Salahaddin University-Erbil
College of Science-Department of Mathematics

Numerical Analysis

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A **finite difference** is a mathematical expression of the form $f(x+b)-f(x+a)$. If a finite difference is divided by $b-a$, one gets a difference quotient. The approximation of derivatives by finite differences plays a central role in finite difference methods for the numerical solution of differential equations, especially boundary value problems.

The difference operator, commonly denoted by Δ is the operator that maps a function f to the function $\Delta[f]$ defined by:

$\Delta[f](x)=f(x+1)-f(x)$.

A difference equation is a functional equation that employs the finite difference operator in the same manner that a differential equation uses derivatives. Difference equations and differential equations share many similarities, particularly in their solving methods. Certain recurrence relations can be expressed as difference equations by swapping iteration notation for finite differences.

In numerical analysis, finite differences are commonly used to approximate derivatives, and the word "finite difference" is sometimes used as an abbreviation for "finite difference approximation of derivatives. In the terminology used above, finite difference approximations are equivalent to finite difference quotients. Brook Taylor established finite differences in 1715, and they have been treated as abstract self-standing mathematical objects by George Boole (1860), L. M. Milne-Thomson (1933), and Károly.

Three basic types are commonly considered: forward, backward, and central finite differences

A forward difference, denoted $\triangle_h(f)$ of a function f is a function defined as

$\triangle_h[f](x)=f(x+h)-f(x)$

A backward difference uses the function values at x and $x - h$, instead of the values at $h + x$ and x:

$$\nabla_h[f](x) = f(x) - f(x - h) = \Delta_h[f](x - h).$$

Finally, the central difference is given by:

$$\delta_h[f](x) = f\left(x + \frac{h}{2}\right) - f\left(x - \frac{h}{2}\right) = \triangle_{h/2}\,[f](x) + \nabla_{h/2}[f](x)$$

Example:

1- Show that $\triangle\, f = 0$ for any constant function f.