A.

Derivative of Trigonometric Functions

In Exercises 1–12, find dy/dx.

1. $y = -10x + 3\cos x$
2. $y = \frac{3}{x} + 5\sin x$
3. $y = \csc x - 4\sqrt{x} + 7$
4. $y = x^2 \cot x - \frac{1}{x^2}$
5. $y = (\sec x + \tan x)(\sec x - \tan x)$
6. $y = (\sin x + \cos x)\sec x$
7. $y = \frac{\cot x}{1 + \cot x}$
8. $y = \frac{\cos x}{1 + \sin x}$
9. $y = \frac{4}{\cos x} + \frac{1}{\tan x}$
10. $y = \frac{\cos x}{x} + \frac{x}{\cos x}$
11. $y = x^2 \sin x + 2x\cos x - 2\sin x$
12. $y = x^2 \cos x - 2x\sin x - 2\cos x$

In Exercises 13–16, find ds/dt.

13. $s = \tan t - t$
14. $s = t^2 - \sec t + 1$
15. $s = \frac{1 + \csc t}{1 - \csc t}$
16. $s = \frac{\sin t}{1 - \cos t}$

In Exercises 17–20, find $dr/d\theta$.

17. $r = 4 - \theta^2 \sin\theta$
18. $r = \theta\sin\theta + \cos\theta$
19. $r = \sec\theta\csc\theta$
20. $r = (1 + \sec\theta)\sin\theta$

In Exercises 21–24, find dp/dq.

21. $p = 5 + \frac{1}{\cot q}$
22. $p = (1 + \csc q)\cos q$
23. $p = \frac{\sin q + \cos q}{\cos q}$
24. $p = \frac{\tan q}{1 + \tan q}$
25. Find y'' if

    a. $y = \csc x$.

    b. $y = \sec x$.

B.

The Chain Rule and Parametric Questions

Derivative Calculations

In Exercises 1–8, given $y = f(u)$ and $u = g(x)$, find $dy/dx = f'(g(x))g'(x)$.

1. $y = 6u - 9, \quad u = (1/2)x^4$
2. $y = 2u^3, \quad u = 8x - 1$
3. $y = \sin u, \quad u = 3x + 1$
4. $y = \cos u, \quad u = -x/3$
5. $y = \cos u, \quad u = \sin x$
6. $y = \sin u, \quad u = x - \cos x$
7. $y = \tan u, \quad u = 10x - 5$
8. $y = -\sec u, \quad u = x^2 + 7x$

In Exercises 9–18, write the function in the form $y = f(u)$ and $u = g(x)$. Then find dy/dx as a function of x.

9. $y = (2x + 1)^5$
10. $y = (4 - 3x)^9$
11. $y = \left(1 - \frac{x}{7}\right)^{-7}$
12. $y = \left(\frac{x}{2} - 1\right)^{-10}$
13. $y = \left(\frac{x^2}{8} + x - \frac{1}{x}\right)^4$
14. $y = \left(\frac{x}{5} + \frac{1}{5x}\right)^5$
15. $y = \sec(\tan x)$
16. $y = \cot\left(\pi - \frac{1}{x}\right)$
17. $y = \sin^3 x$
18. $y = 5\cos^{-4} x$

Find the derivatives of the functions in Exercises 19–38.

19. $p = \sqrt{3 - t}$
20. $q = \sqrt{2r - r^2}$
21. $s = \frac{4}{3\pi}\sin 3t + \frac{4}{5\pi}\cos 5t$
22. $s = \sin\left(\frac{3\pi t}{2}\right) + \cos\left(\frac{3\pi t}{2}\right)$
23. $r = (\csc\theta + \cot\theta)^{-1}$
24. $r = -(\sec\theta + \tan\theta)^{-1}$
25. $y = x^2\sin^4 x + x\cos^{-2} x$
26. $y = \frac{1}{x}\sin^{-5} x - \frac{x}{3}\cos^3 x$
27. $y = \frac{1}{21}(3x - 2)^7 + \left(4 - \frac{1}{2x^2}\right)^{-1}$
28. $y = (5 - 2x)^{-3} + \frac{1}{8}\left(\frac{2}{x} + 1\right)^4$
29. $y = (4x + 3)^4(x + 1)^{-3}$
30. $y = (2x - 5)^{-1}(x^2 - 5x)^6$
31. $h(x) = x\tan\left(2\sqrt{x}\right) + 7$
32. $k(x) = x^2\sec\left(\frac{1}{x}\right)$
33. $f(\theta) = \left(\frac{\sin\theta}{1 + \cos\theta}\right)^2$
34. $g(t) = \left(\frac{1 + \cos t}{\sin t}\right)^{-1}$
35. $r = \sin(\theta^2)\cos(2\theta)$
36. $r = \sec\sqrt{\theta}\tan\left(\frac{1}{\theta}\right)$
37. $q = \sin\left(\frac{t}{\sqrt{t + 1}}\right)$
38. $q = \cot\left(\frac{\sin t}{t}\right)$

In Exercises 39–48, find dy/dt.

39. $y = \sin^2(\pi t - 2)$
40. $y = \sec^2 \pi t$
41. $y = (1 + \cos 2t)^{-4}$
42. $y = (1 + \cot(t/2))^{-2}$
43. $y = \sin(\cos(2t - 5))$
44. $y = \cos\left(5\sin\left(\frac{t}{3}\right)\right)$
45. $y = \left(1 + \tan^4\left(\frac{t}{12}\right)\right)^3$
46. $y = \frac{1}{6}\left(1 + \cos^2(7t)\right)^3$
47. $y = \sqrt{1 + \cos(t^2)}$
48. $y = 4\sin\left(\sqrt{1 + \sqrt{t}}\right)$

Second Derivatives

Find y'' in Exercises 49–52.

49. $y = \left(1 + \frac{1}{x}\right)^3$
50. $y = \left(1 - \sqrt{x}\right)^{-1}$
51. $y = \frac{1}{9}\cot(3x - 1)$
52. $y = 9\tan\left(\frac{x}{3}\right)$

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Derivatives of Rational Powers

Find dy/dx in Exercises 1-10.

1. $y = x^{9/4}$
2. $y = x^{-3/5}$
3. $y = \sqrt[3]{2x}$
4. $y = \sqrt[4]{5x}$
5. $y = 7\sqrt{x+6}$
6. $y = -2\sqrt{x-1}$
7. $y = (2x+5)^{-1/2}$
8. $y = (1-6x)^{2/3}$
9. $y = x(x^2+1)^{1/2}$
10. $y = x(x^2+1)^{-1/2}$

Find the first derivatives of the functions in Exercises 11-18.

11. $s = \sqrt[7]{t^2}$
12. $r = \sqrt[4]{\theta^{-3}}$
13. $y = \sin[(2t+5)^{-2/3}]$
14. $z = \cos[(1-6t)^{2/3}]$
15. $f(x) = \sqrt{1-\sqrt{x}}$
16. $g(x) = 2(2x^{-1/2}+1)^{-1/3}$
17. $h(\theta) = \sqrt[3]{1+\cos(2\theta)}$
18. $k(\theta) = (\sin(\theta+5))^{5/4}$

Differentiating Implicitly

Use implicit differentiation to find dy/dx in Exercises 19-32.

19. $x^2y + xy^2 = 6$
20. $x^3 + y^3 = 18xy$
21. $2xy + y^2 = x + y$
22. $x^3 - xy + y^3 = 1$
23. $x^2(x-y)^2 = x^2 - y^2$
24. $(3xy+7)^2 = 6y$
25. $y^2 = \dfrac{x-1}{x+1}$
26. $x^2 = \dfrac{x-y}{x+y}$
27. $x = \tan y$
28. $xy = \cot(xy)$
29. $x + \tan(xy) = 0$
30. $x + \sin y = xy$
31. $y\sin\left(\dfrac{1}{y}\right) = 1 - xy$
32. $y^2\cos\left(\dfrac{1}{y}\right) = 2x + 2y$

Find $dr/d\theta$ in Exercises 33-36.

33. $\theta^{1/2} + r^{1/2} = 1$
34. $r - 2\sqrt{\theta} = \frac{3}{2}\theta^{2/3} + \frac{4}{3}\theta^{3/4}$
35. $\sin(r\theta) = \frac{1}{2}$
36. $\cos r + \cot\theta = r\theta$

Second Derivatives

In Exercises 37-42, use implicit differentiation to find dy/dx and then d^2y/dx^2.

37. $x^2 + y^2 = 1$
38. $x^{2/3} + y^{2/3} = 1$
39. $y^2 = x^2 + 2x$
40. $y^2 - 2x = 1 - 2y$
41. $2\sqrt{y} = x - y$
42. $xy + y^2 = 1$
43. If $x^3 + y^3 = 16$, find the value of d^2y/dx^2 at the point (2, 2).
44. If $xy + y^2 = 1$, find the value of d^2y/dx^2 at the point $(0, -1)$.

Slopes, Tangents, and Normals

In Exercises 45 and 46, find the slope of the curve at the given points.

45. $y^2 + x^2 = y^4 - 2x$ at $(-2, 1)$ and $(-2, -1)$
46. $(x^2+y^2)^2 = (x-y)^2$ at $(1, 0)$ and $(1, -1)$

In Exercises 47-56, verify that the given point is on the curve and find the lines that are **(a)** tangent and **(b)** normal to the curve at the given point.

47. $x^2 + xy - y^2 = 1$, $(2, 3)$
48. $x^2 + y^2 = 25$, $(3, -4)$
49. $x^2y^2 = 9$, $(-1, 3)$

50. $y^2 - 2x - 4y - 1 = 0$, $(-2, 1)$
51. $6x^2 + 3xy + 2y^2 + 17y - 6 = 0$, $(-1, 0)$
52. $x^2 - \sqrt{3}xy + 2y^2 = 5$, $\left(\sqrt{3}, 2\right)$
53. $2xy + \pi\sin y = 2\pi$, $(1, \pi/2)$
54. $x\sin 2y = y\cos 2x$, $(\pi/4, \pi/2)$
55. $y = 2\sin(\pi x - y)$, $(1, 0)$
56. $x^2\cos^2 y - \sin y = 0$, $(0, \pi)$

D. Related Rates

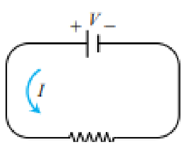
1. **Area** Suppose that the radius r and area $A = \pi r^2$ of a circle are differentiable functions of t. Write an equation that relates dA/dt to dr/dt.

2. **Surface area** Suppose that the radius r and surface area $S = 4\pi r^2$ of a sphere are differentiable functions of t. Write an equation that relates dS/dt to dr/dt.

3. **Volume** The radius r and height h of a right circular cylinder are related to the cylinder's volume V by the formula $V = \pi r^2 h$.
   - **a.** How is dV/dt related to dh/dt if r is constant?
   - **b.** How is dV/dt related to dr/dt if h is constant?
   - **c.** How is dV/dt related to dr/dt and dh/dt if neither r nor h is constant?

4. **Volume** The radius r and height h of a right circular cone are related to the cone's volume V by the equation $V = (1/3)\pi r^2 h$.
   - **a.** How is dV/dt related to dh/dt if r is constant?
   - **b.** How is dV/dt related to dr/dt if h is constant?
   - **c.** How is dV/dt related to dr/dt and dh/dt if neither r nor h is constant?

5. **Changing voltage** The voltage V (volts), current I (amperes), and resistance R (ohms) of an electric circuit like the one shown here are related by the equation $V = IR$. Suppose that V is increasing at the rate of 1 volt/sec while I is decreasing at the rate of 1/3 amp/sec. Let t denote time in seconds.



   - **a.** What is the value of dV/dt?
   - **b.** What is the value of dI/dt?

7.
   - **b.** How is ds/dt related to dx/dt and dy/dt if neither x nor y is constant?
   - **c.** How is dx/dt related to dy/dt if s is constant?

8. **Diagonals** If x, y, and z are lengths of the edges of a rectangular box, the common length of the box's diagonals is $s = \sqrt{x^2 + y^2 + z^2}$.
   - **a.** Assuming that x, y, and z are differentiable functions of t, how is ds/dt related to dx/dt, dy/dt, and dz/dt?
   - **b.** How is ds/dt related to dy/dt and dz/dt if x is constant?
   - **c.** How are dx/dt, dy/dt, and dz/dt related if s is constant?

9. **Area** The area A of a triangle with sides of lengths a and b enclosing an angle of measure θ is

$$A = \frac{1}{2}ab\sin\theta.$$

   - **a.** How is dA/dt related to $d\theta/dt$ if a and b are constant?
   - **b.** How is dA/dt related to $d\theta/dt$ and da/dt if only b is constant?
   - **c.** How is dA/dt related to $d\theta/dt$, da/dt, and db/dt if none of a, b, and θ are constant?

10. **Heating a plate** When a circular plate of metal is heated in an oven, its radius increases at the rate of 0.01 cm/min. At what rate is the plate's area increasing when the radius is 50 cm?

11. **Changing dimensions in a rectangle** The length l of a rectangle is decreasing at the rate of 2 cm/sec while the width w is increasing at the rate of 2 cm/sec. When $l = 12$ cm and $w = 5$ cm, find the rates of change of **(a)** the area, **(b)** the perimeter, and **(c)** the lengths of the diagonals of the rectangle. Which of these quantities are decreasing, and which are increasing?

R

a. What is the value of dV/dt?

b. What is the value of dI/dt?

c. What equation relates dR/dt to dV/dt and dI/dt?

d. Find the rate at which R is changing when $V = 12$ volts and $I = 2$ amp. Is R increasing, or decreasing?

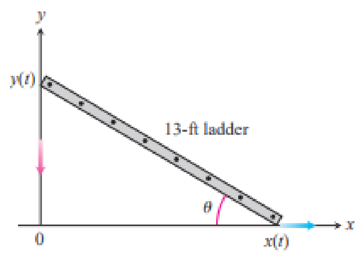
6. **Electrical power** The power P (watts) of an electric circuit is related to the circuit's resistance R (ohms) and current I (amperes) by the equation $P = RI^2$.

a. How are dP/dt, dR/dt, and dI/dt related if none of P, R, and I are constant?

b. How is dR/dt related to dI/dt if P is constant?

7. **Distance** Let x and y be differentiable functions of t and let $s = \sqrt{x^2 + y^2}$ be the distance between the points $(x, 0)$ and $(0, y)$ in the xy-plane.

a. How is ds/dt related to dx/dt if y is constant?



14. **Commercial air traffic** Two commercial airplanes are flying at 40,000 ft along straight-line courses that intersect at right angles. Plane A is approaching the intersection point at a speed of 442 knots (nautical miles per hour; a nautical mile is 2000 yd). Plane B is approaching the intersection at 481 knots. At what rate is the distance between the planes changing when A is 5 nautical miles from the intersection point and B is 12 nautical miles from the intersection point?

15. **Flying a kite** A girl flies a kite at a height of 300 ft, the wind carrying the kite horizontally away from her at a rate of 25 ft/sec. How fast must she let out the string when the kite is 500 ft away from her?

16. **Boring a cylinder** The mechanics at Lincoln Automotive are reboring a 6-in.-deep cylinder to fit a new piston. The machine they are using increases the cylinder's radius one-thousandth of an inch every 3 min. How rapidly is the cylinder volume increasing when the bore (diameter) is 3.800 in.?

17. **A growing sand pile** Sand falls from a conveyor belt at the rate of $10\ \text{m}^3/\text{min}$ onto the top of a conical pile. The height of the pile is always three-eighths of the base diameter. How fast are the **(a)** height and **(b)** radius changing when the pile is 4 m high? Answer

12. **Changing dimensions in a rectangular box** Suppose that the edge lengths x, y, and z of a closed rectangular box are changing at the following rates:

$$\frac{dx}{dt} = 1\ \text{m/sec}, \quad \frac{dy}{dt} = -2\ \text{m/sec}, \quad \frac{dz}{dt} = 1\ \text{m/sec}.$$

Find the rates at which the box's **(a)** volume, **(b)** surface area, and **(c)** diagonal length $s = \sqrt{x^2 + y^2 + z^2}$ are changing at the instant when $x = 4$, $y = 3$, and $z = 2$.

13. **A sliding ladder** A 13-ft ladder is leaning against a house when its base starts to slide away. By the time the base is 12 ft from the house, the base is moving at the rate of 5 ft/sec.

a. How fast is the top of the ladder sliding down the wall then?

b. At what rate is the area of the triangle formed by the ladder, wall, and ground changing then?

c. At what rate is the angle θ between the ladder and the ground changing then?

a. At what rate is the water level changing when the water is 8 m deep?

b. What is the radius r of the water's surface when the water is y m deep?

c. At what rate is the radius r changing when the water is 8 m deep?

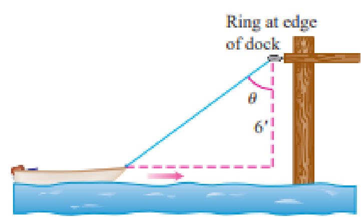
20. **A growing raindrop** Suppose that a drop of mist is a perfect sphere and that, through condensation, the drop picks up moisture at a rate proportional to its surface area. Show that under these circumstances the drop's radius increases at a constant rate.

21. **The radius of an inflating balloon** A spherical balloon is inflated with helium at the rate of $100\pi\ \text{ft}^3/\text{min}$. How fast is the balloon's radius increasing at the instant the radius is 5 ft? How fast is the surface area increasing?

22. **Hauling in a dinghy** A dinghy is pulled toward a dock by a rope from the bow through a ring on the dock 6 ft above the bow. The rope is hauled in at the rate of 2 ft/sec.

a. How fast is the boat approaching the dock when 10 ft of rope are out?

b. At what rate is the angle θ changing then (see the figure)?



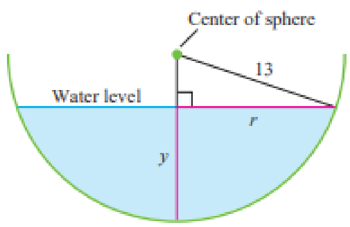
in centimeters per minute.

18. A draining conical reservoir Water is flowing at the rate of $50\ \text{m}^3/\text{min}$ from a shallow concrete conical reservoir (vertex down) of base radius 45 m and height 6 m.

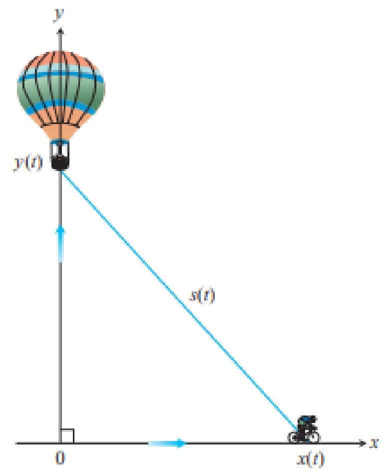
a. How fast (centimeters per minute) is the water level falling when the water is 5 m deep?

b. How fast is the radius of the water's surface changing then? Answer in centimeters per minute.

19. A draining hemispherical reservoir Water is flowing at the rate of $6\ \text{m}^3/\text{min}$ from a reservoir shaped like a hemispherical bowl of radius 13 m, shown here in profile. Answer the following questions, given that the volume of water in a hemispherical bowl of radius R is $V = (\pi/3)y^2(3R - y)$ when the water is y meters deep.



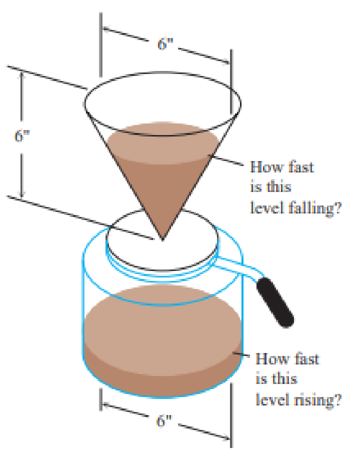
23. A balloon and a bicycle A balloon is rising vertically above a level, straight road at a constant rate of 1 ft/sec. Just when the balloon is 65 ft above the ground, a bicycle moving at a constant rate of 17 ft/sec passes under it. How fast is the distance $s(t)$ between the bicycle and balloon increasing 3 sec later?



24. Making coffee Coffee is draining from a conical filter into a cylindrical coffeepot at the rate of $10\ \text{in}^3/\text{min}$.

a. How fast is the level in the pot rising when the coffee in the cone is 5 in. deep?

b. How fast is the level in the cone falling then?



26. Cost, revenue, and profit A company can manufacture x items at a cost of $c(x)$ thousand dollars, a sales revenue of $r(x)$ thousand dollars, and a profit of $p(x) = r(x) - c(x)$ thousand dollars. Find dc/dt, dr/dt, and dp/dt for the following values of x and dx/dt.

a. $r(x) = 9x$, $c(x) = x^3 - 6x^2 + 15x$, and $dx/dt = 0.1$ when $x = 2$

b. $r(x) = 70x$, $c(x) = x^3 - 6x^2 + 45/x$, and $dx/dt = 0.05$ when $x = 1.5$

27. Moving along a parabola A particle moves along the parabola $y = x^2$ in the first quadrant in such a way that its x-coordinate (measured in meters) increases at a steady 10 m/sec. How fast is the angle of inclination θ of the line joining the particle to the origin changing when $x = 3$ m?

28. Moving along another parabola A particle moves from right to left along the parabolic curve $y = \sqrt{-x}$ in such a way that its x-coordinate (measured in meters) decreases at the rate of 8 m/sec. How fast is the angle of inclination θ of the line joining the particle to the origin changing when $x = -4$?

29. Motion in the plane The coordinates of a particle in the metric xy-plane are differentiable functions of time t with $dx/dt = -1$ m/sec and $dy/dt = -5$ m/sec. How fast is the particle's distance from the origin changing as it passes through the point $(5, 12)$?

25. Cardiac output In the late 1860s, Adolf Fick, a professor of physiology in the Faculty of Medicine in Würzberg, Germany, developed one of the methods we use today for measuring how much blood your heart pumps in a minute. Your cardiac output as you read this sentence is probably about 7 L/min. At rest it is likely to be a bit under 6 L/min. If you are a trained marathon runner running a marathon, your cardiac output can be as high as 30 L/min.

Your cardiac output can be calculated with the formula

$$y = \frac{Q}{D},$$

where Q is the number of milliliters of CO_2 you exhale in a minute and D is the difference between the CO_2 concentration (ml/L) in the blood pumped to the lungs and the CO_2 concentration in the blood returning from the lungs. With $Q = 233$ ml/min and $D = 97 - 56 = 41$ ml/L,

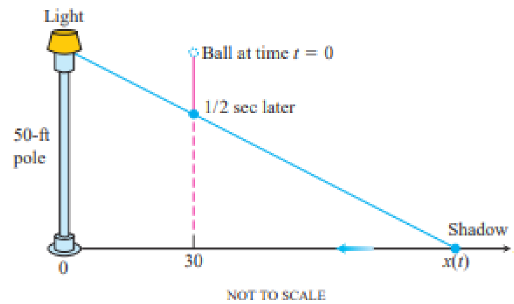
$$y = \frac{233 \text{ ml/min}}{41 \text{ ml/L}} \approx 5.68 \text{ L/min},$$

fairly close to the 6 L/min that most people have at basal (resting) conditions. (Data courtesy of J. Kenneth Herd, M.D., Quillan College of Medicine, East Tennessee State University.)

Suppose that when $Q = 233$ and $D = 41$, we also know that D is decreasing at the rate of 2 units a minute but that Q remains unchanged. What is happening to the cardiac output?

30. A moving shadow A man 6 ft tall walks at the rate of 5 ft/sec toward a streetlight that is 16 ft above the ground. At what rate is the tip of his shadow moving? At what rate is the length of his shadow changing when he is 10 ft from the base of the light?

31. Another moving shadow A light shines from the top of a pole 50 ft high. A ball is dropped from the same height from a point 30 ft away from the light. (See accompanying figure.) How fast is the shadow of the ball moving along the ground 1/2 sec later? (Assume the ball falls a distance $s = 16t^2$ ft in t sec.)



32. Videotaping a moving car You are videotaping a race from a stand 132 ft from the track, following a car that is moving at 180 mi/h (264 ft/sec). How fast will your camera angle θ be changing when the car is right in front of you? A half second later?

E.

Indeterminate Form and L'Hopital's Rule

Finding Limits

In Exercises 1–6, use l'Hôpital's Rule to evaluate the limit. Then evaluate the limit using a method studied in Chapter 2.

1. $\lim_{x\to 2} \frac{x-2}{x^2-4}$

2. $\lim_{x\to 0} \frac{\sin 5x}{x}$

3. $\lim_{x\to\infty} \frac{5x^2-3x}{7x^2+1}$

4. $\lim_{x\to 1} \frac{x^3-1}{4x^3-x-3}$

5. $\lim_{x\to 0} \frac{1-\cos x}{x^2}$

6. $\lim_{x\to\infty} \frac{2x^2+3x}{x^3+x+1}$

Applying l'Hôpital's Rule

Use l'Hôpital's Rule to find the limits in Exercises 7–26.

7. $\lim_{t\to 0} \frac{\sin t^2}{t}$

8. $\lim_{x\to\pi/2} \frac{2x-\pi}{\cos x}$

9. $\lim_{\theta\to\pi} \frac{\sin\theta}{\pi-\theta}$

10. $\lim_{x\to\pi/2} \frac{1-\sin x}{1+\cos 2x}$

11. $\lim_{x\to\pi/4} \frac{\sin x-\cos x}{x-\pi/4}$

12. $\lim_{x\to\pi/3} \frac{\cos x-0.5}{x-\pi/3}$

13. $\lim_{x\to(\pi/2)^-} \left(x-\frac{\pi}{2}\right)\tan x$

14. $\lim_{x\to 0} \frac{2x}{x+7\sqrt{x}}$

15. $\lim_{x\to 1} \frac{2x^2-(3x+1)\sqrt{x}+2}{x-1}$

16. $\lim_{x\to 2} \frac{\sqrt{x^2+5}-3}{x^2-4}$

17. $\lim_{x\to 0} \frac{\sqrt{a(a+x)}-a}{x}, \quad a>0$

18. $\lim_{t\to 0} \frac{10(\sin t-t)}{t^3}$

19. $\lim_{x\to 0} \frac{x(\cos x-1)}{\sin x-x}$

20. $\lim_{h\to 0} \frac{\sin(a+h)-\sin a}{h}$

21. $\lim_{r\to 1} \frac{a(r^n-1)}{r-1}$, n a positive integer

22. $\lim_{x\to 0^+} \left(\frac{1}{x}-\frac{1}{\sqrt{x}}\right)$

23. $\lim_{x\to\infty} (x-\sqrt{x^2+x})$

24. $\lim_{x\to\infty} x\tan\frac{1}{x}$

25. $\lim_{x\to\pm\infty} \frac{3x-5}{2x^2-x+2}$

26. $\lim_{x\to 0} \frac{\sin 7x}{\tan 11x}$

Theory and Applications

L'Hôpital's Rule does not help with the limits in Exercises 27–30. Try it; you just keep on cycling. Find the limits some other way.

27. $\lim_{x\to\infty} \frac{\sqrt{9x+1}}{\sqrt{x+1}}$

28. $\lim_{x\to 0^+} \frac{\sqrt{x}}{\sqrt{\sin x}}$

29. $\lim_{x\to(\pi/2)^-} \frac{\sec x}{\tan x}$

30. $\lim_{x\to 0^+} \frac{\cot x}{\csc x}$

31. Which one is correct, and which one is wrong? Give reasons for your answers.

a. $\lim_{x\to 3} \frac{x-3}{x^2-3} = \lim_{x\to 3} \frac{1}{2x} = \frac{1}{6}$

b. $\lim_{x\to 3} \frac{x-3}{x^2-3} = \frac{0}{6} = 0$

32. **∞/∞ Form** Give an example of two differentiable functions f and g with $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} g(x) = \infty$ that satisfy the following.

a. $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 3$

b. $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$

c. $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \infty$

33. **Continuous extension** Find a value of c that makes the function

$$f(x) = \begin{cases} \dfrac{9x-3\sin 3x}{5x^3}, & x\neq 0 \\ c, & x=0 \end{cases}$$

continuous at $x=0$. Explain why your value of c works.

34. Let

$$f(x) = \begin{cases} x+2, & x\neq 0 \\ 0, & x=0 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} x+1, & x\neq 0 \\ 0, & x=0. \end{cases}$$

a. Show that

$$\lim_{x\to 0} \frac{f'(x)}{g'(x)} = 1 \quad \text{but} \quad \lim_{x\to 0} \frac{f(x)}{g(x)} = 2.$$

b. Explain why this does not contradict l'Hôpital's Rule.

T 35. **0/0 Form** Estimate the value of

$$\lim_{x\to 1} \frac{2x^2-(3x+1)\sqrt{x}+2}{x-1}$$

by graphing. Then confirm your estimate with l'Hôpital's Rule.

T 36. **$\infty-\infty$ Form**

a. Estimate the value of

$$\lim_{x\to\infty} \left(x-\sqrt{x^2+x}\right)$$

by graphing $f(x) = x-\sqrt{x^2+x}$ over a suitably large interval of x-values.

b. Now confirm your estimate by finding the limit with l'Hôpital's Rule. As the first step, multiply $f(x)$ by the fraction $\left(x+\sqrt{x^2+x}\right)/\left(x+\sqrt{x^2+x}\right)$ and simplify the new numerator.

F

Derivatives of Logarithms

In Exercises 5–36, find the derivative of y with respect to x, t, or θ, as appropriate.

5. $y = \ln 3x$

6. $y = \ln kx$, k constant

7. $y = \ln(t^2)$

8. $y = \ln(t^{3/2})$

9. $y = \ln \dfrac{3}{x}$

10. $y = \ln \dfrac{10}{x}$

11. $y = \ln(\theta + 1)$

12. $y = \ln(2\theta + 2)$

13. $y = \ln x^3$

14. $y = (\ln x)^3$

15. $y = t(\ln t)^2$

16. $y = t\sqrt{\ln t}$

17. $y = \dfrac{x^4}{4}\ln x - \dfrac{x^4}{16}$

18. $y = \dfrac{x^3}{3}\ln x - \dfrac{x^3}{9}$

19. $y = \dfrac{\ln t}{t}$

20. $y = \dfrac{1 + \ln t}{t}$

21. $y = \dfrac{\ln x}{1 + \ln x}$

22. $y = \dfrac{x \ln x}{1 + \ln x}$

23. $y = \ln(\ln x)$

24. $y = \ln(\ln(\ln x))$

25. $y = \theta(\sin(\ln \theta) + \cos(\ln \theta))$

26. $y = \ln(\sec\theta + \tan\theta)$

27. $y = \ln \dfrac{1}{x\sqrt{x + 1}}$

28. $y = \dfrac{1}{2}\ln \dfrac{1 + x}{1 - x}$

29. $y = \dfrac{1 + \ln t}{1 - \ln t}$

30. $y = \sqrt{\ln \sqrt{t}}$

31. $y = \ln(\sec(\ln \theta))$

32. $y = \ln\left(\dfrac{\sqrt{\sin\theta\cos\theta}}{1 + 2\ln\theta}\right)$

33. $y = \ln\left(\dfrac{(x^2 + 1)^5}{\sqrt{1 - x}}\right)$

34. $y = \ln\sqrt{\dfrac{(x + 1)^5}{(x + 2)^{20}}}$

35. $y = \displaystyle\int_{x^2/2}^{x^2} \ln\sqrt{t}\,dt$

36. $y = \displaystyle\int_{\sqrt{x}}^{\sqrt[3]{x}} \ln t\,dt$

Integration

Evaluate the integrals in Exercises 37–54.

37. $\displaystyle\int_{-3}^{-2} \frac{dx}{x}$

38. $\displaystyle\int_{-1}^{0} \frac{3\,dx}{3x - 2}$

39. $\displaystyle\int \frac{2y\,dy}{y^2 - 25}$

40. $\displaystyle\int \frac{8r\,dr}{4r^2 - 5}$

41. $\displaystyle\int_0^{\pi} \frac{\sin t}{2 - \cos t}\,dt$

42. $\displaystyle\int_0^{\pi/3} \frac{4\sin\theta}{1 - 4\cos\theta}\,d\theta$

43. $\displaystyle\int_1^2 \frac{2\ln x}{x}\,dx$

44. $\displaystyle\int_2^4 \frac{dx}{x\ln x}$

45. $\displaystyle\int_2^4 \frac{dx}{x(\ln x)^2}$

46. $\displaystyle\int_2^{16} \frac{dx}{2x\sqrt{\ln x}}$

47. $\displaystyle\int \frac{3\sec^2 t}{6 + 3\tan t}\,dt$

48. $\displaystyle\int \frac{\sec y\tan y}{2 + \sec y}\,dy$

49. $\displaystyle\int_0^{\pi/2} \tan\frac{x}{2}\,dx$

50. $\displaystyle\int_{\pi/4}^{\pi/2} \cot t\,dt$

51. $\displaystyle\int_{\pi/2}^{\pi} 2\cot\frac{\theta}{3}\,d\theta$

52. $\displaystyle\int_0^{\pi/12} 6\tan 3x\,dx$

53. $\displaystyle\int \frac{dx}{2\sqrt{x} + 2x}$

54. $\displaystyle\int \frac{\sec x\,dx}{\sqrt{\ln(\sec x + \tan x)}}$

G.

The Natural Function

In Exercises 5–10, solve for y in terms of t or x, as appropriate.

5. $\ln y = 2t + 4$ **6.** $\ln y = -t + 5$

7. $\ln(y - 40) = 5t$ **8.** $\ln(1 - 2y) = t$

9. $\ln(y - 1) - \ln 2 = x + \ln x$

10. $\ln(y^2 - 1) - \ln(y + 1) = \ln(\sin x)$

In Exercises 11 and 12, solve for k.

11. **a.** $e^{2k} = 4$ **b.** $100e^{10k} = 200$ **c.** $e^{k/1000} = a$

12. **a.** $e^{5k} = \frac{1}{4}$ **b.** $80e^{k} = 1$ **c.** $e^{(\ln 0.8)k} = 0.8$

In Exercises 13–16, solve for t.

13. **a.** $e^{-0.3t} = 27$ **b.** $e^{kt} = \frac{1}{2}$ **c.** $e^{(\ln 0.2)t} = 0.4$

14. **a.** $e^{-0.01t} = 1000$ **b.** $e^{kt} = \frac{1}{10}$ **c.** $e^{(\ln 2)t} = \frac{1}{2}$

15. $e^{\sqrt{t}} = x^2$ **16.** $e^{(x^2)}e^{(2x+1)} = e^t$

Derivatives

In Exercises 17–36, find the derivative of y with respect to x, t, or θ, as appropriate.

17. $y = e^{-5x}$ **18.** $y = e^{2x/3}$

19. $y = e^{5-7x}$ **20.** $y = e^{(4\sqrt{x}+x^2)}$

21. $y = xe^x - e^x$ **22.** $y = (1 + 2x)e^{-2x}$

23. $y = (x^2 - 2x + 2)e^x$ **24.** $y = (9x^2 - 6x + 2)e^{3x}$

25. $y = e^{\theta}(\sin\theta + \cos\theta)$ **26.** $y = \ln(3\theta e^{-\theta})$

27. $y = \cos(e^{-\theta^2})$ **28.** $y = \theta^3 e^{-2\theta}\cos 5\theta$

29. $y = \ln(3te^{-t})$ **30.** $y = \ln(2e^{-t}\sin t)$

31. $y = \ln\left(\frac{e^{\theta}}{1 + e^{\theta}}\right)$ **32.** $y = \ln\left(\frac{\sqrt{\theta}}{1 + \sqrt{\theta}}\right)$

33. $y = e^{(\cos t + \ln t)}$ **34.** $y = e^{\sin t}(\ln t^2 + 1)$

35. $y = \int_0^{\ln x} \sin e^t\, dt$ **36.** $y = \int_{e^{4\sqrt{x}}}^{e^{2x}} \ln t\, dt$

In Exercises 37–40, find dy/dx.

37. $\ln y = e^y \sin x$ **38.** $\ln xy = e^{x+y}$

39. $e^{2x} = \sin(x + 3y)$ **40.** $\tan y = e^x + \ln x$

Integrals

Evaluate the integrals in Exercises 41–62.

41. $\int (e^{3x} + 5e^{-x})\, dx$ **42.** $\int (2e^x - 3e^{-2x})\, dx$

43. $\int_{\ln 2}^{\ln 3} e^x\, dx$ **44.** $\int_{-\ln 2}^{0} e^{-x}\, dx$

45. $\int 8e^{(x+1)}\, dx$ **46.** $\int 2e^{(2x-1)}\, dx$

47. $\int_{\ln 4}^{\ln 9} e^{x/2}\, dx$ **48.** $\int_0^{\ln 16} e^{x/4}\, dx$

49. $\int \frac{e^{\sqrt{r}}}{\sqrt{r}}\, dr$ **50.** $\int \frac{e^{-\sqrt{r}}}{\sqrt{r}}\, dr$

51. $\int 2t\, e^{-t^2}\, dt$ **52.** $\int t^3 e^{(t^4)}\, dt$

53. $\int \frac{e^{1/x}}{x^2}\, dx$ **54.** $\int \frac{e^{-1/x^2}}{x^3}\, dx$

55. $\int_0^{\pi/4} (1 + e^{\tan\theta})\sec^2\theta\, d\theta$ **56.** $\int_{\pi/4}^{\pi/2} (1 + e^{\cot\theta})\csc^2\theta\, d\theta$

57. $\int e^{\sec \pi t} \sec \pi t \tan \pi t\, dt$

58. $\int e^{\csc(\pi + t)} \csc(\pi + t)\cot(\pi + t)\, dt$

59. $\int_{\ln(\pi/6)}^{\ln(\pi/2)} 2e^v \cos e^v\, dv$ **60.** $\int_0^{\sqrt{\ln \pi}} 2x\, e^{x^2} \cos(e^{x^2})\, dx$

61. $\int \frac{e^r}{1 + e^r}\, dr$ **62.** $\int \frac{dx}{1 + e^x}$

H.

a^x

and

$\log_a x$

49. $\int_0^1 2^{-\theta}\, d\theta$

50. $\int_{-2}^0 5^{-\theta}\, d\theta$

51. $\int_1^{\sqrt{2}} x2^{(x^2)}\, dx$

52. $\int_1^4 \frac{2^{\sqrt{x}}}{\sqrt{x}}\, dx$

53. $\int_0^{\pi/2} 7^{\cos t} \sin t\, dt$

54. $\int_0^{\pi/4} \left(\frac{1}{3}\right)^{\tan t} \sec^2 t\, dt$

55. $\int_2^4 x^{2x}(1 + \ln x)\, dx$

56. $\int_1^2 \frac{2^{\ln x}}{x}\, dx$

Evaluate the integrals in Exercises 57–60.

57. $\int 3x^{\sqrt{3}}\, dx$

58. $\int x^{\sqrt{2}-1}\, dx$

59. $\int_0^3 (\sqrt{2} + 1)x^{\sqrt{2}}\, dx$

60. $\int_1^e x^{(\ln 2)-1}\, dx$

Evaluate the integrals in Exercises 61–70.

61. $\int \frac{\log_{10} x}{x}\, dx$

62. $\int_1^4 \frac{\log_2 x}{x}\, dx$

63. $\int_1^4 \frac{\ln 2 \log_2 x}{x}\, dx$

64. $\int_1^e \frac{2 \ln 10 \log_{10} x}{x}\, dx$

65. $\int_0^2 \frac{\log_2 (x + 2)}{x + 2}\, dx$

66. $\int_{1/10}^{10} \frac{\log_{10} (10x)}{x}\, dx$

67. $\int_0^9 \frac{2 \log_{10} (x + 1)}{x + 1}\, dx$

68. $\int_2^3 \frac{2 \log_2 (x - 1)}{x - 1}\, dx$

69. $\int \frac{dx}{x \log_{10} x}$

70. $\int \frac{dx}{x(\log_8 x)^2}$

Evaluate the integrals in Exercises 71–74.

71. $\int_1^{\ln x} \frac{1}{t}\, dt, \quad x > 1$

72. $\int_1^{e^x} \frac{1}{t}\, dt$

73. $\int_1^{1/x} \frac{1}{t}\, dt, \quad x > 0$

74. $\frac{1}{\ln a} \int_1^x \frac{1}{t}\, dt, \quad x > 0$

I.

Inverse Trigonometric Functions

Trigonometric Function Values

13. Given that $\alpha = \sin^{-1}(5/13)$, find $\cos\alpha$, $\tan\alpha$, $\sec\alpha$, $\csc\alpha$, and $\cot\alpha$.

14. Given that $\alpha = \tan^{-1}(4/3)$, find $\sin\alpha$, $\cos\alpha$, $\sec\alpha$, $\csc\alpha$, and $\cot\alpha$.

15. Given that $\alpha = \sec^{-1}(-\sqrt{5})$, find $\sin\alpha$, $\cos\alpha$, $\tan\alpha$, $\csc\alpha$, and $\cot\alpha$.

16. Given that $\alpha = \sec^{-1}(-\sqrt{13}/2)$, find $\sin\alpha$, $\cos\alpha$, $\tan\alpha$, $\csc\alpha$, and $\cot\alpha$.

Evaluating Trigonometric and Inverse Trigonometric Terms

Find the values in Exercises 17–28.

17. $\sin\left(\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)\right)$

18. $\sec\left(\cos^{-1}\frac{1}{2}\right)$

19. $\tan\left(\sin^{-1}\left(-\frac{1}{2}\right)\right)$

20. $\cot\left(\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

21. $\csc(\sec^{-1}2) + \cos(\tan^{-1}(-\sqrt{3}))$

22. $\tan(\sec^{-1}1) + \sin(\csc^{-1}(-2))$

23. $\sin\left(\sin^{-1}\left(-\frac{1}{2}\right) + \cos^{-1}\left(-\frac{1}{2}\right)\right)$

24. $\cot\left(\sin^{-1}\left(-\frac{1}{2}\right) - \sec^{-1}2\right)$

25. $\sec(\tan^{-1}1 + \csc^{-1}1)$

26. $\sec(\cot^{-1}\sqrt{3} + \csc^{-1}(-1))$

27. $\sec^{-1}\left(\sec\left(-\frac{\pi}{6}\right)\right)$ (The answer is *not* $-\pi/6$.)

28. $\cot^{-1}\left(\cot\left(-\frac{\pi}{4}\right)\right)$ (The answer is *not* $-\pi/4$.)

Finding Trigonometric Expressions

Evaluate the expressions in Exercises 29–40.

29. $\sec\left(\tan^{-1}\frac{x}{2}\right)$

30. $\sec(\tan^{-1}2x)$

31. $\tan(\sec^{-1}3y)$

32. $\tan\left(\sec^{-1}\frac{y}{5}\right)$

33. $\cos(\sin^{-1}x)$

34. $\tan(\cos^{-1}x)$

35. $\sin(\tan^{-1}\sqrt{x^2-2x}),\quad x \ge 2$

36. $\sin\left(\tan^{-1}\frac{x}{\sqrt{x^2+1}}\right)$

37. $\cos\left(\sin^{-1}\frac{2y}{3}\right)$

38. $\cos\left(\sin^{-1}\frac{y}{5}\right)$

39. $\sin\left(\sec^{-1}\frac{x}{4}\right)$

40. $\sin\sec^{-1}\left(\frac{\sqrt{x^2+4}}{x}\right)$

Finding Derivatives

In Exercises 49–70, find the derivative of y with respect to the appropriate variable.

49. $y = \cos^{-1}(x^2)$

50. $y = \cos^{-1}(1/x)$

51. $y = \sin^{-1}\sqrt{2}\,t$

52. $y = \sin^{-1}(1-t)$

53. $y = \sec^{-1}(2s+1)$

54. $y = \sec^{-1}5s$

55. $y = \csc^{-1}(x^2+1),\quad x > 0$

56. $y = \csc^{-1}\frac{x}{2}$

57. $y = \sec^{-1}\frac{1}{t},\quad 0 < t < 1$

58. $y = \sin^{-1}\frac{3}{t^2}$

59. $y = \cot^{-1}\sqrt{t}$

60. $y = \cot^{-1}\sqrt{t-1}$

61. $y = \ln(\tan^{-1}x)$

62. $y = \tan^{-1}(\ln x)$

63. $y = \csc^{-1}(e^t)$

64. $y = \cos^{-1}(e^{-t})$

65. $y = s\sqrt{1-s^2} + \cos^{-1}s$

66. $y = \sqrt{s^2-1} - \sec^{-1}s$

67. $y = \tan^{-1}\sqrt{x^2-1} + \csc^{-1}x,\quad x > 1$

68. $y = \cot^{-1}\frac{1}{x} - \tan^{-1}x$

69. $y = x\sin^{-1}x + \sqrt{1-x^2}$

70. $y = \ln(x^2+4) - x\tan^{-1}\left(\frac{x}{2}\right)$

Evaluating Integrals

Evaluate the integrals in Exercises 71–94.

71. $\int \frac{dx}{\sqrt{9-x^2}}$

72. $\int \frac{dx}{\sqrt{1-4x^2}}$

73. $\int \frac{dx}{17 + x^2}$ **74.** $\int \frac{dx}{9 + 3x^2}$

75. $\int \frac{dx}{x\sqrt{25x^2 - 2}}$ **76.** $\int \frac{dx}{x\sqrt{5x^2 - 4}}$

77. $\int_0^1 \frac{4\,ds}{\sqrt{4 - s^2}}$ **78.** $\int_0^{3\sqrt{2}/4} \frac{ds}{\sqrt{9 - 4s^2}}$

79. $\int_0^2 \frac{dt}{8 + 2t^2}$ **80.** $\int_{-2}^2 \frac{dt}{4 + 3t^2}$

81. $\int_{-1}^{-\sqrt{2}/2} \frac{dy}{y\sqrt{4y^2 - 1}}$ **82.** $\int_{-2/3}^{-\sqrt{2}/3} \frac{dy}{y\sqrt{9y^2 - 1}}$

83. $\int \frac{3\,dr}{\sqrt{1 - 4(r - 1)^2}}$ **84.** $\int \frac{6\,dr}{\sqrt{4 - (r + 1)^2}}$

85. $\int \frac{dx}{2 + (x - 1)^2}$ **86.** $\int \frac{dx}{1 + (3x + 1)^2}$

87. $\int \frac{dx}{(2x - 1)\sqrt{(2x - 1)^2 - 4}}$

88. $\int \frac{dx}{(x + 3)\sqrt{(x + 3)^2 - 25}}$

89. $\int_{-\pi/2}^{\pi/2} \frac{2\cos\theta\,d\theta}{1 + (\sin\theta)^2}$ **90.** $\int_{\pi/6}^{\pi/4} \frac{\csc^2 x\,dx}{1 + (\cot x)^2}$

91. $\int_0^{\ln\sqrt{3}} \frac{e^x\,dx}{1 + e^{2x}}$ **92.** $\int_1^{e^{\pi/4}} \frac{4\,dt}{t(1 + \ln^2 t)}$

93. $\int \frac{y\,dy}{\sqrt{1 - y^4}}$ **94.** $\int \frac{\sec^2 y\,dy}{\sqrt{1 - \tan^2 y}}$

J.

Antiderivatives

Finding Antiderivatives

In Exercises 1–16, find an antiderivative for each function. Do as many as you can mentally. Check your answers by differentiation.

1. **a.** $2x$ **b.** x^2 **c.** $x^2 - 2x + 1$

2. **a.** $6x$ **b.** x^7 **c.** $x^7 - 6x + 8$

3. **a.** $-3x^{-4}$ **b.** x^{-4} **c.** $x^{-4} + 2x + 3$

4. **a.** $2x^{-3}$ **b.** $\frac{x^{-3}}{2} + x^2$ **c.** $-x^{-3} + x - 1$

5. **a.** $\frac{1}{x^2}$ **b.** $\frac{5}{x^2}$ **c.** $2 - \frac{5}{x^2}$

6. **a.** $-\frac{2}{x^3}$ **b.** $\frac{1}{2x^3}$ **c.** $x^3 - \frac{1}{x^3}$

7. **a.** $\frac{3}{2}\sqrt{x}$ **b.** $\frac{1}{2\sqrt{x}}$ **c.** $\sqrt{x} + \frac{1}{\sqrt{x}}$

8. **a.** $\frac{4}{3}\sqrt[3]{x}$ **b.** $\frac{1}{3\sqrt[3]{x}}$ **c.** $\sqrt[3]{x} + \frac{1}{\sqrt[3]{x}}$

9. **a.** $\frac{2}{3}x^{-1/3}$ **b.** $\frac{1}{3}x^{-2/3}$ **c.** $-\frac{1}{3}x^{-4/3}$

10. **a.** $\frac{1}{2}x^{-1/2}$ **b.** $-\frac{1}{2}x^{-3/2}$ **c.** $-\frac{3}{2}x^{-5/2}$

11. **a.** $-\pi \sin \pi x$ **b.** $3 \sin x$ **c.** $\sin \pi x - 3 \sin 3x$

12. **a.** $\pi \cos \pi x$ **b.** $\frac{\pi}{2} \cos \frac{\pi x}{2}$ **c.** $\cos \frac{\pi x}{2} + \pi \cos x$

13. **a.** $\sec^2 x$ **b.** $\frac{2}{3} \sec^2 \frac{x}{3}$ **c.** $-\sec^2 \frac{3x}{2}$

14. **a.** $\csc^2 x$ **b.** $-\frac{3}{2} \csc^2 \frac{3x}{2}$ **c.** $1 - 8 \csc^2 2x$

15. **a.** $\csc x \cot x$ **b.** $-\csc 5x \cot 5x$ **c.** $-\pi \csc \frac{\pi x}{2} \cot \frac{\pi x}{2}$

16. **a.** $\sec x \tan x$ **b.** $4 \sec 3x \tan 3x$ **c.** $\sec \frac{\pi x}{2} \tan \frac{\pi x}{2}$

Finding Indefinite Integrals

In Exercises 17–54, find the most general antiderivative or indefinite integral. Check your answers by differentiation.

17. $\int (x + 1)\,dx$ **18.** $\int (5 - 6x)\,dx$

19. $\int \left(3t^2 + \frac{t}{2}\right) dt$ **20.** $\int \left(\frac{t^2}{2} + 4t^3\right) dt$

21. $\int (2x^3 - 5x + 7)\,dx$ **22.** $\int (1 - x^2 - 3x^5)\,dx$

23. $\int \left(\frac{1}{x^2} - x^2 - \frac{1}{3}\right) dx$ **24.** $\int \left(\frac{1}{5} - \frac{2}{x^3} + 2x\right) dx$

25. $\int x^{-1/3}\,dx$ **26.** $\int x^{-5/4}\,dx$

27. $\int \left(\sqrt{x} + \sqrt[3]{x}\right) dx$ **28.** $\int \left(\frac{\sqrt{x}}{2} + \frac{2}{\sqrt{x}}\right) dx$

29. $\int \left(8y - \frac{2}{y^{1/4}}\right) dy$ **30.** $\int \left(\frac{1}{7} - \frac{1}{y^{5/4}}\right) dy$

31. $\int 2x(1 - x^{-3})\,dx$ **32.** $\int x^{-3}(x + 1)\,dx$

33. $\int \frac{t\sqrt{t} + \sqrt{t}}{t^2}\,dt$ **34.** $\int \frac{4 + \sqrt{t}}{t^3}\,dt$

35. $\int (-2 \cos t)\,dt$ **36.** $\int (-5 \sin t)\,dt$

37. $\int 7 \sin \frac{\theta}{3}\,d\theta$ **38.** $\int 3 \cos 5\theta\,d\theta$

39. $\int (-3 \csc^2 x)\,dx$ **40.** $\int \left(-\frac{\sec^2 x}{3}\right) dx$

41. $\int \frac{\csc\theta \cot\theta}{2}\,d\theta$ **42.** $\int \frac{2}{5} \sec\theta \tan\theta\,d\theta$

43. $\int (4 \sec x \tan x - 2 \sec^2 x)\, dx$

44. $\int \frac{1}{2}(\csc^2 x - \csc x \cot x)\, dx$

45. $\int (\sin 2x - \csc^2 x)\, dx$

46. $\int (2 \cos 2x - 3 \sin 3x)\, dx$

47. $\int \frac{1 + \cos 4t}{2}\, dt$

48. $\int \frac{1 - \cos 6t}{2}\, dt$

49. $\int (1 + \tan^2 \theta)\, d\theta$

(*Hint:* $1 + \tan^2 \theta = \sec^2 \theta$)

50. $\int (2 + \tan^2 \theta)\, d\theta$

51. $\int \cot^2 x\, dx$

(*Hint:* $1 + \cot^2 x = \csc^2 x$)

52. $\int (1 - \cot^2 x)\, dx$

53. $\int \cos \theta (\tan \theta + \sec \theta)\, d\theta$

54. $\int \frac{\csc \theta}{\csc \theta - \sin \theta}\, d\theta$

K.

The Fund. Theorem of Calculus

Evaluating Integrals

Evaluate the integrals in Exercises 1–26.

1. $\int_{-2}^{0} (2x + 5)\, dx$

2. $\int_{-3}^{4} \left(5 - \frac{x}{2}\right) dx$

3. $\int_{0}^{4} \left(3x - \frac{x^3}{4}\right) dx$

4. $\int_{-2}^{2} (x^3 - 2x + 3)\, dx$

5. $\int_{0}^{1} \left(x^2 + \sqrt{x}\right) dx$

6. $\int_{0}^{5} x^{3/2}\, dx$

7. $\int_{1}^{32} x^{-6/5}\, dx$

8. $\int_{-2}^{-1} \frac{2}{x^2}\, dx$

9. $\int_{0}^{\pi} \sin x\, dx$

10. $\int_{0}^{\pi} (1 + \cos x)\, dx$

11. $\int_{0}^{\pi/3} 2 \sec^2 x\, dx$

12. $\int_{\pi/6}^{5\pi/6} \csc^2 x\, dx$

13. $\int_{\pi/4}^{3\pi/4} \csc\theta \cot\theta\, d\theta$

14. $\int_{0}^{\pi/3} 4 \sec u \tan u\, du$

15. $\int_{\pi/2}^{0} \frac{1 + \cos 2t}{2}\, dt$

16. $\int_{-\pi/3}^{\pi/3} \frac{1 - \cos 2t}{2}\, dt$

17. $\int_{-\pi/2}^{\pi/2} (8y^2 + \sin y)\, dy$

18. $\int_{-\pi/3}^{-\pi/4} \left(4 \sec^2 t + \frac{\pi}{t^2}\right) dt$

19. $\int_{1}^{-1} (r + 1)^2\, dr$

20. $\int_{-\sqrt{3}}^{\sqrt{3}} (t + 1)(t^2 + 4)\, dt$

21. $\int_{\sqrt{2}}^{1} \left(\frac{u^7}{2} - \frac{1}{u^5}\right) du$

22. $\int_{1/2}^{1} \left(\frac{1}{v^3} - \frac{1}{v^4}\right) dv$

23. $\int_{1}^{\sqrt{2}} \frac{s^2 + \sqrt{s}}{s^2}\, ds$

24. $\int_{9}^{4} \frac{1 - \sqrt{u}}{\sqrt{u}}\, du$

25. $\int_{-4}^{4} |x|\, dx$

26. $\int_{0}^{\pi} \frac{1}{2}(\cos x + |\cos x|)\, dx$