Department of …Statistics and Informatics

College of ………Adm. & Eco………………….

University of …SALAHADDIN UNIVERSITY- HAWLER….

Subject: ……Matrices…….

Course Book –(Year 2)

Lecturer's name : Kawthar Saeed Taha (MSc)

Academic Year: 2022/2023

Course Book

1. Course name	**Matrices**
2. Lecturer in charge	**Kawthar Saeed Taha**
3. Department/ College	**Statistics &Information / Adm. & Eco**
4. Contact	**e-mail: http//gmail.com/side/Kawthar** **Tel: (07504703371)**
5. Time (in hours) per week	**For example Theory: 3**
6. Office hours	**Availability of the lecturer to the student during the week**
7. Course code	**SAE208**
8. Teacher's academic profile	**From 1998 until 2003 worked as Assistant Lecture in Statistics Department – Salahaddin University, I have taught the following subject with some MSc. : Statistics, Biostatistics, Analysis of Regression, Mathematical Statistics, Differential Equation. In 2006 I had my MSc. In Statistics from the same university. From 2006 until 2012 worked decision maker at Statistics Department. From 2006 till now I am working as a lecturer in Statistics Department Salahaddin University. I have taught the following subject: Statistics, Word and Windows, linear algebra, Biostatistics, Statgraphics.**
9. Keywords	**Elementary of Matrices: Definition of Matrices, Matrix, Square Matrix, Equal matrix, Zero matrix, Algebraic operations, Addition of matrices, Subtraction of matrices, Multiplication of a matrix by a scalar, Multiplication of two matrices, Type of matrices: Diagonal, ….**
10. Course overview: **The general purpose of this course is to study the basic concepts of this course Matrices is divided into five parts. The first part deals with Matrices , some type of matrix, Algebraic operation and Trace of matrix , the second part deals with Partitioning of matrix and partition algebraic processes, the third part deals with Some type of matrices, the forth part deals with Determinant of a Matrices , and the fifth part deals with adjoint of square matrix and invers of matrix.**	
11. **Course objective:** **Matrices is concerned with finite dimensional vector spaces. Solving systems of linear equations is one of the most important applications of linear algebra. It has been argued that the majority of all mathematical problems encountered in scientific and industrial applications involve solving system at some point. Linear applications arise in such diverse areas as engineering, chemistry, economics, business, ecology, biology and psychology. One of the early goals of this course is to develop algorithm that helps solve larger systems in an orderly manner.**	

12. **Student's obligation**

The student should attend the class at the exact time and place determined. The student should keep his mobile closed and lessen well to all lectures he/she should never loss concentration in the class neither occupying him with any not necessary things. Other important steps they should attend all the exams the exact time and place determined bringing all the requirements like calculator, pen and papers.

13. Forms of teaching

For giving the lecture I will use the Data show and the white board and sometimes I will use the prepared lectures with Data show and white board all together.

I am the only responsible lecturer who gives this subject without the help of any other teaching members.

14. Assessment scheme

The students are obliged to perform at least two closed book exams during the academic year. Quizzes (5%), the exam has 30% besides homework and classroom activities (5%) , and (60%)will be reserved for the final exam.

15. Student learning outcome:

At the end of this course, students are expected to be confidence from analyzing the relationships between all factors that related together in the reality. They will be able to formulate the modeling the relation and distinguish the type of relation and analyzing with interpreting the consequences after that make decisions.

The students should have the ability to work in both public and private sectors as having good skills in analyzing.

16. Course Reading List and References:

1- Strang, G., 1980, Linear algebra and it is application, 2nd edition, Academic Press, New York.

2- S.J. Leon, Linear algebra with applications, Prentice Hall, 6th Edition, 2002.

3- G.H.Golub and C.F.Vantamn. Matrix and application, John Hopkins Univ. Press, 3rd Ed. Baltimore, 1996.

4- Larson R., C. Falvo D.C. Elementary Linear algebra 6th Edition, Houghton Mifflin Harcourt Publishing Company, New York,2009.

1- الناصر، عبد المجيد حمزة ، جواد، لميعة باقر، الجبر الخطي، تموز 1988.

2-العلي،ابراهيم محمد، أسس التحليل الاحصائي متعدد المتغيرات، 2020.

17. The Topics:	**Lecturer's name**

Definition of Matrices, Matrix, Square Matrix, Equal matrix, Zero matrix, Algebraic operations, Addition of matrices, Subtraction of matrices, Multiplication of a matrix by a scalar, Multiplication of two matrices, Type of matrices: Diagonal matrix, Upper triangular matrix, Lower triangular matrix, Scalar matrix, Identity matrix, Anti – commute matrix, Idempotent matrix, Nilpotent matrix, Involutary matrix, Trace of matrix, Partitioning of matrix and partition algebraic processes: Partitioning of matrix, Addition of matrices by partition, Multiplication matrices by partition, Some of types for matrix: the transpose of a matrix, Symmetric of matrices, Skew Symmetric of matrices, Complex numbers, The conjugate Complex numbers, The conjugate of a matrices, The tranjugate of a matrices, Hermitian of a matrices, Skwe Hermitian matrix, Invers of matrix for size (2×2), The determinant of square matrix : Permutations, Determinant, First minor and cofactor, Minor and Algebric complements, The way to find Determinant for size (2×2) and (3×3), The adjoint of a matrix and the way to find invers of matrix,	**Kawthar Saeed Taha** **ex: three hours a week**
18. Practical Topics (If there is any)	
No any one	**Kawthar Saeed Taha** **ex: Three hours a week**

19. Examinations:

1. Compositional: ex: **Show that A.B is Anti – commute matrix if**

$$A=\begin{bmatrix}1 & 1\\ 4 & -1\end{bmatrix} \text{ and } B=\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}$$

<u>solution:</u>

A.B= -B.A

$$\begin{bmatrix}1 & 1\\ 4 & -1\end{bmatrix}\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}=-\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}\begin{bmatrix}1 & 1\\ 4 & -1\end{bmatrix}$$

$$\begin{bmatrix}1+2 & -1-1\\ 4-2 & -4+1\end{bmatrix}=-\begin{bmatrix}1-4 & 1+1\\ 2-4 & 2+1\end{bmatrix}$$

$$\begin{bmatrix}3 & -2\\ 2 & -3\end{bmatrix}=-\begin{bmatrix}-3 & 2\\ -2 & 3\end{bmatrix}$$

$$\begin{bmatrix}3 & -2\\ 2 & -3\end{bmatrix}=\begin{bmatrix}3 & -2\\ 2 & -3\end{bmatrix}$$

Ex: If $A=\begin{bmatrix} 2-i & 3+i \\ 2+i & 2-i \end{bmatrix}$ find A^*.

<u>Solution:</u> $A^* = (\overline{A})'$

$$\overline{A} = \overline{\begin{bmatrix} 2-i & 3+i \\ 2+i & 2-i \end{bmatrix}} = \begin{bmatrix} 2+i & 3-i \\ 2-i & 2+i \end{bmatrix}$$

$$(\overline{A})' = \begin{bmatrix} 2+i & 3-i \\ 2-i & 2+i \end{bmatrix}' = \begin{bmatrix} 2+i & 2-i \\ 3-i & 2+i \end{bmatrix}$$

Ex: Let $A=\begin{bmatrix} 5 & 3 \\ 2 & -1 \end{bmatrix} \Rightarrow B = \begin{bmatrix} 3 & 5 \\ -1 & 2 \end{bmatrix}$ show that |B| = -|A|

Solution:

$$|A| = \begin{vmatrix} 5 & 3 \\ 2 & -1 \end{vmatrix} = -5-6 = -11$$

$$|B| = \begin{vmatrix} 3 & 5 \\ -1 & 2 \end{vmatrix} = 6-(-5) = 6+5 = 11$$

$$|B| = -|A| \Rightarrow 11 = -(-11) \Rightarrow 11 = 11$$

Ex: Find inverse of matrix A by using adjoint method $A = \begin{bmatrix} 5 & 3 \\ -2 & 7 \end{bmatrix}$.

<u>Solution:</u>

$$|A| = \begin{vmatrix} 5 & 3 \\ -2 & 7 \end{vmatrix} = 35-(-6) = 35+6 = 41 \neq 0$$

$\therefore$ المعكوسة الى (A) موجودة وتساوي:

$$A^{-1} = \frac{adj(A)}{|A|}$$

$$adj(A) = \begin{bmatrix} a_{22} & -a_{12} \\ a_{21} & a_{11} \end{bmatrix} = \begin{bmatrix} 7 & -3 \\ 2 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{\begin{bmatrix} 7 & -3 \\ 2 & 5 \end{bmatrix}}{41}$$

$$= \begin{bmatrix} \frac{7}{41} & \frac{-3}{41} \\ \frac{2}{41} & \frac{5}{41} \end{bmatrix}$$

Ex: Find the determinant of A if $A = \begin{bmatrix} 5 & -2 & 1 & 0 \\ 4 & 3 & -2 & 5 \\ -3 & 6 & 9 & -1 \\ 8 & 1 & 0 & -2 \end{bmatrix}$ by using Laplace method.

<u>Solution</u>:

افضل عدد يؤخذ للحذف من الصفوف هو الصف الاول و الثاني($i_1, i_2 = 1, 2$) أما الأعمدة فتأخذ شكل اختيار (j_1, j_2) و الاختبارات الممكنة المختلفة هي:

$$j_1, j_2 = (1,2), (1,3), (1,4), (2,3), (2,4), (3,4)$$

لذا فإن المحدد الى A تساوي

$$|A| = (-1)^{\overbrace{1+2}^{row}+\overbrace{1+2}^{col}} \begin{vmatrix} 5 & -2 \\ 4 & 3 \end{vmatrix} \begin{vmatrix} 9 & -1 \\ 0 & -2 \end{vmatrix} + (-1)^{1+2+1+3} \begin{vmatrix} 5 & 1 \\ 4 & -2 \end{vmatrix} \begin{vmatrix} 6 & -1 \\ 1 & -2 \end{vmatrix} + (-1)^{1+2+1+4} \begin{vmatrix} 5 & 0 \\ 4 & 5 \end{vmatrix} \begin{vmatrix} 6 & 9 \\ 1 & 0 \end{vmatrix} +$$

$$(-1)^{1+2+2+3} \begin{vmatrix} -2 & 1 \\ 3 & -2 \end{vmatrix} \begin{vmatrix} -3 & -1 \\ 8 & -2 \end{vmatrix} + (-1)^{1+2+2+4} \begin{vmatrix} -2 & 0 \\ 3 & 5 \end{vmatrix} \begin{vmatrix} -3 & 9 \\ 8 & 0 \end{vmatrix} + (-1)^{1+2+3+4} \begin{vmatrix} 1 & 0 \\ -2 & 5 \end{vmatrix} \begin{vmatrix} -3 & 6 \\ 8 & 1 \end{vmatrix}$$

$$= (15+8)(-18) - (-10-4)(-12+1) + (25)(-9) + (4-3)(6+8) - (-10)(-72) + (5)(-3-48)$$

$$= 23(-18) + (-14)(11) - 225 + 14 - 720 + 5(-51)$$

$$= -414 - 154 - 225 + 14 - 720 - 255$$

$$= -1754$$

Ex: Find inverse of matrix B by using adjoint method $B = \begin{bmatrix} 6 & -2 & 2 \\ 0 & 4 & 1 \\ 0 & 5 & -3 \end{bmatrix}$.

Solution: We use column one to find determinant of matrix B. Then

$$|B| = 6\begin{vmatrix} 4 & 1 \\ 5 & -3 \end{vmatrix} - 0\begin{vmatrix} -2 & 2 \\ 5 & -3 \end{vmatrix} + 0\begin{vmatrix} -2 & 2 \\ 4 & 1 \end{vmatrix}$$

$$= 6(-12-5) - 0 + 0 = 6(-17) = -102 \neq 0$$

$$B^{-1} = \frac{adj(B)}{|B|}$$

$$adj(B) = \begin{bmatrix} \alpha_{11} & \alpha_{21} & \alpha_{31} \\ \alpha_{12} & \alpha_{22} & \alpha_{32} \\ \alpha_{13} & \alpha_{23} & \alpha_{33} \end{bmatrix}$$

$$\alpha_{ij} = (-1)^{i+j} |M_{ij}|$$

$$\alpha_{11} = (-1)^{1+1}|M_{11}| = (-1)^2 \begin{vmatrix} 4 & 1 \\ 5 & -3 \end{vmatrix} = -12 - 5 = -17$$

$$\alpha_{12} = (-1)^{1+2}|M_{12}| = (-1)^3 \begin{vmatrix} 0 & 1 \\ 0 & -3 \end{vmatrix} = 0 - 0 = 0$$

$$\alpha_{13} = (-1)^{1+3}|M_{13}| = (-1)^4 \begin{vmatrix} 0 & 4 \\ 0 & 5 \end{vmatrix} = 0 - 0 = 0$$

$$\alpha_{21} = (-1)^{2+1}|M_{21}| = (-1)^3 \begin{vmatrix} -2 & 2 \\ 5 & -3 \end{vmatrix} = -(6 - 10) = 4$$

$$\alpha_{22} = (-1)^{2+2}|M_{22}| = (-1)^4 \begin{vmatrix} 6 & 2 \\ 0 & -3 \end{vmatrix} = -18 - 0 = -18$$

$$\alpha_{23} = (-1)^{2+3}|M_{23}| = (-1)^5 \begin{vmatrix} 6 & -2 \\ 0 & 5 \end{vmatrix} = -(30 - 0) = -30$$

$$\alpha_{31} = (-1)^{3+1}|M_{31}| = (-1)^4 \begin{vmatrix} -2 & 2 \\ 4 & 1 \end{vmatrix} = -2 - 8 = -10$$

$$\alpha_{32} = (-1)^{3+2}|M_{32}| = (-1)^5 \begin{vmatrix} 6 & 2 \\ 0 & 1 \end{vmatrix} = -(6 - 0) = -6$$

$$\alpha_{33} = (-1)^{3+3}|M_{33}| = (-1)^6 \begin{vmatrix} 6 & -2 \\ 0 & 4 \end{vmatrix} = 24 + 0 = 24$$

$$adj(B) = \begin{bmatrix} \alpha_{11} & \alpha_{21} & \alpha_{31} \\ \alpha_{12} & \alpha_{22} & \alpha_{32} \\ \alpha_{13} & \alpha_{23} & \alpha_{33} \end{bmatrix} = \begin{bmatrix} -17 & 4 & -10 \\ 0 & -18 & -6 \\ 0 & -30 & 24 \end{bmatrix}$$

$$B^{-1} = \frac{adj(B)}{|B|}$$

$$= \frac{-1}{102} \begin{bmatrix} -17 & 4 & -10 \\ 0 & -18 & -6 \\ 0 & -30 & 24 \end{bmatrix}$$

20. Extra notes:لا يوجد ملاحظة

21. Peer review