University of Salahaddin
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Experimental Design and Analysis
L4

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Latin Square Design (LSD)

Properties:

1- LSD can be use if the difference among experiment units is in two directions.

2- The number of treatments equal to number of blocks, (number of replication) and equal to number of columns

3- The number of treatments must be (4 or more) must donates as (4*4), (5*5) and the maximum number (8 or 12).

4- The dfe must be (six or more) at least, since decrease in dfe causes increase in Mse value then decrease in accuracy.

5- This design not widely use in the field because the increase in the number of treatments two times causes the increase in experimental units four times.

6- The relative efficiency of (LSD) is more than (CRD and RCBD).

Construction of LSD

Suppose we have 4 treatments and the experimental units are differ in two direction, in this case we must be using LSD design for construction

	C1	C2	C3	C4
R1	**A (T1)**	**B (T2)**	**C (T3)**	**D (T4)**
R2	**B (T2)**	**C (T3)**	**D (T4)**	**A (T1)**
R3	**C (T3)**	**D (T4)**	**A (T1)**	**B (T 2)**
R4	**D (T4)**	**A (T1)**	**B (T2)**	**C (T3)**

The treatments are located on experiment units by using special tables or by using alphabetic (A, B, C, ….) method, the treatments must be repeat once in Row and Column.

ANOVA table for Latin square design.

S.O.V	df	SS	MS	Cal. F
Treat	Same as RCBD			
Row	Same as RCBD			
Column	C – 1	$SSC = \frac{\Sigma C1^2 + \Sigma C2^2 + \Sigma C3^2 + ...\Sigma Cn^2}{t} - C.F$	= SSC / dfc	= MSc / MSe
Error	(t – 1) (t – 2)	SST – SSt – SSc - SSr	Same as RCBD	
total	t^2 - 1	Same as RCBD		

Missing value in LSD

Use this equation for finding missing value in LSD design

Where:

$$Xij = \frac{r(R + C + T) - 2G}{(t-1)(t-2)}$$

r = number of replications

R = sum of Row which contain missing value

C= sum of Column which contain missing value

T = sum of treatment which contain missing value

G = sum of data before calculating missing value

The other steps are similar to the steps used in RCBD.

Example1:-

The following tables represent the results of Latin square design experiment if the data belong to (T_1 C_1 R_2) was lost, calculate missing value, if G = 8371

T2 (400)	T3 (672)	T4 (440)	T1(451)
T1 ------	T2 (600)	T3 (625)	T4 (550)
T3 (529)	T4 (600)	T1 (596)	T2 (500)
T4 (589)	T1 (632)	T2 (587)	T3 (600)

	1	2	3	4
ΣC	1518	2504	2248	2101
ΣR	1963	1775	2225	2408
ΣT	1679	2087	2426	2179

Latin Square Design (LSD):

Exp.1/ From the following data complete ANOVA Table and compare between the insecticides under $_{0.05,}$ if the Statistical Model is:

$$x_{ijk} = \mu + \tau_i + R_j + C_k + \varepsilon_{ijk}$$
$$i = 1,2,3,4$$
$$i = j = k$$

	Col.1	Col.2	Col.3	Col.4	∑rows	∑treats
Row1	A_1 *14*	B_1 11	C_1 **12**	D_1 11	48	∑A=60
Row2	B_2 12	C_2 **13**	D_2 18	A_2 *15*	58	∑B=46
Row3	C_3 **15**	D_3 15	A_3 *14*	B_3 11	55	∑C=57
row4	D_4 10	A_4 *17*	B_4 12	C_4 **17**	56	∑D=54
∑columns	51	56	56	54	*G*=217	

C.F.= $(217)^2/16=2943.0625$ $\sum x^2_{ijk}=(14)^2+\ldots+(17)^2=3033$

SST=3033-2943.0625=89.94

$$SS_{treat.} = \frac{(60)^2 + \ldots + (54)^2}{4} - 2943.0625 = 2970.25 - 2943.0625 = 27.2$$

$$SS_{Block} = \frac{(48)^2 + \ldots + (56)^2}{4} - 2943.0625 = 2957.25 - 2943.0625 = 14.2$$

$$SS_{Column} = \frac{(51)^2 + \ldots + (54)^2}{4} - 2943.0625 = 2947.25 - 2943.0625 = 4.2$$

$$SS_{Error} = 89.94 - 27.2 - 14.2 - 4.2 = 44.34$$

$$MS_t = 27.2/3 = 9.07 \qquad MS_{Row} = 14.2/3 = 4.7$$

$$MS_{Col.} = 4.2/3 = 1.4 \qquad MS_E = 44.34/6 = 7.4$$

S.O.V.	df	SS	MS	Cal.F
Treats	3	27.2	9.07	1.22 n.s.
Block	3	14.2	4.7	
Column	3	4.2	1.4	
Error	6	44.34	7.4	
Total	15	89.94		

Since cal.F less than Tab.F , it means that there are no sig. differences between the treatments.

Exp.2/ An experiment conducted to study the effect of (6) types of chemical fertilizers on yield of sugar cane (ton/ acer) .Complete ANOVA then compare between treatments and control (A treatment)using the suitable design at level of significant 0.05.

	C1	C2	C 3	C 4	C 5	C 6	∑rows	∑treat
R1	C 29.6	D 26.4	E 33.2	F 30.5	A 31.8	B 33.0	184.5	176.8
2	D 28.8	E 29.4	F 29.9	A 25.9	B 27.4	C 30.1	171.5	176.9
3	E 30.6	F 30.8	A 28.3	B 30.4	C 32.0	D 21.7	173.8	184.0
4	F 26.7	A 29.0	B 29.2	C 32.0	D 26.6	E 28.9	172.4	155.1
5	A 31.4	B 28.5	C 30.8	D 23.9	E 31.3	F 33.2	179.1	183.0
R6	B 28.4	C 29.5	D 27.7	E 29.6	F 32.1	A 30.4	177.7	183.2
∑col.	175.5	173.6	179.1	172.3	181.2	177.3	G=1059	

$$C.F. = 3115225 \qquad SS_{Total} = 31374.3 - 31152.25 = 222.05$$

$$SS_{treat.} = 3125252 - 3115225 = 100.27$$

$$SS_{Row} = 311725 - 31152.25 = 20.25$$

$$SS_{Column} = 3116167 - 3115225 = 9.42$$

$$SS_{Error} = 92.11$$

ANOVA Table

S.O.V	df	SS	MS	Cal.F	Tab-F
Treat.	5	100.27	20.054	4.35*	2.71
Row	5	20.25	4.05		
Column	5	9.42	1.884		
Error	20	92.11	4.605		
Total	35	222.05			

$$\bar{t}_2 - \bar{t}_1 = 29.5 - 29.5 = 0.0$$
$$\bar{t}_3 - \bar{t}_1 = 30.7 - 29.5 = 1.2$$
$$\bar{t}_4 - \bar{t}_1 = 25.85 - 29.5 = -3.65^*$$
$$\bar{t}_5 - \bar{t}_1 = 30.5 - 29.5 = 1.0$$
$$\bar{t}_6 - \bar{t}_1 = 30.53 - 29.5 = 1.03$$

$$D_t = tab.t_{D(\alpha, df_t, df_E)} \sqrt{2MS_E / r} = tab.t_{D(0.05,5,20)} \sqrt{2(4.605)/6}$$
$$D_t = 2.81(1.24) = \mathbf{3.5}$$

- ***Example 4:***
- An experiment was conducted for comparing between the effects of (P) Fertilizer on the cotton yield by LSD the following information was obtained.
- 1- $\Sigma t_1 = 32$ $\Sigma t_2 = 24$ $\Sigma t_3 = 20$
- 2- C.F = 625 3- $Cal.F_t = 9.5$ 4- dfT = 15
- 5- SSC = 12 6- SSR = 8
-

dfT = n – 1 n = dfT + 1 n = 15 + 1 = 16

n = t * r in LSD t = r so n = t * t $n = t^2$

t = √n t = √16 = 4 t= r = c = 4

C.F = $(G)^2 / n$ $625 = (G)^2/16 =$ $G^2 = 625 * 16$

$G^2 = 10000$ G = √10000 G = 100

G = Σt1 + Σt2 + Σt3 + Σt4 Σt4 = G – (Σt1 + Σt2 + Σt3)

Σt4 = 100 – (32 + 24 + 20) = 24

$$SSt = \frac{\sum t1^2 + ... + \sum tn^2}{r} - C.F$$

$$SSt = \frac{(32)^2 + (24)^2 + (20)^2 + (24)^2}{4} - 625$$

SSt = 19

MSt = SSt / dft = 19 / 3= 6.33

Cal. Ft = MSt / MSe = 9.5 = 6.33 / MSe MSE = 6.33 / 9.5 = 0.66

MSe = SSe / dfe SSe = MSe * dfe = 0.66 *6 = 3.96

SST = SSt + SSc + SSr + SSe SST = 19 + 8 + 12 + 3.96 = 42.96

MSC = SSC / dfc = 12 / 3 = 4

MSR = SSR / dfr = 8 / 3 = 5

Cal. FC = MSC / MSe = 4/ 0.66 = 9.92

Cal. Ft = MSt / MSe = 2.66/ 0.66 = 4.03

S.O.V	df	SS	Ms	CalF
treat	3	19	6.33	9.5
row	3	8	2.66	4.03
column	3	12	4	6.06
error	6	3.96	0.66	
Total	15	42.96		

• ***Example 2:***

• An experiment was conducted in the field, if the data belong to (T3 C1 R2) was lost, calculate it if you know that:

• 1- C.F = 100

• 2- dfC = 3

• 3- Mean t_1 = 2 Mean t_2 = 3 Mean t_4 = 1

• 4- ΣR1 = 11 ΣR3 = 9 ΣR4 = 8 5- ΣC2 = 13

• ΣC3 = 9 ΣC4 = 9

- ***Relative Efficiency (RE):***

Q/ From the following ANOVA calculate the RE:

1) Approximately

RCBD ANOVA Table

s.o.v.	df	SS	MS	
Treat.	3	135.2	45.1	→
Block	4	21.5	5.4	→
Error	12	26.3	2.2	→
Total	19	183		

CRD ANOVA Table

s.o.v.	df	SS	MS
Treat.	3	135.2	45.1
Error	16	47.8	2.98
Total	19	183	

$$RE(RCBD:CRD)=\frac{MSE_{CRD}}{MSER_{CBD}}*100=(2.98/2.2)*100=135.4\%$$

RCBD more efficiency than CRD about 35.4%, using RCBD increase the efficiency of experiment about 35.4%

- Rep. =5 in RCBD
- 5*1.354 =6.77 in CRD

2) $MSE_{CRD} = \frac{SSB + (df_t + df_E)\,MSE}{df_T} = \frac{21.5 + (3+12)*2.2}{19} = 2.87$

RE(RCBD:CRD)=(2.87/2.2)*100=130.4%

It means RCBD more efficient than CRD about 30.4%

Q/ from the following ANOVA table calculate the RE:

s.o.v.	df	SS	MS		s.o.v.	df	SS	MS
Block	4	2.04	0.51	→	Treat	4	1.32	0.33
Treat	4	1.32	0.33	→				
Error	16	1.005	0.063	→	Error	20	3.045	0.152
Total	24	4.365			Total	24	4.365	

$$RE(RCBD:CRD) = \frac{MSE_{CRD}}{MSE_{RCBD}} * \frac{(n_1+1)(n_2+3)}{(n_1+3)(n_2+1)} * 100$$

$$= \frac{0.152}{0.063} * \frac{(16+1)(20+3)}{(16+3)(20+1)} * 100 = 236.5\% \quad RCBD \text{ more efficient than CRD}$$

$$236.5/100 = 2.365 \Rightarrow 5*2.365 = 11.83 \text{ rep. in CRD}$$

Example 2:-

Calculate the relative efficiency of RCBD: CRD from the information below:

S.O.V	df	SS	MS
Block	2	44.67	22.33
Treat	3	518.25	172.75
Error	6	28.003	4.67
Total	11	590.923	

Answer:

dft = t – 1 3 = t – 1 t = 3 + 1 = 4

dfb = r – 1 2 = r – 1 r = 2 + 1 = 3

MSe CRD = [(dfb * MSb) + (dft + dfe) * MSe]/ df total

= [(2 * 22.33) + (3 + 6) * 4.67] / 11 = 7.88

dfe CRD = t (r – 1)= 4 (3 – 1) = 8

$$RE(RCBD:CRD) = \frac{MSe}{MSe}\frac{CRD}{RCBD} X \frac{(n1+1)(n2+3)}{(n1+3)(n2+1)} X100$$

Q/If you are given the following LSD ANOVA, calculate the RE (LSD:CRD) and RE (LSD: RCBD):

S.O.V	df	SS	MS	
Column		4	2.18	0.54
Block (Row)	4	2.4	0.6	
Treat	4	1.92	0.48	
Error	12	1.07	0.1	
Total	24	7.57		

$$RE(LSD:CRD)=\frac{MS_r+MS_c+(r-1)MS_E}{(r+1)MS_E}=\frac{0.6+0.54+(4)(0.1)}{(6)(0.1)}*100$$

$$=\frac{1.54}{0.6}*100=256.6\% \quad \text{LSD more efficient than CRD about } 156.6\%$$

$$RE(LSD:RCBD)=\frac{MS_c+(r-1)MS_E}{rMS_E}=\frac{0.54+(4)(0.1)}{(5)(0.1)}*100$$

$$=188\% \quad \text{LSD more efficient than RCBD when rows becomes Blocks.}$$

$$RE(LSD:RCBD)=\frac{MS_r+(r-1)MS_E}{rMS_E}=\frac{0.6+(4)(0.1)}{(5)(0.1)}*100$$

$$=48\% \quad \text{LSD less efficient than RCBD when columns becoms blocks.}$$

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