A.

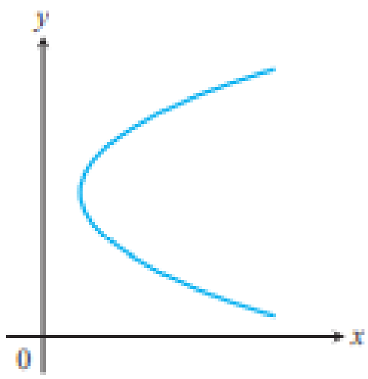
Functions and Graphs

In Exercises 1–6, find the domain and range of each function.

1. $f(x) = 1 + x^2$
2. $f(x) = 1 - \sqrt{x}$
3. $F(t) = \dfrac{1}{\sqrt{t}}$
4. $F(t) = \dfrac{1}{1 + \sqrt{t}}$
5. $g(z) = \sqrt{4 - z^2}$
6. $g(z) = \dfrac{1}{\sqrt{4 - z^2}}$

In Exercises 7 and 8, which of the graphs are graphs of functions of x, and which are not? Give reasons for your answers.

7. a.



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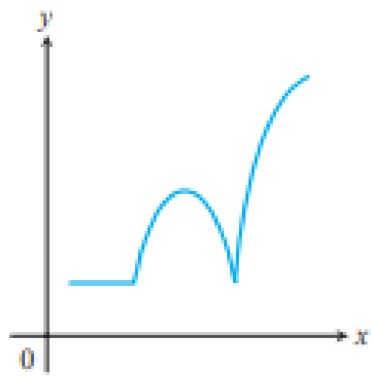
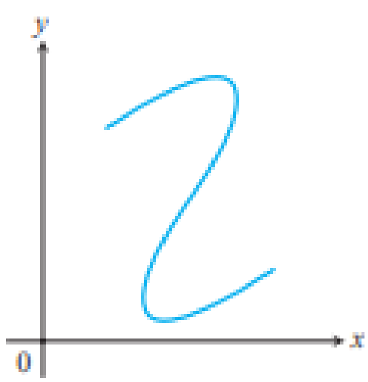
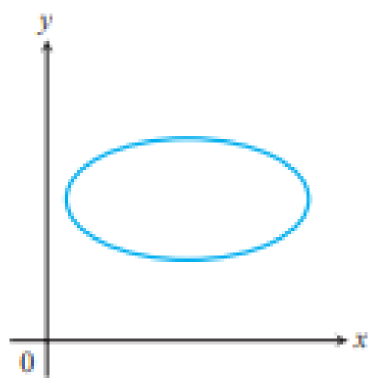


8. a.



b.



9. Consider the function $y = \sqrt{(1/x) - 1}$.
   - **a.** Can x be negative?
   - **b.** Can $x = 0$?
   - **c.** Can x be greater than 1?
   - **d.** What is the domain of the function?
10. Consider the function $y = \sqrt{2 - \sqrt{x}}$.
   - **a.** Can x be negative?
   - **b.** Can $\sqrt{x}$ be greater than 2?
   - **c.** What is the domain of the function?

Functions and Graphs

Find the domain and graph the functions in Exercises 15–20.

15. $f(x) = 5 - 2x$

16. $f(x) = 1 - 2x - x^2$

17. $g(x) = \sqrt{|x|}$

18. $g(x) = \sqrt{-x}$

19. $F(t) = t/|t|$

20. $G(t) = 1/|t|$

21. Graph the following equations and explain why they are not graphs of functions of x.

a. $|y| = x$ **b.** $y^2 = x^2$

22. Graph the following equations and explain why they are not graphs of functions of x.

a. $|x| + |y| = 1$ **b.** $|x + y| = 1$

B.

Composite of Functions

Sums, Differences, Products, and Quotients

In Exercises 1 and 2, find the domains and ranges of f, g, $f + g$, and $f \cdot g$.

1. $f(x) = x$, $g(x) = \sqrt{x - 1}$

2. $f(x) = \sqrt{x + 1}$, $g(x) = \sqrt{x - 1}$

In Exercises 3 and 4, find the domains and ranges of f, g, f/g, and g/f.

3. $f(x) = 2$, $g(x) = x^2 + 1$

4. $f(x) = 1$, $g(x) = 1 + \sqrt{x}$

Composites of Functions

5. If $f(x) = x + 5$ and $g(x) = x^2 - 3$, find the following.

a. $f(g(0))$ **b.** $g(f(0))$

c. $f(g(x))$ **d.** $g(f(x))$

e. $f(f(-5))$ **f.** $g(g(2))$

g. $f(f(x))$ **h.** $g(g(x))$

6. If $f(x) = x - 1$ and $g(x) = 1/(x + 1)$, find the following.

a. $f(g(1/2))$ **b.** $g(f(1/2))$

c. $f(g(x))$ **d.** $g(f(x))$

e. $f(f(2))$ **f.** $g(g(2))$

g. $f(f(x))$ **h.** $g(g(x))$

7. If $u(x) = 4x - 5$, $v(x) = x^2$, and $f(x) = 1/x$, find formulas for the following.

a. $u(v(f(x)))$ **b.** $u(f(v(x)))$

c. $v(u(f(x)))$ **d.** $v(f(u(x)))$

e. $f(u(v(x)))$ **f.** $f(v(u(x)))$

8. If $f(x) = \sqrt{x}$, $g(x) = x/4$, and $h(x) = 4x - 8$, find formulas for the following.

a. $h(g(f(x)))$ **b.** $h(f(g(x)))$

c. $g(h(f(x)))$ **d.** $g(f(h(x)))$

e. $f(g(h(x)))$ **f.** $f(h(g(x)))$

Let $f(x) = x - 3$, $g(x) = \sqrt{x}$, $h(x) = x^3$, and $j(x) = 2x$. Express each of the functions in Exercises 9 and 10 as a composite involving one or more of f, g, h, and j.

9. a. $y = \sqrt{x} - 3$ **b.** $y = 2\sqrt{x}$

c. $y = x^{1/4}$ **d.** $y = 4x$

e. $y = \sqrt{(x - 3)^3}$ **f.** $y = (2x - 6)^3$

10. a. $y = 2x - 3$ **b.** $y = x^{3/2}$

c. $y = x^9$ **d.** $y = x - 6$

e. $y = 2\sqrt{x - 3}$ **f.** $y = \sqrt{x^3 - 3}$

11. Copy and complete the following table.

	$g(x)$	$f(x)$	$(f \circ g)(x)$
a.	$x - 7$	$\sqrt{x}$	
b.	$x + 2$	$3x$	
c.		$\sqrt{x - 5}$	$\sqrt{x^2 - 5}$
d.	$\frac{x}{x - 1}$	$\frac{x}{x - 1}$	
e.		$1 + \frac{1}{x}$	x
f.	$\frac{1}{x}$		x

12. Copy and complete the following table.

	$g(x)$	$f(x)$	$(f \circ g)(x)$
a.	$\frac{1}{x - 1}$	$\lvert x \rvert$	?
b.	?	$\frac{x - 1}{x}$	$\frac{x}{x + 1}$
c.	?	$\sqrt{x}$	$\lvert x \rvert$
d.	$\sqrt{x}$	?	$\lvert x \rvert$

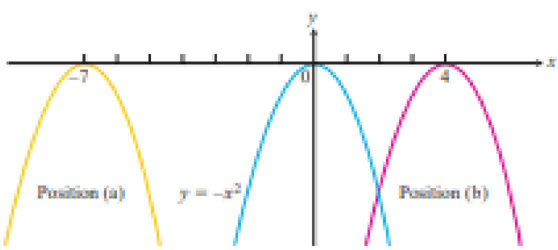
In Exercises 13 and 14, **(a)** write a formula for $f \circ g$ and $g \circ f$ and find the **(b)** domain and **(c)** range of each.

13. $f(x) = \sqrt{x + 1}, \quad g(x) = \frac{1}{x}$

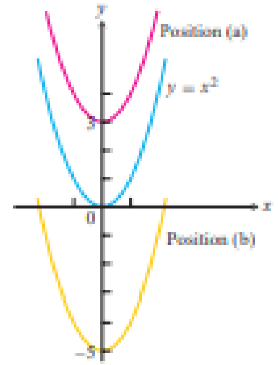
14. $f(x) = x^2, \quad g(x) = 1 - \sqrt{x}$

Shifting Graphs

15. The accompanying figure shows the graph of $y = -x^2$ shifted to two new positions. Write equations for the new graphs.



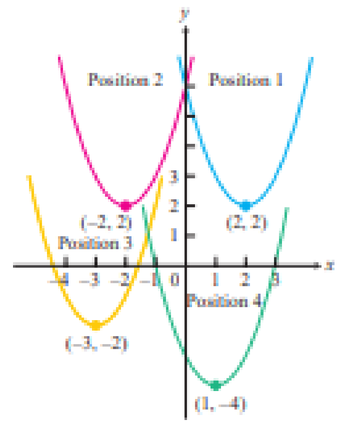
16. The accompanying figure shows the graph of $y = x^2$ shifted to two new positions. Write equations for the new graphs.



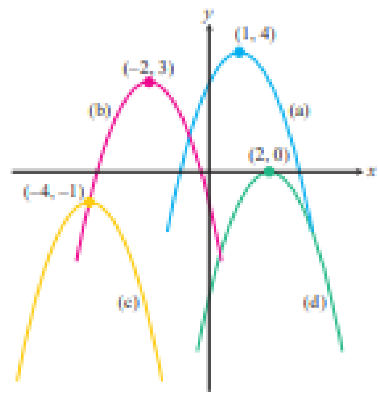
17. Match the equations listed in parts (a)–(d) to the graphs in the accompanying figure.

a. $y = (x - 1)^2 - 4$ **b.** $y = (x - 2)^2 + 2$

c. $y = (x + 2)^2 + 2$ **d.** $y = (x + 3)^2 - 2$



18. The accompanying figure shows the graph of $y = -x^2$ shifted to four new positions. Write an equation for each new graph.



Exercises 19–28 tell how many units and in what directions the graphs of the given equations are to be shifted. Give an equation for the shifted graph. Then sketch the original and shifted graphs together, labeling each graph with its equation.

19. $x^2 + y^2 = 49$ Down 3, left 2

20. $x^2 + y^2 = 25$ Up 3, left 4

21. $y = x^3$ Left 1, down 1

22. $y = x^{2/3}$ Right 1, down 1

23. $y = \sqrt{x}$ Left 0.81

24. $y = -\sqrt{x}$ Right 3

25. $y = 2x - 7$ Up 7

26. $y = \frac{1}{2}(x + 1) + 5$ Down 5, right 1

27. $y = 1/x$ Up 1, right 1

28. $y = 1/x^2$ Left 2, down 1

Graph the functions in Exercises 29–48.

29. $y = \sqrt{x + 4}$

30. $y = \sqrt{9 - x}$

31. $y = |x - 2|$

32. $y = |1 - x| - 1$

33. $y = 1 + \sqrt{x - 1}$

34. $y = 1 - \sqrt{x}$

35. $y = (x + 1)^{2/3}$

36. $y = (x - 8)^{2/3}$

37. $y = 1 - x^{2/3}$

38. $y + 4 = x^{2/3}$

39. $y = \sqrt[3]{x - 1} - 1$

40. $y = (x + 2)^{3/2} + 1$

41. $y = \frac{1}{x - 2}$

42. $y = \frac{1}{x} - 2$

43. $y = \frac{1}{x} + 2$

44. $y = \frac{1}{x + 2}$

45. $y = \frac{1}{(x - 1)^2}$

46. $y = \frac{1}{x^2} - 1$

47. $y = \frac{1}{x^2} + 1$

48. $y = \frac{1}{(x + 1)^2}$

C.

Trigonometric Functions

Radians, Degrees, and Circular Arcs

1. On a circle of radius 10 m, how long is an arc that subtends a central angle of (a) $4\pi/5$ radians? (b) $110°$?
2. A central angle in a circle of radius 8 is subtended by an arc of length 10π. Find the angle's radian and degree measures.
3. You want to make an $80°$ angle by marking an arc on the perimeter of a 12-in.-diameter disk and drawing lines from the ends of the arc to the disk's center. To the nearest tenth of an inch, how long should the arc be?
4. If you roll a 1-m-diameter wheel forward 30 cm over level ground, through what angle will the wheel turn? Answer in radians (to the nearest tenth) and degrees (to the nearest degree).

Evaluating Trigonometric Functions

5. Copy and complete the following table of function values. If the function is undefined at a given angle, enter "UND." Do not use a calculator or tables.

θ	$-\pi$	$-2\pi/3$	0	$\pi/2$	$3\pi/4$
$\sin\theta$					
$\cos\theta$					
$\tan\theta$					
$\cot\theta$					
$\sec\theta$					
$\csc\theta$					

6. Copy and complete the following table of function values. If the function is undefined at a given angle, enter "UND." Do not use a calculator or tables.

θ	$-3\pi/2$	$-\pi/3$	$-\pi/6$	$\pi/4$	$5\pi/6$
$\sin\theta$					
$\cos\theta$					
$\tan\theta$					
$\cot\theta$					
$\sec\theta$					
$\csc\theta$					

In Exercises 7–12, one of $\sin x$, $\cos x$, and $\tan x$ is given. Find the other two if x lies in the specified interval.

7. $\sin x = \frac{3}{5}, \quad x \in \left[\frac{\pi}{2}, \pi\right]$
8. $\tan x = 2, \quad x \in \left[0, \frac{\pi}{2}\right]$
9. $\cos x = \frac{1}{3}, \quad x \in \left[-\frac{\pi}{2}, 0\right]$
10. $\cos x = -\frac{5}{13}, \quad x \in \left[\frac{\pi}{2}, \pi\right]$
11. $\tan x = \frac{1}{2}, \quad x \in \left[\pi, \frac{3\pi}{2}\right]$
12. $\sin x = -\frac{1}{2}, \quad x \in \left[\pi, \frac{3\pi}{2}\right]$

Graphing Trigonometric Functions

Graph the functions in Exercises 13–22. What is the period of each function?

13. $\sin 2x$
14. $\sin (x/2)$

D.

Inverse trigonometric Functions

Common Values of Inverse Trignonometric Functions

Use reference triangles like those in Examples 1–3 to find the angles in Exercises 1–12.

1. a. $\tan^{-1} 1$ b. $\tan^{-1}(-\sqrt{3})$ c. $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$
2. a. $\tan^{-1}(-1)$ b. $\tan^{-1}\sqrt{3}$ c. $\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$
3. a. $\sin^{-1}\left(\frac{-1}{2}\right)$ b. $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ c. $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$
4. a. $\sin^{-1}\left(\frac{1}{2}\right)$ b. $\sin^{-1}\left(\frac{-1}{\sqrt{2}}\right)$ c. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$
5. a. $\cos^{-1}\left(\frac{1}{2}\right)$ b. $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$ c. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$
6. a. $\cos^{-1}\left(\frac{-1}{2}\right)$ b. $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ c. $\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$
7. a. $\sec^{-1}(-\sqrt{2})$ b. $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ c. $\sec^{-1}(-2)$
8. a. $\sec^{-1}\sqrt{2}$ b. $\sec^{-1}\left(\frac{-2}{\sqrt{3}}\right)$ c. $\sec^{-1} 2$
9. a. $\csc^{-1}\sqrt{2}$ b. $\csc^{-1}\left(\frac{-2}{\sqrt{3}}\right)$ c. $\csc^{-1} 2$
10. a. $\csc^{-1}(-\sqrt{2})$ b. $\csc^{-1}\left(\frac{2}{\sqrt{3}}\right)$ c. $\csc^{-1}(-2)$
11. a. $\cot^{-1}(-1)$ b. $\cot^{-1}(\sqrt{3})$ c. $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$
12. a. $\cot^{-1}(1)$ b. $\cot^{-1}(-\sqrt{3})$ c. $\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Trigonometric Function Values

13. Given that $\alpha = \sin^{-1}(5/13)$, find $\cos\alpha$, $\tan\alpha$, $\sec\alpha$, $\csc\alpha$, and $\cot\alpha$.
14. Given that $\alpha = \tan^{-1}(4/3)$, find $\sin\alpha$, $\cos\alpha$, $\sec\alpha$, $\csc\alpha$, and $\cot\alpha$.
15. Given that $\alpha = \sec^{-1}(-\sqrt{5})$, find $\sin\alpha$, $\cos\alpha$, $\tan\alpha$, $\csc\alpha$, and $\cot\alpha$.
16. Given that $\alpha = \sec^{-1}(-\sqrt{13}/2)$, find $\sin\alpha$, $\cos\alpha$, $\tan\alpha$, $\csc\alpha$, and $\cot\alpha$.

Evaluating Trigonometric and Inverse Trigonometric Terms

Find the values in Exercises 17–28.

17. $\sin\left(\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)\right)$

18. $\sec\left(\cos^{-1}\frac{1}{2}\right)$

19. $\tan\left(\sin^{-1}\left(-\frac{1}{2}\right)\right)$

20. $\cot\left(\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

21. $\csc(\sec^{-1}2) + \cos(\tan^{-1}(-\sqrt{3}))$

22. $\tan(\sec^{-1}1) + \sin(\csc^{-1}(-2))$

23. $\sin\left(\sin^{-1}\left(-\frac{1}{2}\right) + \cos^{-1}\left(-\frac{1}{2}\right)\right)$

24. $\cot\left(\sin^{-1}\left(-\frac{1}{2}\right) - \sec^{-1}2\right)$

25. $\sec(\tan^{-1}1 + \csc^{-1}1)$

26. $\sec(\cot^{-1}\sqrt{3} + \csc^{-1}(-1))$

27. $\sec^{-1}\left(\sec\left(-\frac{\pi}{6}\right)\right)$ (The answer is *not* $-\pi/6$.)

28. $\cot^{-1}\left(\cot\left(-\frac{\pi}{4}\right)\right)$ (The answer is *not* $-\pi/4$.)

Finding Trigonometric Expressions

Evaluate the expressions in Exercises 29–40.

29. $\sec\left(\tan^{-1}\frac{x}{2}\right)$

30. $\sec(\tan^{-1}2x)$

31. $\tan(\sec^{-1}3y)$

32. $\tan\left(\sec^{-1}\frac{y}{5}\right)$

33. $\cos(\sin^{-1}x)$

34. $\tan(\cos^{-1}x)$

35. $\sin(\tan^{-1}\sqrt{x^2 - 2x}),\quad x \geq 2$

36. $\sin\left(\tan^{-1}\frac{x}{\sqrt{x^2+1}}\right)$

37. $\cos\left(\sin^{-1}\frac{2y}{3}\right)$

38. $\cos\left(\sin^{-1}\frac{y}{5}\right)$

39. $\sin\left(\sec^{-1}\frac{x}{4}\right)$

40. $\sin\sec^{-1}\left(\frac{\sqrt{x^2+4}}{x}\right)$

E.

Hyperbolic Functions

Hyperbolic Function Values and Identities

Each of Exercises 1–4 gives a value of $\sinh x$ or $\cosh x$. Use the definitions and the identity $\cosh^2 x - \sinh^2 x = 1$ to find the values of the remaining five hyperbolic functions.

1. $\sinh x = -\frac{3}{4}$
2. $\sinh x = \frac{4}{3}$
3. $\cosh x = \frac{17}{15}, \quad x > 0$
4. $\cosh x = \frac{13}{5}, \quad x > 0$

Rewrite the expressions in Exercises 5–10 in terms of exponentials and simplify the results as much as you can.

5. $2\cosh(\ln x)$
6. $\sinh(2\ln x)$
7. $\cosh 5x + \sinh 5x$
8. $\cosh 3x - \sinh 3x$
9. $(\sinh x + \cosh x)^4$
10. $\ln(\cosh x + \sinh x) + \ln(\cosh x - \sinh x)$

11. Use the identities

$$\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$$
$$\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$$

to show that

a. $\sinh 2x = 2\sinh x \cosh x$

b. $\cosh 2x = \cosh^2 x + \sinh^2 x$.

12. Use the definitions of $\cosh x$ and $\sinh x$ to show that

$$\cosh^2 x - \sinh^2 x = 1.$$

F.

Exponential Functions

Algebraic Calculations with the Exponential and Logarithm

Find simpler expressions for the quantities in Exercises 1–4.

1. a. $e^{\ln 7.2}$ b. $e^{-\ln x^2}$ c. $e^{\ln x - \ln y}$
2. a. $e^{\ln(x^2+y^2)}$ b. $e^{-\ln 0.3}$ c. $e^{\ln \pi x - \ln 2}$
3. a. $2\ln\sqrt{e}$ b. $\ln(\ln e^e)$ c. $\ln(e^{-x^2-y^2})$
4. a. $\ln(e^{\sec\theta})$ b. $\ln(e^{(e^x)})$ c. $\ln(e^{2\ln x})$

Solving Equations with Logarithmic or Exponential Terms

In Exercises 5–10, solve for y in terms of t or x, as appropriate.

5. $\ln y = 2t + 4$
6. $\ln y = -t + 5$
7. $\ln(y - 40) = 5t$
8. $\ln(1 - 2y) = t$
9. $\ln(y - 1) - \ln 2 = x + \ln x$
10. $\ln(y^2 - 1) - \ln(y + 1) = \ln(\sin x)$

In Exercises 11 and 12, solve for k.

11. a. $e^{2k} = 4$ b. $100e^{10k} = 200$ c. $e^{k/1000} = a$
12. a. $e^{5k} = \frac{1}{4}$ b. $80e^k = 1$ c. $e^{(\ln 0.8)k} = 0.8$

In Exercises 13–16, solve for t.

13. a. $e^{-0.3t} = 27$ b. $e^{kt} = \frac{1}{2}$ c. $e^{(\ln 0.2)t} = 0.4$
14. a. $e^{-0.01t} = 1000$ b. $e^{kt} = \frac{1}{10}$ c. $e^{(\ln 2)t} = \frac{1}{2}$
15. $e^{\sqrt{t}} = x^2$
16. $e^{(x^2)}e^{(2x+1)} = e^t$

G.

Logarithmic Functions

Algebraic Calculations With a^x and $\log_a x$

Simplify the expressions in Exercises 1–4.

1. a. $5^{\log_5 7}$ b. $8^{\log_8 \sqrt{2}}$ c. $1.3^{\log_{1.3} 75}$

   d. $\log_4 16$ e. $\log_3 \sqrt{3}$ f. $\log_4 \left(\frac{1}{4}\right)$

2. a. $2^{\log_2 3}$ b. $10^{\log_{10}(1/2)}$ c. $\pi^{\log_\pi 7}$

   d. $\log_{11} 121$ e. $\log_{121} 11$ f. $\log_3 \left(\frac{1}{9}\right)$

3. a. $2^{\log_4 x}$ b. $9^{\log_3 x}$ c. $\log_2 \left(e^{(\ln 2)(\sin x)}\right)$

4. a. $25^{\log_5 (3x^2)}$ b. $\log_e (e^x)$ c. $\log_4 \left(2^{e^x \sin x}\right)$

Express the ratios in Exercises 5 and 6 as ratios of natural logarithm and simplify.

5. a. $\frac{\log_2 x}{\log_3 x}$ b. $\frac{\log_2 x}{\log_8 x}$ c. $\frac{\log_x a}{\log_{x^2} a}$

6. a. $\frac{\log_9 x}{\log_3 x}$ b. $\frac{\log_{\sqrt{10}} x}{\log_{\sqrt{2}} x}$ c. $\frac{\log_a b}{\log_b a}$

Solve the equations in Exercises 7–10 for x.

7. $3^{\log_3 (7)} + 2^{\log_2 (5)} = 5^{\log_5 (x)}$

8. $8^{\log_8 (3)} - e^{\ln 5} = x^2 - 7^{\log_7 (3x)}$

9. $3^{\log_3 (x^2)} = 5e^{\ln x} - 3 \cdot 10^{\log_{10} (2)}$

10. $\ln e + 4^{-2 \log_4 (x)} = \frac{1}{x} \log_{10} (100)$

Answers of questions

A.

1. domain $= (-\infty, \infty)$; range $= [1, \infty)$

2. domain $= [0, \infty)$; range $= (-\infty, 1]$

3. domain $= (0, \infty)$; y in range $\Rightarrow$ $y = \frac{1}{\sqrt{t}}, t > 0$ $\Rightarrow$ $y^2 = \frac{1}{t}$ and $y > 0$ $\Rightarrow$ y can be any positive real number $\Rightarrow$ range $= (0, \infty)$.

4. domain $= [0, \infty)$; y in range $\Rightarrow$ $y = \frac{1}{1+\sqrt{t}}$, $t > 0$. If $t = 0$, then $y = 1$ and as t increases, y becomes a smaller and smaller positive real number $\Rightarrow$ range $= (0, 1]$.

5. $4 - z^2 = (2 - z)(2 + z) \geq 0$ $\Leftrightarrow$ $z \in [-2, 2] =$ domain. Largest value is $g(0) = \sqrt{4} = 2$ and smallest value is $g(-2) = g(2) = \sqrt{0} = 0$ $\Rightarrow$ range $= [0, 2]$.

6. domain $= (-2, 2)$ from Exercise 5; smallest value is $g(0) = \frac{1}{2}$ and as $0 < z$ increases to 2, g(z) gets larger and larger (also true as $z < 0$ decreases to -2) $\Rightarrow$ range $= \left[\frac{1}{2}, \infty\right)$.

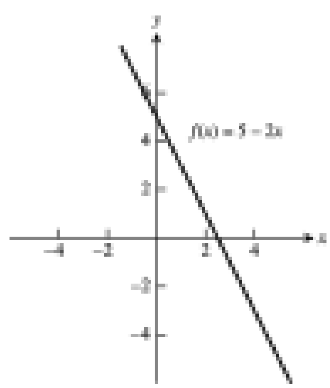
7. (a) Not the graph of a function of x since it fails the vertical line test.
   (b) Is the graph of a function of x since any vertical line intersects the graph at most once.

8. (a) Not the graph of a function of x since it fails the vertical line test.
   (b) Not the graph of a function of x since it fails the vertical line test.

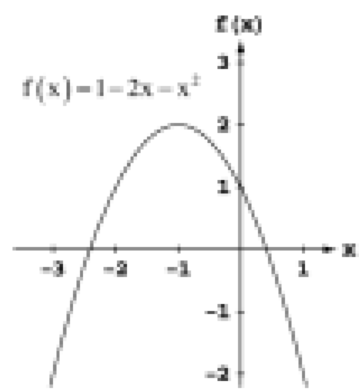
9. $y = \sqrt{\left(\frac{1}{x}\right) - 1} \Rightarrow \frac{1}{x} - 1 \geq 0 \Rightarrow x \leq 1$ and $x > 0$. So,
   (a) No $(x > 0)$;
   (b) No; division by 0 undefined;
   (c) No; if $x \geq 1$, $\frac{1}{x} < 1 \Rightarrow \frac{1}{x} - 1 < 0$;
   (d) $(0, 1]$

10. $y = \sqrt{2 - \sqrt{x}} \Rightarrow 2 - \sqrt{x} \geq 0 \Rightarrow \sqrt{x} \geq 0$ and $\sqrt{x} \leq 2$. $\sqrt{x} \geq 0 \Rightarrow x \geq 0$ and $\sqrt{x} \leq 2$ $\Rightarrow x \leq 4$. So, $0 \leq x \leq$
   (a) No; (b) No; (c) $[0, 4]$

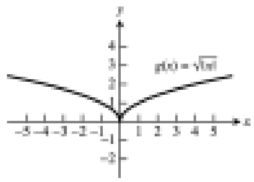
15. The domain is $(-\infty, \infty)$.



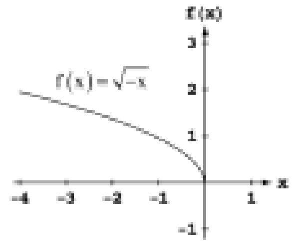
16. The domain is $(-\infty, \infty)$.



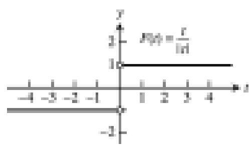
17. The domain is $(-\infty, \infty)$.



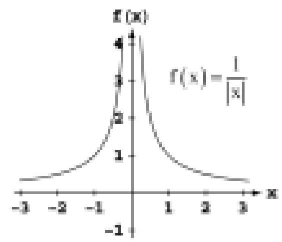
18. The domain is $(-\infty, 0]$.



19. The domain is $(-\infty, 0) \cup (0, \infty)$.

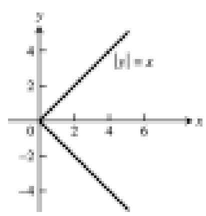


20. The domain is $(-\infty, 0) \cup (0, \infty)$.

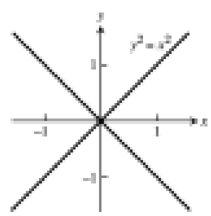


21. Neither graph passes the vertical line test

(a)

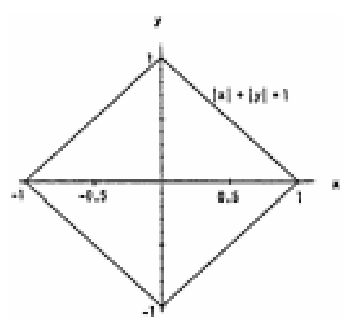


(b)

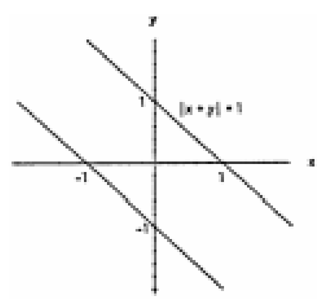


22. Neither graph passes the vertical line test

(a)



(b)



$$|x+y| = 1 \Leftrightarrow \left\{ \begin{array}{c} x+y=1 \\ \text{or} \\ x+y=-1 \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{c} y = 1-x \\ \text{or} \\ y=-1-x \end{array} \right\}$$

B.

1. $D_f: -\infty < x < \infty, D_g: x \geq 1 \Rightarrow D_{f+g} = D_{fg}: x \geq 1.$ $R_f: -\infty < y < \infty, R_g: y \geq 0, R_{f+g}: y \geq 1, R_{fg}: y \geq 0$

2. $D_f: x+1 \geq 0 \Rightarrow x \geq -1, D_g: x-1 \geq 0 \Rightarrow x \geq 1.$ Therefore $D_{f+g} = D_{fg}: x \geq 1.$
   $R_f = R_g: y \geq 0, R_{f+g}: y \geq \sqrt{2}, R_{fg}: y \geq 0$

3. $D_f: -\infty < x < \infty, D_g: -\infty < x < \infty \Rightarrow D_{f/g}: -\infty < x < \infty$ since $g(x) \neq 0$ for any x; $D_{g/f}: -\infty < x < \infty$ since $f(x) \neq 0$ for any x. $R_f: y = 2, R_g: y \geq 1, R_{f/g}: 0 < y \leq 2, R_{g/f}: y \geq \frac{1}{2}$

4. $D_f: -\infty < x < \infty, D_g: x \geq 0 \Rightarrow D_{f/g}: x \geq 0$ since $g(x) \neq 0$ for any $x \geq 0$; $D_{g/f}: x \geq 0$ since $f(x) \neq 0$ for any $x \geq 0$. $R_f: y = 1, R_g: y \geq 1, R_{f/g}: 0 < y \leq 1, R_{g/f}: y \geq 1$

5. (a) $f(g(0)) = f(-3) = 2$
   (b) $g(f(0)) = g(5) = 22$
   (c) $f(g(x)) = f(x^2-3) = x^2 - 3 + 5 = x^2 + 2$
   (d) $g(f(x)) = g(x+5) = (x+5)^2 - 3 = x^2 + 10x + 22$
   (e) $f(f(-5)) = f(0) = 5$
   (f) $g(g(2)) = g(1) = -2$
   (g) $f(f(x)) = f(x+5) = (x+5) + 5 = x + 10$
   (h) $g(g(x)) = g(x^2-3) = (x^2-3)^2 - 3 = x^4 - 6x^2 + 6$

6. (a) $f\left(g\left(\frac{1}{2}\right)\right) = f\left(\frac{2}{3}\right) = -\frac{1}{3}$
   (b) $g\left(f\left(\frac{1}{2}\right)\right) = g\left(-\frac{1}{2}\right) = 2$
   (c) $f(g(x)) = f\left(\frac{1}{x+1}\right) = \frac{1}{x+1} - 1 = \frac{-x}{x+1}$
   (d) $g(f(x)) = g(x-1) = \frac{1}{(x-1)+1} = \frac{1}{x}$
   (e) $f(f(2)) = f(1) = 0$
   (f) $g(g(2)) = g\left(\frac{1}{3}\right) = \frac{1}{\frac{4}{3}} = \frac{3}{4}$
   (g) $f(f(x)) = f(x-1) = (x-1) - 1 = x - 2$

7. (a) $u(v(f(x))) = u\left(v\left(\frac{1}{x}\right)\right) = u\left(\frac{1}{x^2}\right) = 4\left(\frac{1}{x}\right)^2 - 5 = \frac{4}{x^2} - 5$
   (b) $u(f(v(x))) = u\left(f\left(x^2\right)\right) = u\left(\frac{1}{x^2}\right) = 4\left(\frac{1}{x^2}\right) - 5 = \frac{4}{x^2} - 5$
   (c) $v(u(f(x))) = v\left(u\left(\frac{1}{x}\right)\right) = v\left(4\left(\frac{1}{x}\right) - 5\right) = \left(\frac{4}{x} - 5\right)^2$
   (d) $v(f(u(x))) = v(f(4x-5)) = v\left(\frac{1}{4x-5}\right) = \left(\frac{1}{4x-5}\right)^2$
   (e) $f(u(v(x))) = f\left(u\left(x^2\right)\right) = f\left(4\left(x^2\right) - 5\right) = \frac{1}{4x^2-5}$
   (f) $f(v(u(x))) = f(v(4x-5)) = f\left((4x-5)^2\right) = \frac{1}{(4x-5)^2}$

8. (a) $h(g(f(x))) = h\left(g\left(\sqrt{x}\right)\right) = h\left(\frac{\sqrt{x}}{4}\right) = 4\left(\frac{\sqrt{x}}{4}\right) - 8 = \sqrt{x} - 8$
   (b) $h(f(g(x))) = h\left(f\left(\frac{x}{4}\right)\right) = h\left(\sqrt{\frac{x}{4}}\right) = 4\sqrt{\frac{x}{4}} - 8 = 2\sqrt{x} - 8$
   (c) $g(h(f(x))) = g\left(h\left(\sqrt{x}\right)\right) = g\left(4\sqrt{x} - 8\right) = \frac{4\sqrt{x}-8}{4} = \sqrt{x} - 2$
   (d) $g(f(h(x))) = g(f(4x-8)) = g\left(\sqrt{4x-8}\right) = \frac{\sqrt{4x-8}}{4} = \frac{\sqrt{x-2}}{2}$
   (e) $f(g(h(x))) = f(g(4x-8)) = f\left(\frac{4x-8}{4}\right) = f(x-2) = \sqrt{x-2}$
   (f) $f(h(g(x))) = f\left(h\left(\frac{x}{4}\right)\right) = f\left(4\left(\frac{x}{4}\right) - 8\right) = f(x-8) = \sqrt{x-8}$

9. (a) $y = f(g(x))$
   (b) $y = j(g(x))$
   (c) $y = g(g(x))$
   (d) $y = j(j(x))$
   (e) $y = g(h(f(x)))$
   (f) $y = h(j(f(x)))$

10. (a) $y = f(j(x))$
    (b) $y = h(g(x)) = g(h(x))$
    (c) $y = h(h(x))$
    (d) $y = f(f(x))$
    (e) $y = j(g(f(x)))$
    (f) $y = g(f(h(x)))$

11.

	g(x)	f(x)	(f ∘ g)(x)
(a)	$x-7$	$\sqrt{x}$	$\sqrt{x-7}$
(b)	$x+2$	$3x$	$3(x+2)=3x+6$
(c)	x^2	$\sqrt{x-5}$	$\sqrt{x^2-5}$
(d)	$\frac{x}{x-1}$	$\frac{x}{x-1}$	$\frac{\frac{x}{x-1}}{\frac{x}{x-1}-1}=\frac{x}{x-(x-1)}=x$
(e)	$\frac{1}{x-1}$	$1+\frac{1}{x}$	x
(f)	$\frac{1}{x}$	$\frac{1}{x}$	x

12. (a) $(f\circ g)(x)=\lvert g(x)\rvert=\frac{1}{\lvert x-1\rvert}$.

(b) $(f\circ g)(x)=\frac{g(x)-1}{g(x)}=\frac{x}{x+1}\Rightarrow 1-\frac{1}{g(x)}=\frac{x}{x+1}\Rightarrow 1-\frac{x}{x+1}=\frac{1}{g(x)}\Rightarrow\frac{1}{x+1}=\frac{1}{g(x)}$, so $g(x)=x+1$.

(c) Since $(f\circ g)(x)=\sqrt{g(x)}=\lvert x\rvert$, $g(x)=x^2$.

(d) Since $(f\circ g)(x)=f(\sqrt{x})=\lvert x\rvert$, $f(x)=x^2$. (Note that the domain of the composite is $[0,\infty)$.)

The completed table is shown. Note that the absolute value sign in part (d) is optional.

g(x)	f(x)	(f∘g)(x)
$\frac{1}{x-1}$	$\lvert x\rvert$	$\frac{1}{\lvert x-1\rvert}$
$x+1$	$\frac{x-1}{x}$	$\frac{x}{x+1}$
x^2	$\sqrt{x}$	$\lvert x\rvert$
$\sqrt{x}$	x^2	$\lvert x\rvert$

13. (a) $f(g(x))=\sqrt{\frac{1}{x}+1}=\sqrt{\frac{1+x}{x}}$

$g(f(x))=\frac{1}{\sqrt{x+1}}$

(b) Domain (f∘g): $(0,\infty)$, domain (g∘f): $(-1,\infty)$

(c) Range (f∘g): $(1,\infty)$, range (g∘f): $(0,\infty)$

14. (a) $f(g(x))=1-2\sqrt{x}+x$

$g(f(x))=1-\lvert x\rvert$

(b) Domain (f∘g): $(0,\infty)$, domain (g∘f): $(0,\infty)$

(c) Range (f∘g): $(0,\infty)$, range (g∘f): $(-\infty,1)$

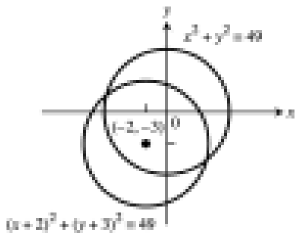
15. (a) $y=-(x+7)^2$ (b) $y=-(x-4)^2$

16. (a) $y=x^2+3$ (b) $y=x^2-5$

17. (a) Position 4 (b) Position 1 (c) Position 2 (d) Position 3

18. (a) $y=-(x-1)^2+4$ (b) $y=-(x+2)^2+3$ (c) $y=-(x+4)^2-1$ (d) $y=-(x-2)^2$

19.



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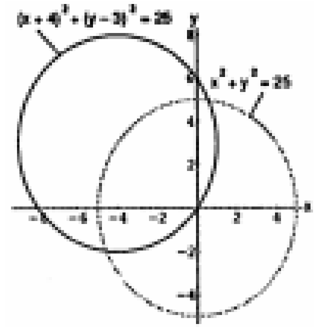

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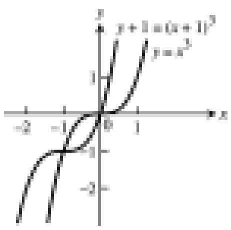

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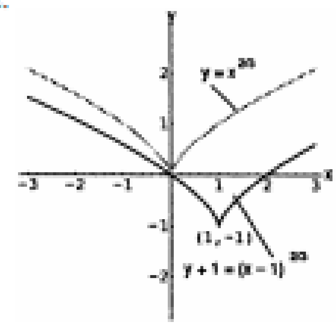

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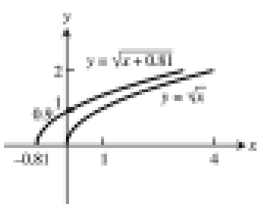

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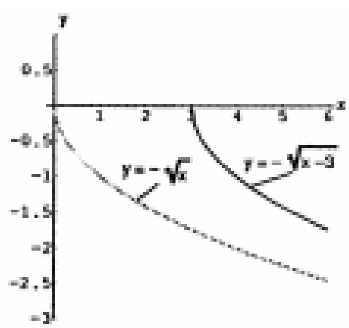

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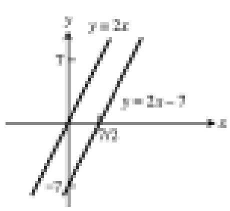

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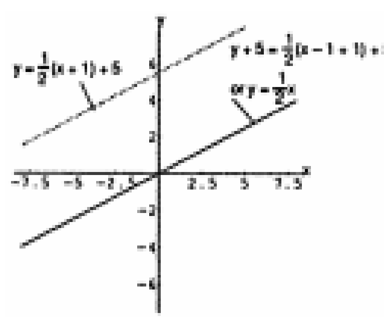

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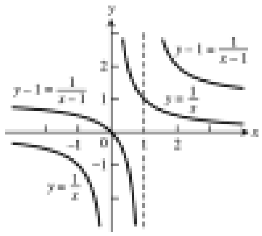

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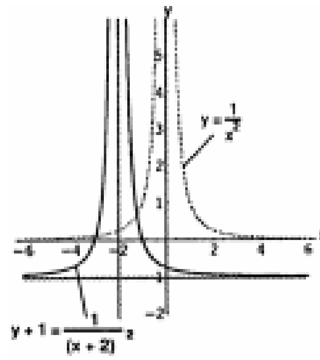

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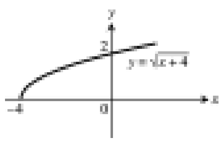

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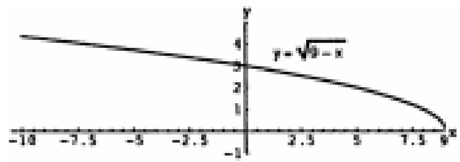

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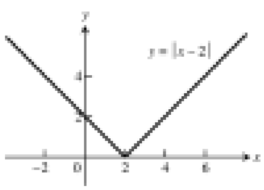

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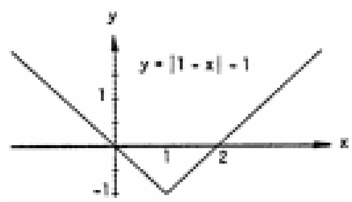

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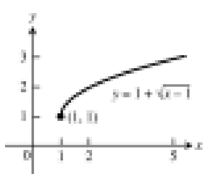

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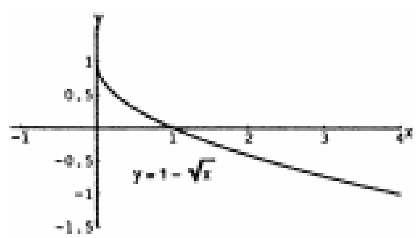

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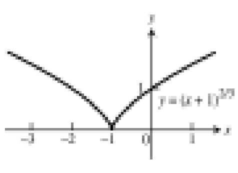

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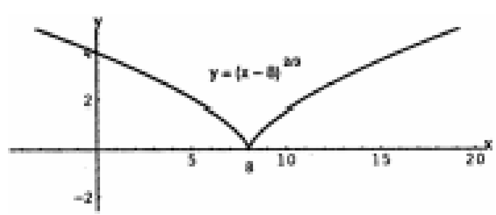

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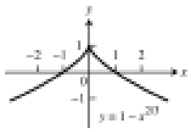

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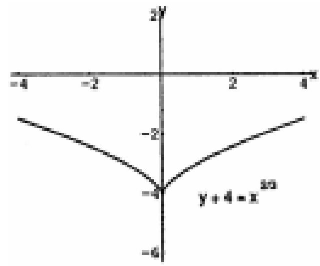

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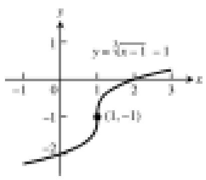

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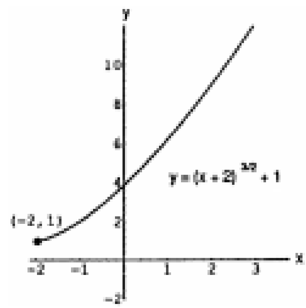

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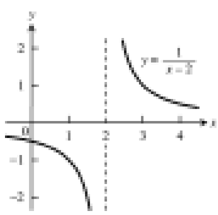

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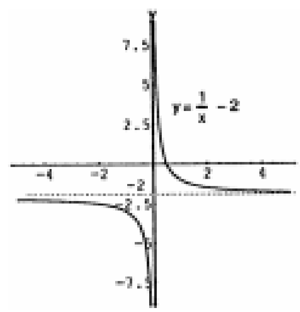

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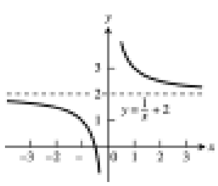

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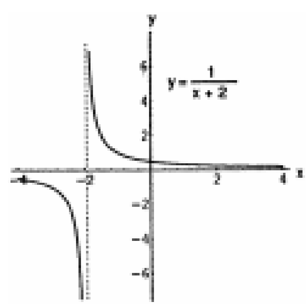

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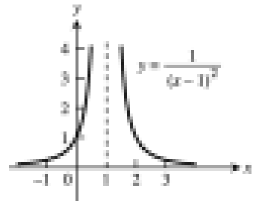

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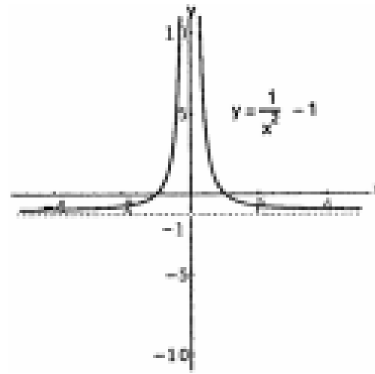

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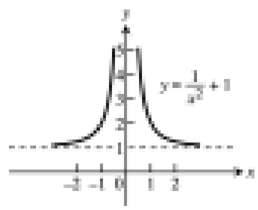

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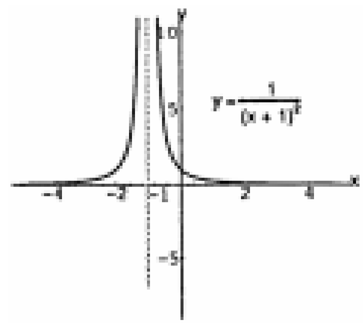


C.

1. (a) $s = r\theta = (10)\left(\frac{4\pi}{5}\right) = 8\pi$ m (b) $s = r\theta = (10)(110°)\left(\frac{\pi}{180°}\right) = \frac{110\pi}{18} = \frac{55\pi}{9}$ m

2. $\theta = \frac{s}{r} = \frac{10\pi}{8} = \frac{5\pi}{4}$ radians and $\frac{5\pi}{4}\left(\frac{180°}{\pi}\right) = 225°$

3. $\theta = 80° \Rightarrow \theta = 80°\left(\frac{\pi}{180°}\right) = \frac{4\pi}{9} \Rightarrow s = (6)\left(\frac{4\pi}{9}\right) = 8.4$ in. (since the diameter = 12 in. $\Rightarrow$ radius = 6 in.)

4. d = 1 meter $\Rightarrow$ r = 50 cm $\Rightarrow$ $\theta = \frac{s}{r} = \frac{30}{50} = 0.6$ rad or $0.6\left(\frac{180°}{\pi}\right) \approx 34°$

5.

θ	$-\pi$	$-\frac{2\pi}{3}$	0	$\frac{\pi}{2}$	$\frac{3\pi}{4}$
$\sin\theta$	0	$-\frac{\sqrt{3}}{2}$	0	1	$\frac{1}{\sqrt{2}}$
$\cos\theta$	-1	$-\frac{1}{2}$	1	0	$-\frac{1}{\sqrt{2}}$
$\tan\theta$	0	$\sqrt{3}$	0	und.	-1
$\cot\theta$	und.	$\frac{1}{\sqrt{3}}$	und.	0	-1
$\sec\theta$	-1	-2	1	und.	$-\sqrt{2}$
$\csc\theta$	und.	$-\frac{2}{\sqrt{3}}$	und.	1	$\sqrt{2}$

6.

θ	$-\frac{3\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{5\pi}{6}$
$\sin\theta$	1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\cos\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$
$\tan\theta$	und.	$-\sqrt{3}$	$-\frac{1}{\sqrt{3}}$	1	$-\frac{1}{\sqrt{3}}$
$\cot\theta$	0	$-\frac{1}{\sqrt{3}}$	$-\sqrt{3}$	1	$-\sqrt{3}$
$\sec\theta$	und.	2	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	$-\frac{2}{\sqrt{3}}$
$\csc\theta$	1	$-\frac{2}{\sqrt{3}}$	-2	$\sqrt{2}$	2

7. $\cos x = -\frac{4}{5}$, $\tan x = -\frac{3}{4}$

8. $\sin x = \frac{2}{\sqrt{5}}$, $\cos x = \frac{1}{\sqrt{5}}$

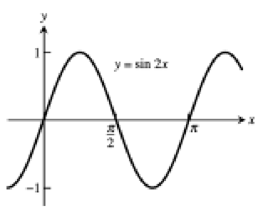
9. $\sin x = -\frac{\sqrt{8}}{3}$, $\tan x = -\sqrt{8}$

10. $\sin x = \frac{12}{13}$, $\tan x = -\frac{12}{5}$

11. $\sin x = -\frac{1}{\sqrt{5}}$, $\cos x = -\frac{2}{\sqrt{5}}$

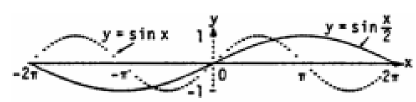
12. $\cos x = -\frac{\sqrt{3}}{2}$, $\tan x = \frac{1}{\sqrt{3}}$

13.



period $= \pi$

14.



period $= 4\pi$

D.

1. (a) $\frac{\pi}{4}$ (b) $-\frac{\pi}{3}$ (c) $\frac{\pi}{6}$

2. (a) $-\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $-\frac{\pi}{6}$

3. (a) $-\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $-\frac{\pi}{3}$

4. (a) $\frac{\pi}{6}$ (b) $-\frac{\pi}{4}$ (c) $\frac{\pi}{3}$

5. (a) $\frac{\pi}{3}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{6}$

6. (a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{5\pi}{6}$

7. (a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{2\pi}{3}$

8. (a) $\frac{\pi}{4}$ (b) $\frac{5\pi}{6}$ (c) $\frac{\pi}{3}$

9. (a) $\frac{\pi}{4}$ (b) $-\frac{\pi}{3}$ (c) $\frac{\pi}{6}$

10. (a) $-\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $-\frac{\pi}{6}$

11. (a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{2\pi}{3}$

12. (a) $\frac{\pi}{4}$ (b) $\frac{5\pi}{6}$ (c) $\frac{\pi}{3}$

13. $\alpha = \sin^{-1}\left(\frac{5}{13}\right) \Rightarrow \cos\alpha = \frac{12}{13}$, $\tan\alpha = \frac{5}{12}$, $\sec\alpha = \frac{13}{12}$, $\csc\alpha = \frac{13}{5}$, and $\cot\alpha = \frac{12}{5}$

14. $\alpha = \tan^{-1}\left(\frac{4}{3}\right) \Rightarrow \sin\alpha = \frac{4}{5}$, $\cos\alpha = \frac{3}{5}$, $\sec\alpha = \frac{5}{3}$, $\csc\alpha = \frac{5}{4}$, and $\cot\alpha = \frac{3}{4}$

15. $\alpha = \sec^{-1}\left(-\sqrt{5}\right) \Rightarrow \sin\alpha = \frac{2}{\sqrt{5}}$, $\cos\alpha = -\frac{1}{\sqrt{5}}$, $\tan\alpha = -2$, $\csc\alpha = \frac{\sqrt{5}}{2}$, and $\cot\alpha = -\frac{1}{2}$

16. $\alpha = \sec^{-1}\left(-\frac{\sqrt{13}}{2}\right) \Rightarrow \sin\alpha = \frac{3}{\sqrt{13}}$, $\cos\alpha = -\frac{2}{\sqrt{13}}$, $\tan\alpha = -\frac{3}{2}$, $\csc\alpha = \frac{\sqrt{13}}{3}$, and $\cot\alpha = -\frac{2}{3}$

17. $\sin\left(\cos^{-1}\frac{\sqrt{2}}{2}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

18. $\sec\left(\cos^{-1}\frac{1}{2}\right) = \sec\left(\frac{\pi}{3}\right) = 2$

19. $\tan\left(\sin^{-1}\left(-\frac{1}{2}\right)\right) = \tan\left(-\frac{\pi}{6}\right) = -\frac{1}{\sqrt{3}}$

20. $\cot\left(\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right) = \cot\left(-\frac{\pi}{3}\right) = -\frac{1}{\sqrt{3}}$

21. $\csc\left(\sec^{-1}2\right) + \cos\left(\tan^{-1}\left(-\sqrt{3}\right)\right) = \csc\left(\cos^{-1}\left(\frac{1}{2}\right)\right) + \cos\left(-\frac{\pi}{3}\right) = \csc\left(\frac{\pi}{3}\right) + \cos\left(-\frac{\pi}{3}\right) = \frac{2}{\sqrt{3}} + \frac{1}{2} = \frac{4+\sqrt{3}}{2\sqrt{3}}$

22. $\tan\left(\sec^{-1}1\right) + \sin\left(\csc^{-1}(-2)\right) = \tan\left(\cos^{-1}\frac{1}{1}\right) + \sin\left(\sin^{-1}\left(-\frac{1}{2}\right)\right) = \tan(0) + \sin\left(-\frac{\pi}{6}\right) = 0 + \left(-\frac{1}{2}\right) = -\frac{1}{2}$

23. $\sin\left(\sin^{-1}\left(-\frac{1}{2}\right) + \cos^{-1}\left(-\frac{1}{2}\right)\right) = \sin\left(-\frac{\pi}{6} + \frac{2\pi}{3}\right) = \sin\left(\frac{\pi}{2}\right) = 1$

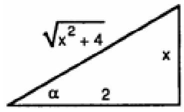
24. $\cot\left(\sin^{-1}\left(-\frac{1}{2}\right) - \sec^{-1}2\right) = \cot\left(-\frac{\pi}{6} - \cos^{-1}\left(\frac{1}{2}\right)\right) = \cot\left(-\frac{\pi}{6} - \frac{\pi}{3}\right) = \cot\left(-\frac{\pi}{2}\right) = 0$

25. $\sec\left(\tan^{-1}1 + \csc^{-1}1\right) = \sec\left(\frac{\pi}{4} + \sin^{-1}\frac{1}{1}\right) = \sec\left(\frac{\pi}{4} + \frac{\pi}{2}\right) = \sec\left(\frac{3\pi}{4}\right) = -\sqrt{2}$

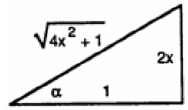
26. $\sec\left(\cot^{-1}\sqrt{3} + \csc^{-1}(-1)\right) = \sec\left(\frac{\pi}{6} + \sin^{-1}\left(\frac{1}{-1}\right)\right) = \sec\left(\frac{\pi}{2} - \frac{\pi}{3} - \frac{\pi}{2}\right) = \sec\left(-\frac{\pi}{3}\right) = 2$

27. $\sec^{-1}\left(\sec\left(-\frac{\pi}{6}\right)\right) = \sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

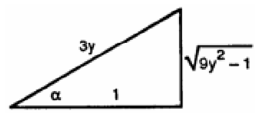
28. $\cot^{-1}\left(\cot\left(-\frac{\pi}{4}\right)\right) = \cot^{-1}(-1) = \frac{3\pi}{4}$

29. $\alpha = \tan^{-1} \frac{x}{2}$ indicates the diagram 

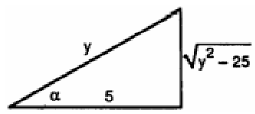
$\Rightarrow \sec\left(\tan^{-1} \frac{x}{2}\right) = \sec \alpha = \frac{\sqrt{x^2+4}}{2}$

30. $\alpha = \tan^{-1} 2x$ indicates the diagram 

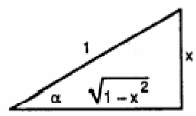
$\Rightarrow \sec\left(\tan^{-1} 2x\right) = \sec \alpha = \sqrt{4x^2+1}$

31. $\alpha = \sec^{-1} 3y$ indicates the diagram 

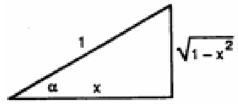
$\Rightarrow \tan\left(\sec^{-1} 3y\right) = \tan \alpha = \sqrt{9y^2-1}$

32. $\alpha = \sec^{-1} \frac{y}{5}$ indicates the diagram 

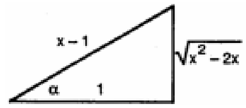
$\Rightarrow \tan\left(\sec^{-1} \frac{y}{5}\right) = \tan \alpha = \frac{\sqrt{y^2-25}}{5}$

33. $\alpha = \sin^{-1} x$ indicates the diagram 

$\Rightarrow \cos\left(\sin^{-1} x\right) = \cos \alpha = \sqrt{1-x^2}$

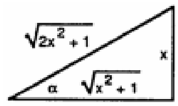
34. $\alpha = \cos^{-1} x$ indicates the diagram 

$\Rightarrow \tan(\cos^{-1} x) = \tan \alpha = \frac{\sqrt{1-x^2}}{x}$

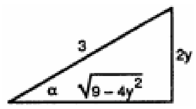
35. $\alpha = \tan^{-1} \sqrt{x^2 - 2x}$ indicates the diagram 

$\Rightarrow \sin\left(\tan^{-1} \sqrt{x^2 - 2x}\right)$

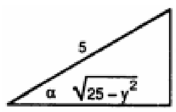
$= \sin \alpha = \frac{\sqrt{x^2 - 2x}}{x - 1}$

36. $\alpha = \tan^{-1} \frac{x}{\sqrt{x^2+1}}$ indicates the diagram 

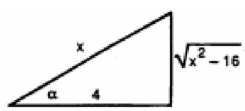
$\Rightarrow \sin\left(\tan^{-1} \frac{x}{\sqrt{x^2+1}}\right) = \sin \alpha = \frac{x}{\sqrt{2x^2+1}}$

37. $\alpha = \sin^{-1} \frac{2y}{3}$ indicates the diagram 

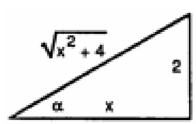
$\Rightarrow \cos\left(\sin^{-1} \frac{2y}{3}\right) = \cos \alpha = \frac{\sqrt{9-4y^2}}{3}$

38. $\alpha = \sin^{-1} \frac{y}{5}$ indicates the diagram 

$\Rightarrow \cos\left(\sin^{-1} \frac{y}{5}\right) = \cos \alpha = \frac{\sqrt{25-y^2}}{5}$

39. $\alpha = \sec^{-1} \frac{x}{4}$ indicates the diagram 

$\Rightarrow \sin\left(\sec^{-1} \frac{x}{4}\right) = \sin \alpha = \frac{\sqrt{x^2-16}}{x}$

40. $\alpha = \sec^{-1} \frac{\sqrt{x^2+4}}{x}$ indicates the diagram 

$\Rightarrow \sin\left(\sec^{-1} \frac{\sqrt{x^2+4}}{x}\right) = \sin \alpha = \frac{2}{\sqrt{x^2+4}}$

E.

1. $\sinh x = -\frac{3}{4} \Rightarrow \cosh x = \sqrt{1+\sinh^2 x} = \sqrt{1+\left(-\frac{3}{4}\right)^2} = \sqrt{1+\frac{9}{16}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$, $\tanh x = \frac{\sinh x}{\cosh x} = \frac{\left(-\frac{3}{4}\right)}{\left(\frac{5}{4}\right)} = -\frac{3}{5}$, $\coth x = \frac{1}{\tanh x} = -\frac{5}{3}$, $\operatorname{sech} x = \frac{1}{\cosh x} = \frac{4}{5}$, and $\operatorname{csch} x = \frac{1}{\sin x} = -\frac{4}{3}$

2. $\sinh x = \frac{4}{3} \Rightarrow \cosh x = \sqrt{1+\sinh^2 x} = \sqrt{1+\frac{16}{9}} = \sqrt{\frac{25}{9}} = \frac{5}{3}$, $\tanh x = \frac{\sinh x}{\cosh x} = \frac{\left(\frac{4}{3}\right)}{\left(\frac{5}{3}\right)} = \frac{4}{5}$, $\coth x = \frac{1}{\tanh x} = \frac{5}{4}$, $\operatorname{sech} x = \frac{1}{\cosh x} = \frac{3}{5}$, and $\operatorname{csch} x = \frac{1}{\sinh x} = \frac{3}{4}$

3. $\cosh x = \frac{17}{15}, x > 0 \Rightarrow \sinh x = \sqrt{\cosh^2 x - 1} = \sqrt{\left(\frac{17}{15}\right)^2 - 1} = \sqrt{\frac{289}{225} - 1} = \sqrt{\frac{64}{225}} = \frac{8}{15}$, $\tanh x = \frac{\sinh x}{\cosh x} = \frac{\left(\frac{8}{15}\right)}{\left(\frac{17}{15}\right)} = \frac{8}{17}$, $\coth x = \frac{1}{\tanh x} = \frac{17}{8}$, $\operatorname{sech} x = \frac{1}{\cosh x} = \frac{15}{17}$, and $\operatorname{csch} x = \frac{1}{\sinh x} = \frac{15}{8}$

4. $\cosh x = \frac{13}{5}, x > 0 \Rightarrow \sinh x = \sqrt{\cosh^2 x - 1} = \sqrt{\frac{169}{25} - 1} = \sqrt{\frac{144}{25}} = \frac{12}{5}$, $\tanh x = \frac{\sinh x}{\cosh x} = \frac{\left(\frac{12}{5}\right)}{\left(\frac{13}{5}\right)} = \frac{12}{13}$, $\coth x = \frac{1}{\tanh x} = \frac{13}{12}$, $\operatorname{sech} x = \frac{1}{\cosh x} = \frac{5}{13}$, and $\operatorname{csch} x = \frac{1}{\sinh x} = \frac{5}{12}$

5. $2\cosh(\ln x) = 2\left(\frac{e^{\ln x} + e^{-\ln x}}{2}\right) = e^{\ln x} + \frac{1}{e^{\ln x}} = x + \frac{1}{x}$

6. $\sinh(2\ln x) = \frac{e^{2\ln x} - e^{-2\ln x}}{2} = \frac{e^{\ln x^2} - e^{\ln x^{-2}}}{2} = \frac{\left(x^2 - \frac{1}{x^2}\right)}{2} = \frac{x^4 - 1}{2x^2}$

7. $\cosh 5x + \sinh 5x = \frac{e^{5x} + e^{-5x}}{2} + \frac{e^{5x} - e^{-5x}}{2} = e^{5x}$

8. $\cosh 3x - \sinh 3x = \frac{e^{3x} + e^{-3x}}{2} - \frac{e^{3x} - e^{-3x}}{2} = e^{-3x}$

9. $(\sinh x + \cosh x)^4 = \left(\frac{e^x - e^{-x}}{2} + \frac{e^x + e^{-x}}{2}\right)^4 = (e^x)^4 = e^{4x}$

10. $\ln(\cosh x + \sinh x) + \ln(\cosh x - \sinh x) = \ln(\cosh^2 x - \sinh^2 x) = \ln 1 = 0$

11. (a) $\sinh 2x = \sinh(x + x) = \sinh x \cosh x + \cosh x \sinh x = 2\sinh x \cosh x$
    (b) $\cosh 2x = \cosh(x + x) = \cosh x \cosh x + \sinh x \sin x = \cosh^2 x + \sinh^2 x$

12. $\cosh^2 x - \sinh^2 x = \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2 = \frac{1}{4}\left[(e^x + e^{-x}) + (e^x - e^{-x})\right]\left[(e^x + e^{-x}) - (e^x - e^{-x})\right]$
$= \frac{1}{4}(2e^x)(2e^{-x}) = \frac{1}{4}(4e^0) = \frac{1}{4}(4) = 1$

F.

1. (a) $e^{\ln 7.2} = 7.2$ (b) $e^{-\ln x^2} = \frac{1}{e^{\ln x^2}} = \frac{1}{x^2}$ (c) $e^{\ln x - \ln y} = e^{\ln(x/y)} = \frac{x}{y}$

2. (a) $e^{\ln(x^2+y^2)} = x^2 + y^2$ (b) $e^{-\ln 0.3} = \frac{1}{e^{\ln 0.3}} = \frac{1}{0.3}$ (c) $e^{\ln \pi x - \ln 2} = e^{\ln(\pi x/2)} = \frac{\pi x}{2}$

3. (a) $2 \ln \sqrt{e} = 2 \ln e^{1/2} = (2)\left(\frac{1}{2}\right) \ln e = 1$ (b) $\ln(\ln e^e) = \ln(e \ln e) = \ln e = 1$
   (c) $\ln e^{(-x^2-y^2)} = (-x^2 - y^2) \ln e = -x^2 - y^2$

4. (a) $\ln\left(e^{\sec\theta}\right) = (\sec\theta)(\ln e) = \sec\theta$ (b) $\ln e^{(e^x)} = (e^x)(\ln e) = e^x$
   (c) $\ln\left(e^{2\ln x}\right) = \ln\left(e^{\ln x^2}\right) = \ln x^2 = 2 \ln x$

5. $\ln y = 2t + 4 \Rightarrow e^{\ln y} = e^{2t+4} \Rightarrow y = e^{2t+4}$

6. $\ln y = -t + 5 \Rightarrow e^{\ln y} = e^{-t+5} \Rightarrow y = e^{-t+5}$

7. $\ln(y - 40) = 5t \Rightarrow e^{\ln(y-40)} = e^{5t} \Rightarrow y - 40 = e^{5t} \Rightarrow y = e^{5t} + 40$

8. $\ln(1 - 2y) = t \Rightarrow e^{\ln(1-2y)} = e^t \Rightarrow 1 - 2y = e^t \Rightarrow -2y = e^t - 1 \Rightarrow y = -\left(\frac{e^t - 1}{2}\right)$

9. $\ln(y-1) - \ln 2 = x + \ln x \Rightarrow \ln(y-1) - \ln 2 - \ln x = x \Rightarrow \ln\left(\frac{y-1}{2x}\right) = x \Rightarrow e^{\ln\left(\frac{y-1}{2x}\right)} = e^x \Rightarrow \frac{y-1}{2x} = e^x$
   $\Rightarrow y - 1 = 2xe^x \Rightarrow y = 2xe^x + 1$

10. $\ln(y^2 - 1) - \ln(y + 1) - \ln(\sin x) \rightarrow \ln\left(\frac{y^2-1}{y+1}\right) - \ln(\sin x) \rightarrow \ln(y - 1) - \ln(\sin x) \rightarrow e^{\ln(y-1)} - e^{\ln(\sin x)}$
   $\Rightarrow y - 1 = \sin x \Rightarrow y = \sin x + 1$

11. (a) $e^{2k} = 4 \Rightarrow \ln e^{2k} = \ln 4 \Rightarrow 2k \ln e = \ln 2^2 \Rightarrow 2k = 2 \ln 2 \Rightarrow k = \ln 2$
   (b) $100e^{10k} = 200 \Rightarrow e^{10k} = 2 \Rightarrow \ln e^{10k} = \ln 2 \Rightarrow 10k \ln e = \ln 2 \Rightarrow 10k = \ln 2 \Rightarrow k = \frac{\ln 2}{10}$
   (c) $e^{k/1000} = a \Rightarrow \ln e^{k/1000} = \ln a \Rightarrow \frac{k}{1000} \ln e = \ln a \Rightarrow \frac{k}{1000} = \ln a \Rightarrow k = 1000 \ln a$

12. (a) $e^{5k} = \frac{1}{4} \Rightarrow \ln e^{5k} = \ln 4^{-1} \Rightarrow 5k \ln e = -\ln 4 \Rightarrow 5k = -\ln 4 \Rightarrow k = -\frac{\ln 4}{5}$
   (b) $80e^k = 1 \Rightarrow e^k = 80^{-1} \Rightarrow \ln e^k = \ln 80^{-1} \Rightarrow k \ln e = -\ln 80 \Rightarrow k = -\ln 80$
   (c) $e^{(\ln 0.8)k} = 0.8 \Rightarrow \left(e^{\ln 0.8}\right)^k = 0.8 \Rightarrow (0.8)^k = 0.8 \Rightarrow k = 1$

13. (a) $e^{-0.3t} = 27 \Rightarrow \ln e^{-0.3t} = \ln 3^3 \Rightarrow (-0.3t) \ln e = 3 \ln 3 \Rightarrow -0.3t = 3 \ln 3 \Rightarrow t = -10 \ln 3$
   (b) $e^{kt} = \frac{1}{2} \Rightarrow \ln e^{kt} = \ln 2^{-1} = kt \ln e = -\ln 2 \Rightarrow t = -\frac{\ln 2}{k}$
   (c) $e^{(\ln 0.2)t} = 0.4 \Rightarrow \left(e^{\ln 0.2}\right)^t = 0.4 \Rightarrow 0.2^t = 0.4 \Rightarrow \ln 0.2^t = \ln 0.4 \Rightarrow t \ln 0.2 = \ln 0.4 \Rightarrow t = \frac{\ln 0.4}{\ln 0.2}$

14. (a) $e^{-0.01t} = 1000 \Rightarrow \ln e^{-0.01t} = \ln 1000 \Rightarrow (-0.01t) \ln e = \ln 1000 \Rightarrow -0.01t = \ln 1000 \Rightarrow t = -100 \ln 1000$
   (b) $e^{kt} = \frac{1}{10} \Rightarrow \ln e^{kt} = \ln 10^{-1} = kt \ln e = -\ln 10 \Rightarrow kt = -\ln 10 \Rightarrow t = -\frac{\ln 10}{k}$
   (c) $e^{(\ln 2)t} = \frac{1}{2} \Rightarrow \left(e^{\ln 2}\right)^t = 2^{-1} \Rightarrow 2^t = 2^{-1} \Rightarrow t = -1$

15. $e^{\sqrt{t}} = x^2 \Rightarrow \ln e^{\sqrt{t}} = \ln x^2 \Rightarrow \sqrt{t} = 2 \ln x \Rightarrow t = 4(\ln x)^2$

16. $e^{x^2} e^{2x+1} = e^t \Rightarrow e^{x^2+2x+1} = e^t \Rightarrow \ln e^{x^2+2x+1} = \ln e^t \Rightarrow t = x^2 + 2x + 1$

G.

1. (a) $5^{\log_5 7} = 7$ (b) $8^{\log_8 \sqrt{2}} = \sqrt{2}$ (c) $1.3^{\log_{1.3} 75} = 75$
   (d) $\log_4 16 = \log_4 4^2 = 2 \log_4 4 = 2 \cdot 1 = 2$ (e) $\log_3 \sqrt{3} = \log_3 3^{1/2} = \frac{1}{2} \log_3 3 = \frac{1}{2} \cdot 1 = \frac{1}{2} = 0.5$
   (f) $\log_4 \left(\frac{1}{4}\right) = \log_4 4^{-1} = -1 \log_4 4 = -1 \cdot 1 = -1$

2. (a) $2^{\log_2 3} = 3$ (b) $10^{\log_{10}(1/2)} = \frac{1}{2}$ (c) $\pi^{\log_\pi 7} = 7$
   (d) $\log_{11} 121 = \log_{11} 11^2 = 2 \log_{11} 11 = 2 \cdot 1 = 2$
   (e) $\log_{121} 11 = \log_{121} 121^{1/2} = \left(\frac{1}{2}\right) \log_{121} 121 = \left(\frac{1}{2}\right) \cdot 1 = \frac{1}{2}$
   (f) $\log_3 \left(\frac{1}{9}\right) = \log_3 3^{-2} = -2 \log_3 3 = -2 \cdot 1 = -2$

3. (a) Let $z = \log_4 x \Rightarrow 4^z = x \Rightarrow 2^{2z} = x \Rightarrow (2^z)^2 = x \Rightarrow 2^z = \sqrt{x}$
   (b) Let $z = \log_3 x \Rightarrow 3^z = x \Rightarrow (3^z)^2 = x^2 \Rightarrow 3^{2z} = x^2 \Rightarrow 9^z = x^2$
   (c) $\log_2 \left(e^{(\ln 2) \sin x}\right) = \log_2 2^{\sin x} = \sin x$

4. (a) Let $z = \log_5 (3x^2) \Rightarrow 5^z = 3x^2 \Rightarrow 25^z = 9x^4$
   (b) $\log_e (e^x) = x$
   (c) $\log_4 \left(2^{e^x \sin x}\right) = \log_4 4^{(e^x \sin x)/2} = \frac{e^x \sin x}{2}$

5. (a) $\frac{\log_2 x}{\log_3 x} = \frac{\ln x}{\ln 2} \div \frac{\ln x}{\ln 3} = \frac{\ln x}{\ln 2} \cdot \frac{\ln 3}{\ln x} = \frac{\ln 3}{\ln 2}$ (b) $\frac{\log_2 x}{\log_8 x} = \frac{\ln x}{\ln 2} \div \frac{\ln x}{\ln 8} = \frac{\ln x}{\ln 2} \cdot \frac{\ln 8}{\ln x} = \frac{3 \ln 2}{\ln 2} = 3$
   (c) $\frac{\log_x a}{\log_{x^2} a} = \frac{\ln a}{\ln x} \div \frac{\ln a}{\ln x^2} = \frac{\ln a}{\ln x} \cdot \frac{\ln x^2}{\ln a} = \frac{2 \ln x}{\ln x} = 2$

6. (a) $\frac{\log_9 x}{\log_3 x} = \frac{\ln x}{\ln 9} \div \frac{\ln x}{\ln 3} = \frac{\ln x}{2 \ln 3} \cdot \frac{\ln 3}{\ln x} = \frac{1}{2}$
   (b) $\frac{\log_{\sqrt{10}} x}{\log_{\sqrt{2}} x} = \frac{\ln x}{\ln \sqrt{10}} \div \frac{\ln x}{\ln \sqrt{2}} = \frac{\ln x}{\left(\frac{1}{2}\right) \ln 10} \cdot \frac{\left(\frac{1}{2}\right) \ln 2}{\ln x} = \frac{\ln 2}{\ln 10}$
   (c) $\frac{\log_a b}{\log_b a} = \frac{\ln b}{\ln a} \div \frac{\ln a}{\ln b} = \frac{\ln b}{\ln a} \cdot \frac{\ln b}{\ln a} = \left(\frac{\ln b}{\ln a}\right)^2$

7. $3^{\log_3(7)} + 2^{\log_2(5)} = 5^{\log_5(x)} \Rightarrow 7 + 5 = x \Rightarrow x = 12$

8. $8^{\log_8(3)} - e^{\ln 5} = x^2 - 7^{\log_7(3x)} \Rightarrow 3 - 5 = x^2 - 3x \Rightarrow 0 = x^2 - 3x + 2 = (x-1)(x-2) \Rightarrow x = 1$ or $x = 2$

9. $3^{\log_3(x^2)} = 5e^{\ln x} - 3 \cdot 10^{\log_{10}(2)} \Rightarrow x^2 = 5x - 6 \Rightarrow x^2 - 5x + 6 = 0 \Rightarrow (x-2)(x-3) = 0 \Rightarrow x = 2$ or $x = 3$

10. $\ln e + 4^{-2\log_4(x)} = \frac{1}{x} \log_{10} 100 \Rightarrow 1 + 4^{\log_4(x^{-2})} = \frac{1}{x} \log_{10} 10^2 \Rightarrow 1 + x^{-2} = \left(\frac{1}{x}\right)(2) \Rightarrow 1 + \frac{1}{x^2} - \frac{2}{x} = 0$
    $\Rightarrow x^2 - 2x + 1 = 0 \Rightarrow (x-1)^2 = 0 \Rightarrow x = 1$