Department of …Statistics and Informatics

College of ………Adm. & Eco………………….

University of …SALAHADDIN UNIVERSITY- HAWLER….

Subject: …… Linear Algebra With Application….

Course Book –(Year 2)

Lecturer's name : Kawthar Saeed Taha (MSc)

Academic Year: 2022/2023

Course Book

1. Course name	**Linear Algebra with Application**
2. Lecturer in charge	**Kawthar Saeed Taha**
3. Department/ College	**Statistics &Information / Adm. & Eco**
4. Contact	**e-mail: http//gmail.com/side/Kawthar** **Tel: (07504703371)**
5. Time (in hours) per week	**For example Theory: 3**
6. Office hours	**Availability of the lecturer to the student during the week**
7. Course code	**SAE208**
8. Teacher's academic profile	**From 1998 until 2003 worked as Assistant Lecture in Statistics Department – Salahaddin University, I have taught the following subject with some MSc. : Statistics, Biostatistics, Analysis of Regression, Mathematical Statistics, Differential Equation. In 2006 I had my MSc. In Statistics from the same university. From 2006 until 2012 worked decision maker at Statistics Department. From 2006 till now I am working as a lecturer in Statistics Department Salahaddin University. I have taught the following subject: Statistics, Word and Windows, linear algebra, Biostatistics, Statgraphics.**
9. Keywords	**Elementary of Linear algebra, Definition of Linear algebra, Linear equation, Linear systems, Substitution method, Elimination method, Matrix method (Invers method), Liner equations if A is square matrix, Elementary transformations and Equivalent matrices, Rank of matrix, Elementary transformations, Equivalent Matrix, Canonical Form , Normal Form , Inverse of matrix by used elementary transformations, Liner equations if A is non-square matrix, Orthogonal Matrix, Orthonormal Matrix, Eigenvalues, Eigenvectors and Characteristic Equation of a matrix, some application in linear algebra**

10. Course overview:

The general purpose of this course is to study the basic concepts of this course Linear algebra is divided into five parts. The first part deals with liner Algebra and the way to solve system of the linear equations, the second part deals with Elementary transformations and Equivalent matrices, and the third part deals with the Orthogonal Matrix, Orthonormal_Matrix, and the four part deals with the Eigenvectors and Characteristic Equation of a matrix , and the five part deals with some application in linear algebra …

11. **Course objective:**

Linear algebra is concerned with finite dimensional vector spaces. Solving systems of linear equations is one of the most important applications of linear algebra. It has been argued that the majority of all mathematical problems encountered in scientific and industrial applications involve solving system at some point. Linear applications arise in such diverse areas as engineering, chemistry, economics, business, ecology, biology and psychology. One of the early goals of this course is to develop algorithm that helps solve larger systems in an orderly manner.

12. **Student's obligation**

The student should attend the class at the exact time and place determined. The student should keep his mobile closed and lessen well to all lectures he/she should never loss concentration in the class neither occupying him with any not necessary things. Other important steps they should attend all the exams the exact time and place determined bringing all the requirements like calculator, pen and papers.

13. Forms of teaching

For giving the lecture I will use the Data show and the white board and sometimes I will use the prepared lectures with Data show and white board all together.

I am the only responsible lecturer who gives this subject without the help of any other teaching members.

14. Assessment scheme

The students are obliged to perform at least two closed book exams during the academic year. Quizzes (5%), the exam has 30% besides homework and classroom activities (5%) , and (60%)will be reserved for the final exam.

15. Student learning outcome:

At the end of this course, students are expected to be confidence from analyzing the relationships between all factors that related together in the reality. They will be able to formulate the modeling the relation and distinguish the type of relation and analyzing with interpreting the consequences after that make decisions.

The students should have the ability to work in both public and private sectors as having good skills in analyzing.

16. Course Reading List and References:

1- Strang, G., 1980, Linear algebra and it is application, 2nd edition, Academic Press, New York.

2- S.J. Leon, Linear algebra with applications, Prentice Hall, 6th Edition, 2002.

3- G.H.Golub and C.F.Vantamn. Matrix and application, John Hopkins Univ. Press, 3rd

Ed. Baltimore, 1996.
4- Larson R., C. Falvo D.C. Elementary Linear algebra 6th Edition, Houghton Mifflin Harcourt Publishing Company, New York,2009.

1- الناصر، عبد المجيد حمزة ، جواد، لميعة باقر، الجبر الخطي، تموز 1988.

2-العلي،ابراهيم محمد، أسس التحليل الاحصائي متعدد المتغيرات، 2020.

17. The Topics:	**Lecturer's name**
Definition of Linear algebra, Definition of Linear algebra, Linear equation, Linear systems, Substitution method, Elimination method, Matrix method (Invers method), Liner equations if A is square matrix, Elementary transformations and Equivalent matrices, Rank of matrix, Elementary transformations , Equivalent Matrix, Canonical Form, Normal Form, Inverse of matrix by used elementary transformations, Liner equations if A is non-square matrix, Orthogonal Matrix, Orthonormal Matrix, Eigenvalues, Eigenvectors and Characteristic Equation of a matrix, some application in linear algebra.	**Kawthar Saeed Taha** **ex: three hours a week**

18. Practical Topics (If there is any)	
No any one	**Kawthar Saeed Taha** **ex: Three hours a week**

19. Examinations:

1. Compositional:

<u>*Ex1*</u>: Solve these systems of the linear equations using Substitution method:

$$x - 3y = -3 \quad \cdots 1$$
$$2x + y = 8 \quad \cdots 2$$

Solution:

$x \ -3y=-3$

$x=-3+3y \ \dots \ 3$

$by\, sub\ 3 in 2$

$2x \ + y=8$

$2(-3+3y)+y=8$

$-6+6y+y=8$

$7y=8+6$

$7y=14 \ \Rightarrow y=2$

$by\, sub\ y\ in 3$

$x=-3+3y$

$x=-3+3(2)$

$x=-3+6 \ \Rightarrow \ x=3$

Ex: Let $A=\begin{bmatrix} 5 & -1 & 0 \\ 5 & -1 & 4 \end{bmatrix}$ find rank(A)?

Solution:

$$|A|=\begin{vmatrix} 5 & -1 \\ 5 & -1 \end{vmatrix}=-5+5=0$$

$$|A|=\begin{vmatrix} 5 & 0 \\ 5 & 4 \end{vmatrix}=20-0=20\neq 0$$

Then rank(A)=2

Ex: Let $A=\begin{bmatrix} 5 & 3 & -1 \\ 0 & 2 & -2 \\ -1 & 1 & 0 \end{bmatrix}$ make the elementary transformations in all arranged way:

1) H_{12} 2) $K_3(-2)$ 3) $H_{13}(4)$ 4) $K_{21}(-3)$

$$A=\begin{bmatrix}5&3&-1\\0&2&-2\\-1&1&0\end{bmatrix}\xrightarrow{H_{12}}\begin{bmatrix}0&2&-2\\5&3&-1\\-1&1&0\end{bmatrix}\xrightarrow{K_3(-2)}\begin{bmatrix}0&2&4\\5&3&2\\-1&1&0\end{bmatrix}$$

$$\xrightarrow{H_{13}(4)}\begin{bmatrix}0&2&4\\5&3&2\\-1&9&16\end{bmatrix}\xrightarrow{K_{21}(-3)}\begin{bmatrix}-6&2&4\\-4&3&2\\-28&9&16\end{bmatrix}$$

Ex: Let , find rank of A by used the canonical form transformations way?

Solution:

$$A=\begin{bmatrix}-2&4\\0&-1\\5&1\end{bmatrix}\overset{H_1(\frac{-1}{2})}{\sim}\begin{bmatrix}1&-2\\0&-1\\5&1\end{bmatrix}\overset{H_{13}(-5)}{\sim}\begin{bmatrix}1&-2\\0&-1\\0&11\end{bmatrix}\overset{H_2(-1)}{\sim}\begin{bmatrix}1&-2\\0&1\\0&11\end{bmatrix}$$

$$\overset{H_{23}(-11)}{\sim}\begin{bmatrix}1&-2\\0&1\\0&0\end{bmatrix}\overset{H_{21}(2)}{\sim}\begin{bmatrix}1&0\\0&1\\0&0\end{bmatrix}=C$$

$Then\, rank(A)=2\,, the\, number\, of\, rows\, of\, C\, not\, all\, zeros.$

Ex: Transform this matrix $A=\begin{pmatrix}1&2&-1&4\\2&4&3&5\\-1&-2&6&-7\end{pmatrix}$ to normal form then find rank(A)?

Solution:

$$A=\begin{pmatrix}1&2&-1&4\\2&4&3&5\\-1&-2&6&-7\end{pmatrix}\underset{H_{13}(1)}{\overset{H_{12}(-2)}{\sim}}\begin{pmatrix}1&2&-1&4\\0&0&5&-3\\0&0&5&-3\end{pmatrix}\overset{H_{23}(-1)}{\sim}\begin{pmatrix}1&2&-1&4\\0&0&5&-3\\0&0&0&0\end{pmatrix}$$

$$\underset{\substack{K_{13}(1)\\K_{14}(-4)}}{\overset{K_{12}(-2)}{\sim}}\begin{pmatrix}1&0&0&0\\0&0&5&-3\\0&0&0&0\end{pmatrix}\overset{K_3(\frac{1}{5})}{\sim}\begin{pmatrix}1&0&0&0\\0&0&1&-3\\0&0&0&0\end{pmatrix}\overset{K_{34}(3)}{\sim}\begin{pmatrix}1&0&0&0\\0&0&1&0\\0&0&0&0\end{pmatrix}$$

$$\overset{K_{23}}{\sim}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\end{pmatrix}$$

$$=\begin{pmatrix}I_2&0\\0&0\end{pmatrix}=N\ .\quad Then\ \ rank(A)=2$$

Ex: Let $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 5\\ 3 & 5 & 6\end{bmatrix}$ find Inverse of matrix by used elementary transformations.

Solution:

$$(A/I)=\left[\begin{array}{ccc|ccc}1 & 2 & 3 & 1 & 0 & 0\\ 2 & 4 & 5 & 0 & 1 & 0\\ 3 & 5 & 6 & 0 & 0 & 1\end{array}\right]\underset{H_{13}(-3)}{\overset{H_{12}(-2)}{\sim}}\left[\begin{array}{ccc|ccc}1 & 2 & 3 & 1 & 0 & 0\\ 0 & 0 & -1 & -2 & 1 & 0\\ 0 & -1 & -3 & -3 & 0 & 1\end{array}\right]$$

$$\overset{H_{23}}{\sim}\left[\begin{array}{ccc|ccc}1 & 2 & 3 & 1 & 0 & 0\\ 0 & -1 & -3 & -3 & 0 & 1\\ 0 & 0 & -1 & -2 & 1 & 0\end{array}\right]\underset{H_{3}(-1)}{\overset{H_{2}(-1)}{\sim}}\left[\begin{array}{ccc|ccc}1 & 2 & 3 & 1 & 0 & 0\\ 0 & 1 & 3 & 3 & 0 & -1\\ 0 & 0 & 1 & 2 & -1 & 0\end{array}\right]$$

$$\underset{H_{32}(-3)}{\overset{H_{31}(-3)}{\sim}}\left[\begin{array}{ccc|ccc}1 & 2 & 0 & -5 & 3 & 0\\ 0 & 1 & 0 & -3 & 3 & -1\\ 0 & 0 & 1 & 2 & -1 & 0\end{array}\right]\overset{H_{21}(-2)}{\sim}\left[\begin{array}{ccc|ccc}1 & 0 & 0 & 1 & -3 & 2\\ 0 & 1 & 0 & -3 & 3 & -1\\ 0 & 0 & 1 & 2 & -1 & 0\end{array}\right]=(I/A^{-1})$$

$$\therefore A^{-1}=\begin{bmatrix}1 & -3 & 0\\ -3 & 3 & -1\\ 2 & -1 & 0\end{bmatrix}$$

Ex: Solve the system of the liner equation by using Gramer method:

$$x_1-3x_2=5$$
$$-x_2+x_3=-1$$
$$6x_1+2x_3=0$$

Solution:

$$A=\begin{bmatrix}1 & -3 & 0\\ 0 & -1 & 1\\ 6 & 0 & 2\end{bmatrix},\quad X=\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix},\quad B=\begin{bmatrix}5\\ -1\\ 0\end{bmatrix}$$

$$|A|=\begin{vmatrix}1 & -3 & 0 & \vdots & 1 & -3\\ 0 & -1 & 1 & \vdots & 0 & -1\\ 6 & 0 & 2 & \vdots & 6 & 0\end{vmatrix}=-2-18+0-(0+0+0)=-20\neq 0$$

$$x_1=\frac{|A_1|}{|A|}\ ,\ |A_1|=\begin{vmatrix}5 & -3 & 0 & \vdots & 5 & -3\\ -1 & -1 & 1 & \vdots & -1 & -1\\ 0 & 0 & 2 & \vdots & 0 & 0\end{vmatrix}=-10+0+0-(6+0+0)=-16$$

$$x_1=\frac{-16}{-20}=\frac{4}{5}$$

$$x_2=\frac{|A_2|}{|A|}\ ,\ |A_2|=\begin{vmatrix}1 & 5 & 0 & \vdots & 1 & 5\\ 0 & -1 & 1 & \vdots & 0 & -1\\ 6 & 0 & 2 & \vdots & 6 & 0\end{vmatrix}=-2+30+0-(0+0+0)=28$$

$$x_2=\frac{28}{-20}=\frac{-7}{5}$$

$$x_3=\frac{|A_3|}{|A|}\ ,\ |A_3|=\begin{vmatrix}1 & -3 & 5 & \vdots & 1 & -3\\ 0 & -1 & -1 & \vdots & 0 & -1\\ 6 & 0 & 0 & \vdots & 6 & 0\end{vmatrix}=0+18+0-(0+0-30)=48$$

$$x_3=\frac{48}{-20}=\frac{-12}{5}$$

$$X=\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}4/5\\ -7/5\\ -12/5\end{bmatrix}$$

Ex: Solve this system of the liner equations by using elementary transformation:

$$x_1-2x_2+3x_3=4$$

$$x_1+x_2+2x_3=4$$

Solution:

$$A=\begin{bmatrix}1 & -2 & 3\\ 1 & 1 & 2\end{bmatrix},\quad B=\begin{bmatrix}4\\4\end{bmatrix},\quad X=\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$$

$$|A|=\begin{vmatrix}1 & -2\\ 1 & 1\end{vmatrix}=1+2=3 \qquad \Rightarrow \qquad rank(A)=2$$

$$(A/B)=\left[\begin{array}{ccc|c}1 & -2 & 3 & 4\\ 1 & 1 & 2 & 4\end{array}\right]\overset{H_{12}(-1)}{\sim}\left[\begin{array}{ccc|c}1 & -2 & 3 & 4\\ 0 & 3 & -1 & 0\end{array}\right]\overset{H_2(\frac{1}{3})}{\sim}\left[\begin{array}{ccc|c}1 & -2 & 3 & 4\\ 0 & 1 & -1/3 & 0\end{array}\right]\overset{H_{21}(2)}{\sim}\left[\begin{array}{ccc|c}1 & 0 & 7/3 & 4\\ 0 & 1 & -1/3 & 0\end{array}\right]$$

$$=(C/K) \Rightarrow CX=K$$

$\because rank(A)=2 \text{ and } rank(A/B)=2 \text{ then}$

Ex: Show that $A=\begin{bmatrix}\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}}\\ 0 & 1 & 0\end{bmatrix}$ is a orthogonal matrix.

Solution: (A is orthogonal matrix if $A.A'=A'.A=I$)

Then we must to show $A.A'=A'.A=I$

$$A'=\begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0\\ 0 & 0 & 1\\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0\end{bmatrix}$$

$$A'A=\begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0\\ 0 & 0 & 1\\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}}\\ 0 & 1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}\frac{1}{2}+\frac{1}{2}+0 & 0+0+0 & \frac{1}{2}-\frac{1}{2}+0\\ 0+0+0 & 0+0+1 & 0+0+0\\ \frac{1}{2}-\frac{1}{2}+0 & 0+0+0 & \frac{1}{2}+\frac{1}{2}+0\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$AA' = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2}+0+\frac{1}{2} & \frac{1}{2}+0-\frac{1}{2} & 0+0+0 \\ \frac{1}{2}+0-\frac{1}{2} & \frac{1}{2}+0+\frac{1}{2} & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+1+0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$then\, AA' = A'A = I$

$there\ fore\ A\, is\, an\, orthogonal\, matrix.$

Ex: Show that $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ is orthonormal matrix.

Solution:

$$A^{-1} = \frac{1}{|A|} adj(A)$$

$$|A| = \begin{vmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{vmatrix} = (\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}}) - (\frac{-1}{\sqrt{2}}.\frac{1}{\sqrt{2}}) = \frac{1}{2} + \frac{1}{2} = 1$$

$$adj(A) = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$A^{-1} = \frac{1}{1} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$A' = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\therefore A' = A^{-1} \Rightarrow A\, is\ orthonormal\ matrix$$

Ex: Find the eigenvalue & eigenvector of A if $A = \begin{bmatrix} -2 & 1 \\ 12 & -3 \end{bmatrix}$

Solution:

$$A-\lambda I=\begin{bmatrix}-2 & 1\\ 12 & -3\end{bmatrix}-\lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}-2 & 1\\ 12 & -3\end{bmatrix}-\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}$$

$$=\begin{bmatrix}-2-\lambda & 1\\ 12 & -3-\lambda\end{bmatrix}$$

$|A-\lambda I|=0$ *eigenvalue*

$$\begin{vmatrix}-2-\lambda & 1\\ 12 & -3-\lambda\end{vmatrix}=0$$

$(-2-\lambda)(-3-\lambda)-12=0$

$6+2\lambda+3\lambda+\lambda^2 12=0$

$\lambda^2+5\lambda-6=0$

$(\lambda+6)(\lambda-1)=0$

$$\left.\begin{array}{l}\lambda-1=0 \Rightarrow \lambda_1=1\\ \lambda+6=0 \Rightarrow \lambda_2=-6\end{array}\right\}\ eigenvalue$$

<u>for $\lambda=1$</u>:

$(A-\lambda I)X=0$ *eigenvector*

$$\begin{bmatrix}-2-\lambda & 1\\ 12 & -3-\lambda\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}-2-1 & 1\\ 12 & -3-1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}-3 & 1\\ 12 & -4\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}-3x+y\\ 12x-4y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}\Rightarrow -3x+y=0\Rightarrow x=\frac{1}{3}y$$

$Let\ y=t \Rightarrow x=\frac{1}{3}t$

$$\underline{X}=\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}\frac{1}{3}t\\ t\end{bmatrix}=t\begin{bmatrix}\frac{1}{3}\\ 1\end{bmatrix}\quad eigenvector$$

<u>For $\lambda=-6$</u>:

$(A-\lambda I)X=0$

$$\begin{bmatrix}-2-\lambda & 1\\ 12 & -3-\lambda\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}-2+6 & 1\\ 12 & -3+6\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}4 & 1\\ 12 & 3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}4x+y\\ 12x+3y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}\Rightarrow 4x+y=0\Rightarrow x=\frac{-1}{4}y$$

$Let\ y=t \Rightarrow x=\frac{-1}{4}t$

$$\underline{X}=\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}\frac{-1}{4}t\\ t\end{bmatrix}=t\begin{bmatrix}\frac{-1}{4}\\ 1\end{bmatrix}\quad eigenvector$$

20. Extra notes:لا يوجد ملاحظة

21. Peer review