Salahaddin University

College of Agricultural Engineering Sciences

Soil and Water Department

Second Stage

Lecture – 3 -

Land Levelling

By

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Centroid

Centroid is the geometric center of a geometric object: a one-dimensional curve, a two-dimensional area or a three-dimensional volume. Centroids are useful for many situations in Statics, including the analysis of distributed forces, beam bending, and shaft torsion.

Two related concepts are the ***center of gravity***, which is the average location of an object's *weight*, and the ***center of mass*** which is the average location of an object's *mass*.

Centroid Definition

The centroid is the center point of the object

Types of Centroid

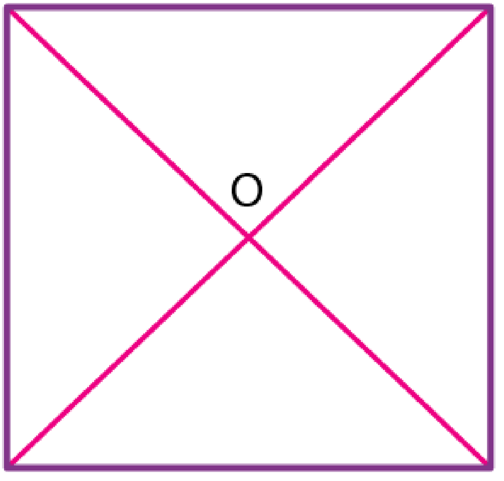
1. **Simple Centroid Shapes**
2. **Compound Centroid Shapes**
3. **Irregular Shapes**

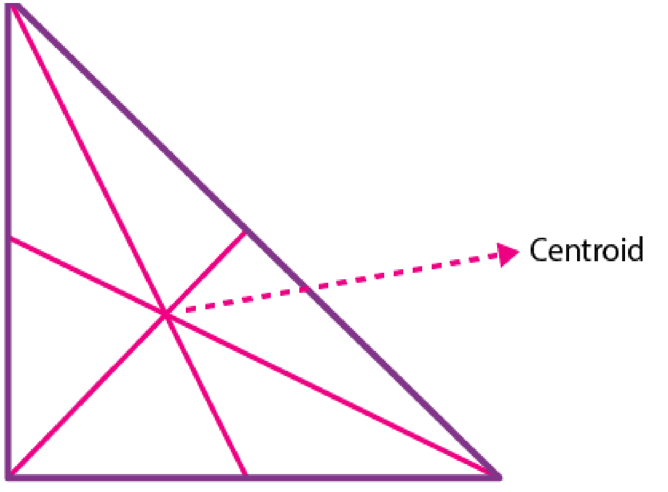
Properties of Centroid

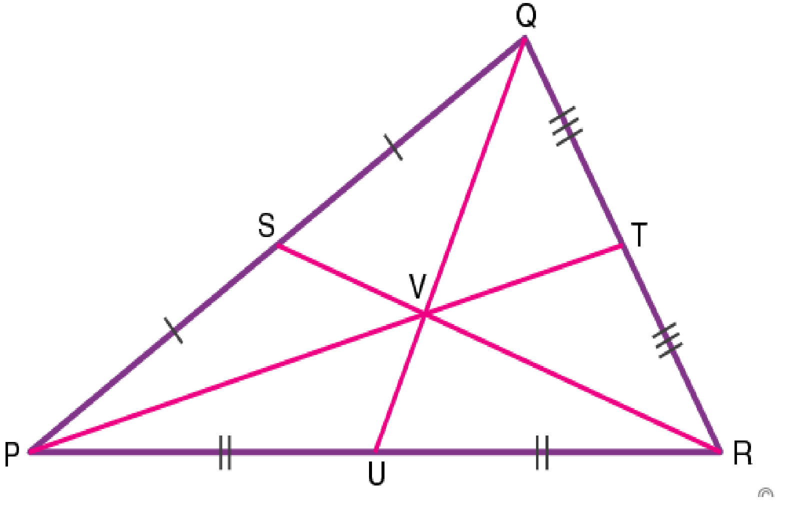
The properties of the centroid are as follows:

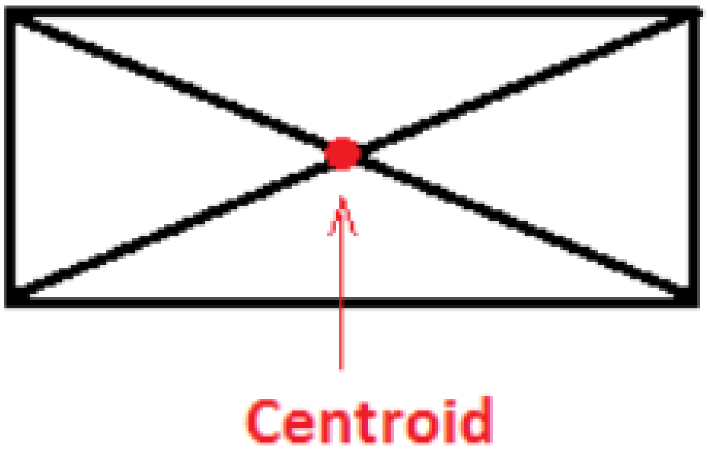
1. The centroid is the centre of the object.
2. It is the centre of gravity.
3. It should always lie inside the object.
4. It is the point of concurrency of the medians.

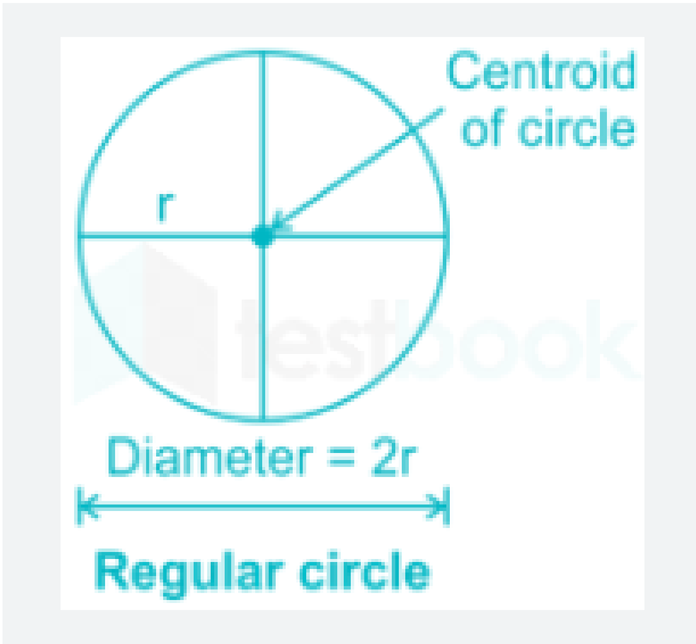
1. Simple Centroid Shapes

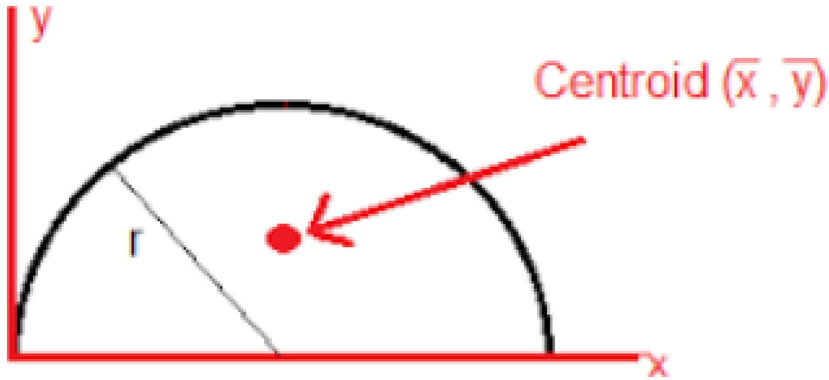




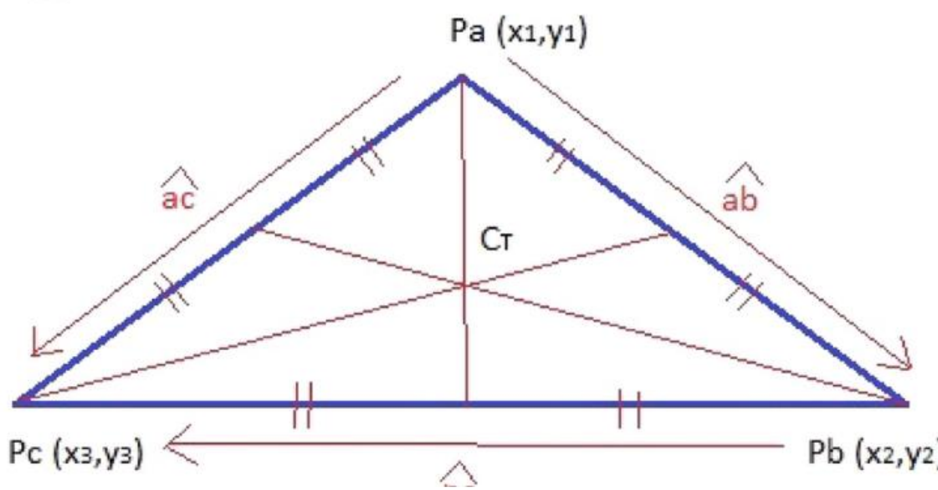



Centroid

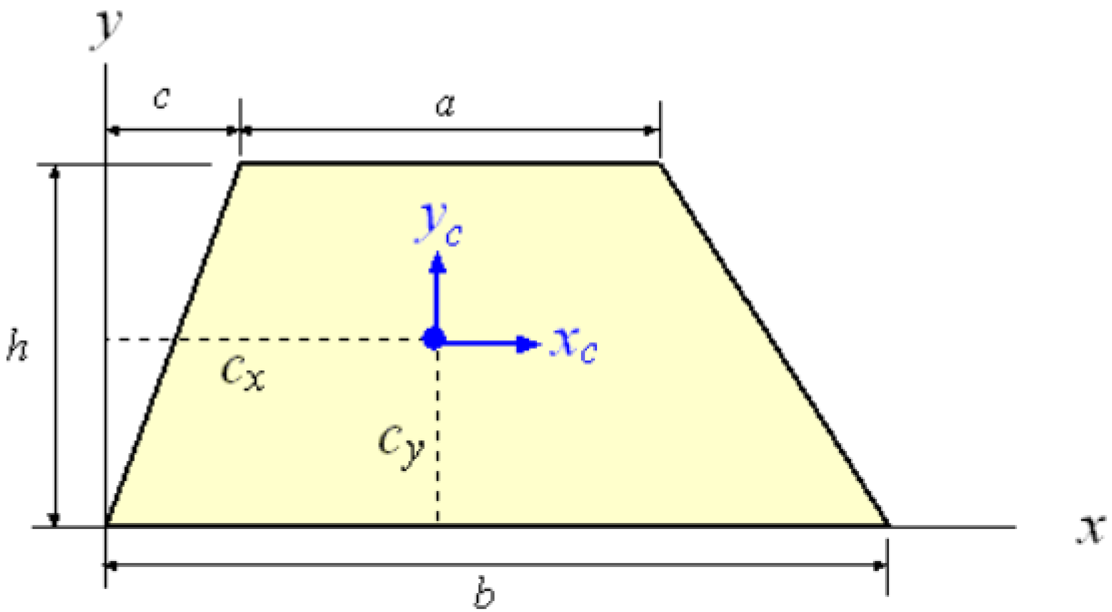

Centroid
of circle
r
Diameter = 2r
Regular circle



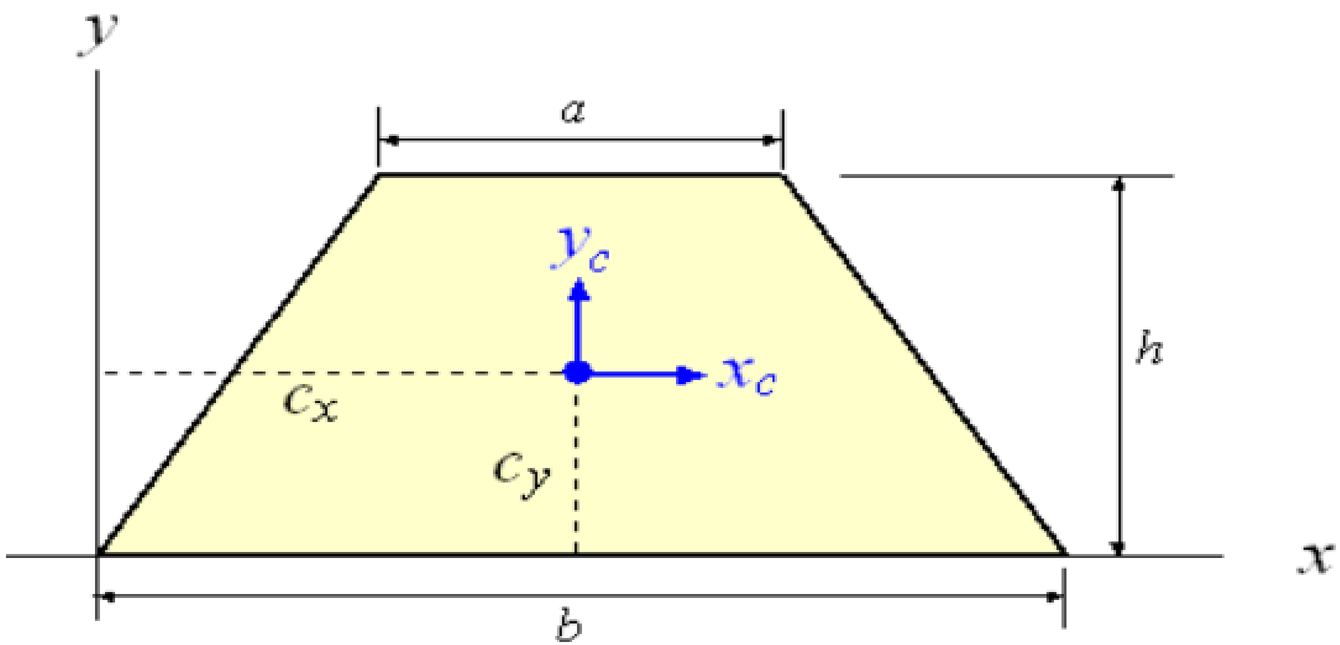
$$\bar{y} = \frac{4r}{3\pi} \qquad \bar{x} = r$$



2. Compound Centroid Shapes

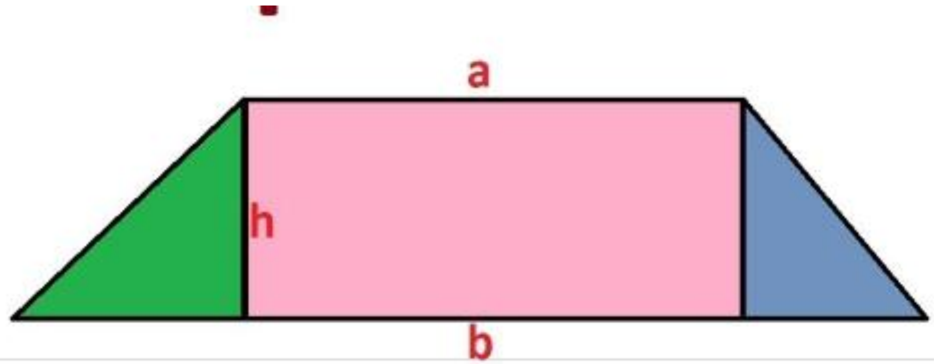


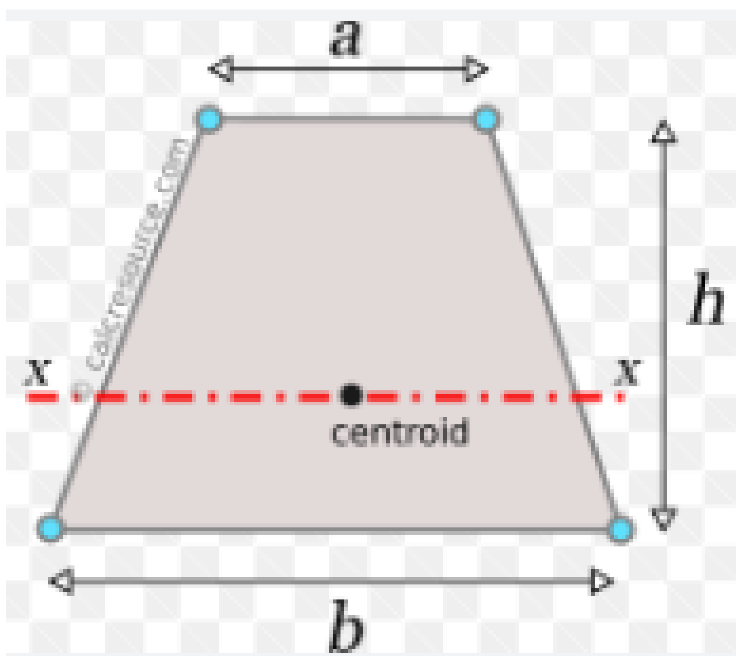
C_x	C_y	Area
$\dfrac{2ac + a^2 + cb + ab + b^2}{3(a+b)}$	$\dfrac{h(2a+b)}{3(a+b)}$	$\dfrac{h(a+b)}{2}$

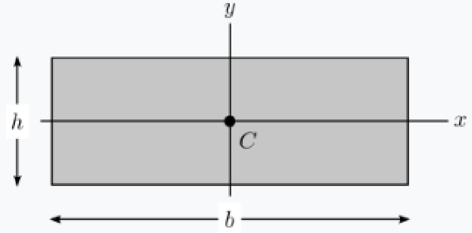
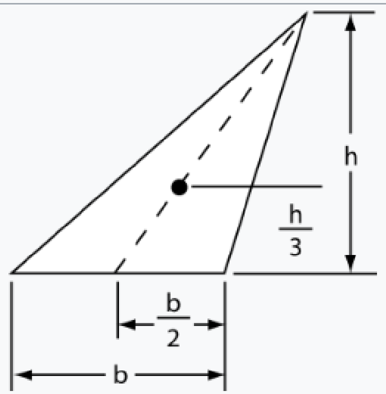
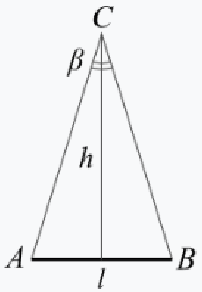


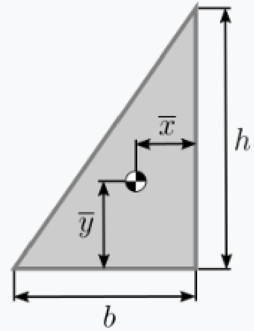
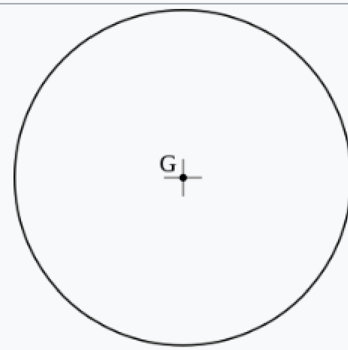
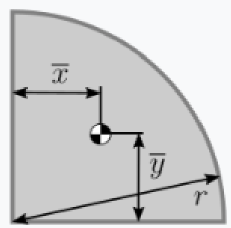
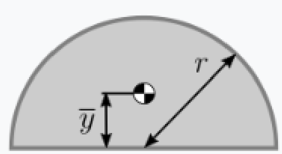
C_x	C_y	Area
$\frac{b}{2}$	$\frac{h}{3}\left(\frac{2a+b}{a+b}\right)$	$\frac{h}{2}(a+b)$

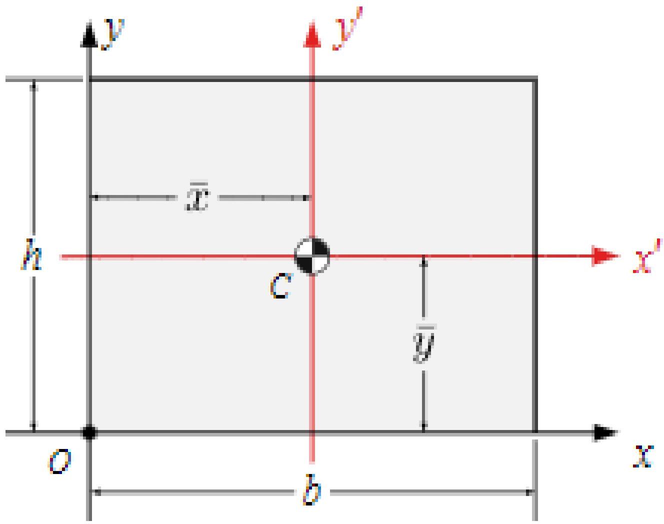
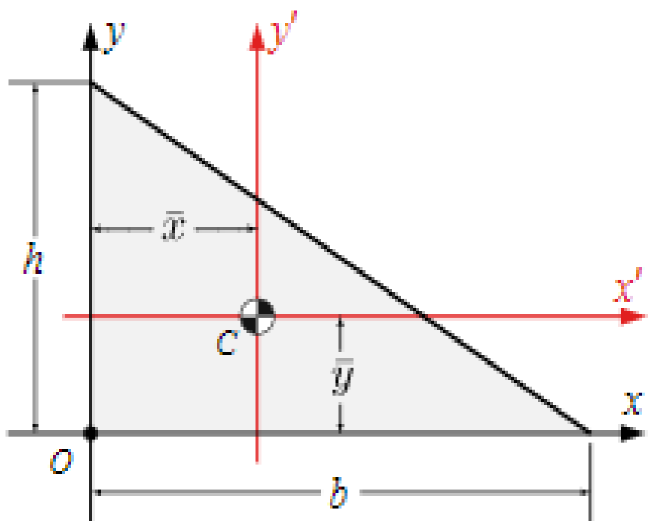
$$Y = \frac{h\,(b+2a)}{3\,(b+a)}$$

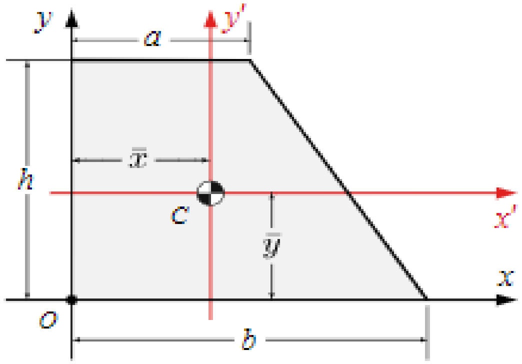
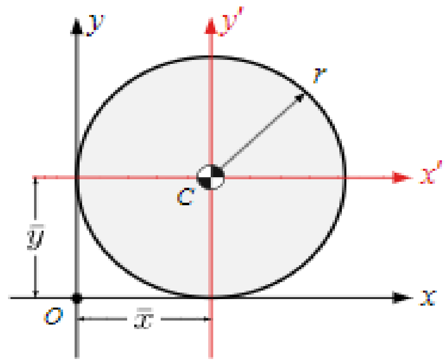
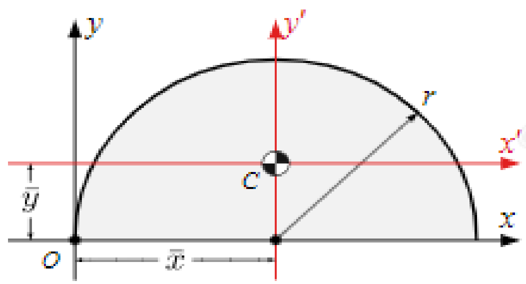
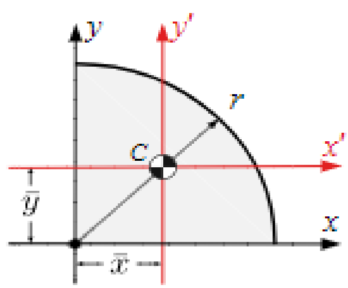


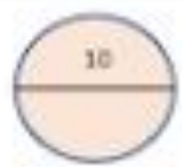
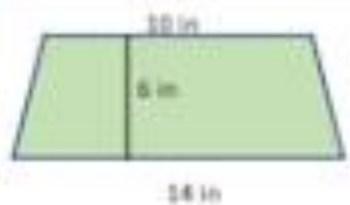


Shape	Figure	$\bar{x}$	$\bar{y}$	Area
rectangle area		$\frac{b}{2}$	$\frac{h}{2}$	bh
General triangular area		$\frac{x_1 + x_2 + x_3}{3}$ [1]	$\frac{h}{3}$	$\frac{bh}{2}$
Isosceles-triangular area		$\frac{l}{2}$	$\frac{h}{3}$	$\frac{lh}{2}$

Right-triangular area		$\frac{b}{3}$	$\frac{h}{3}$	$\frac{bh}{2}$
Circular area		0	0	πr^2
Quarter-circular area[2]		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area[3]		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$

Shape	Area	$\bar{x}$	$\bar{y}$
	$A = bh$	$b/2$	$h/2$
	$\dfrac{bh}{2}$	$b/3$	$h/3$

	$\dfrac{(a+b)h}{2}$	$\dfrac{a^2+ab+b^2}{3(a+b)}$	$\dfrac{h(2a+b)}{3(a+b)}$
	πr^2	r	r
	$\dfrac{\pi r^2}{2}$	r	$\dfrac{4r}{3\pi}$
	$\dfrac{\pi r^2}{4}$	$\dfrac{4r}{3\pi}$	$\dfrac{4r}{3\pi}$

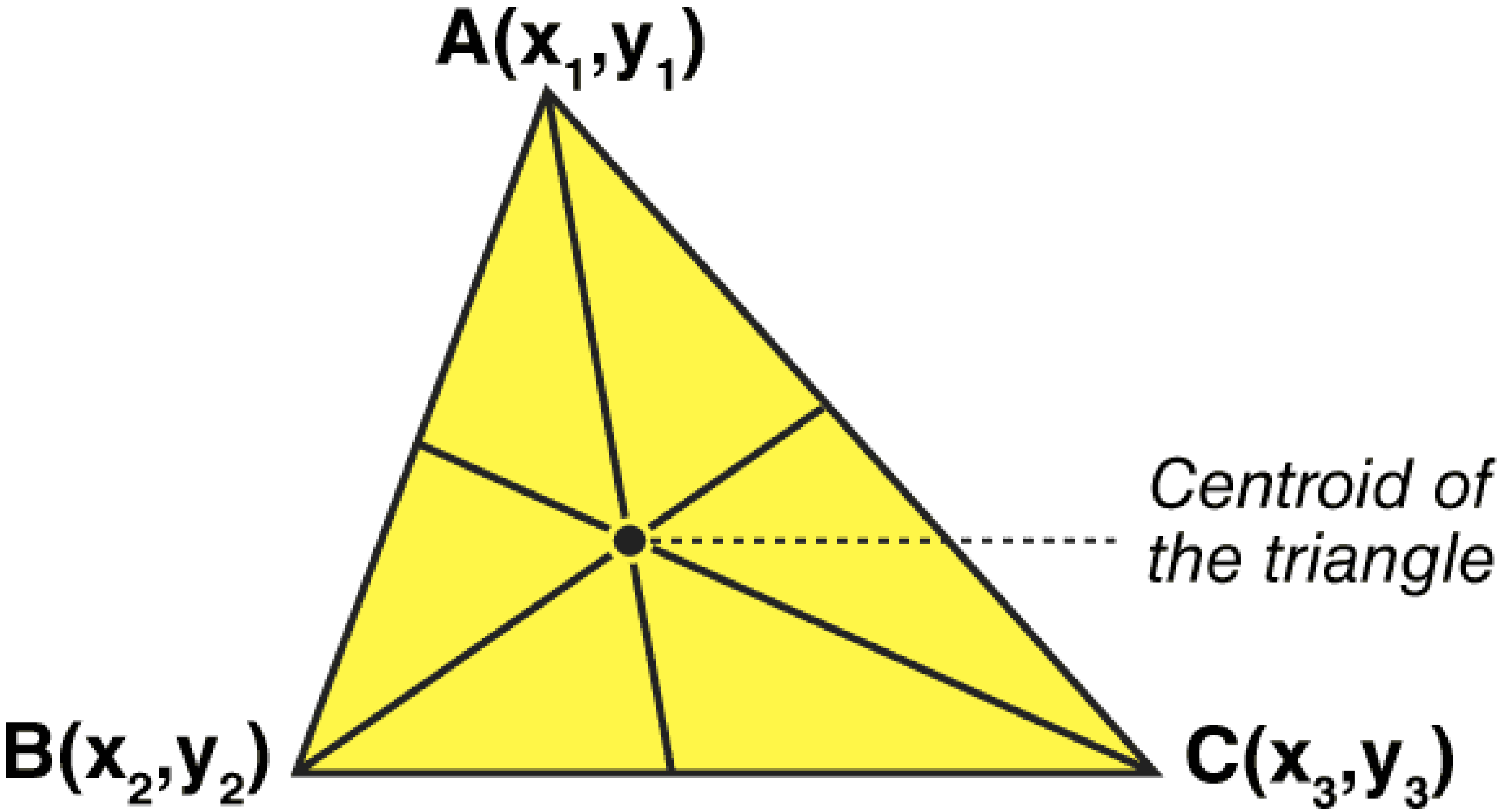
Shape Name	Diagram	Area
Square		$A = s^2$ $A = 4^2$ A = 16 sq inches
Triangle		$A = \frac{1}{2} bh$ $A = \frac{1}{2} 10 \times 3$ $A = \frac{1}{2}\ 30$ A = 15 sq. units
Rectangle		A = L x W A = 12 x 4 A = 48 sq. cm
Circle		$A = \pi r^2$ $A = 3.14 \times 5^2$ A = 3.14 x 25 A = 78.5 sq. units
Trapezoid		$A = \frac{1}{2}\ h(b_1 + b_2)$ $A = \frac{1}{2}\ 6(10 + 14)$ $A = \frac{1}{2}\ 6(24)$ A = 3 x 24 A =72 sq. in

Centroid Formula

Centroid Formula

Let's consider a triangle. If the three vertices of the triangle are A(x_1, y_1), B(x_2, y_2), C(x_3, y_3), then the centroid of a triangle can be calculated by taking the average of X and Y coordinate points of all three vertices. Therefore, the centroid of a triangle can be written as:

Centroid of a triangle = $((x_1+x_2+x_3)/3, (y_1+y_2+y_3)/3)$


A(x_1,y_1)
B(x_2,y_2)
C(x_3,y_3)
Centroid of the triangle

Examples on Calculating Centroid

Find the solved examples below, to find the centroid of triangles with the given values of vertices.

Question 1: Find the centroid of the triangle whose vertices are A (2, 6), B (4, 9), and

C (6,15).

Solution:

A (x_1, y_1) = A (2, 6)

B (x_2, y_2) = B (4,9)

C (x_3, y_3) = C (6,15)

We know that the formula to find the centroid of a triangle is = $((x_1+x_2+x_3)/3, (y_1+y_2+y_3)/3)$

Centroid of a triangle = ((2+4+6)/3, (6+9+15)/3) = (12/3, 30/3) = (4, 10)

Therefore, the centroid of the triangle for the given vertices A (2, 6), B (4,9), and C (6,15) is (4, 10).

Question 2: Find the centroid of the triangle whose vertices are A (1, 5), B (2, 6), and C (4, 10).

Solution: Given, A (1, 5), B (2, 6), and C (4, 10) are the vertices of a triangle ABC.

By the formula of the centroid we know;

Centroid = $((x_1+x_2+x_3)/3, (y_1+y_2+y_3)/3)$

Centroid = (1+2+4)/3, (5+6+10)/3 = (7/3, 21/3) = (7/3,7)

Hence, the centroid of the triangle having vertices

A (1, 5), B (2, 6), and C (4, 10) is (7/3, 7).

Question 3: If the vertices of a triangle are (2, 1), (3, 2) and (-2, 4). Then find the centroid of it.

Solution: Given, (2, 1), (3, 2) and (-2, 4) are the vertices of triangle

Centroid = $((x_1+x_2+x_3)/3, (y_1+y_2+y_3)/3)$

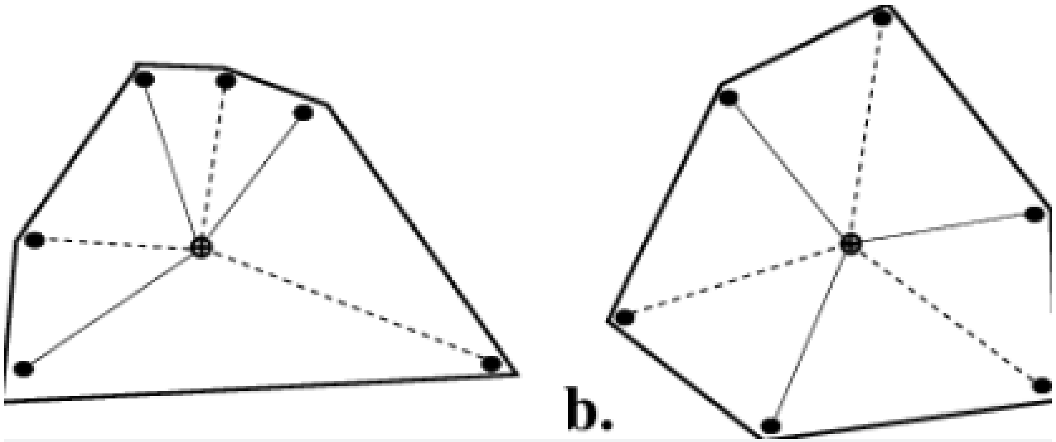
Putting the values, we get;

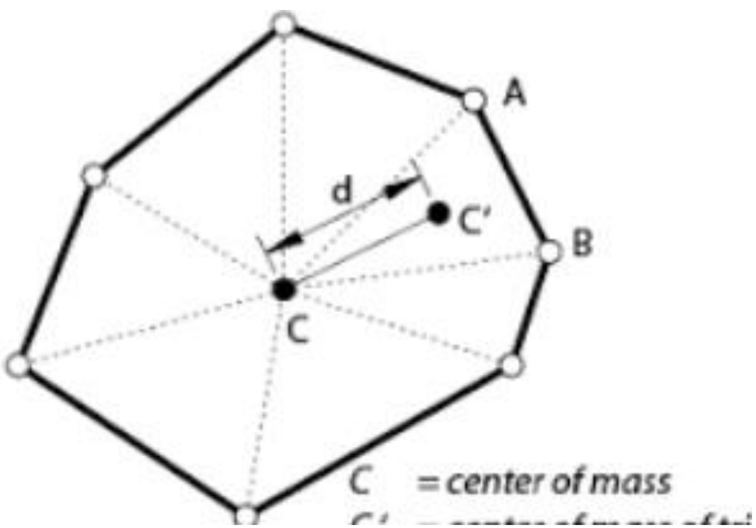
Centroid, O = (2+3-2)/3, (1+2+4)/3 O = (3/3, 7/3)

O = (1, 7/3)

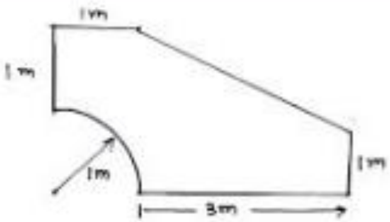
Hence, the centroid of the triangle having vertices (2, 1), (3, 2) and (-2, 4) is (1, 7/3).

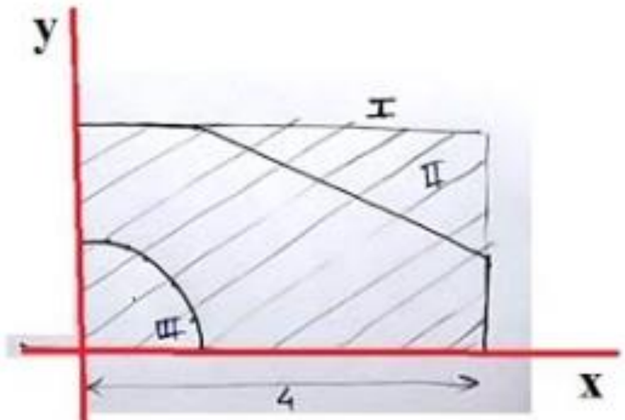
3. Irregular Centroid Shapes



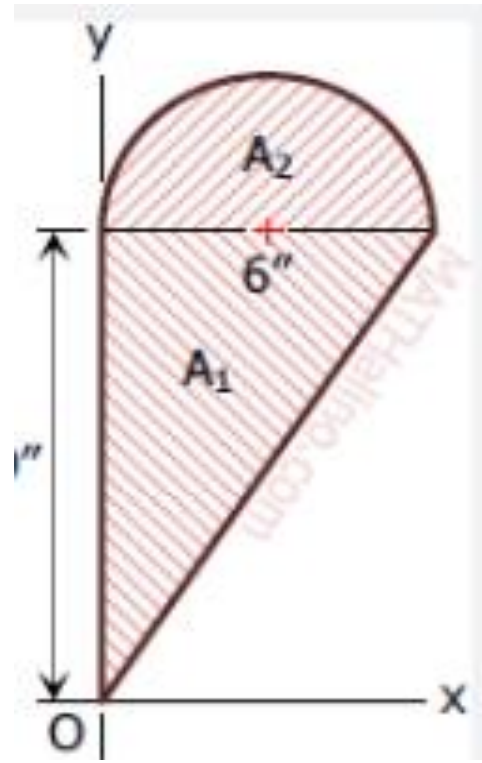


Centroid Example - 1





Centroid of an irregular shape plane



Irregular shapes

For irregular 3D shapes you can also separate the object into segments, determine the cross-sectional area of each piece and then plot the cross-sectional area vs. position to determine the centroid of the shape. The centroid of each area segment must be determined.

The expressions are modified as follows:

$$\bar{X} = \frac{\sum x_c(A\Delta x)}{\sum A\Delta x} = \frac{\sum x_{c_i} V_i}{\sum V_i}$$

$$\bar{Y} = \frac{\sum y_c(A\Delta x)}{\sum A\Delta x} = \frac{\sum y_{c_i} V_i}{\sum V_i}$$

$$\bar{Z} = \frac{\sum z_c(A\Delta x)}{\sum A\Delta x} = \frac{\sum z_{c_i} V_i}{\sum V_i}$$