Stage: Second
Subject:Linear Algebra with Application

2022-2023

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Chapter Three

1- Orthogonal Matrix المصفوفة المتعامدة:

Definition: A square matrix A is called orthogonal matrix if

$$A.A' = A'.A = I$$

Defn: **An orthogonal matrix is a matrix whose row vectors are orthonormal (يعني العمود نضربها بنفسها كصف)=** $A_1 A_1' = 1$**. then** $AA' = A'A = I$

Ex1: Show that $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \\ 0 & 1 & 0 \end{bmatrix}$ is a orthogonal matrix.

Solution: (A is orthogonal matrix if $A.A' = A'.A = I$)

Then we must to show $A.A' = A'.A = I$

$$A' = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \end{bmatrix}$$

$$A'A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2}+\frac{1}{2}+0 & 0+0+0 & \frac{1}{2}-\frac{1}{2}+0 \\ 0+0+0 & 0+0+1 & 0+0+0 \\ \frac{1}{2}-\frac{1}{2}+0 & 0+0+0 & \frac{1}{2}+\frac{1}{2}+0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$AA' = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2}+0+\frac{1}{2} & \frac{1}{2}+0-\frac{1}{2} & 0+0+0 \\ \frac{1}{2}+0-\frac{1}{2} & \frac{1}{2}+0+\frac{1}{2} & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+1+0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$then\ AA' = A'A = I$

$there\ fore\ A\ is\ an\ orthogonal\ matrix.$

Properties of orthogonal matrix:

Let A be an_orthogonal matrix

1- $AA' = I$

2- $AA^{-1} = I \quad \Rightarrow \quad A' = A^{-1}$

3- Product of two_orthogonal matrices is a orthogonal matrix.

4- Transpose of orthogonal matrices is a orthogonal matrix.

5- Invers of orthogonal matrices is a orthogonal matrix.

6- If A is orthogonal matrix then $|A| = \pm 1$.

Ex2: Prove that the following matrix is orthogonal matrix and hens find B^{-1}

$$B = \frac{1}{3}\begin{bmatrix} -2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{bmatrix}$$

Solution: (A is orthogonal matrix if $B.B' = B'.B = I$)

Then we must to show $B.B' = B'.B = I \quad \Rightarrow B' = B^{-1}$

$$B' = \frac{1}{3}\begin{bmatrix} -2 & 2 & 1 \\ 1 & 2 & -2 \\ 2 & 1 & 2 \end{bmatrix}$$

$$B.B' = \frac{1}{3}\begin{bmatrix} -2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{bmatrix} \frac{1}{3}\begin{bmatrix} -2 & 2 & 1 \\ 1 & 2 & -2 \\ 2 & 1 & 2 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 4+1+4 & -4+2+2 & -2-2+4 \\ -4+2+2 & 4+4+1 & 2-4+2 \\ -2-2+4 & 2-4+2 & 1+4+4 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$then\, B.B' = I$

$there\ fore\, \mathbf{B}\, is\, an\, orthogonal\, matrix.$

$\because \mathbf{B}\ is\, an\, orthogonal\, matrix\ then\ B.B' = I\ \ and\ \ BB^{-1} = I \ \ \Rightarrow B' = B^{-1}$

$$\therefore\ B^{-1} = \frac{1}{3}\begin{bmatrix} -2 & 2 & 1 \\ 1 & 2 & -2 \\ 2 & 1 & 2 \end{bmatrix}$$

<u>Ex3:</u> Show that $C = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ is orthogonal matrix .

<u>Solution:</u> (A is orthogonal matrix if $B.B' = B'.B = I$)

Then we must to show $B.B' = B'.B = I \ \ \Rightarrow B' = B^{-1}$

$$B' = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

$B.B' =$

<u>Ex4:</u> Show that $A = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$ is orthogonal matrix.

Solution: (A is orthogonal matrix if $A.A' = A'.A = I$)
Then we must to show $A.A' = A'.A = I$

$$A' = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$A.A' = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & 0 & -\sin\theta\cos\theta + \cos\theta\sin\theta \\ 0 & 1 & 0 \\ -\sin\theta\cos\theta + \cos\theta\sin\theta & 0 & \cos^2\theta + \sin^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$then\, A.A' = I$

$there\ fore\ A\ is\ an\ orthogonal\ matrix$

Ex5: Show that $A = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & \frac{-2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \end{bmatrix}$ is orthogonal matrix.

2- Orthonormal Matrix:

Definition: A square matrix A is called orthonormal matrix if

$$A^{-1}=A' \quad \det(A)=\pm 1$$

We have

$$AA^{-1}=A^{-1}A=I \quad and \quad AA'=A'A=I$$

$$\det(AA^{-1})=\det(I)$$

Ex1: Show that $A=\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ is orthonormal matrix.

Solution:

$$A^{-1}=\frac{1}{|A|}adj(A)$$

$$|A|=\begin{vmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{vmatrix}=(\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}})-(\frac{-1}{\sqrt{2}}.\frac{1}{\sqrt{2}})=\frac{1}{2}+\frac{1}{2}=1$$

$$adj(A)=\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$A^{-1}=\frac{1}{1}\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}=\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$A'=\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\therefore A'=A^{-1} \Rightarrow A\, is\ orthonormal\, matrix$$

Ex2: Show that $B=\begin{bmatrix} \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{-2}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{-2}{3} \end{bmatrix}$ is orthonormal matrix.

Solution:

$$B^{-1}=\frac{1}{|B|}adj(B)$$

$$|B|=\frac{2}{3}\begin{vmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{-2}{3} \end{vmatrix}-\frac{1}{3}\begin{vmatrix} \frac{-2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{-2}{3} \end{vmatrix}+\frac{2}{3}\begin{vmatrix} \frac{-2}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} \end{vmatrix}=$$

Chapter Four

Eigenvalues, Eigenvectors and Charateristic Equation of a matrix القيم و المتجهات الذاتية لمصفوفة و المعادلة المميزة لها

Definition: If A is $(n \times n)$ matrix, Then a non-zero vector $\underline{X}$ is called eigenvector of A if $(A\,X)$ is a scalar multiple (مضاعفاً عددياً) of X; that is $(A\,X = \lambda\,X)$ for some scalar λ.

The *scalar* λ *is* called an eigenvalue (القيم الذاتية) of A, and $\underline{X}$ is said to be an eigenvector (متجه ذاتي) corresponding (مناظر) to λ. المعادلة المميزة

To find the eigenvalue:

$$A\,X = \lambda\,X$$

$$A\,X - \lambda\,X = 0$$

$$(A - \lambda I)X = 0$$

$$\det(A - \lambda I) = 0 \qquad \text{Characteristic Equation}$$

$$or\,|A - \lambda I| = 0 \Leftarrow \text{المعادلة المميزة}$$

λ : هي الجذور المميزة eigenvalue (الكامنة) للمصفوفة A، يعني λ هي حل حقيقي للمعادلة المميزة.

<u>Ex:</u> Find the eigenvalue & eigenvector of A if $A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$

<u>Solution</u>:

$$A - \lambda I = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3-\lambda & 2 \\ -1 & -\lambda \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 3-\lambda & 2 \\ -1 & -\lambda \end{vmatrix} = 0$$

$$-\lambda(3-\lambda) + 2 = 0$$

$$-3\lambda + \lambda^2 + 2 = 0$$

$$\lambda^2 - 3\lambda + 2 = 0$$

$$(\lambda - 1)(\lambda - 2) = 0$$

$$\lambda - 1 = 0 \Rightarrow \lambda_1 = 1$$

$$\lambda - 2 = 0 \Rightarrow \lambda_2 = 2$$

$for\ \underline{\lambda = 1}:$

$$(A - \lambda I)X = 0$$

$$\begin{bmatrix} 3-\lambda & 2 \\ -1 & -\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3-1 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2x+2y \\ -x-y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow 2x + 2y = 0 \Rightarrow y = -x$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ -x \end{bmatrix} = x\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$ifor\ \underline{\lambda = 2}:$

$$(A - \lambda I)X = 0$$

$$\begin{bmatrix} 3-\lambda & 2 \\ -1 & -\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3-2 & 2 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x+2y \\ -x-2y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow x + 2y = 0 \Rightarrow y = \frac{-1}{2}x$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ \frac{-1}{2}x \end{bmatrix} = x\begin{bmatrix} 1 \\ \frac{-1}{2} \end{bmatrix}$$

Ex: If $B=\begin{bmatrix}3 & -2\\ 1 & 0\end{bmatrix}$, $X=\begin{bmatrix}2\\ 1\end{bmatrix}$ Find λ.

Solution:

$$BX=\lambda X$$

$$\begin{bmatrix}3 & -2\\ 1 & 0\end{bmatrix}\begin{bmatrix}2\\ 1\end{bmatrix}=\lambda\begin{bmatrix}2\\ 1\end{bmatrix}$$

$$\begin{bmatrix}6-2\\ 2+0\end{bmatrix}=\lambda\begin{bmatrix}2\\ 1\end{bmatrix}$$

$$\begin{bmatrix}4\\ 2\end{bmatrix}=\lambda\begin{bmatrix}2\\ 1\end{bmatrix}$$

$$2\begin{bmatrix}2\\ 1\end{bmatrix}=\lambda\begin{bmatrix}2\\ 1\end{bmatrix}\Rightarrow\lambda=2$$

Ex: Let $A=\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}$, $x_1=\begin{bmatrix}1\\ 1\end{bmatrix}$ *and* $x_2=\begin{bmatrix}2\\ -1\end{bmatrix}$ Find the eigenvalue λ_1 *and* λ_2.

Solution:

$$AX_1=\lambda_1 X_1$$

$$\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix}=\lambda_1\begin{bmatrix}1\\ 1\end{bmatrix}$$

$$\begin{bmatrix}1+4\\ 2+3\end{bmatrix}=\lambda_1\begin{bmatrix}1\\ 1\end{bmatrix}$$

$$\begin{bmatrix}5\\ 5\end{bmatrix}=\lambda_1\begin{bmatrix}1\\ 1\end{bmatrix}$$

$$5\begin{bmatrix}1\\ 1\end{bmatrix}=\lambda_1\begin{bmatrix}1\\ 1\end{bmatrix}\Rightarrow\lambda_1=5$$

2) $AX_2=\lambda_2 X_2$

$$\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}\begin{bmatrix}2\\ -1\end{bmatrix}=\lambda_2\begin{bmatrix}2\\ -1\end{bmatrix}$$

$$\begin{bmatrix}2-4\\ 4-3\end{bmatrix}=\lambda_2\begin{bmatrix}2\\ -1\end{bmatrix}$$

$$\begin{bmatrix}-2\\ 1\end{bmatrix}=\lambda_2\begin{bmatrix}2\\ -1\end{bmatrix}$$

$$-1\begin{bmatrix}2\\ -1\end{bmatrix}=\lambda_2\begin{bmatrix}2\\ -1\end{bmatrix}\Rightarrow\lambda_2=-1$$

Ex: Find the eigenvalue & eigenvector of A if $A=\begin{bmatrix}-2 & 1\\ 12 & -3\end{bmatrix}$

Solution:

$$A-\lambda I=\begin{bmatrix}-2 & 1\\ 12 & -3\end{bmatrix}-\lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}-2 & 1\\ 12 & -3\end{bmatrix}-\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}$$

$$=\begin{bmatrix}-2-\lambda & 1\\ 12 & -3-\lambda\end{bmatrix}$$

$|A-\lambda I|=0$ *eigenvalue*

$$\begin{vmatrix}-2-\lambda & 1\\ 12 & -3-\lambda\end{vmatrix}=0$$

$(-2-\lambda)(-3-\lambda)-12=0$

$6+2\lambda+3\lambda+\lambda^2 12=0$

$\lambda^2+5\lambda-6=0$

$(\lambda+6)(\lambda-1)=0$

$\left.\begin{array}{l}\lambda-1=0 \Rightarrow \lambda_1=1\\ \lambda+6=0 \Rightarrow \lambda_2=-6\end{array}\right\}$ *eigenvalue*

$\underline{\text{for } \lambda=1}$:

$(A-\lambda I)X=0$ *eigenvector*

$$\begin{bmatrix}-2-\lambda & 1\\ 12 & -3-\lambda\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}-2-1 & 1\\ 12 & -3-1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}-3 & 1\\ 12 & -4\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}-3x+y\\ 12x-4y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}\Rightarrow -3x+y=0\Rightarrow x=\frac{1}{3}y$$

$Let\ y=t \Rightarrow x=\frac{1}{3}t$

$$\underline{X}=\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}\frac{1}{3}t\\ t\end{bmatrix}=t\begin{bmatrix}\frac{1}{3}\\ 1\end{bmatrix} \quad eigenvector$$

$\underline{For\ \lambda=-6}$:

$(A-\lambda I)X=0$

$$\begin{bmatrix}-2-\lambda & 1\\ 12 & -3-\lambda\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}-2+6 & 1\\ 12 & -3+6\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}4 & 1\\ 12 & 3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}4x+y\\ 12x+3y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}\Rightarrow 4x+y=0\Rightarrow x=\frac{-1}{4}y$$

$Let\ y=t \Rightarrow x=\frac{-1}{4}t$

$$\underline{X}=\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}\frac{-1}{4}t\\ t\end{bmatrix}=t\begin{bmatrix}\frac{-1}{4}\\ 1\end{bmatrix} \quad eigenvector$$

Ex: Find the eigenvalue & eigenvector of A if $A=\begin{bmatrix}3 & -2 & -5\\ 4 & -1 & -5\\ -2 & -1 & -3\end{bmatrix}$

Solution:

$$A-\lambda I=\begin{bmatrix}3 & -2 & -5\\ 4 & -1 & -5\\ -2 & -1 & -3\end{bmatrix}-\lambda\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}3 & -2 & -5\\ 4 & -1 & -5\\ -2 & -1 & -3\end{bmatrix}-\left[\begin{bmatrix}\lambda & 0 & 0\\ 0 & \lambda & 0\\ 0 & 0 & \lambda\end{bmatrix}\right]$$

$$=\begin{bmatrix}3-\lambda & -2 & -5\\ 4 & -1-\lambda & -5\\ -2 & -1 & -3-\lambda\end{bmatrix}$$

$$|A-\lambda I|=0 \quad eigenvalue$$

$$\begin{vmatrix}3-\lambda & -2 & -5\\ 4 & -1-\lambda & -5\\ -2 & -1 & -3-\lambda\end{vmatrix}=0$$

$$\begin{vmatrix}3-\lambda & -2 & -5\\ 4 & -1-\lambda & -5\\ -2 & -1 & -3-\lambda\end{vmatrix}\begin{matrix}3-\lambda & -1\\ 4 & -1-\lambda\\ -2 & -1\end{matrix}=0$$

$$(3-\lambda)(-1-\lambda)(-3-\lambda)-20+20-[-8(-3-\lambda)+5(3-\lambda)+10(-1-\lambda)]=0$$

$$(3-\lambda)(3+\lambda)(1+\lambda)-[-10-10\lambda+15-5\lambda+24+8\lambda]=0$$

$$(9-\lambda^2)(1+\lambda)+29+7\lambda=0$$

$$9+9\lambda-\lambda^2-\lambda^3+29+7\lambda=0$$

$$[-\lambda^3-\lambda^2+16\lambda-20=0]\times(-1)$$

$$\lambda^3+\lambda^2-16\lambda+20=0$$

هاوكۆلكەی ژمارە (معامل العدد) 20 بریتیە لە: 1 ±، 2 ±، 4 ±، 5 ±، 10 ±، 20 ±
مجموعة حل المعادلة هي مجموعة قيم λ التي تحقق المعادلة والتي يقبل القسمة على (20) القسمة على كل منهما بدون باقي

$$if\ \lambda=1 \Rightarrow 1+1-16+20\neq 0$$

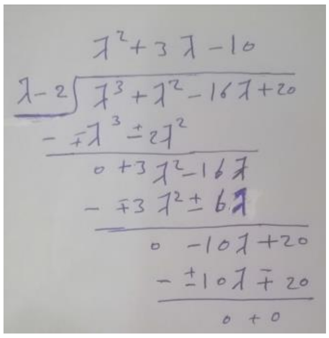
$$\lambda=-1 \Rightarrow -1+1+16+20\neq 0$$

$$\lambda=2 \Rightarrow 8+4-32+20=0 \Rightarrow 32-32=0$$

$$\lambda^3+\lambda^2-16\lambda+20=0$$

$$(\lambda-2)(\lambda^2+3\lambda-10)=0$$

$$(\lambda-2)(\lambda-2)(\lambda+5)=0 \Rightarrow \lambda=2,\ \lambda=2,\ \lambda=-5 \quad eigenvalue$$



$$When\ \lambda=2 \Rightarrow A-\lambda I=\begin{bmatrix}3-\lambda & -2 & -5\\ 4 & -1-\lambda & -5\\ -2 & -1 & -3-\lambda\end{bmatrix}=\begin{bmatrix}3-2 & -2 & -5\\ 4 & -1-2 & -5\\ -2 & -1 & -3-2\end{bmatrix}=\begin{bmatrix}1 & -2 & -5\\ 4 & -3 & -5\\ -2 & -1 & -5\end{bmatrix}$$

$$(A-\lambda I)X=0 \Rightarrow \begin{bmatrix}1 & -2 & -5\\ 4 & -3 & -5\\ -2 & -1 & -5\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix} \Rightarrow \begin{bmatrix}x-2y-5z\\ 4x-3y-5z\\ -2x-y-5z\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

$$x-2y-5z=0 \quad \cdots(1)$$
$$4x-3y-5z=0 \quad \cdots(2)$$
$$-2x-y-5z=0 \quad \cdots(3)$$

equation(1) − *equation*(2)

$$\begin{array}{r} x-2y-5z=0 \quad \cdots(1) \\ -\ \mp 4x \pm 3y \pm 5z = 0 \quad \cdots(2) \\ \hline -3x+y=0 \ \Rightarrow \ y=3x \end{array}$$

$$-2x-y-5z=0 \quad \cdots(3)$$
$$-2x-3x-5z=0 \Rightarrow -5x-5z=0 \Rightarrow z=-x$$

$$\underline{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ 3x \\ -x \end{bmatrix} = x\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} \quad \textit{eigenvector}$$

$$When\ \lambda=-5 \Rightarrow A-\lambda I = \begin{bmatrix} 3-\lambda & -2 & -5 \\ 4 & -1-\lambda & -5 \\ -2 & -1 & -3-\lambda \end{bmatrix} = \begin{bmatrix} 3+5 & -2 & -5 \\ 4 & -1+5 & -5 \\ -2 & -1 & -3+5 \end{bmatrix} = \begin{bmatrix} 8 & -2 & -5 \\ 4 & 4 & -5 \\ -2 & -1 & 2 \end{bmatrix}$$

$$(A-\lambda I)\,X=0 \ \Rightarrow \begin{bmatrix} 8 & -2 & -5 \\ 4 & 4 & -5 \\ -2 & -1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \ \Rightarrow \begin{bmatrix} 8x-2y-5z \\ 4x+4y-5z \\ -2x-y+2z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$8x-2y-5z=0 \quad \cdots(1)$$
$$4x+4y-5z=0 \quad \cdots(2)$$
$$-2x-y+2z=0 \quad \cdots(3)$$

equation(1) − *equation*(2)

$$\begin{array}{r} 8x-2y-5z=0 \quad \cdots(1) \\ -\ \mp 4x \mp 4y \pm 5z = 0 \quad \cdots(2) \\ \hline 4x+6y=0 \ \Rightarrow \qquad\qquad \end{array}$$

$$2x-3y=0 \ \Rightarrow y=\frac{2}{3}x$$

$$-2x-y+2z=0 \quad \cdots(3)$$
$$\left[-2x-\frac{2}{3}x+2z=0\right]\times(\frac{3}{2}) \Rightarrow -3x+x+3z=0$$
$$-4x+3z=0 \Rightarrow z=\frac{4}{3}x$$

$$\underline{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ \frac{2}{3}x \\ \frac{4}{3}x \end{bmatrix} = x\begin{bmatrix} 1 \\ \frac{2}{3} \\ \frac{4}{3} \end{bmatrix} \quad \textit{eigenvector}$$