

Erbil Algebra Project (EAP)

Groups, Rings, Fields and Galois Theory “multiple hands” course in Erbil 2023-2029

This is a proposal for organizing a course on *Groups, Rings, fields and Galois Theory* at the bachelor (B.Sc.) and master (M.Sc.) levels at **Salahaddin University-Erbil**.

The project has a **span of six years** starting in Fall of 2023, ending in the summer of 2029. Each year, **one course of 50 hours** will be offered at **Salahaddin University-Erbil** by several lecturers from developed countries.

The program will last for 6 years, and involve two different courses. In odd years there will be a general course in Algebra (groups, rings and fields). In even years there will a course in Galois Theory. The course is for 12 consecutive weeks. It will be divided into 6 modules each of two weeks. Each module will be taught by a different team of 1 or 2 individuals. All the teachers will follow strictly the same book/notes so that each module will be able to count on the materials that will have been covered in its previous module(s). We think of 6 hours of teaching per week, 2 of which are exercise sessions. Every module will be taught by a different lecturer according to a rigid schedule that is outlined below.

The first edition of the course in 2023 will stick to the following schedule:

Module	Weeks	Topics	Lecturer
I	04/9 – 18/9	Relations and Functions, The Integers Modulo n , Equivalence Relations and Partitions, Semigroup and Monoid, Definition and Examples of Groups, Group Tables, Elementary Properties, Abelian and Cyclic Groups, Subgroups, Symmetric, Alternating and Dihedral Groups.	
II	18/09-02/10	Cosets and Lagrange Theorem, Normal Subgroup and Factor Group, Homomorphism and Isomorphism, Fundamental of Homomorphism Theorem for Groups, Cayley’ s Theorem, Sylow Theorems and Applications.	
III	02/10-16/10	Definition and Examples of Rings, Fields and Integral Domains, Units in a Ring, Integral Domains and Fields, Characteristic of a Ring, Subrings, Ideals, Quotient Rings, Principal and Maximal Ideals, The Isomorphism Theorems, The Field of Quotients of an Integral Domain.	
Midterm	22/10/2023	two-hour midterm exam	
IV	23/10 –06/11	The Division Algorithm, Principal Ideal Domains, Factorization of Polynomials, Prime and Irreducible Elements, Unique factorization Domains, Euclidean Domains. Algebraic Extensions, Finite Fields.	
V	06/11 –20/11	Construction of some extension fields, Stem fields, Algebraic and transcendental elements, Transcendental numbers, algebraically closed fields, Maps from simple extensions.	
VI	20/11-04/12	Splitting fields, Multiple roots, Groups of automorphisms of fields, Separable, and normal extensions.	
Final	20/12/2023	two-hour final exam	

The first edition of the course in 2024 will stick to the following schedule:

Module	Weeks	Topics	Lecturer
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I	04/9 – 18/9	Revision of ring theory: polynomials, Euclidean algorithm, unique factorization, Prime subfield, characteristic, symmetric polynomials. Field extensions: degree, simple extensions, algebraic and finitely generate extensions. Splitting fields, normal extensions, normal closure.
II	18/09-02/10	Galois extensions, The fundamental theorem of Galois theory, The fundamental theorem of Galois theory (FTGT), Examples and applications of FTGT, Constructions with straight-edge and compass, The Galois group of a polynomial, Solvability of equations.
III	02/10-16/10	Computing Galois Groups, When is G_f contained in A_n ? When is G_f transitive? Polynomials of degree at most three, Quartic polynomials, Examples of polynomials with S_p as Galois group over \mathbf{Q} , Computing Galois groups over \mathbf{Q} .
Midterm 22/10/2023 two-hour midterm exam		
IV	23/10 – 06/11	Applications of Galois Theory: Primitive element theorem, Fundamental Theorem of Algebra, Cyclotomic extensions, Dedekind's theorem on the independence of characters, Normal basis theorem, Hilbert's Theorem.
V	06/11 – 20/11	Cyclic extensions, Kummer theory, Proof of Galois's solvability theorem, Symmetric polynomials, General polynomial of degree n , Norms and traces.
VI	20/11-04/12	Integral Extensions, Quadratic Extensions of the Rationals, Norms and Traces, The Discriminant, Noetherian and Artinian Modules and Rings, Fractional Ideals, Unique Factorization of Ideals in a Dedekind Domain
Final 20/12/2023 two-hour final exam		

Lecturers

Each lecturer will lecture one or more modules. Each module lasts two weeks. The participation in the project is purely voluntary and the lecturers will receive a salary of 100€/week from Salahaddin University-Erbil, which is comparable to the salary of an Iraqi lecturer. The lecturers for the first edition of the course have already been selected.

- They have all agreed to give lectures according to the above scheduled;
- They are all experienced teachers with experience in developing countries;
- They have all agreed to the spirit and goals of AP;
- All of them (except one) have already visited Salahaddin University-Erbil and given lecture there in the past;
- They have all agreed to commit themselves for several years.

For the following years, in case of the necessity of some substitutions, the project will be advertised and more availabilities of lecturers will be collected. Priority will be given to those interested, willing to give lectures for a longer time, willing to commit for several years and with more experience.

Some Iraqi faculty will start to participate actively in the lectures. The role of Iraqi participants will grow every year so that, after the project is completed, a local teacher can continue give the same course in the following years.

Scientific and Didactic aspects

The course book will be: *Fields and Galois Theory* by J.S. Milne available online at

<https://www.jmilne.org/math/CourseNotes/FTe6.pdf>

The course will be evaluated according to the rules of the other courses at Salahaddin Univeisty-Erbil, and the final exam is scheduled after the last day of classes, possibly in the week after.

Every week the students will be assigned to take home exercises. They shall email the solutions, according to given deadlines, to an address at the Università Roma Tre where some doctoral students shall mark them and mail back corrected assignments to the students and grades to the teachers. The doctoral students of Roma Tre shall do their marking activity on a voluntary basis and will not be paid for their work.

A web page will be maintained at the Università Roma Tre with the diary of the lectures, the solutions of the assignments and other information for the students.

Commitment of Salahaddin University-Erbil

A contract will be signed between Salahaddin University-Erbil and the organizers. In the contract the following commitments will be listed

- List this course as one of the regular courses of B.Sc. and, possible, master's.
- Provide classroom space and secretarial assistance for the course.
- Select the Iraqi faculties that will participate in the project.
- Pay the lecturers a salary equivalent to 140€/week.
- Communicate the list of students in time.
- Provide assistance and actively help to obtain multiple Iraqi entry visas (possibly for five years) for those lecturers that will be teaching for more than one year.

Preliminary Budget

14,685€ each year for a total of **88,110€**

COSTS OUT		
Transportation to Erbil (6 Air tickets x 1000€)		6,000€
Local expenses (12 weeks x 7 days x 70€)		5,880€
Salaries (12 x 140€)		1,680€
Visas (70€ x 6)		420€
Local Transportation (20€ x 25 trips)		500€
Course Material		205€
TOTAL		14,685€
FUNDS IN		
CIMPA	Two Modules	4335€
IMP (VLP)	Two Modules	4335€
INDAM	Two Modules	4335€
Salahaddin University-Erbil	Salaries	1.680€
TOTAL		14,685€

Proponents

- Herish Omer Abdullah (Salahaddin University-Erbil)
- Karzan Ahmed Berdawood (Salahaddin University-Erbil)
- Andam Ali Mustafa (Salahaddin University-Erbil)
- Francesco Pappalardi (Università Roma Tre-Italy)
- Peter Stevenhagen (Universiteit Leiden- Netherlands)
- Valerio Talamanca (Università Roma Tre-Italy)
- Michel Waldschmidt (Sorbonne Université-France)

Lectures for the first edition of the course:

- Francesco Amoroso (Université de Caen-France)
- Kamran Divaani-Aazar (Alzahra University-Iran)
- Laura Geatti (Universita' di Roma 2 Tor Vergata-Italy)
- Francesco Pappalardi (Università Roma Tre-Italy)
- René Schoof (Universita' di Roma 2 Tor Vergata-Italy)
- Peter Stevenhagen (Universiteit Leiden- Netherlands)
- Lea Terracini (Università di Torino-Italy)
- Michel Waldschmidt (Sorbonne Université-France)

Replacement Lectures in case of need:

- Valerio Talamanca (Università Roma Tre-Italy)