, p. 178)



2.2- Markup: Setting the Regular Price Markup):

is the process of taking a product's cost and increasing it by some amount to arrive at a selling price.

The Components in a Selling Price

Before you learn to calculate markup, you first have to understand the various components of a selling price. Then, in the next section, markup and its various methods of calculation will become much clearer.

When your business acquires merchandise for resale, this is a monetary outlay representing a cost. When you then resell the product, the price you charge must recover more than just the product cost. You must also recover all the selling and operating expenses associated with the product, which is often referred to as overhead. Ultimately, you also need to make some money, or profit, as a result of the whole process.

Most people think that marking up a product must be a fairly complex process. It is not. Formula 2.5 illustrates the relationship between the three components of cost, expenses (also known as overhead), and profits in calculating the selling price.

The Selling Price of a Product:

$$S = C + E + P \text{ (2.5)}$$

where

S is Selling Price: Once you calculate what the business paid for the product (cost), the bills it needs to cover (expenses), and how much money it needs to earn (profit), you arrive at a selling price by summing the three components.

C is Cost: The cost is the amount of money that the business must pay to purchase or manufacture the product.

E is Expenses: Expenses (also called overhead) are the financial outlays involved in selling the product. Beyond just purchasing the product,

P is Profit: Profit is the amount of money that remains after a business pays all of its costs and expenses. (Bluman, 2014, p. 106)

Important Notes

In applying Formula 2.5 you must adhere to the basic rule of linear equations requiring all terms to be in the same unit. That is, you could use Formula 2.5 to solve for the selling price of an individual product, where the three components are the unit cost, unit expenses (unit overhead), and unit profit. When you add these, you calculate the unit selling price. Alternatively, you could use Formula 2.5 in an aggregate form where the three components are total cost, total expenses (total overhead), and total profit. In this case, the selling price is a total selling price, which is more commonly known as total revenue. But you cannot mix individual components with aggregate components.

Example 2.5: Setting a Price on Fashion in Dollars

Mary's Boutique purchases a dress for resale for \$23.67. The owner determines that each dress must contribute \$5.42 to the expenses of the store. The owner also wants this dress to earn \$6.90 toward profit. What is the regular selling price for the dress?

Solution: plan: You are looking for the regular selling price for the dress, or S .

Understand: What You Already Know

Step 1: The unit cost of the dress and the unit expense and the unit profit are all known: $C = \$23.67$, $E = \$5.42$, $P = \$6.90$

How You Will Get There

Step 2: Apply Formula 2.2.1.

Perform:

$$\text{Step 2: } S = \$23.67 + \$5.42 + \$6.90 = \$35.99$$

Present: Mary's Boutique will set the regular price of the dress at \$35.99.

Calculating the Markup Dollars

Most companies sell more than one product, each of which has different price components with varying costs, expenses, and profits. Can you imagine trying to compare 50 different products, each with three different components? You would have to juggle 150 numbers! To make merchandising decisions more manageable and comparable, many companies combine expenses and profit into a single quantity, either as a dollar amount or a percentage. This section focuses on the markup as a dollar amount. And One of the most basic ways a business simplifies its merchandising is by combining the dollar amounts of its expenses and profits as expressed in Formula 2.6.

Markup Amount: $M\$ = E + P$ (2.6)

where

M\$ is Markup Amount: Markup is taking the cost of a product and converting it into a selling price. The **markup amount** represents the dollar amount difference between the cost and the selling price.

Note that since the markup amount (M\$) represents the expenses (E) and profit (P) combined, you can substitute the variable for the markup amount into Formula 2.5 to create Formula 2.7, which calculates the regular selling price.

Selling Price Using Markup: $S = C + M\$$ (2.7)

where

S is Selling Price: The regular selling price of the product.

Example 2.6: Markup as a Dollar Amount

A cellular retail store purchases an iPhone with an MSRP of \$779 less a trade discount of 35% and a volume discount of 8%. The store sells the phone at the MSRP.

- What is the markup amount?
- If the store knows that its expenses are 20% of the cost, what is the store's profit?

Solution: plan: First of all, you need to calculate the markup amount (M\$). You also want to find the store's profit (P).

Understand: What You Already Know

Step 1: The smartphone MSRP and the two discounts are known, along with the expenses and selling price:

$$L = \$270 \quad d_1 = 0.35 \quad d_2 = 0.08$$

$$E = 20\% \text{ of cost, or } 0.2C \quad S = \$779$$

How You Will Get There

Step 1 (continued): Calculate the cost of the iPhone by applying Formula 2.1.3.

Step 2: Calculate the markup amount using Formula 2.2.3.

Step 2 (continued): Calculate the profit by applying Formula 2.2.2 rearranging for P.

$$\text{Perform: Step 1 (continued): } N = \$779.00 \times (1 - 0.35) \times (1 - 0.08)$$

$$N = \$779.00 \times 0.65 \times 0.92 = \$465.84 = C$$

$$\text{Step 2: } \$779.00 = \$465.84 + M\$$$

$$\$313.16 = M\$$$

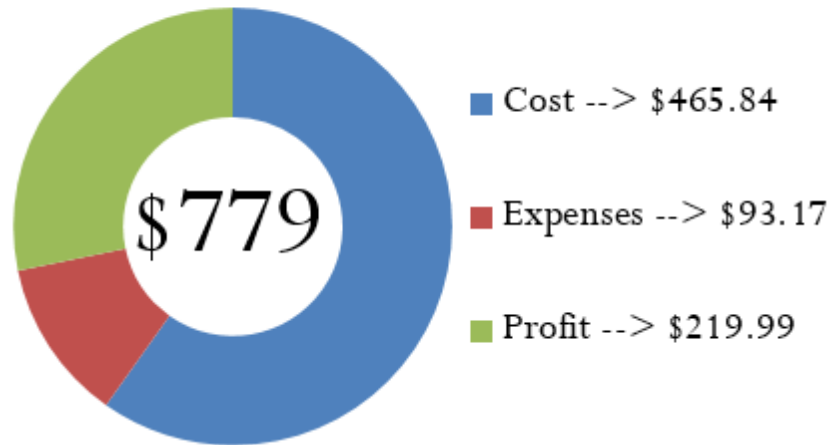
$$\text{Step 2 (continued): } \$313.16 = 0.2(\$465.84) + P$$

$$\$313.16 = \$93.17 + P$$

\$219.99 = P

present: The markup amount for the iPhone is \$313.16. When the store sells the phone for \$779.00, its profit is \$219.99.

The Price Components of an Apple iPhone



Calculating the Markup Percent

It is important to understand markup in terms of the actual dollar amount; however, it is more common in business practice to calculate the markup as a percentage. There are three benefits to converting the markup dollar amount into a percentage:

1. Easy comparison of different products having vastly different price levels and costs, to help you see how each product contributes toward the financial success of the company. For example, if a chocolate bar has a 50¢ markup included in a selling price of \$1, while a car has a \$1,000 markup included in a selling price of \$20,000, it is difficult to compare the profitability of these items. If these numbers were expressed as a percentage of the selling price such that the chocolate bar has a 50% markup and the car has a 5% markup, it is clear that more of every dollar sold for chocolate bars goes toward list profitability.

2. Simplified translation of costs into a regular selling price—a task that must be done for each product, making it helpful to have an easy formula, especially when a company carries hundreds, thousands, or even tens of thousands of products. For

example, if all products are to be marked up by 50% of the cost, an item with a \$100 cost can be quickly converted into a selling price of \$150.

3. An increased understanding of the relationship between costs, selling prices, and the list profitability for any given

product. For example, if an item selling for \$25 includes a markup on the selling price of 40% (which is \$10), then you can determine that the cost is 60% of the selling price (\$15) and that \$10 of every \$25 item sold goes toward list profits. You can translate the markup dollars into a percentage using two methods, which express the amount either as a

percentage of cost or as a percentage of selling price:

1. Method 1: Markup as a Percentage of Cost. This method expresses the markup rate using cost as the base. Many companies use this technique internally because most accounting is based on cost information. The result, known as the markup on cost percentage, allows a reseller to convert easily from a product's cost to its regular unit selling price.

2. Method 2: Markup as a Percentage of Selling Price. This method expresses the markup rate using the regular selling price as the

base. Many other companies use this method, known as the markup on selling price percentage since it allows for a quick understanding of the portion of the selling price that remains after the cost of the product has been recovered. This percentage represents the list profits before the deduction of expenses and therefore is also referred to as the list profit margin.

Both formulas are versions of Formula * on rate, portion, and base. The markup on the cost percentage is expressed in Formula 2.8, while the markup on the selling price percentage is expressed in Formula 2.9.

$$\text{Markup On Cost Percentage: } MoC\% = \frac{M\$}{C} \times 100 \text{ (2.8)}$$

where

MoC% is Markup on Cost Percentage: This is the percentage by which the cost of the product needs to be increased to arrive at the selling price. and The markup on cost is always a percentage.

$$\text{Markup On Selling Price Percentage: } MoS\% = \frac{M\%}{S} \times 100 \text{ (2.9)}$$

where

MoS% is Markup on Selling Price Percentage: This is the percentage of the selling price that remains available as list profits after the cost of the product is recovered.

Example2.7: Markup as a Percentage

A large national retailer wants to price a Texas Instruments BAI Plus calculator at the MSRP of \$39.99. The retailer can acquire the calculator for \$17.23.

- a. What is the markup on cost percentage?
- b. What is the markup on the selling price percentage?

Solution: plan: You have been asked to solve for the markup on cost percentage (MoC%) and the markup on selling price percentage (MoS%).

Understand: What You Already Know

Step 1: The regular unit selling price and the cost are provided:

$$S = \$39.99, C = \$17.23$$

How You Will Get There

Step 1 (continued): You need the markup dollars. Apply Formula 2.2.3, rearranging for M\$.

Step 2: To calculate markup on cost percentage, apply Formula 2.2.4.

Step 2 (continued): To calculate markup on selling price percentage, apply Formula 2.2.5.

Perform: Step 1: (continued): $\$39.99 = \$17.23 + M\$$

$\$22.76 = M\$$

$$\text{Step 2: } MoC\% = \frac{\$22.76}{\$17.23} \times 100 = 132.0952\%$$

Step 2 (continued):

$$MoS\% = \frac{\$22.76}{\$39.99} \times 100 = 56.9142\%$$

CALCULATOR INSTRUCTIONS

CST	SEL	MAR
17.23	39.99	Answer:
		56.9142

Present: The markup on cost

percentage is 132.0952%. The markup on the selling price percentage is 56.9142%.

(Bluman, 2014, p. 97)

2.3-Markdown: Setting the Sale Price

Markdown:

is a reduction from the regular selling price of a product resulting in a lower price.

This lower price is called the sale price to distinguish it from the selling price.

The Importance of Markdowns

Many people perceive markdowns as a sign of bad business management decisions.

However, in most situations, this is not true. Companies must always attempt to forecast the future. To stock products, a reseller must estimate the number of units that might sell shortly for every product that it carries. This is both an art and a

science. While businesses use statistical techniques that predict future sales with a relative degree of accuracy, consumers are fickle and regularly change their shopping habits. Markdowns most commonly occur under four circumstances:

1. **Clearing Out Excess or Unwanted Inventory.** In these situations, the business thought it could sell 100 units; however, consumers purchased only 20 units. In the case of seasonal inventory, such as Christmas items on Boxing Day, the retailer wishes to avoid packing up and storing the inventory until the next season.

2. **Clearing Out Damaged or Discontinued Items.** Selling a damaged product at a discount is better than not selling it at all. When products are discontinued, this leaves shelf space underused, so it is better to clear the item out altogether to make profitable items that can keep the shelves fully stocked.

3. **Increasing Sales Volumes.** Sales attract customers because almost everyone loves a deal. Though special marketing events such as a 48-hour sale reduce the profitability per unit, by increasing the volume sold these sales can lead to a greater profit overall.

4. **Promoting Add-On Purchases.** Having items on sale attracts customers to the store. Many times customers will not only purchase the item on sale but also, as long as they are on the premises, grab a few other items, which are regularly priced and very profitable. Like many others, you may have walked into Target to buy one item but left with five instead. And Markdowns are no different from offering a discount. Recall from Section 2.1 that one of the types of discounts is known as a sale discount. The only difference here lies in the choice of language. Markdowns are common, so you will find it handy to adapt the discount formulas to the application of markdowns, replacing the symbols with ones that are meaningful in merchandising. Formula 2.1, calculates the net price for a product after it receives a single discount:

$$N = L \times (1 - d)$$

Formula 2.10, adapts this formula for use in markdown situations.

$$\text{The Sale Price of a Product: } S_{\text{onsale}} = S \times (1 - d) \text{ (2.10)}$$

where

S_{onsale} is Sale Price: The sale price is the price of the product after reduction by the markdown percent. Conceptually, the sale price is the same as the net price (Cleaves.etc., 2012, p. 106).

In markdown situations, the selling price and the sale price are different variables. The sale price is always less than the selling price. If a regular selling price has more than one markdown percent applied to it, you can extend Formula 2.10 in the same manner that Formula 2.3 calculated multiple discounts.

If you are interested in the markdown amount in dollars, recall that Formula 2.2 calculates the discount amount in dollars. Depending on what information is known, the formula has two variations:

$$\text{Formula 2.2a: } D\$ = L \times d$$

$$\text{Formula 2.2b: } D\$ = L - N$$

Formulas 2.11a and 2.11b adapt these formulas to markdown situations.

$$\text{Markdown Amount: } D\$ = S \times d \text{ (2.11a)}$$

$$\text{Markdown Amount: } D\$ = S - S_{\text{onsale}} \text{ (2.12b)}$$

where

D\$ is Markdown Amount: You determine the markdown amount using either formula depending on what information is known. If you know the selling price and

markdown percent, apply Formula 2.11a. If you know the selling price and sale price, apply Formula 2.11b.

The final markdown formula reflects the tendency of businesses to express markdowns as percentages, facilitating easy comprehension and comparison. Recall Formula 2.5 from Section 2.1.2, which calculated a markup on the selling price percent:

$$MoS\% = \frac{M\$}{S} \times 100$$

Formula 2.12, adapts this formula to markdown situations.

$$\text{Markdown Percentage: } d = \frac{D\$}{S} \times 100 \quad (2.12)$$

where

d is Markdown Rate: the percentage of the selling price to be deducted (in decimal format). In this case, because you are interested in figuring out how much the percentage is worth, you do not take it away from 1 as in Formula 2.10.

Example 2.8: Determining the Sale Price and Markdown Amount

The MSRP for the “Guitar Hero: World Tour” video game is \$189.99. Most retail stores sell this product at a price in line with the MSRP. You have just learned that a local electronics retailer is selling the game for 45% off. What is the sale price for the video game and what dollar amount is saved?

Solution: plan: There are two unknown variables. The first is the video game's sale price (S_{onsale}). The second is the markdown amount ($D\$$) that is realized at that sale price.

Understand : What You Already Know

Step 1: The regular selling price for the video game and the markdown rate are known: $S = \$189.99$, $d = 0.45$

How You Will Get There

Step 2: Calculate the sale price by applying Formula 2.10.

Step 2 (continued): Calculate the markdown amount by applying Formula 2.3.2b.

$$\begin{aligned}\text{Perfum: step2: } \text{Sonsale} &= \$189.99 \times (1 - 0.45) \\ &= \$189.99 \times 0.55 = \$104.49\end{aligned}$$

$$\text{Step 2 (continued): } D\$ = \$189.99 - \$104.49 = \$85.50$$

Present: The sale price for the video game is \$104.49. When purchased on sale, “Guitar Hero: World Tour” is \$85.50 off of its regular price.

Example 2.9: Markdown Requiring Selling Price Calculation

A reseller acquires an Apple iPad for \$650. Expenses are planned at 20% of the cost, and profits are set at 15% of the cost. During a special promotion, the iPad is advertised at \$100 off. What is the sale price and markdown percent?

Solution: Plan: The unknown variables for the iPad are the sale price (Sonsale) and the markdown rate (d).

Understand: What You Already Know

Step 1: The pricing elements of the iPad along with the markdown dollars are known:

$$C = \$650 \quad , \quad E = 0.2C$$

$$P = 0.15C \quad , \quad D\$ = \$100$$

How You Will Get There

Step 1 (continued): Calculate the selling price of the product by applying Formula 2.10.

Step 2: Calculate the markdown percent by applying Formula 2.12:

Step 2 (continued): Calculate the sale price by applying Formula 2.11b, rearranging for Sonsale.

Perform

$$\text{Step 1 (continued): } S = \$650 + 0.2(\$650) + 0.15(\$650) = \$877.50$$

$$\text{Step 2: } d = \$100 / (\$877.50) \times 100 = 11.396\%$$

$$\text{Step 2 (continued): } \$100 = \$877.50 - \text{Sonsale}$$

$$\text{Sonsale} = \$777.50$$

Present: When the iPad is advertised at \$100 off, it receives an 11.396% markdown and it will retail at a sale price of \$777.50. (Cleaves.etc., 2012, p. 304).

Conclusion

The incorporation of mathematical frameworks into marketing techniques provides vital insights and solutions for effectively navigating the intricacies of the current data-driven commercial landscape. By using quantitative analysis and statistical methodologies, marketers may improve their comprehension of customer behavior, market dynamics, and the efficacy of their campaigns. Businesses may enhance profitability and gain a competitive edge by using mathematical models to determine pricing strategies, analyze costs, apply discounts, set markups, and implement markdowns. Adopting data-driven methods is essential for firms that want to succeed in a highly competitive market. This highlights the importance of marketing mathematics as a fundamental tool for making strategic decisions and achieving long-term success.

Reference:

- 1-Vince, R. (1992). The Mathematics of Money Management: Risk Analysis Techniques for Traders (Vol. 18). John Wiley & Sons.
- 2-Bluman, A. G. (2014). Business Math Demystified. McGraw-Hill Education.
- 3-Cleaves, C., Hobbs, M., & Noble, J. (2012). Business Math.
- 4-Davis, M., 2001. Mathematics of financial markets. Mathematics unlimited—2001 and beyond, pp.361-380.
- 5-Olivier, J.P., 2018. Business Math: A Step-by-Step Handbook.
- 6-Francis, A. and Mousley, B., 1998. Business mathematics and statistics. Letts Educational.

پوخته

بیرکاری بازارکردن، که تیکه له یه ک له بنه ما ته قلیدیبه کانی بازارکردن و مؤدیلی بیرکاری پیشکەوتوو، له دیمه نی بازارگانی ئەمڕۆدا که به داتا کانه وه به پڕیوه ده چیت، شتیکی سه ره کییه. ئەم توئیزینه وه یه قوول ده بیته وه له به کارهینانی چوارچۆه بیرکاریبه کان له بازارکردندا، گرنگی به ستراتیژیبه کانی نرخدانان، شیکاری تێچوون، داشکاندن، زیادکردن و دابه زاندنی نرخه کان ده دات. له پڕیگه ی شیکاری چه ندایه تی و ته کنیکه ئاماریبه کانه وه، بازارگانه کان ده توانن تیروانینیکی قوولتر له ره فتاری به کاربه ر، ره وتی بازار و ئەدای که مپه یین به ده سته مپین. توئیزینه وه که به دوادا چوون بۆ فورموله ی بیرکاری و میتۆدۆلۆژیای جۆراوجۆر ده کات که له پڕۆسه کانی پریاردانی بازارکردندا به کارده هینرین، جهخت له سه ر گرنگی ستراتیژیبه کانی داتا-بزۆینه ر

■ ده کاته وه بۆ سه رکه وتنی بازارگانی

ملخص

تعد رياضيات التسويق، وهي عبارة عن مزيج من مبادئ التسويق التقليدية والنماذج الرياضية المتقدمة، أمرًا محوريًا في مشهد الأعمال المعتمد على البيانات اليوم. تتعمق هذه الورقة في تطبيق الأطر الرياضية في التسويق، مع التركيز على استراتيجيات التسعير، وتحليل التكاليف، والخصومات، وهوامش الربح، وتخفيضات الأسعار. من خلال التحليل الكمي والتقنيات الإحصائية، يمكن للمسوقين الحصول على رؤى أعمق حول سلوك المستهلك واتجاهات السوق وأداء الحملة. تستكشف الورقة مختلف الصيغ الرياضية والمنهجيات المستخدمة في عمليات صنع القرار التسويقي، مع التركيز على أهمية الاستراتيجيات المبنية على البيانات لنجاح الأعمال.