Department: Statistics & Informatics

Stage: Second

Subject: Matrices

2022-2023

Lecturer Kawther Saeed Taha

Chapter one//

Matrices(المصفوفات) (ريزكراوه)

Matrix: An (m×n) real (complex) matrix A is an array of real (complex) numbers a_{ij} $(1 \le i \le m \ , \ 1 \le j \le n)$ arranged (يرتب) in (m) rows and (n) columns, and enclosed by brackets (الاقواس), as follows:

$$A=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m\times n} \quad \textbf{or} \quad A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m\times n}$$

A matrix can contain any number or any function as its elements and each unique (فر يد من نوعه ، بيَهاوتا) data (any number, symbol (رمز ،هيَما) or expression (روخسار)) is called the elements of the matrix.

Notes:

a) **If A is (m×n) , then m is number of rows in the array (مساحه).**
b) **If A is (m×n) , then n is number of columns in the array.**
c) **The size (or order) of the matrix is (m×n).**
d) **The a_{ij} appears(يظهر) in the ith rows and jth columns.**
e) **The numbers of are called the elements (عناصر) of the matrix.**
f) **The notation is sometimes abbreviated(يختصر) to $[a_{ij}]$, or $[a_{ij}]_{m\times n}$ if we wish to specify(يفصل) the size of the array.**

Ex:

$A=\begin{bmatrix} 1 & 2 & -3 \\ 4 & 0 & 6 \end{bmatrix}_{2\times 3}$ **is a 2×3 matrix in which $a_{11}=1$, $a_{12}=2$, $a_{13}=-3$, $a_{21}=4$, $a_{22}=0$, $a_{23}=6$**

$B=\begin{bmatrix} 1 & -2 \\ 3 & 4 \\ 5 & 7 \end{bmatrix}_{3\times 2}$ **is a 3×2 matrix in which $b_{11}=1$, $b_{12}=-2$, $b_{21}=3$, $b_{22}=4$, $b_{31}=5$, $b_{32}=7$**

Note: قد تشكل عناصر المصفوفة في بعض الاحيان دالة وفي هذه الحالة يمكن استخراج جميع العناصر بسهولة

هەندێك جار لەوانەیە یەکەکانی ڕیزکراوە نەخشە بێت، لەم کاتدا دەتوانین هەموو یەکەکانی بە ئاسانی بدۆزینەوە

Ex: find the elements of matrix A=(a_{ij}) for size 3×2. Where $a_{ij}=i^2+3j$

Solution:

$$A_{3\times 2}=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}_{3\times 2}$$

$a_{ij}=i^2+3j$
$a_{11}=(1)^2+3(1)=1+3=4$
$a_{12}=(1)^2+3(2)=1+6=7$
$a_{21}=(2)^2+3(1)=4+3=7$
$a_{22}=(2)^2+3(2)=4+6=10$
$a_{31}=(3)^2+3(1)=9+3=12$
$a_{32}=(3)^2+3(2)=9+6=15$

$$\Rightarrow \therefore A=\begin{bmatrix} 4 & 7 \\ 7 & 10 \\ 12 & 15 \end{bmatrix}_{3\times 2}$$

Ex: find the elements of matrix B=(b_{ij}) for size 3×3. Where b_{ij}=ì -2j^2

Solution:

$$B_{3\times3}=\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}_{3\times3}$$

b_{ij}=ì -2j^2

b_{11}= 1- 2(1)^2 = 1-2 = -1

b_{12}= 1- 2(2)^2 = 1-8 = -7

b_{13}= 1- 2(3)^2 =1-18=-17

b_{21}= 2- 2(1)^2 = 2-2 = 0

b_{22}= 2- 2(2)^2 = 2-8 = -6

b_{23}= 2- 2(3)^2 =2-18=-16

b_{31}= 3- 2(1)^2 = 3-2 = 1

b_{32}= 3- 2(2)^2 = 3-6 = -3

b_{33}= 3- 2(3)^2 =3-18=-15

$$\Rightarrow B_{3\times3}=\begin{bmatrix} -1 & -7 & -17 \\ 0 & -6 & -16 \\ 1 & -3 & -15 \end{bmatrix}_{3\times2}$$

Types of matrix:

1- **Row matrix:** If a matrix has only one row. $A_{1\times3}=\begin{bmatrix} 2 & -1 & 3 \end{bmatrix}_{1\times3}$

2- **column matrix:** If a matrix has only one column. $A_{2\times1}=\begin{bmatrix} 2 \\ 1 \end{bmatrix}_{2\times1}$

3- **Square matrix (المصفوفة المربعة):**

If number of rows (m) and columns (n) in any matrix are equal (m=n) we said this matrix is Square matrix.

Or An (m×n) matrix A is Square if (m=n), that is if A has the same number of rows and columns. In a Square matrix A=$[a_{ij}]_{m\times n}$, a_{11}, a_{22} , … , a_{nn} are called the element of the main (رئيسي) (or leading القيادة) diagonal. Or A_n

Ex:

$C=\begin{bmatrix} -3 \end{bmatrix}_{1\times1}$ A is square matrix.

$B=\begin{bmatrix} 2 & -1 \\ 3 & 8 \end{bmatrix}_{2\times2}$ B is square matrix.

$C=\begin{bmatrix} 2 & 5 & 4 \\ -3 & 2 & 7 \\ 0 & -6 & 9 \end{bmatrix}_{3\times3}$ C is square matrix.

4- **Equal matrix:**

Two matrices A=$[a_{ij}]_{m\times n}$, B=$[b_{ij}]_{r\times s}$ are equal if (m=r , n=s) and a_{ij}= b_{ij} $1\le i\le m(=r)$, $1\le j\le n(=s)$; that is , if they have the same number of rows, the same number of columns, and corresponding elements are equal. *or* Two matrix equal if and only if they have exactly the same elements.

Ex: If $A=\begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix}$, $B=\begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix}$ then are A=B?

Solution: $\begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix}$ $a_{11}=b_{11}$, $a_{12}=b_{12}$, $a_{21}=b_{21}$ **and** $a_{22}=b_{22}$ **,then A=B because A and B are the same number of rows and columns and the corresponding elements are equal.**

The properties of equal matrix:

1) **A=A for all matrix A.**
2) **A=B then B=A for all A , B matrix.**
3) **If A=B and B=C then A=C for all A, B C matrix.**

5- **Zero matrix:**

A is matrix if all elements are zero then A is called zero matrix. We denote such a matrix by ($0_{m\times n}$) or simply by($\underline{0}$) . If there can be confusion about its size.

Ex: A and B matrix are a zero matrix.

$$A=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad , \qquad B=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

6- **Diagonal matrix:(المصفوفة القطرية)**

A square matrix $A=[a_{ij}]_{n\times n}$ is called diagonal matrix if all nondiagonal entries are zero [$a_{ij}=0$ for all $i \neq j$]. We write diagonal matrix diag(a_{11} , a_{22} , … , a_{nn})

Ex: $A=\begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

7- **Upper triangular matrix: (مصفوفة مثلث علوي)**

A square matrix $A=[a_{ij}]_{n\times n}$ echelon (مستوى) form (شكل) is upper triangular (that is, $a_{ij}=0$ if $i>j$) or ($a_{ij}\neq 0$ if $i<j$), or if all element under diagonal are zero.

Ex: $A=\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & 5 \\ 0 & 0 & 2 \end{bmatrix}$

8- **Lower triangular matrix: (مصفوفة مثلث سفلي)**

A square matrix $A=[a_{ij}]_{n\times n}$ echelon (مستوى) form (شكل) is Lower triangular (that is, $a_{ij}=0$ if $i<j$) or ($a_{ij}\neq 0$ if $i>j$), or if all element above diagonal are zero.

ex: $A=\begin{bmatrix} 1 & 0 & 0 \\ 2 & -3 & 0 \\ 6 & 3 & 2 \end{bmatrix}$

9- **Scalar matrix : المصفوفة الثابتة**

It is a diagonal matrix that all elements of diagonal are equal to real number. (When $a_{ij}=0$ for all $i \neq j$ and $a_{ii}=k$ for all $i=j$, where k is real number).

Ex: $A=\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}$

10-**Identity matrix (Unit matrix): (المصفوفة الاحادية) وحدة المصفوفة**

It is a diagonal matrix that all elements of diagonal are equal to number one.
Or it is a scalar matrix that all elements of diagonal are equal to number one.
(When $a_{ij}=0$ for all $i \neq j$ and $a_{ii}=1$ for all $i=j$). We write I_n if we need to emphasize (تأكد) that we are referring (الاحالة) to the (n×n) identity matrix.

Ex: $I_3=I_{3\times3}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$, $I_2=I_{2\times2}=\begin{bmatrix}1&0\\0&1\end{bmatrix}$, $I_1=I_{1\times1}=[1]$

Algebraic operations:العمليات الجبرية

1- **Addition of matrices:(جمع المصفوفات)**

If **$A=[a_{ij}]_{m\times n}$ and $B=[b_{ij}]_{m\times n}$ are two (m×n) matrices, their sum (A+B) is defined to be the matrix $[a_{ij}+b_{ij}]_{m\times n}$, where C=A+B**

$$[c_{ij}]_{m\times n}=[a_{ij}+b_{ij}]_{m\times n}$$

Notes: that this sum is defined only when the two matrices have the same number of rows and the same number of columns. The sum is formed by adding (يجمع) corresponding elements, and produces a matrix of the same size as A and B.

2- **Subtraction of matrices (طرح المصفوفات)**

If **$A=[a_{ij}]_{m\times n}$ and $B=[b_{ij}]_{m\times n}$ are two (m×n) matrices, their Subtraction (A-B) is defined to be the matrix $[a_{ij}-b_{ij}]_{m\times n}$, where**

A – B = A + (-B)

$$C=[a_{ij}-b_{ij}]_{m\times n}$$

Ex: If $A=\begin{bmatrix}3&-1\\2&5\end{bmatrix}$ *and* $B=\begin{bmatrix}-2&3\\1&-4\end{bmatrix}$ **find 1) A+B 2) A-B**

Solution:

1) A+B = $\begin{bmatrix}3&-1\\2&5\end{bmatrix}+\begin{bmatrix}-2&3\\1&-4\end{bmatrix}=$

2) A-B = $\begin{bmatrix}3&-1\\2&5\end{bmatrix}-\begin{bmatrix}-2&3\\1&-4\end{bmatrix}=$

The properties:

The properties of Addition of matrices:

Let it all of A, B and C are the matrices acceptable (ملائم) (suitable) for addition in size (m×n) then the law is correct:

1- **Commutative law(قانون التبديل)**

A + B = B + A

2- **Associative law: (قانون التجميع)**

(A + B) + C = A + (B + C)

3- **A + <u>0</u> = <u>0</u> + A = A**

4- **A + (-A) = -A + A = <u>0</u>**

5- **A + C = B + C** ***if*** **A=B**

6- ***If*** **A + C = B + C then A=B**

<u>Ex:</u> Let $A=\begin{bmatrix}1 & 4 & -2\\ 3 & 4 & 5\end{bmatrix}$, $B=\begin{bmatrix}-2 & 4 & 3\\ 2 & 5 & -1\end{bmatrix}$**. Find all the:**

1) **A + B=**

2) **A – B=**

3- <u>Multiplication of a matrix by a scalar:</u>

Let A=(a_{ij}) of a matrix of size (m×n) and (k) be a scalar number then:

k.A = A.k = (k a_{ij})

<u>Ex:</u> Let $A=\begin{bmatrix}2 & 3\\ -1 & 4\\ 7 & 0\end{bmatrix}$ **and k=3, then find k.A?**

<u>Solution:</u>

$$\mathbf{k.A} = 3\begin{bmatrix}2 & 3\\ -1 & 4\\ 7 & 0\end{bmatrix}=\begin{bmatrix}3(2) & 3(3)\\ 3(-1) & 3(4)\\ 3(7) & 3(0\_\end{bmatrix}=\begin{bmatrix}6 & 9\\ -3 & 12\\ 21 & 0\end{bmatrix}$$

<u>The law for Multiplication of a matrix by a scalar:</u>

Let A, B be the matrix for size (m×n) and α (الفا), β (بيتا) are scalar, then:

1- **1.A = A.1 = A**

2- **α (A+B) = αA + αB**

3- **(α + β) A = αA + βA**

4- **α (βA) = (α β)A**

5- **<u>0</u> .A = A. <u>0</u> = <u>0</u>**

4- <u>Multiplication of two matrices(ضرب المصفوفات)</u>

Let A=(a_{ij}) for size (m×n) and B=(b_{ij}) for size (n×p), then A.B is defined if and only if (n=n).

$\mathbf{A_{m\times n} . B_{n\times p} = C_{m\times p}}$

- ***<u>Notes</u>*****: This product is defined only when the number of columns of matrix A is equal to the number of rows of matrix B .**

<u>Lows of multiplication of matrices:</u>

Let A, B and C be any real (complex)(مركب) matrices, and let (α) be any real number. When all the following sums and products are defined, matrix multiplication satisfies (يرضى ، يكفي) the following properties:

1- **(A.B)C = A(B.C)**

2- **A(B+C) = A.B + A.C**

3- (A+B)C = A.C + B.C

4- α (A.B) = (αA)B = A(αB)

5- $A.B \neq B.A$ (بشکل عام) به شیوه یەکی گشتی **In general**

6- **If A.B = 0 does not necessarily** (بالضرورة) **imply** (يدل على) **that** $A \neq 0\ ,\ B \neq 0$.

7- **If A.B = A.C does not necessarily** (بالضرورة) **imply** (يدل على) **that B = C.**

8- **<u>Power of the matrices رفع المصفوفات:</u>**

Let A is a Square matrix and (r) is positive real number then:

1- $A^r = A.A. \dots A$

2- $(A.B)^r = A^r.B^r$ **if A.B=B.A**

3- **In general** $(A \pm B)^2 \neq A^2 \pm 2A.B + B^2$

4- **If and only if A.B=B.A then** $(A \pm B)^2 = A^2 \pm 2A.B + B^2$

5- **If and only if A.B=B.A then** $A^2 - B^2 = (A - B)\ (A + B)$

<u>Ex:</u> let $A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 3 \\ 2 & 1 & 2 \end{bmatrix}\ ,\ B = \begin{bmatrix} 1 & -1 \\ 3 & 0 \\ 2 & 2 \end{bmatrix}$ **find all:**

1- **A.B =** $\begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 3 \\ 2 & 1 & 2 \end{bmatrix} . \begin{bmatrix} 1 & -1 \\ 3 & 0 \\ 2 & 2 \end{bmatrix} =$

2- $A^2 =$ $\begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 3 \\ 2 & 1 & 2 \end{bmatrix} . \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 3 \\ 2 & 1 & 2 \end{bmatrix} =$

<u>Ex:</u> let $A = \begin{bmatrix} -4 & 0 \\ 2 & 0 \end{bmatrix}\ ,\ B = \begin{bmatrix} 6 & -1 \\ 5 & 2 \end{bmatrix}\ ,\ C = \begin{bmatrix} 6 & -1 \\ 4 & 9 \end{bmatrix}$ **Show that A.B=A.C**

<u>Solution:</u>

A.B = A.C

$$\begin{bmatrix} -4 & 0 \\ 2 & 0 \end{bmatrix} . \begin{bmatrix} 6 & -1 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 2 & 0 \end{bmatrix} . \begin{bmatrix} 6 & -1 \\ 4 & 9 \end{bmatrix}$$

<u>H.W:</u>

<u>Ex:</u> 1- let $A = \begin{bmatrix} 3 & 0 \\ 7 & 0 \end{bmatrix}\ ,\ B = \begin{bmatrix} 0 & 0 \\ -2 & 5 \end{bmatrix}$ **Show that A.B = <u>0</u>**

<u>Solution:</u>

A.B = $\begin{bmatrix} 3 & 0 \\ 7 & 0 \end{bmatrix} . \begin{bmatrix} 0 & 0 \\ -2 & 5 \end{bmatrix} =$

Ex: 2-If A.B = B.A show that $(A.B)^4 = A^4.B^4$

Solution:

$(A.B)^4$ = (A.B) (A.B) (A.B) (A.B)

= A.B A.B A.B A.B , ∵ A.B = B.A

= A.A.B.B.A.B.A.B , ∵ A.B = B.A

= A.A.B.A.B.B.A.B , 〃

= A.AA.B.B.B.A.B , 〃

= A.AA.B.B.A.B.B , 〃

= A.AA.B.A.B.B.B , 〃

= A.AA.A.B.B.B.B , 〃

Some Type of matrices: (بعض أنواع المصفوفات)

1- **Idempotent matrix:**(المصفوفة الصماء)

If A= $[a_{ij}]_{n\times n}$ we called A is idempotent matrix if ($A^2 = A$)

Ex: Let $A=\begin{bmatrix} \frac{4}{5} & \frac{-2}{5} \\ \frac{-2}{5} & \frac{1}{5} \end{bmatrix}$ **show that A is idempotent matrix.**

Solution: $A^2 = A$

$$A^2=A.A=\begin{bmatrix} \frac{4}{5} & \frac{-2}{5} \\ \frac{-2}{5} & \frac{1}{5} \end{bmatrix} . \begin{bmatrix} \frac{4}{5} & \frac{-2}{5} \\ \frac{-2}{5} & \frac{1}{5} \end{bmatrix} =$$

2- **Nilpotent matrix:(مصفوفة معدومة القوى)**

We called for A=$[a_{ij}]_{n\times n}$ Nilpotent matrix when: ($A^2 = 0$)

Ex: Let $A=\begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix}$ **show that A is nilpotent matrix.**

Solution: $A^2 = 0$

$$A^2=A.A=\begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix} . \begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix} =$$

3- **Involutary matrix:(المصفوفة الملتفية (الاحادية)**

We say for matrix A=$[a_{ij}]_{n\times n}$ Involutary matrix if ($A^2 = I$)

$A.A^{-1} = A^{-1}.A=I$ ملاحظة: أن المصفوفة الملتفية هي معكوس لنفسها أي:

Ex: Let $A=\begin{bmatrix} -3 & -6 & 2 \\ 2 & 4 & -1 \\ 2 & 3 & 0 \end{bmatrix}$ **Is A Involutary matrix or not?**

Solution: $A^2 = I$

s $A^2=A.A=\begin{bmatrix} -3 & -6 & 2 \\ 2 & 4 & -1 \\ 2 & 3 & 0 \end{bmatrix} . \begin{bmatrix} -3 & -6 & 2 \\ 2 & 4 & -1 \\ 2 & 3 & 0 \end{bmatrix} =$

The properties of square matrix, diagonal matrix and Identity matrix:

1- **Let A is a matrix for any number (size):**
   A.I = I.A = A that is:
   a) If A= $[a_{ij}]_{n\times n}$ then $A. I_n= I_n . A = A$
   b) If A= $[a_{ij}]_{m\times n}$ then $I_m . A = A . I_n = A$
2- $I^r = I . I . I \ldots I = I$
3- **If A= $[a_{ij}]_{n\times n}$ and S is a scalar matrix for the same size then A . S= S . A**
4- **If A= $[a_{ij}]_{n\times n}$ and D is a diagonal matrix not scalar matrix for the same size then $A . D \neq D . A$**
5- **If A and B are the diagonal matrix then: A .B = B .A= diagonal matrix.**

Ex1: **If** $A=\begin{bmatrix} 5 & 4 & -2 \\ 4 & 5 & -2 \\ -2 & -2 & 2 \end{bmatrix}$, **Show that** $A^2 - 11A + 10I = \underline{0}$

Solution: $A^2 - 11A + 10\,I = \begin{bmatrix} 5 & 4 & -2 \\ 4 & 5 & -2 \\ -2 & -2 & 2 \end{bmatrix} . \begin{bmatrix} 5 & 4 & -2 \\ 4 & 5 & -2 \\ -2 & -2 & 2 \end{bmatrix} -11 \begin{bmatrix} 5 & 4 & -2 \\ 4 & 5 & -2 \\ -2 & -2 & 2 \end{bmatrix} +10 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

=

Ex2: **Let** $A=\begin{bmatrix} 6 & 0 \\ 0 & -2 \end{bmatrix}$ **and** $B=\begin{bmatrix} x & y \\ p & z \end{bmatrix}$, **if A.B=B.A= I_2. Find the elements of matrix B.**

Solution: $A.B = I_2$

$\begin{bmatrix} 6 & 0 \\ 0 & -2 \end{bmatrix} . \begin{bmatrix} x & y \\ p & z \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Ex: **Find the value of (x) if** $\begin{bmatrix} x & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = \underline{0}$

4- **Anti – commute matrix:(تبديلتان عكسياً)**

We called for A , B matrix Anti – commute matrix if (A.B = -B.A)

Ex: **Let** $A=\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $B=\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ **show that A.B is Anti – commute matrix.**

Solution: **A.B = -B. A**

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}=-\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

5- **Trace of matrix (أثر المصفوفة):**

هو مجموع العناصر القطر الرئيسي في المصفوفة المربعة.

Is a sum for element main diagonal in square matrix. When:

If A= $[a_{ij}]_{n\times n}$ **then:**

$tr(A)=a_{11} + a_{22} + a_{33} + \dots + a_{nn}$

$$= \sum_{i=1}^{n} a_{ii}$$

The properties of trace matrix:

Let A, B are two matrix for size n then:

1- **tr(A + B) = tr(A) + tr(B)**
2- **If A=** $(a_{ij})_{m\times n}$ **and B=** $(b_{ij})_{n\times m}$ **then:**
   a) **tr(AB) = tr(BA)**
   b) **tr(AB) ≠ tr(A). tr(B) In general.**
3- **For all A, B and C are diagonal matrix then:**
   tr(ABC) = tr(ACB) = tr(CAB) = tr(CBA) = tr(BAC) = tr(BCA)

Ex: **Find the trace for matrix B if** $B=\begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 0 \\ 0 & 4 & 6 \end{bmatrix}$

Solution:

tr(B)= $\sum_{i=1}^{n} a_{ii} = \sum_{i=1}^{3} a_{ii} = a_{11} + a_{22} + a_{33}$ **= 1+(-1) + 6 = 6**

Ex: **Find the trace for matrix C if** $C=\begin{bmatrix} -1 & 2 & 3 & 5 \\ 2 & 2 & 4 & 3 \\ -3 & 4 & 2 & 0 \\ 1 & 3 & -2 & 5 \end{bmatrix}$

Solution:

tr(C)=

Chapter two//

Partitioning of matrix and partition algebraic processes:تجزئة المصفوفات وعمليات الجبرية بطريقة التجزئة

- **Partitioning of matrixتجزئة المصفوفات:**

  It could be distinction(تفريق، فصل) any matrix to the small partitions named with (sub matrices), by doing vertical and horizontal lines among the matrix columns and rows it capital symbolized (يرمز الى) to the sub matrix with capital letters first one for rows and the second for columns.

يمكن تجزئة أي مصفوفة بأمرار خطوط افقية و عمودية بين صفوف و أعمدة المصفوفة فتقسم الى أجزاء تسمى مصفوفات جزئية(sub matrices) ويرمز لها بحروف كبيرة مؤشرة بمؤشريين الاول لصفوف و الثاني للأعمدة فتكون (A_{ij} , B_{ij} , C_{ij} , …)

Ex: $A=\begin{bmatrix} 5 & 3 & -2 & 2 \\ 0 & 7 & 3 & 2 \\ -2 & -1 & 0 & 6 \end{bmatrix}$

- **Addition of matrices by partition:(جمع المصفوفة بالتجزئة)**

ملاحظة: يتم جمع المصفوفتين إذا كان من نفس الدرجة ومجزئة بنفس الشكل.

دوو ڕیزکراوه کۆدەکرێنەوە ئەگەر بێت و هەمان قەبارەیان هەبێت وە وەك یەکتر بەش کرابن.

Let A= ((a$_{ij}$)) and B= ((b$_{ij}$)) of the order (m×n). and let A is partition by:

$$A=\begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1r} \\ A_{21} & A_{22} & \cdots & A_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{sr} \end{bmatrix}$$

and the order of each sub matrices is: $\begin{bmatrix} m_1\times n_1 & m_1\times n_2 & \cdots & m_1\times n_r \\ m_2\times n_1 & m_2\times n_2 & \cdots & m_2\times n_r \\ \vdots & \vdots & \ddots & \vdots \\ m_s\times n_1 & m_s\times n_2 & \cdots & m_s\times n_r \end{bmatrix}$

and let B is partition by:

$$B=\begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1r} \\ B_{21} & B_{22} & \cdots & B_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ B_{s1} & B_{s2} & \cdots & B_{sr} \end{bmatrix}$$

and the order of each sub matrices is: $\begin{bmatrix} m_1\times n_1 & m_1\times n_2 & \cdots & m_1\times n_r \\ m_2\times n_1 & m_2\times n_2 & \cdots & m_2\times n_r \\ \vdots & \vdots & \ddots & \vdots \\ m_s\times n_1 & m_s\times n_2 & \cdots & m_s\times n_r \end{bmatrix}$

then A + B= $\begin{bmatrix} A_{11}+B_{11} & A_{12}+B_{12} & \cdots & A_{1r}+B_{1r} \\ A_{21}+B_{21} & A_{22}+B_{22} & \cdots & A_{2r}+B_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1}+B_{s1} & A_{s2}+B_{s2} & \cdots & A_{sr}+B_{sr} \end{bmatrix}$ **of size (m×n)**

<u>Ex:</u> Let $A=\begin{bmatrix}4 & 3 & -3 & 2\\ 1 & -2 & 0 & 5\\ 4 & 3 & 2 & 8\\ -1 & 3 & 2 & 5\end{bmatrix}$ **and** $B=\begin{bmatrix}2 & -1 & 2 & 4\\ 6 & -2 & 3 & 1\\ 3 & 2 & 0 & -1\\ -4 & 2 & -1 & 4\end{bmatrix}$ **find (A +B) by partition way.**

<u>Solution:</u>

A+B= $\begin{bmatrix}4 & 3 & -3 & 2\\ 1 & -2 & 0 & 5\\ 4 & 3 & 2 & 8\\ -1 & 3 & 2 & 5\end{bmatrix}+\begin{bmatrix}2 & -1 & 2 & 4\\ 6 & -2 & 3 & 1\\ 3 & 2 & 0 & -1\\ -4 & 2 & -1 & 4\end{bmatrix}$

- **<u>Multiplication matrices by partition:(ضرب المصفوفة بالتجزئة)</u>**

ملاحظة: يتم ضرب المصفوفتين إذا كان عدد الاعمدة في المصفوفة الاولى مساوٍ,, الى عدد الصفوف في المصفوفة الثانية.

Let A=(a_{ij}) for size (m×p) and B=(b_{ij}) for size (p×n), then A.B is defined if and only if, and matrix A is partined by:

$$A=\begin{bmatrix}A_{11} & A_{12} & \dots & A_{1r}\\ A_{21} & A_{22} & \cdots & A_{2r}\\ \vdots & \vdots & \ddots & \vdots\\ A_{s1} & A_{s2} & \cdots & A_{sr}\end{bmatrix}=\begin{bmatrix}m_1\times p_1 & m_1\times p_2 & \dots & m_1\times p_r\\ m_2\times p_1 & m_2\times p_2 & \cdots & m_2\times p_r\\ \vdots & \vdots & \ddots & \vdots\\ m_s\times p_1 & m_s\times p_2 & \cdots & m_s\times p_r\end{bmatrix}$$

and matrix B is partined by:

$$B=\begin{bmatrix}B_{11} & B_{12} & \dots & B_{1k}\\ B_{21} & B_{22} & \cdots & B_{2k}\\ \vdots & \vdots & \ddots & \vdots\\ B_{r1} & B_{r2} & \cdots & B_{rk}\end{bmatrix}=\begin{bmatrix}p_1\times n_1 & p_1\times n_2 & \dots & p_1\times n_k\\ p_2\times n_1 & p_2\times n_2 & \cdots & p_2\times n_k\\ \vdots & \vdots & \ddots & \vdots\\ p_s\times n_1 & p_s\times n_2 & \cdots & p_s\times n_k\end{bmatrix}$$

ملاحظة: لكي يتم ضرب المصفوفة يجب أن تكون الخطوط العمودية للمصفوفة الاولى مساوية للخطوط الافقية للمصفوفه الثانية. فإن $A\times B$ يكون بشكل التالي:

$$A\,B=\begin{bmatrix}A_{11}B_{11}+A_{12}B_{21}+\dots+A_{1r}B_{r1}, & \dots, A_{11}B_{1k}+A_{12}B_{2k}+ & \dots & +A_{1r}B_{rk}\\ A_{21}B_{11}+A_{22}B_{21}+\dots+A_{2r}B_{r1}, & \dots, A_{21}B_{1k}+A_{22}B_{2k}+ & \cdots & +A_{2r}B_{rk}\\ \vdots & \vdots & \ddots & \vdots\\ A_{s1}B_{11}+A_{s2}B_{21}+\dots+A_{sr}B_{r1}, & \dots, A_{s1}B_{1k}+A_{s2}B_{2k}+ & \cdots & A_{sr}B_{rk}\end{bmatrix}$$

<u>Or:</u>

$$A\times B=\begin{bmatrix}a & \vdots & b\\ \cdots & \vdots & \cdots\\ c & \vdots & d\end{bmatrix}\begin{bmatrix}e & \vdots & f\\ \cdots & \vdots & \cdots\\ g & \vdots & h\end{bmatrix}=\begin{bmatrix}ae+bg & \vdots & af+bh\\ \cdots & \vdots & \cdots\\ ce+dg & \vdots & cf+dh\end{bmatrix}$$

Ex: Let $A=\begin{bmatrix}2 & 1 & 0\\3 & 2 & 0\\1 & 0 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 1 & 1 & 0\\2 & 1 & 1 & 0\\2 & 3 & 1 & 2\end{bmatrix}$ find (A×B) **by partition way.**

Solution:

$$A=\begin{bmatrix}A_{11} & A_{12}\\A_{21} & A_{22}\end{bmatrix}=\begin{bmatrix}2 & 1 & \vdots & 0\\3 & 2 & \vdots & 0\\\cdots & \cdots & \cdots & \cdots\\1 & 0 & \vdots & 1\end{bmatrix} \qquad B=\begin{bmatrix}B_{11} & B_{12}\\B_{21} & B_{22}\end{bmatrix}=\begin{bmatrix}1 & 1 & 1 & \vdots & 0\\2 & 1 & 1 & \vdots & 0\\\cdots & \cdots & \cdots & \cdots & \cdots\\2 & 3 & 1 & \vdots & 2\end{bmatrix}$$

$$A.B=\begin{bmatrix}A_{11} & A_{12}\\A_{21} & A_{22}\end{bmatrix}\begin{bmatrix}B_{11} & B_{12}\\B_{21} & B_{22}\end{bmatrix}=\begin{bmatrix}A_{11}B_{11}+A_{12}B_{21} & A_{11}B_{12}+A_{12}B_{22}\\A_{21}B_{11}+A_{22}B_{21} & A_{21}B_{12}+A_{22}B_{22}\end{bmatrix}$$

Ex: Let $A=\begin{bmatrix}3 & 1 & 2 & 0\\-1 & 2 & -2 & 1\\2 & 1 & 3 & -1\\2 & 1 & -1 & 2\end{bmatrix}$ and $B=\begin{bmatrix}2 & 1 & 3\\1 & -1 & 0\\2 & 3 & 1\\0 & -1 & 1\end{bmatrix}$ find (A×B) **by partition way.**

Solution:

Chapter three // Same type of matrices

1-The transpose of matrix

The transpose, A^T , of an $(m \times n)$ matrix A is the $(n \times m)$ matrix obtained by interchanging the rows and columns of A, that is, if:

$$A = \left[a_{ij}\right]_{m\times n} \quad then$$

$$A^T = A' = \left[a_{ji}\right]_{n\times m}$$

Ex: 1- If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then $A' = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

2-If $B = \begin{bmatrix} 1 & 2 \end{bmatrix}$ then $B' = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

3-If $C = \begin{bmatrix} 2 & 1 & -3 \\ 1 & 4 & 6 \end{bmatrix}$ then $C' = \begin{bmatrix} 2 & 1 \\ 1 & 4 \\ -3 & 6 \end{bmatrix}$

The properties of the transpose of matrix:

Let A and B be two matrices suitable for adding and multiplying, and α be constant then:

1- $(A')' = A$

2- $(\alpha A)' = \alpha A'$

3- $(A+B)' = A' + B'$

4- $(AB)' = B'A'$

$\neq A'B'$

Ex: If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and α=4 show that: 1) $(A')' = A$ 2) $(\alpha A)' = \alpha A'$

Solution:

1) $(A')' = A$

2) $(\alpha A)' = \alpha A'$

Ex: If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -2 & 3 \\ 3 & -1 & -3 \end{bmatrix}$ show that $(A+B)' = A' + B'$

Solution:

Ex: If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ 3 & -2 \\ 1 & -1 \end{bmatrix}$ show that $(AB)' = B'A'$

Ex: If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} -2 \\ 3 \\ 4 \end{bmatrix}$ show that $(AB)' = B'A'$

2-The symmetric of matrix:

A square matrix A is said to be symmetric if and only if:

$A' = A$ where $a_{ij} = a_{ji}$ $\forall\, i \text{ and } j$

Ex: Is $A = \begin{bmatrix} 5 & -2 \\ -2 & 3 \end{bmatrix}$ a symmetric matrix or not?

Solution: $A' = A$

ملاحظة// تكون المصفوفة متماثلة اذا كان:

1- في المصفوفة المربعة عناصر أعلى القطر وأسفل القطر متساوية مع بعضها.

2- إذا كانت المصفوفة قطرية.

ڕیزکراوە (symmetric) دەبێت ئەگەر بێت و:

1- لە ڕیزکراوەی چوار گۆشەدا (element)ی سەرەوەی چەق و خوارەوەی چەق یەکسان بن.

2- ئەگەر ڕیزکراوەکە ڕیزکراوەی چەق بێت.

Ex:

$$B = \begin{bmatrix} 0 & -2 & 4 \\ -2 & 5 & -3 \\ 4 & -3 & 9 \end{bmatrix} \quad , \quad C = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3-Skew symmetric of matrix:المصفوفة الملتوية التماثل (متماثلة تخالفية)

A square matrix A is said to be skew symmetric matrix if and only if:

$A' = -A$ where $a_{ij} = -a_{ji}$ $\forall\, i \text{ and } j$

ملاحظة// هناك شرطان لمعرفة المصفوفة ملتوية التماثل:

1- عناصر القطر الرئيسي اصفاراً.

2- تكون عناصر أعلى القطر الرئيسي نفس عناصر أسفل القطر الرئيسي لكن بعكس الاشارة.

// دوو مەرج هەیە بۆ ئەوەی ڕیزکراوە (Skew symmetric) بێت:

1- (element)ی چەق هەمووی صفر بێت.

2-(element)ی سەرەوەی چەق هەمان (element)ی خوارەوەی چەق بێت بەڵام بە پێچەوانەی نیشانە.

Ex: Is this matrix symmetric matrix or Skew symmetric matrix:

1- $A = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & 4 \\ -3 & -4 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 5 & -4 \\ -5 & 0 & 2 \\ 4 & -2 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} -2 & 8 \\ -8 & 0 \end{bmatrix}$

Solution:

Theorems of the Symmetric and Skew symmetric matrix:

1- For any square matrix A:

a) $(A+A')'=A'+A$ symmetric.

b) $(A-A')'=-(A-A')$ skew symmetric.

2- For all matrix A : $A.A'$ *and* $A'.A$ symmetric matrix.

3- If A and B are symmetric matrix then:

a) $(\alpha A)'=\alpha A$

b) $(A+B)'=A+B$

c) If AB =BA then $(AB)'=A.B$

4- If A and B skew symmetric matrix then:

a) $(\alpha A)'=-\alpha A$

b) $(A+B)'=-(A+B)$

c) $(AB)'=A.B$ if A.B=B.A

5- If $A'=A$, $B'=-B$ and A.B=B.A then A.B skew symmetric matrix.

The complex numbers ژمارەی ئاوێته (المعقدة) الاعداد المركبة :

If **a** , b are real number, then a+ib is named **complex number ,** where **(** $i=\sqrt{-1}$ **).** Then **a** is named ***real part(الجزء الحقيقي)*** of complex number and **ib** is named ***imaginary part (الجزء الخيالي)*** of complex number.

حقيقي خيالي

Z= a + ib

Ex: Z= 3+ 2i , r = 3i , q = -1+4i

ملاحظة:

1- جميع الاعداد الحقيقية هي أعداد مركبة فيها الجزء الخيالي مساوٍ للصفر أي أن (b) يساوي صفر(b=0).

2- جميع الاعداد الخيالية هي أعداد مركبة فيها الجزء الحقيقي مساوياً بالصفر يعني (a=0).

Algebric operation of the complex number العمليات الجبرية للاعداد المركبة

Let $Z_1 = a + bi$

$Z_2 = c + di$

1) **Addition of the two complex number جمع الاعداد المركبة :**

$Z_1 + Z_2 = (a + bi) + (c + di)$

$= (a + c) + (bi + di)$

$= (a + c) + (b + d)i$

2) **Subtraction ofthe two complex number طرح الاعداد المركبة :**

$Z_1 - Z_2 = (a + bi) - (c + di)$

$= (a - c) + (bi - di)$

$= (a - c) + (b - d)i$

3) **Multiplication of the two complex number ضرب الاعداد المركبة :**

$Z_1 . Z_2 = (a + bi) . (c + di)$

$= (a.c - bd) + (cb + a d)i$

Ex: Let $Z_1 = 4 + 3i$ and $Z_2 = 5i – 2$ find

1) $Z_1 + Z_2$ 2) $Z_1 - Z_2$ 3) $Z_1 . Z_2$

Solution:

1) $Z_1 + Z_2 = (4 + 3i) + (5i – 2) =$

2) $Z_1 - Z_2 = (4 + 3i) - (5i – 2) =$

3) $Z_1 . Z_2 = (4 + 3i) .(5i – 2) =$

4) **Multipl the complex number by real number ضرب الاعداد المركبة بعدد حقيقي :**

Let Z= a + bi and *k* is real number then:

$k.Z = k(a+bi)$

$= ka + kbi$

Ex: Let Z = 3 +6i and k=3 find k.Z

Solution:

k.Z= 3(3 +6i)=

The conjugate of the complex number للاعداد المركبة (ئاوهڵ)مرافق:

If Z= a + bi is complex number, then conjugate of Z is complex number (a-bi) and is symbol(يرمز) $\overline{Z}$.

$Z = a + bi$

$\overline{Z} = a – bi$

Ex: If Z= 5+3i find conjugate of Z.

Solution:

$\overline{Z} = \overline{5 + 3i} = 5 - 3i$

Theorem of conjugate complex number:

Let Z_1 and Z_2 are two complex number, then:

1) $\overline{\overline{Z_1}} = Z_1$

2) $\overline{Z_1 + Z_2} = \overline{Z_1} + \overline{Z_2}$

3) $\overline{Z_1 . Z_2} = \overline{Z_1} . \overline{Z_2}$

4) $Z = \overline{Z}$, if Z is real number

5) $-Z = \overline{Z}$, if and only if Z=0 or Z ***is imaginary number.***

Complex matrix المصفوفة المعقدة

It is the matrix that contain element in the complex number.

Ex: $A=\begin{bmatrix} 4+5i & 3-2i \\ 1+2i & 5-3i \end{bmatrix}$ $B=\begin{bmatrix} 4 & 3 \\ 1 & 5 \end{bmatrix}$ $C=\begin{bmatrix} 3i & 2i \\ -2i & 6i \end{bmatrix}$

المصفوفة المعقدة: هي المصفوفة التي تحتوي عناصرها على شكل أعداد المعقدة.

ڕیزکراوەی ئاوێتەیی: ئەو ڕیزکراوەیە کە (element)ەکاني بریتیە لە ژمارەی ئاوێتە.

The conjugate of a matrix مرافقة المصفوفة

A matrix $A=((a_{ij}))$ of order (m×n) its named the conjugate matrix and symbolized by $\overline{A}=((\overline{a}_{ij}))$ of order (m×n).

Ex: 1) Let $A=\begin{bmatrix} 4+5i & 3-2i \\ 1+2i & 5-3i \end{bmatrix}$ then the conjugate of A is

$$\overline{A}=\overline{\begin{bmatrix} 4+5i & 3-2i \\ 1+2i & 5-3i \end{bmatrix}}=\begin{bmatrix} 4-5i & 3+2i \\ 1-2i & 5+3i \end{bmatrix}$$

Theorem of The conjugate of a matrix:

If $\overline{A}$, $\overline{B}$ are conjugate of matrix A , B matrix and α is constant. Then:

1- $\overline{\overline{A}}=A$

2- $\overline{(\alpha A)}=\alpha.\overline{A}$

3- $\overline{(A+B)}=\overline{A}+\overline{B}$

4- $\overline{(A.B)}=\overline{A}.\overline{B}$

5- $\overline{(A')}=(\overline{A})'$

Ex: Let $A=\begin{bmatrix} 5-3i & 2-i \\ 3+2i & 2-3i \end{bmatrix}$, $B=\begin{bmatrix} 1-3i & i \\ 5 & 2-3i \end{bmatrix}$ and α=2, find:

1- $\overline{\overline{A}}=A$

2- $\overline{(\alpha A)}=\alpha.\overline{A}$

3- $\overline{(A+B)}=\overline{A}+\overline{B}$

4- $\overline{(A.B)}=\overline{A}.\overline{B}$

5- $\overline{(A')}=(\overline{A})'$

The tranjugate of matrix:(مبدلة المرافقة(مرافقة لمبدلة المصفوفة

A matrix $A=((a_{ij}))$ of order (m×n) its named the tranjugate of A and symbolized by A^*,

$$A^* = (\overline{A})^T = (\overline{A})'$$
$$= \overline{(A')}$$

Ex: If $A=\begin{bmatrix} 2-3i & 3+i \\ 2+2i & 2-i \end{bmatrix}$ find A^*.

Solution:

$$A^* = (\overline{A})^T = \overline{(A')} = (\overline{\begin{bmatrix} 2-3i & 3+i \\ 2+2i & 2-i \end{bmatrix}})' =$$

The properties of tranjugate of matrix:

1- $(A^*)^* = A$

2- $(kA)^* = k\ A^*$

3- $(A+B)^* = A^* + B^*$

4- $(A.B)^* = B^*.A^*$

Hermitian matrices المصفوفات الهيرمتية:

The square matrix $A=((a_{ij}))_{n\times n}$ is Hermitian matrix if:

$A^* = A$ where $A^* = (\overline{A})^T \overset{or}{=} \overline{(A')}$ where $a_{ij}=a_{ji}$ $\forall\, i$ *and* j

ملاحظة: المصفوفه تكون هيرمتيه إذا كان القطر يقسمها الى مثلثين أحداهما مرافق الاخر، وعناصر القطر الرئيسي أعداد حقيقية فقط.

تێبيني: ڕيزكراوة هيرمتين دەبێت ئەگەر بێت و چەق دابەشی بكات بۆ دوو سێگۆشة يەكێكيان ئاوەڵی ئەوی تر بێت، وە ئێليمێنتي چەق تەنها ژمارەی ڕاستی بێت.

Ex: If $A=\begin{bmatrix} 6 & 5+i \\ 5-i & -5 \end{bmatrix}$ is A Hermitian matrix or not?

Solution: $A^* = A$

Skew Hermitian matrices المصفوفات الهيرمتية الملتوية:

The square matrix $A=((a_{ij}))_{n\times n}$ is skew hermitian matrix if:

$A^* = -A$ where $A^* = (\overline{A})^T \overset{or}{=} \overline{(A')}$ where $a_{ij}=-a_{ji}$ $\forall\, i$ *and* j

ملاحظة: لكل مصفوفه هيرمتيه ملتوية يجب أن يكون:

1- عناصر القطر عبارة عن أصفار أو أعداد خيالية.

2- أن تكون عناصر المثلث العلوي الجزء الحقيقي فيها تساوي نفس العناصر المثلث السفلي ولكن بعكس الاشارة.

Ex: If $B=\begin{bmatrix} 0 & -3+5i \\ 3+5i & 3i \end{bmatrix}$ is B skew hermitian matrix or not?

Solution: $A^* = -A$

The properties of H. matrix and skew H. matrix:

1- For any matrix $A = ((a_{ij}))_{n \times n}$, then:

a) $A + A^*$ Herm.

b) $A - A^*$ Skew Herm.

2- For any matrix A then:

a) $A.A^*$ Herm.

b) $A^*.A$ Herm.

3- If A and B herm. matrix, then:

a) k.A Herm. If k≠0 real number.

b) k.A Skew Herm. If k ***imaginary*** number.

c) A.B Herm if AB=BA

d) A+B Herm.

4- If A and B Skew herm. matrix, then:

a) k.A Skew Herm. If k≠0 real number.

b) k.A Herm. If k ***imaginary*** number.

c) $(A+B)^* = -(A+B)$ Skew Herm.

d) A.B Herm. if AB=BA

5- If A Herm. and B Skew Herm. and AB=BA, then AB Skew Herm.matrix.