Derivation of most basic transport equation $u_t + cu_x = 0$

1) Consider water flowing through a horizontal pipe of fixed cross section in the positive x direction (right) at a constant rate c centimeter/second. A substance, say salt or any other pollutant, is suspended in the water. Assume diffusion is negligible. Let $u(x, t)$ be its concentration in grams/centimeter at time t. Derive a PDE model to describe the transport of the pollutant.
2) Let's take a look at the pollutant in the section of the pipe from 0 to x for an arbitrary x.
3) What's the amount of pollutant in $[0, x]$ at time t?
4) $M = \int_0^x u(\xi, t) d\xi$

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5) What happen to these molecules of pollutant at a later time $t+h$?
6) Move right by ch centimeters!
7) Thus, $M = \int_0^x u(\xi, t)d\xi = \int_{ch}^{x+ch} u(\xi, t+h)d\xi$
8) Using fundamental theorem of calculus
9) Differentiate both sides with respect to x, we get $u(x, t) = u(x+ch, t+h)$
10) Differentiate both sides of the previous equation with respect to h via the chain rule, to obtain $0 = cu_x(x+ch, t+h) + u_t(x+ch, t+h)$

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11) Let $h \to 0$. Then $0 = cu_x(x,t) + u_t(x,t)$
12) Since x and t are arbitrary, then $cu_x + u_t = 0$
13) The general solution $u(x,t) = f(x - ct)$