Answers of questions:

A

1. $y = -10x + 3\cos x \Rightarrow \frac{dy}{dx} = -10 + 3\frac{d}{dx}(\cos x) = -10 - 3\sin x$

2. $y = \frac{3}{x} + 5\sin x \Rightarrow \frac{dy}{dx} = \frac{-3}{x^2} + 5\frac{d}{dx}(\sin x) = \frac{-3}{x^2} + 5\cos x$

3. $y = \csc x - 4\sqrt{x} + 7 \Rightarrow \frac{dy}{dx} = -\csc x \cot x - \frac{4}{2\sqrt{x}} + 0 = -\csc x \cot x - \frac{2}{\sqrt{x}}$

4. $y = x^2 \cot x - \frac{1}{x^2} \Rightarrow \frac{dy}{dx} = x^2 \frac{d}{dx}(\cot x) + \cot x \cdot \frac{d}{dx}(x^2) + \frac{2}{x^3} = -x^2 \csc^2 x + (\cot x)(2x) + \frac{2}{x^3}$
$= -x^2 \csc^2 x + 2x \cot x + \frac{2}{x^3}$

5. $y = (\sec x + \tan x)(\sec x - \tan x) \Rightarrow \frac{dy}{dx} = (\sec x + \tan x)\frac{d}{dx}(\sec x - \tan x) + (\sec x - \tan x)\frac{d}{dx}(\sec x + \tan x)$
$= (\sec x + \tan x)(\sec x \tan x - \sec^2 x) + (\sec x - \tan x)(\sec x \tan x + \sec^2 x)$
$= (\sec^2 x \tan x + \sec x \tan^2 x - \sec^3 x - \sec^2 x \tan x) + (\sec^2 x \tan x - \sec x \tan^2 x + \sec^3 x - \tan x \sec^2 x) = 0.$
$\left(\text{Note also that } y = \sec^2 x - \tan^2 x = (\tan^2 x + 1) - \tan^2 x = 1 \Rightarrow \frac{dy}{dx} = 0.\right)$

6. $y = (\sin x + \cos x)\sec x \Rightarrow \frac{dy}{dx} = (\sin x + \cos x)\frac{d}{dx}(\sec x) + \sec x \frac{d}{dx}(\sin x + \cos x)$
$= (\sin x + \cos x)(\sec x \tan x) + (\sec x)(\cos x - \sin x) = \frac{(\sin x + \cos x)\sin x}{\cos^2 x} + \frac{\cos x - \sin x}{\cos x}$
$= \frac{\sin^2 x + \cos x \sin x + \cos^2 x - \cos x \sin x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$
$\left(\text{Note also that } y = \sin x \sec x + \cos x \sec x = \tan x + 1 \Rightarrow \frac{dy}{dx} = \sec^2 x.\right)$

7. $y = \frac{\cot x}{1 + \cot x} \Rightarrow \frac{dy}{dx} = \frac{(1 + \cot x)\frac{d}{dx}(\cot x) - (\cot x)\frac{d}{dx}(1 + \cot x)}{(1 + \cot x)^2} = \frac{(1 + \cot x)(-\csc^2 x) - (\cot x)(-\csc^2 x)}{(1 + \cot x)^2}$
$= \frac{-\csc^2 x - \csc^2 x \cot x + \csc^2 x \cot x}{(1 + \cot x)^2} = \frac{-\csc^2 x}{(1 + \cot x)^2}$

8. $y = \frac{\cos x}{1 + \sin x} \Rightarrow \frac{dy}{dx} = \frac{(1 + \sin x)\frac{d}{dx}(\cos x) - (\cos x)\frac{d}{dx}(1 + \sin x)}{(1 + \sin x)^2} = \frac{(1 + \sin x)(-\sin x) - (\cos x)(\cos x)}{(1 + \sin x)^2}$
$= \frac{-\sin x - \sin^2 x - \cos^2 x}{(1 + \sin x)^2} = \frac{-\sin x - 1}{(1 + \sin x)^2} = \frac{-(1 + \sin x)}{(1 + \sin x)^2} = \frac{-1}{1 + \sin x}$

9. $y = \frac{4}{\cos x} + \frac{1}{\tan x} = 4\sec x + \cot x \Rightarrow \frac{dy}{dx} = 4\sec x \tan x - \csc^2 x$

10. $y = \frac{\cos x}{x} + \frac{x}{\cos x} \Rightarrow \frac{dy}{dx} = \frac{x(-\sin x) - (\cos x)(1)}{x^2} + \frac{(\cos x)(1) - x(-\sin x)}{\cos^2 x} = \frac{-x \sin x - \cos x}{x^2} + \frac{\cos x + x \sin x}{\cos^2 x}$

11. $y = x^2 \sin x + 2x \cos x - 2\sin x \Rightarrow \frac{dy}{dx} = (x^2 \cos x + (\sin x)(2x)) + ((2x)(-\sin x) + (\cos x)(2)) - 2\cos x$
$= x^2 \cos x + 2x \sin x - 2x \sin x + 2\cos x - 2\cos x = x^2 \cos x$

12. $y = x^2 \cos x - 2x \sin x - 2\cos x \Rightarrow \frac{dy}{dx} = (x^2(-\sin x) + (\cos x)(2x)) - (2x \cos x + (\sin x)(2)) - 2(-\sin x)$

13. $s = \tan t - t \Rightarrow \frac{ds}{dt} = \frac{d}{dt}(\tan t) - 1 = \sec^2 t - 1 = \tan^2 t$

14. $s = t^2 - \sec t + 1 \Rightarrow \frac{ds}{dt} = 2t - \frac{d}{dt}(\sec t) = 2t - \sec t \tan t$

15. $s = \frac{1 + \csc t}{1 - \csc t} \Rightarrow \frac{ds}{dt} = \frac{(1 - \csc t)(-\csc t \cot t) - (1 + \csc t)(\csc t \cot t)}{(1 - \csc t)^2}$
$= \frac{-\csc t \cot t + \csc^2 t \cot t - \csc t \cot t - \csc^2 t \cot t}{(1 - \csc t)^2} = \frac{-2 \csc t \cot t}{(1 - \csc t)^2}$

16. $s = \frac{\sin t}{1 - \cos t} \Rightarrow \frac{ds}{dt} = \frac{(1 - \cos t)(\cos t) - (\sin t)(\sin t)}{(1 - \cos t)^2} = \frac{\cos t - \cos^2 t - \sin^2 t}{(1 - \cos t)^2} = \frac{\cos t - 1}{(1 - \cos t)^2} = -\frac{1}{1 - \cos t}$
$= \frac{1}{\cos t - 1}$

17. $r = 4 - \theta^2 \sin \theta \Rightarrow \frac{dr}{d\theta} = -\left(\theta^2 \frac{d}{d\theta}(\sin \theta) + (\sin \theta)(2\theta)\right) = -(\theta^2 \cos \theta + 2\theta \sin \theta) = -\theta(\theta \cos \theta + 2 \sin \theta)$

18. $r = \theta \sin \theta + \cos \theta \Rightarrow \frac{dr}{d\theta} = (\theta \cos \theta + (\sin \theta)(1)) - \sin \theta = \theta \cos \theta$

19. $r = \sec \theta \csc \theta \Rightarrow \frac{dr}{d\theta} = (\sec \theta)(-\csc \theta \cot \theta) + (\csc \theta)(\sec \theta \tan \theta)$
$= \left(\frac{-1}{\cos \theta}\right)\left(\frac{1}{\sin \theta}\right)\left(\frac{\cos \theta}{\sin \theta}\right) + \left(\frac{1}{\sin \theta}\right)\left(\frac{1}{\cos \theta}\right)\left(\frac{\sin \theta}{\cos \theta}\right) = \frac{-1}{\sin^2 \theta} + \frac{1}{\cos^2 \theta} = \sec^2 \theta - \csc^2 \theta$

20. $r = (1 + \sec \theta) \sin \theta \Rightarrow \frac{dr}{d\theta} = (1 + \sec \theta) \cos \theta + (\sin \theta)(\sec \theta \tan \theta) = (\cos \theta + 1) + \tan^2 \theta = \cos \theta + \sec^2 \theta$

21. $p = 5 + \frac{1}{\cot q} = 5 + \tan q \Rightarrow \frac{dp}{dq} = \sec^2 q$

22. $p = (1 + \csc q) \cos q \Rightarrow \frac{dp}{dq} = (1 + \csc q)(-\sin q) + (\cos q)(-\csc q \cot q) = (-\sin q - 1) - \cot^2 q = -\sin q - \csc^2 q$

23. $p = \frac{\sin q + \cos q}{\cos q} \Rightarrow \frac{dp}{dq} = \frac{(\cos q)(\cos q - \sin q) - (\sin q + \cos q)(-\sin q)}{\cos^2 q}$
$= \frac{\cos^2 q - \cos q \sin q + \sin^2 q + \cos q \sin q}{\cos^2 q} = \frac{1}{\cos^2 q} = \sec^2 q$

24. $p = \frac{\tan q}{1 + \tan q} \Rightarrow \frac{dp}{dq} = \frac{(1 + \tan q)(\sec^2 q) - (\tan q)(\sec^2 q)}{(1 + \tan q)^2} = \frac{\sec^2 q + \tan q \sec^2 q - \tan q \sec^2 q}{(1 + \tan q)^2} = \frac{\sec^2 q}{(1 + \tan q)^2}$

25. (a) $y = \csc x \Rightarrow y' = -\csc x \cot x \Rightarrow y'' = -\left((\csc x)(-\csc^2 x) + (\cot x)(-\csc x \cot x)\right) = \csc^3 x + \csc x \cot^2 x$
$= (\csc x)(\csc^2 x + \cot^2 x) = (\csc x)(\csc^2 x + \csc^2 x - 1) = 2\csc^3 x - \csc x$

(b) $y = \sec x \Rightarrow y' = \sec x \tan x \Rightarrow y'' = (\sec x)(\sec^2 x) + (\tan x)(\sec x \tan x) = \sec^3 x + \sec x \tan^2 x$
$= (\sec x)(\sec^2 x + \tan^2 x) = (\sec x)(\sec^2 x + \sec^2 x - 1) = 2\sec^3 x - \sec x$

B

1. $f(u) = 6u - 9 \Rightarrow f'(u) = 6 \Rightarrow f'(g(x)) = 6$; $g(x) = \frac{1}{2}x^4 \Rightarrow g'(x) = 2x^3$; therefore $\frac{dy}{dx} = f'(g(x))g'(x)$
$= 6 \cdot 2x^3 = 12x^3$

2. $f(u) = 2u^3 \Rightarrow f'(u) = 6u^2 \Rightarrow f'(g(x)) = 6(8x-1)^2$; $g(x) = 8x - 1 \Rightarrow g'(x) = 8$; therefore $\frac{dy}{dx} = f'(g(x))g'(x)$
$= 6(8x-1)^2 \cdot 8 = 48(8x-1)^2$

3. $f(u) = \sin u \Rightarrow f'(u) = \cos u \Rightarrow f'(g(x)) = \cos(3x+1)$; $g(x) = 3x + 1 \Rightarrow g'(x) = 3$; therefore $\frac{dy}{dx} = f'(g(x))g'(x)$
$= (\cos(3x+1))(3) = 3\cos(3x+1)$

4. $f(u) = \cos u \Rightarrow f'(u) = -\sin u \Rightarrow f'(g(x)) = -\sin\left(\frac{-x}{3}\right)$; $g(x) = \frac{-x}{3} \Rightarrow g'(x) = -\frac{1}{3}$; therefore $\frac{dy}{dx} = f'(g(x))g'(x)$
$= -\sin\left(\frac{-x}{3}\right) \cdot \left(\frac{-1}{3}\right) = \frac{1}{3}\sin\left(\frac{-x}{3}\right)$

5. $f(u) = \cos u \Rightarrow f'(u) = -\sin u \Rightarrow f'(g(x)) = -\sin(\sin x)$; $g(x) = \sin x \Rightarrow g'(x) = \cos x$; therefore
$\frac{dy}{dx} = f'(g(x))g'(x) = -(\sin(\sin x))\cos x$

6. $f(u) = \sin u \Rightarrow f'(u) = \cos u \Rightarrow f'(g(x)) = \cos(x - \cos x)$; $g(x) = x - \cos x \Rightarrow g'(x) = 1 + \sin x$; therefore
$\frac{dy}{dx} = f'(g(x))g'(x) = (\cos(x - \cos x))(1 + \sin x)$

7. $f(u) = \tan u \Rightarrow f'(u) = \sec^2 u \Rightarrow f'(g(x)) = \sec^2(10x - 5)$; $g(x) = 10x - 5 \Rightarrow g'(x) = 10$; therefore
$\frac{dy}{dx} = f'(g(x))g'(x) = (\sec^2(10x-5))(10) = 10\sec^2(10x-5)$

8. $f(u) = -\sec u \Rightarrow f'(u) = -\sec u \tan u \Rightarrow f'(g(x)) = -\sec(x^2+7x)\tan(x^2+7x)$; $g(x) = x^2 + 7x$
$\Rightarrow g'(x) = 2x + 7$; therefore $\frac{dy}{dx} = f'(g(x))g'(x) = -(2x+7)\sec(x^2+7x)\tan(x^2+7x)$

9. With $u = (2x+1)$, $y = u^5$: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = 5u^4 \cdot 2 = 10(2x+1)^4$

10. With $u = (4-3x)$, $y = u^9$: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = 9u^8 \cdot (-3) = -27(4-3x)^8$

11. With $u = \left(1 - \frac{x}{7}\right)$, $y = u^{-7}$: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = -7u^{-8} \cdot \left(-\frac{1}{7}\right) = \left(1 - \frac{x}{7}\right)^{-8}$

12. With $u = \left(\frac{x}{2} - 1\right)$, $y = u^{-10}$: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = -10u^{-11} \cdot \left(\frac{1}{2}\right) = -5\left(\frac{x}{2} - 1\right)^{-11}$

13. With $u = \left(\frac{x^2}{8} + x - \frac{1}{x}\right)$, $y = u^4$: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = 4u^3 \cdot \left(\frac{x}{4} + 1 + \frac{1}{x^2}\right) = 4\left(\frac{x^2}{8} + x - \frac{1}{x}\right)^3\left(\frac{x}{4} + 1 + \frac{1}{x^2}\right)$

14. With $u = \left(\frac{x}{5} + \frac{1}{5x}\right)$, $y = u^5$: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = 5u^4 \cdot \left(\frac{1}{5} - \frac{1}{5x^2}\right) = \left(\frac{x}{5} + \frac{1}{5x}\right)^4\left(1 - \frac{1}{x^2}\right)$

15. With $u = \tan x$, $y = \sec u$: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = (\sec u \tan u)(\sec^2 x) = (\sec(\tan x)\tan(\tan x))\sec^2 x$

16. With $u = \pi - \frac{1}{x}$, $y = \cot u$: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = (-\csc^2 u)\left(\frac{1}{x^2}\right) = -\frac{1}{x^2}\csc^2\left(\pi - \frac{1}{x}\right)$

17. With $u = \sin x$, $y = u^3$: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = 3u^2\cos x = 3(\sin^2 x)(\cos x)$

18. With $u = \cos x$, $y = 5u^{-4}$: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = (-20u^{-5})(-\sin x) = 20(\cos^{-5} x)(\sin x)$

19. $p = \sqrt{3 - t} = (3 - t)^{1/2} \Rightarrow \frac{dp}{dt} = \frac{1}{2}(3 - t)^{-1/2} \cdot \frac{d}{dt}(3 - t) = -\frac{1}{2}(3 - t)^{-1/2} = \frac{-1}{2\sqrt{3 - t}}$

20. $q = \sqrt{2r - r^2} = (2r - r^2)^{1/2} \Rightarrow \frac{dq}{dr} = \frac{1}{2}(2r - r^2)^{-1/2} \cdot \frac{d}{dr}(2r - r^2) = \frac{1}{2}(2r - r^2)^{-1/2}(2 - 2r) = \frac{1 - r}{\sqrt{2r - r^2}}$

21. $s = \frac{4}{3\pi}\sin 3t + \frac{4}{5\pi}\cos 5t \Rightarrow \frac{ds}{dt} = \frac{4}{3\pi}\cos 3t \cdot \frac{d}{dt}(3t) + \frac{4}{5\pi}(-\sin 5t) \cdot \frac{d}{dt}(5t) = \frac{4}{\pi}\cos 3t - \frac{4}{\pi}\sin 5t$
$= \frac{4}{\pi}(\cos 3t - \sin 5t)$

22. $s = \sin\left(\frac{3\pi t}{2}\right) + \cos\left(\frac{3\pi t}{2}\right) \Rightarrow \frac{ds}{dt} = \cos\left(\frac{3\pi t}{2}\right) \cdot \frac{d}{dt}\left(\frac{3\pi t}{2}\right) - \sin\left(\frac{3\pi t}{2}\right) \cdot \frac{d}{dt}\left(\frac{3\pi t}{2}\right) = \frac{3\pi}{2}\cos\left(\frac{3\pi t}{2}\right) - \frac{3\pi}{2}\sin\left(\frac{3\pi t}{2}\right)$
$= \frac{3\pi}{2}\left(\cos\frac{3\pi t}{2} - \sin\frac{3\pi t}{2}\right)$

23. $r = (\csc\theta + \cot\theta)^{-1} \Rightarrow \frac{dr}{d\theta} = -(\csc\theta + \cot\theta)^{-2}\frac{d}{d\theta}(\csc\theta + \cot\theta) = \frac{\csc\theta\cot\theta + \csc^2\theta}{(\csc\theta + \cot\theta)^2} = \frac{\csc\theta(\cot\theta + \csc\theta)}{(\csc\theta + \cot\theta)^2}$
$= \frac{\csc\theta}{\csc\theta + \cot\theta}$

24. $r = -(\sec\theta + \tan\theta)^{-1} \Rightarrow \frac{dr}{d\theta} = (\sec\theta + \tan\theta)^{-2}\frac{d}{d\theta}(\sec\theta + \tan\theta) = \frac{\sec\theta\tan\theta + \sec^2\theta}{(\sec\theta + \tan\theta)^2} = \frac{\sec\theta(\tan\theta + \sec\theta)}{(\sec\theta + \tan\theta)^2}$
$= \frac{\sec\theta}{\sec\theta + \tan\theta}$

25. $y = x^2\sin^4 x + x\cos^{-2} x \Rightarrow \frac{dy}{dx} = x^2\frac{d}{dx}(\sin^4 x) + \sin^4 x \cdot \frac{d}{dx}(x^2) + x\frac{d}{dx}(\cos^{-2} x) + \cos^{-2} x \cdot \frac{d}{dx}(x)$
$= x^2\left(4\sin^3 x\frac{d}{dx}(\sin x)\right) + 2x\sin^4 x + x\left(-2\cos^{-3} x \cdot \frac{d}{dx}(\cos x)\right) + \cos^{-2} x$
$= x^2(4\sin^3 x\cos x) + 2x\sin^4 x + x((-2\cos^{-3} x)(-\sin x)) + \cos^{-2} x$
$= 4x^2\sin^3 x\cos x + 2x\sin^4 x + 2x\sin x\cos^{-3} x + \cos^{-2} x$

26. $y = \frac{1}{x}\sin^{-5} x - \frac{x}{3}\cos^3 x \Rightarrow y' = \frac{1}{x}\frac{d}{dx}(\sin^{-5} x) + \sin^{-5} x \cdot \frac{d}{dx}\left(\frac{1}{x}\right) - \frac{x}{3}\frac{d}{dx}(\cos^3 x) - \cos^3 x \cdot \frac{d}{dx}\left(\frac{x}{3}\right)$
$= \frac{1}{x}(-5\sin^{-6} x\cos x) + (\sin^{-5} x)\left(-\frac{1}{x^2}\right) - \frac{x}{3}((3\cos^2 x)(-\sin x)) - (\cos^3 x)\left(\frac{1}{3}\right)$
$= -\frac{5}{x}\sin^{-6} x\cos x - \frac{1}{x^2}\sin^{-5} x + x\cos^2 x\sin x - \frac{1}{3}\cos^3 x$

27. $y = \frac{1}{21}(3x - 2)^7 + \left(4 - \frac{1}{2x^2}\right)^{-1} \Rightarrow \frac{dy}{dx} = \frac{7}{21}(3x - 2)^6 \cdot \frac{d}{dx}(3x - 2) + (-1)\left(4 - \frac{1}{2x^2}\right)^{-2} \cdot \frac{d}{dx}\left(4 - \frac{1}{2x^2}\right)$
$= \frac{7}{21}(3x - 2)^6 \cdot 3 + (-1)\left(4 - \frac{1}{2x^2}\right)^{-2}\left(\frac{1}{x^3}\right) = (3x - 2)^6 - \frac{1}{x^3\left(4 - \frac{1}{2x^2}\right)^2}$

28. $y = (5 - 2x)^{-3} + \frac{1}{8}\left(\frac{2}{x} + 1\right)^4 \Rightarrow \frac{dy}{dx} = -3(5 - 2x)^{-4}(-2) + \frac{4}{8}\left(\frac{2}{x} + 1\right)^3\left(-\frac{2}{x^2}\right) = 6(5 - 2x)^{-4} - \left(\frac{1}{x^2}\right)\left(\frac{2}{x} + 1\right)^3$
$= \frac{6}{(5 - 2x)^4} - \frac{\left(\frac{2}{x} + 1\right)^3}{x^2}$

29. $y = (4x + 3)^4(x + 1)^{-3} \Rightarrow \frac{dy}{dx} = (4x + 3)^4(-3)(x + 1)^{-4} \cdot \frac{d}{dx}(x + 1) + (x + 1)^{-3}(4)(4x + 3)^3 \cdot \frac{d}{dx}(4x + 3)$
$= (4x + 3)^4(-3)(x + 1)^{-4}(1) + (x + 1)^{-3}(4)(4x + 3)^3(4) = -3(4x + 3)^4(x + 1)^{-4} + 16(4x + 3)^3(x + 1)^{-3}$
$= \frac{(4x + 3)^3}{(x + 1)^4}[-3(4x + 3) + 16(x + 1)] = \frac{(4x + 3)^3(4x + 7)}{(x + 1)^4}$

30. $y = (2x - 5)^{-1}(x^2 - 5x)^6 \Rightarrow \frac{dy}{dx} = (2x - 5)^{-1}(6)(x^2 - 5x)^5(2x - 5) + (x^2 - 5x)^6(-1)(2x - 5)^{-2}(2)$
$= 6(x^2 - 5x)^5 - \frac{2(x^2 - 5x)^6}{(2x - 5)^2}$

31. $h(x) = x\tan\left(2\sqrt{x}\right) + 7 \Rightarrow h'(x) = x\frac{d}{dx}\left(\tan\left(2x^{1/2}\right)\right) + \tan\left(2x^{1/2}\right) \cdot \frac{d}{dx}(x) + 0$
$= x\sec^2\left(2x^{1/2}\right) \cdot \frac{d}{dx}\left(2x^{1/2}\right) + \tan\left(2x^{1/2}\right) = x\sec^2\left(2\sqrt{x}\right) \cdot \frac{1}{\sqrt{x}} + \tan\left(2\sqrt{x}\right) = \sqrt{x}\sec^2\left(2\sqrt{x}\right) + \tan\left(2\sqrt{x}\right)$

32. $k(x) = x^2 \sec\left(\frac{1}{x}\right) \Rightarrow k'(x) = x^2 \frac{d}{dx}\left(\sec \frac{1}{x}\right) + \sec\left(\frac{1}{x}\right) \cdot \frac{d}{dx}\left(x^2\right) = x^2 \sec\left(\frac{1}{x}\right) \tan\left(\frac{1}{x}\right) \cdot \frac{d}{dx}\left(\frac{1}{x}\right) + 2x \sec\left(\frac{1}{x}\right)$
$= x^2 \sec\left(\frac{1}{x}\right) \tan\left(\frac{1}{x}\right) \cdot \left(-\frac{1}{x^2}\right) + 2x \sec\left(\frac{1}{x}\right) = 2x \sec\left(\frac{1}{x}\right) - \sec\left(\frac{1}{x}\right) \tan\left(\frac{1}{x}\right)$

33. $f(\theta) = \left(\frac{\sin\theta}{1+\cos\theta}\right)^2 \Rightarrow f'(\theta) = 2\left(\frac{\sin\theta}{1+\cos\theta}\right) \cdot \frac{d}{d\theta}\left(\frac{\sin\theta}{1+\cos\theta}\right) = \frac{2\sin\theta}{1+\cos\theta} \cdot \frac{(1+\cos\theta)(\cos\theta)-(\sin\theta)(-\sin\theta)}{(1+\cos\theta)^2}$
$= \frac{(2\sin\theta)(\cos\theta+\cos^2\theta+\sin^2\theta)}{(1+\cos\theta)^3} = \frac{(2\sin\theta)(\cos\theta+1)}{(1+\cos\theta)^3} = \frac{2\sin\theta}{(1+\cos\theta)^2}$

34. $g(t) = \left(\frac{1+\cos t}{\sin t}\right)^{-1} \Rightarrow g'(t) = -\left(\frac{1+\cos t}{\sin t}\right)^{-2} \cdot \frac{d}{dt}\left(\frac{1+\cos t}{\sin t}\right) = -\frac{\sin^2 t}{(1+\cos t)^2} \cdot \frac{(\sin t)(-\sin t)-(1+\cos t)(\cos t)}{(\sin t)^2}$
$= \frac{-(-\sin^2 t - \cos t - \cos^2 t)}{(1+\cos t)^2} = \frac{1}{1+\cos t}$

35. $r = \sin\left(\theta^2\right)\cos(2\theta) \Rightarrow \frac{dr}{d\theta} = \sin\left(\theta^2\right)(-\sin 2\theta)\frac{d}{d\theta}(2\theta) + \cos(2\theta)\left(\cos\left(\theta^2\right)\right) \cdot \frac{d}{d\theta}\left(\theta^2\right)$
$= \sin\left(\theta^2\right)(-\sin 2\theta)(2) + (\cos 2\theta)\left(\cos\left(\theta^2\right)\right)(2\theta) = -2\sin\left(\theta^2\right)\sin(2\theta) + 2\theta\cos(2\theta)\cos\left(\theta^2\right)$

36. $r = \left(\sec\sqrt{\theta}\right)\tan\left(\frac{1}{\theta}\right) \Rightarrow \frac{dr}{d\theta} = \left(\sec\sqrt{\theta}\right)\left(\sec^2\frac{1}{\theta}\right)\left(-\frac{1}{\theta^2}\right) + \tan\left(\frac{1}{\theta}\right)\left(\sec\sqrt{\theta}\tan\sqrt{\theta}\right)\left(\frac{1}{2\sqrt{\theta}}\right)$
$= -\frac{1}{\theta^2}\sec\sqrt{\theta}\sec^2\left(\frac{1}{\theta}\right) + \frac{1}{2\sqrt{\theta}}\tan\left(\frac{1}{\theta}\right)\sec\sqrt{\theta}\tan\sqrt{\theta} = \left(\sec\sqrt{\theta}\right)\left[\frac{\tan\sqrt{\theta}\tan\left(\frac{1}{\theta}\right)}{2\sqrt{\theta}} - \frac{\sec^2\left(\frac{1}{\theta}\right)}{\theta^2}\right]$

37. $q = \sin\left(\frac{t}{\sqrt{t+1}}\right) \Rightarrow \frac{dq}{dt} = \cos\left(\frac{t}{\sqrt{t+1}}\right) \cdot \frac{d}{dt}\left(\frac{t}{\sqrt{t+1}}\right) = \cos\left(\frac{t}{\sqrt{t+1}}\right) \cdot \frac{\sqrt{t+1}(1)-t\cdot\frac{d}{dt}\left(\sqrt{t+1}\right)}{\left(\sqrt{t+1}\right)^2}$
$= \cos\left(\frac{t}{\sqrt{t+1}}\right) \cdot \frac{\sqrt{t+1}-\frac{t}{2\sqrt{t+1}}}{t+1} = \cos\left(\frac{t}{\sqrt{t+1}}\right)\left(\frac{2(t+1)-t}{2(t+1)^{3/2}}\right) = \left(\frac{t+2}{2(t+1)^{3/2}}\right)\cos\left(\frac{t}{\sqrt{t+1}}\right)$

38. $q = \cot\left(\frac{\sin t}{t}\right) \Rightarrow \frac{dq}{dt} = -\csc^2\left(\frac{\sin t}{t}\right) \cdot \frac{d}{dt}\left(\frac{\sin t}{t}\right) = \left(-\csc^2\left(\frac{\sin t}{t}\right)\right)\left(\frac{t\cos t - \sin t}{t^2}\right)$

39. $y = \sin^2(\pi t - 2) \Rightarrow \frac{dy}{dt} = 2\sin(\pi t - 2) \cdot \frac{d}{dt}\sin(\pi t - 2) = 2\sin(\pi t - 2) \cdot \cos(\pi t - 2) \cdot \frac{d}{dt}(\pi t - 2)$
$= 2\pi\sin(\pi t - 2)\cos(\pi t - 2)$

40. $y = \sec^2 \pi t \Rightarrow \frac{dy}{dt} = (2\sec\pi t) \cdot \frac{d}{dt}(\sec\pi t) = (2\sec\pi t)(\sec\pi t\tan\pi t) \cdot \frac{d}{dt}(\pi t) = 2\pi\sec^2\pi t\tan\pi t$

41. $y = (1+\cos 2t)^{-4} \Rightarrow \frac{dy}{dt} = -4(1+\cos 2t)^{-5} \cdot \frac{d}{dt}(1+\cos 2t) = -4(1+\cos 2t)^{-5}(-\sin 2t) \cdot \frac{d}{dt}(2t) = \frac{8\sin 2t}{(1+\cos 2t)^5}$

42. $y = \left(1+\cot\left(\frac{t}{2}\right)\right)^{-2} \Rightarrow \frac{dy}{dt} = -2\left(1+\cot\left(\frac{t}{2}\right)\right)^{-3} \cdot \frac{d}{dt}\left(1+\cot\left(\frac{t}{2}\right)\right) = -2\left(1+\cot\left(\frac{t}{2}\right)\right)^{-3} \cdot \left(-\csc^2\left(\frac{t}{2}\right)\right) \cdot \frac{d}{dt}\left(\frac{t}{2}\right)$
$= \frac{\csc^2\left(\frac{t}{2}\right)}{\left(1+\cot\left(\frac{t}{2}\right)\right)^3}$

43. $y = \sin(\cos(2t-5)) \Rightarrow \frac{dy}{dt} = \cos(\cos(2t-5)) \cdot \frac{d}{dt}\cos(2t-5) = \cos(\cos(2t-5)) \cdot (-\sin(2t-5)) \cdot \frac{d}{dt}(2t-5)$
$= -2\cos(\cos(2t-5))(\sin(2t-5))$

44. $y = \cos\left(5\sin\left(\frac{t}{3}\right)\right) \Rightarrow \frac{dy}{dt} = -\sin\left(5\sin\left(\frac{t}{3}\right)\right) \cdot \frac{d}{dt}\left(5\sin\left(\frac{t}{3}\right)\right) = -\sin\left(5\sin\left(\frac{t}{3}\right)\right)\left(5\cos\left(\frac{t}{3}\right)\right) \cdot \frac{d}{dt}\left(\frac{t}{3}\right)$
$= -\frac{5}{3}\sin\left(5\sin\left(\frac{t}{3}\right)\right)\left(\cos\left(\frac{t}{3}\right)\right)$

45. $y = \left[1+\tan^4\left(\frac{t}{12}\right)\right]^3 \Rightarrow \frac{dy}{dt} = 3\left[1+\tan^4\left(\frac{t}{12}\right)\right]^2 \cdot \frac{d}{dt}\left[1+\tan^4\left(\frac{t}{12}\right)\right] = 3\left[1+\tan^4\left(\frac{t}{12}\right)\right]^2\left[4\tan^3\left(\frac{t}{12}\right) \cdot \frac{d}{dt}\tan\left(\frac{t}{12}\right)\right]$
$= 12\left[1+\tan^4\left(\frac{t}{12}\right)\right]^2\left[\tan^3\left(\frac{t}{12}\right)\sec^2\left(\frac{t}{12}\right) \cdot \frac{1}{12}\right] = \left[1+\tan^4\left(\frac{t}{12}\right)\right]^2\left[\tan^3\left(\frac{t}{12}\right)\sec^2\left(\frac{t}{12}\right)\right]$

46. $y = \frac{1}{6}\left[1+\cos^2(7t)\right]^3 \Rightarrow \frac{dy}{dt} = \frac{3}{6}\left[1+\cos^2(7t)\right]^2 \cdot 2\cos(7t)(-\sin(7t))(7) = -7\left[1+\cos^2(7t)\right]^2(\cos(7t)\sin(7t))$

47. $y=(1+\cos(t^2))^{1/2} \Rightarrow \frac{dy}{dt}=\frac{1}{2}(1+\cos(t^2))^{-1/2}\cdot\frac{d}{dt}(1+\cos(t^2))=\frac{1}{2}(1+\cos(t^2))^{-1/2}\left(-\sin(t^2)\cdot\frac{d}{dt}(t^2)\right)$
$=-\frac{1}{2}(1+\cos(t^2))^{-1/2}(\sin(t^2))\cdot 2t=-\frac{t\sin(t^2)}{\sqrt{1+\cos(t^2)}}$

48. $y=4\sin\left(\sqrt{1+\sqrt{t}}\right) \Rightarrow \frac{dy}{dt}=4\cos\left(\sqrt{1+\sqrt{t}}\right)\cdot\frac{d}{dt}\left(\sqrt{1+\sqrt{t}}\right)=4\cos\left(\sqrt{1+\sqrt{t}}\right)\cdot\frac{1}{2\sqrt{1+\sqrt{t}}}\cdot\frac{d}{dt}(1+\sqrt{t})$
$=\frac{2\cos\left(\sqrt{1+\sqrt{t}}\right)}{\sqrt{1+\sqrt{t}}\cdot 2\sqrt{t}}=\frac{\cos\left(\sqrt{1+\sqrt{t}}\right)}{\sqrt{t+\sqrt{t}}}$

49. $y=\left(1+\frac{1}{x}\right)^3 \Rightarrow y'=3\left(1+\frac{1}{x}\right)^2\left(-\frac{1}{x^2}\right)=-\frac{3}{x^2}\left(1+\frac{1}{x}\right)^2 \Rightarrow y''=\left(-\frac{3}{x^2}\right)\cdot\frac{d}{dx}\left(1+\frac{1}{x}\right)^2-\left(1+\frac{1}{x}\right)^2\cdot\frac{d}{dx}\left(\frac{3}{x^2}\right)$
$=\left(-\frac{3}{x^2}\right)\left(2\left(1+\frac{1}{x}\right)\left(-\frac{1}{x^2}\right)\right)+\left(\frac{6}{x^3}\right)\left(1+\frac{1}{x}\right)^2=\frac{6}{x^4}\left(1+\frac{1}{x}\right)+\frac{6}{x^3}\left(1+\frac{1}{x}\right)^2=\frac{6}{x^3}\left(1+\frac{1}{x}\right)\left(\frac{1}{x}+1+\frac{1}{x}\right)$
$=\frac{6}{x^3}\left(1+\frac{1}{x}\right)\left(1+\frac{2}{x}\right)$

50. $y=\left(1-\sqrt{x}\right)^{-1} \Rightarrow y'=-\left(1-\sqrt{x}\right)^{-2}\left(-\frac{1}{2}x^{-1/2}\right)=\frac{1}{2}\left(1-\sqrt{x}\right)^{-2}x^{-1/2}$
$\Rightarrow y''=\frac{1}{2}\left[\left(1-\sqrt{x}\right)^{-2}\left(-\frac{1}{2}x^{-3/2}\right)+x^{-1/2}(-2)\left(1-\sqrt{x}\right)^{-3}\left(-\frac{1}{2}x^{-1/2}\right)\right]$
$=\frac{1}{2}\left[\frac{-1}{2}x^{-3/2}\left(1-\sqrt{x}\right)^{-2}+x^{-1}\left(1-\sqrt{x}\right)^{-3}\right]=\frac{1}{2}x^{-1}\left(1-\sqrt{x}\right)^{-3}\left[-\frac{1}{2}x^{-1/2}\left(1-\sqrt{x}\right)+1\right]$
$=\frac{1}{2x}\left(1-\sqrt{x}\right)^{-3}\left(-\frac{1}{2\sqrt{x}}+\frac{1}{2}+1\right)=\frac{1}{2x}\left(1-\sqrt{x}\right)^{-3}\left(\frac{3}{2}-\frac{1}{2\sqrt{x}}\right)$

51. $y=\frac{1}{9}\cot(3x-1) \Rightarrow y'=-\frac{1}{9}\csc^2(3x-1)(3)=-\frac{1}{3}\csc^2(3x-1) \Rightarrow y''=\left(-\frac{2}{3}\right)\left(\csc(3x-1)\cdot\frac{d}{dx}\csc(3x-1)\right)$
$=-\frac{2}{3}\csc(3x-1)(-\csc(3x-1)\cot(3x-1)\cdot\frac{d}{dx}(3x-1))=2\csc^2(3x-1)\cot(3x-1)$

52. $y=9\tan\left(\frac{x}{3}\right) \Rightarrow y'=9\left(\sec^2\left(\frac{x}{3}\right)\right)\left(\frac{1}{3}\right)=3\sec^2\left(\frac{x}{3}\right) \Rightarrow y''=3\cdot 2\sec\left(\frac{x}{3}\right)\left(\sec\left(\frac{x}{3}\right)\tan\left(\frac{x}{3}\right)\right)\left(\frac{1}{3}\right)=2\sec^2\left(\frac{x}{3}\right)\tan\left(\frac{x}{3}\right)$

C

1. $y = x^{9/4} \Rightarrow \frac{dy}{dx} = \frac{9}{4}x^{5/4}$

2. $y = x^{-3/5} \Rightarrow \frac{dy}{dx} = -\frac{3}{5}x^{-8/5}$

3. $y = \sqrt[3]{2x} = (2x)^{1/3} \Rightarrow \frac{dy}{dx} = \frac{1}{3}(2x)^{-2/3} \cdot 2 = \frac{2^{1/3}}{3x^{2/3}}$

4. $y = \sqrt[4]{5x} = (5x)^{1/4} \Rightarrow \frac{dy}{dx} = \frac{1}{4}(5x)^{-3/4} \cdot 5 = \frac{5^{1/4}}{4x^{3/4}}$

5. $y = 7\sqrt{x+6} = 7(x+6)^{1/2} \Rightarrow \frac{dy}{dx} = \frac{7}{2}(x+6)^{-1/2} = \frac{7}{2\sqrt{x+6}}$

6. $y = -2\sqrt{x-1} = -2(x-1)^{1/2} \Rightarrow \frac{dy}{dx} = -1(x-1)^{-1/2} = -\frac{1}{\sqrt{x-1}}$

7. $y = (2x+5)^{-1/2} \Rightarrow \frac{dy}{dx} = -\frac{1}{2}(2x+5)^{-3/2} \cdot 2 = -(2x+5)^{-3/2}$

8. $y = (1-6x)^{2/3} \Rightarrow \frac{dy}{dx} = \frac{2}{3}(1-6x)^{-1/3}(-6) = -4(1-6x)^{-1/3}$

9. $y = x\left(x^2+1\right)^{1/2} \Rightarrow y' = x \cdot \frac{1}{2}\left(x^2+1\right)^{-1/2}(2x) + \left(x^2+1\right)^{1/2} \cdot 1 = \left(x^2+1\right)^{-1/2}\left(x^2+x^2+1\right) = \frac{2x^2+1}{\sqrt{x^2+1}}$

10. $y = x\left(x^2+1\right)^{-1/2} \Rightarrow y' = x \cdot \left(-\frac{1}{2}\right)\left(x^2+1\right)^{-3/2}(2x) + \left(x^2+1\right)^{-1/2} \cdot 1 = \left(x^2+1\right)^{-3/2}\left(-x^2+x^2+1\right) = \frac{1}{\left(x^2+1\right)^{3/2}}$

11. $s = \sqrt[7]{t^2} = t^{2/7} \Rightarrow \frac{ds}{dt} = \frac{2}{7}t^{-5/7}$

12. $r = \sqrt[4]{\theta^{-3}} = \theta^{-3/4} \Rightarrow \frac{dr}{d\theta} = -\frac{3}{4}\theta^{-7/4}$

13. $y = \sin\left((2t+5)^{-2/3}\right) \Rightarrow \frac{dy}{dt} = \cos\left((2t+5)^{-2/3}\right) \cdot \left(-\frac{2}{3}\right)(2t+5)^{-5/3} \cdot 2 = -\frac{4}{3}(2t+5)^{-5/3}\cos\left((2t+5)^{-2/3}\right)$

14. $z = \cos\left((1-6t)^{2/3}\right) \Rightarrow \frac{dz}{dt} = -\sin\left((1-6t)^{2/3}\right) \cdot \frac{2}{3}(1-6t)^{-1/3}(-6) = 4(1-6t)^{-1/3}\sin\left((1-6t)^{2/3}\right)$

15. $f(x) = \sqrt{1-\sqrt{x}} = \left(1-x^{1/2}\right)^{1/2} \Rightarrow f'(x) = \frac{1}{2}\left(1-x^{1/2}\right)^{-1/2}\left(-\frac{1}{2}x^{-1/2}\right) = \frac{-1}{4\left(\sqrt{1-\sqrt{x}}\right)\sqrt{x}} = \frac{-1}{4\sqrt{x\left(1-\sqrt{x}\right)}}$

16. $g(x) = 2\left(2x^{-1/2}+1\right)^{-1/3} \Rightarrow g'(x) = -\frac{2}{3}\left(2x^{-1/2}+1\right)^{-4/3} \cdot (-1)x^{-3/2} = \frac{2}{3}\left(2x^{-1/2}+1\right)^{-4/3}x^{-3/2}$

17. $h(\theta) = \sqrt[3]{1+\cos(2\theta)} = (1+\cos 2\theta)^{1/3} \Rightarrow h'(\theta) = \frac{1}{3}(1+\cos 2\theta)^{-2/3} \cdot (-\sin 2\theta) \cdot 2 = -\frac{2}{3}(\sin 2\theta)(1+\cos 2\theta)^{-2/3}$

18. $k(\theta) = (\sin(\theta+5))^{5/4} \Rightarrow k'(\theta) = \frac{5}{4}(\sin(\theta+5))^{1/4} \cdot \cos(\theta+5) = \frac{5}{4}\cos(\theta+5)(\sin(\theta+5))^{1/4}$

19. $x^2y + xy^2 = 6$:

Step 1: $\left(x^2\frac{dy}{dx} + y \cdot 2x\right) + \left(x \cdot 2y\frac{dy}{dx} + y^2 \cdot 1\right) = 0$

Step 2: $x^2\frac{dy}{dx} + 2xy\frac{dy}{dx} = -2xy - y^2$

Step 3: $\frac{dy}{dx}\left(x^2+2xy\right) = -2xy - y^2$

Step 4: $\frac{dy}{dx} = \frac{-2xy-y^2}{x^2+2xy}$

20. $x^3 + y^3 = 18xy \Rightarrow 3x^2 + 3y^2\frac{dy}{dx} = 18y + 18x\frac{dy}{dx} \Rightarrow \left(3y^2 - 18x\right)\frac{dy}{dx} = 18y - 3x^2 \Rightarrow \frac{dy}{dx} = \frac{6y-x^2}{y^2-6x}$

21. $2xy + y^2 = x + y$:

Step 1: $\left(2x\frac{dy}{dx} + 2y\right) + 2y\frac{dy}{dx} = 1 + \frac{dy}{dx}$

Step 2: $2x\,\frac{dy}{dx} + 2y\,\frac{dy}{dx} - \frac{dy}{dx} = 1 - 2y$

Step 3: $\frac{dy}{dx}(2x + 2y - 1) = 1 - 2y$

Step 4: $\frac{dy}{dx} = \frac{1-2y}{2x+2y-1}$

22. $x^3 - xy + y^3 = 1 \Rightarrow 3x^2 - y - x\,\frac{dy}{dx} + 3y^2\,\frac{dy}{dx} = 0 \Rightarrow (3y^2 - x)\,\frac{dy}{dx} = y - 3x^2 \Rightarrow \frac{dy}{dx} = \frac{y-3x^2}{3y^2-x}$

23. $x^2(x-y)^2 = x^2 - y^2$:

Step 1: $x^2\left[2(x-y)\left(1-\frac{dy}{dx}\right)\right] + (x-y)^2(2x) = 2x - 2y\,\frac{dy}{dx}$

Step 2: $-2x^2(x-y)\,\frac{dy}{dx} + 2y\,\frac{dy}{dx} = 2x - 2x^2(x-y) - 2x(x-y)^2$

Step 3: $\frac{dy}{dx}\left[-2x^2(x-y) + 2y\right] = 2x\left[1 - x(x-y) - (x-y)^2\right]$

Step 4: $\frac{dy}{dx} = \frac{2x\left[1-x(x-y)-(x-y)^2\right]}{-2x^2(x-y)+2y} = \frac{x\left[1-x(x-y)-(x-y)^2\right]}{y-x^2(x-y)} = \frac{x\left(1-x^2+xy-x^2+2xy-y^2\right)}{x^2y-x^3+y}$

$= \frac{x-2x^3+3x^2y-xy^2}{x^2y-x^3+y}$

24. $(3xy+7)^2 = 6y \Rightarrow 2(3xy+7)\cdot\left(3x\,\frac{dy}{dx} + 3y\right) = 6\,\frac{dy}{dx} \Rightarrow 2(3xy+7)(3x)\,\frac{dy}{dx} - 6\,\frac{dy}{dx} = -6y(3xy+7)$

$\Rightarrow \frac{dy}{dx}\left[6x(3xy+7) - 6\right] = -6y(3xy+7) \Rightarrow \frac{dy}{dx} = -\frac{y(3xy+7)}{x(3xy+7)-1} = \frac{3xy^2+7y}{1-3x^2y-7x}$

25. $y^2 = \frac{x-1}{x+1} \Rightarrow 2y\,\frac{dy}{dx} = \frac{(x+1)-(x-1)}{(x+1)^2} = \frac{2}{(x+1)^2} \Rightarrow \frac{dy}{dx} = \frac{1}{y(x+1)^2}$

26. $x^2 = \frac{x-y}{x+y} \Rightarrow x^3 + x^2y = x - y \Rightarrow 3x^2 + 2xy + x^2y' = 1 - y' \Rightarrow (x^2+1)\,y' = 1 - 3x^2 - 2xy \Rightarrow y' = \frac{1-3x^2-2xy}{x^2+1}$

27. $x = \tan y \Rightarrow 1 = (\sec^2 y)\,\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{\sec^2 y} = \cos^2 y$

28. $xy = \cot(xy) \Rightarrow x\frac{dy}{dx} + y = -\csc^2(xy)\left(x\frac{dy}{dx} + y\right) \Rightarrow x\frac{dy}{dx} + x\csc^2(xy)\frac{dy}{dx} = -y\csc^2(xy) - y$

$\Rightarrow \frac{dy}{dx}\left[x + x\csc^2(xy)\right] = -y\left[\csc^2(xy) + 1\right] \Rightarrow \frac{dy}{dx} = \frac{-y\left[\csc^2(xy)+1\right]}{x\left[1+\csc^2(xy)\right]} = -\frac{y}{x}$

29. $x + \tan(xy) = 0 \Rightarrow 1 + \left[\sec^2(xy)\right]\left(y + x\,\frac{dy}{dx}\right) = 0 \Rightarrow x\sec^2(xy)\,\frac{dy}{dx} = -1 - y\sec^2(xy) \Rightarrow \frac{dy}{dx} = \frac{-1-y\sec^2(xy)}{x\sec^2(xy)}$

$= \frac{-1}{x\sec^2(xy)} - \frac{y}{x} = \frac{-\cos^2(xy)}{x} - \frac{y}{x} = \frac{-\cos^2(xy) - y}{x}$

30. $x + \sin y = xy \Rightarrow 1 + (\cos y)\,\frac{dy}{dx} = y + x\,\frac{dy}{dx} \Rightarrow (\cos y - x)\,\frac{dy}{dx} = y - 1 \Rightarrow \frac{dy}{dx} = \frac{y-1}{\cos y - x}$

31. $y\sin\left(\frac{1}{y}\right) = 1 - xy \Rightarrow y\left[\cos\left(\frac{1}{y}\right)\cdot(-1)\,\frac{1}{y^2}\cdot\frac{dy}{dx}\right] + \sin\left(\frac{1}{y}\right)\cdot\frac{dy}{dx} = -x\,\frac{dy}{dx} - y \Rightarrow$

$\frac{dy}{dx}\left[-\frac{1}{y}\cos\left(\frac{1}{y}\right) + \sin\left(\frac{1}{y}\right) + x\right] = -y \Rightarrow \frac{dy}{dx} = \frac{-y}{-\frac{1}{y}\cos\left(\frac{1}{y}\right)+\sin\left(\frac{1}{y}\right)+x} = \frac{-y^2}{y\sin\left(\frac{1}{y}\right)-\cos\left(\frac{1}{y}\right)+xy}$

32. $y^2\cos\left(\frac{1}{y}\right) = 2x + 2y \Rightarrow y^2\left[-\sin\left(\frac{1}{y}\right)\cdot(-1)\,\frac{1}{y^2}\cdot\frac{dy}{dx}\right] + \cos\left(\frac{1}{y}\right)\cdot 2y\,\frac{dy}{dx} = 2 + 2\,\frac{dy}{dx} \Rightarrow$

$\frac{dy}{dx}\left[\sin\left(\frac{1}{y}\right) + 2y\cos\left(\frac{1}{y}\right) - 2\right] = 2 \Rightarrow \frac{dy}{dx} = \frac{2}{\sin\left(\frac{1}{y}\right)+2y\cos\left(\frac{1}{y}\right)-2}$

33. $\theta^{1/2} + r^{1/2} = 1 \Rightarrow \frac{1}{2}\,\theta^{-1/2} + \frac{1}{2}\,r^{-1/2}\cdot\frac{dr}{d\theta} = 0 \Rightarrow \frac{dr}{d\theta}\left[\frac{1}{2\sqrt{r}}\right] = \frac{-1}{2\sqrt{\theta}} \Rightarrow \frac{dr}{d\theta} = -\frac{2\sqrt{r}}{2\sqrt{\theta}} = -\frac{\sqrt{r}}{\sqrt{\theta}}$

34. $r - 2\sqrt{\theta} = \frac{3}{2}\theta^{2/3} + \frac{4}{3}\theta^{3/4} \Rightarrow \frac{dr}{d\theta} - \theta^{-1/2} = \theta^{-1/3} + \theta^{-1/4} \Rightarrow \frac{dr}{d\theta} = \theta^{-1/2} + \theta^{-1/3} + \theta^{-1/4}$

35. $\sin(r\theta) = \frac{1}{2} \Rightarrow [\cos(r\theta)]\left(r + \theta\frac{dr}{d\theta}\right) = 0 \Rightarrow \frac{dr}{d\theta}[\theta\cos(r\theta)] = -r\cos(r\theta) \Rightarrow \frac{dr}{d\theta} = \frac{-r\cos(r\theta)}{\theta\cos(r\theta)} = -\frac{r}{\theta}$, $\cos(r\theta) \neq 0$

36. $\cos r + \cot\theta = r\theta \Rightarrow (-\sin r)\frac{dr}{d\theta} - \csc^2\theta = r + \theta\frac{dr}{d\theta} \Rightarrow \frac{dr}{d\theta}[-\sin r - \theta] = r + \csc^2\theta \Rightarrow \frac{dr}{d\theta} = -\frac{r + \csc^2\theta}{\sin r + \theta}$

37. $x^2 + y^2 = 1 \Rightarrow 2x + 2yy' = 0 \Rightarrow 2yy' = -2x \Rightarrow \frac{dy}{dx} = y' = -\frac{x}{y}$; now to find $\frac{d^2y}{dx^2}$, $\frac{d}{dx}(y') = \frac{d}{dx}\left(-\frac{x}{y}\right)$
$\Rightarrow y'' = \frac{y(-1)+xy'}{y^2} = \frac{-y+x\left(-\frac{x}{y}\right)}{y^2}$ since $y' = -\frac{x}{y} \Rightarrow \frac{d^2y}{dx^2} = y'' = \frac{-y^2-x^2}{y^3} = \frac{-y^2-(1-y^2)}{y^3} = \frac{-1}{y^3}$

38. $x^{2/3} + y^{2/3} = 1 \Rightarrow \frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx}\left[\frac{2}{3}y^{-1/3}\right] = -\frac{2}{3}x^{-1/3} \Rightarrow y' = \frac{dy}{dx} = -\frac{x^{-1/3}}{y^{-1/3}} = -\left(\frac{y}{x}\right)^{1/3}$;
Differentiating again, $y'' = \frac{x^{1/3}\cdot\left(-\frac{1}{3}y^{-2/3}\right)y' + y^{1/3}\left(\frac{1}{3}x^{-2/3}\right)}{x^{2/3}} = \frac{x^{1/3}\cdot\left(-\frac{1}{3}y^{-2/3}\right)\left(-\frac{y^{1/3}}{x^{1/3}}\right) + y^{1/3}\left(\frac{1}{3}x^{-2/3}\right)}{x^{2/3}}$
$\Rightarrow \frac{d^2y}{dx^2} = \frac{1}{3}x^{-2/3}y^{-1/3} + \frac{1}{3}y^{1/3}x^{-4/3} = \frac{y^{1/3}}{3x^{4/3}} + \frac{1}{3y^{1/3}x^{2/3}}$

39. $y^2 = x^2 + 2x \Rightarrow 2yy' = 2x + 2 \Rightarrow y' = \frac{2x+2}{2y} = \frac{x+1}{y}$; then $y'' = \frac{y-(x+1)y'}{y^2} = \frac{y-(x+1)\left(\frac{x+1}{y}\right)}{y^2}$
$\Rightarrow \frac{d^2y}{dx^2} = y'' = \frac{y^2-(x+1)^2}{y^3}$

40. $y^2 - 2x = 1 - 2y \Rightarrow 2y\cdot y' - 2 = -2y' \Rightarrow y'(2y+2) = 2 \Rightarrow y' = \frac{1}{y+1} = (y+1)^{-1}$; then $y'' = -(y+1)^{-2}\cdot y'$
$= -(y+1)^{-2}(y+1)^{-1} \Rightarrow \frac{d^2y}{dx^2} = y'' = \frac{-1}{(y+1)^3}$

41. $2\sqrt{y} = x - y \Rightarrow y^{-1/2}y' = 1 - y' \Rightarrow y'\left(y^{-1/2}+1\right) = 1 \Rightarrow \frac{dy}{dx} = y' = \frac{1}{y^{-1/2}+1} = \frac{\sqrt{y}}{\sqrt{y}+1}$; we can
differentiate the equation $y'\left(y^{-1/2}+1\right) = 1$ again to find y'': $y'\left(-\frac{1}{2}y^{-3/2}y'\right) + \left(y^{-1/2}+1\right)y'' = 0$
$\Rightarrow \left(y^{-1/2}+1\right)y'' = \frac{1}{2}[y']^2y^{-3/2} \Rightarrow \frac{d^2y}{dx^2} = y'' = \frac{\frac{1}{2}\left(\frac{1}{y^{-1/2}+1}\right)^2y^{-3/2}}{\left(y^{-1/2}+1\right)} = \frac{1}{2y^{3/2}\left(y^{-1/2}+1\right)^3} = \frac{1}{2\left(1+\sqrt{y}\right)^3}$

42. $xy + y^2 = 1 \Rightarrow xy' + y + 2yy' = 0 \Rightarrow xy' + 2yy' = -y \Rightarrow y'(x+2y) = -y \Rightarrow y' = \frac{-y}{(x+2y)}$; $\frac{d^2y}{dx^2} = y''$
$= \frac{-(x+2y)y' + y(1+2y')}{(x+2y)^2} = \frac{-(x+2y)\left[\frac{-y}{(x+2y)}\right] + y\left[1+2\left(\frac{-y}{(x+2y)}\right)\right]}{(x+2y)^2} = \frac{\frac{1}{(x+2y)}\left[y(x+2y) + y(x+2y) - 2y^2\right]}{(x+2y)^2}$
$= \frac{2y(x+2y) - 2y^2}{(x+2y)^3} = \frac{2y^2 + 2xy}{(x+2y)^3} = \frac{2y(x+y)}{(x+2y)^3}$

43. $x^3 + y^3 = 16 \Rightarrow 3x^2 + 3y^2y' = 0 \Rightarrow 3y^2y' = -3x^2 \Rightarrow y' = -\frac{x^2}{y^2}$; we differentiate $y^2y' = -x^2$ to find y'':
$y^2y'' + y'[2y\cdot y'] = -2x \Rightarrow y^2y'' = -2x - 2y[y']^2 \Rightarrow y'' = \frac{-2x - 2y\left(-\frac{x^2}{y^2}\right)^2}{y^2} = \frac{-2x - \frac{2x^4}{y^3}}{y^2}$
$= \frac{-2xy^3 - 2x^4}{y^5} \Rightarrow \left.\frac{d^2y}{dx^2}\right|_{(2,2)} = \frac{-32-32}{32} = -2$

44. $xy + y^2 = 1 \Rightarrow xy' + y + 2yy' = 0 \Rightarrow y'(x+2y) = -y \Rightarrow y' = \frac{-y}{(x+2y)} \Rightarrow y'' = \frac{(x+2y)(-y') - (-y)(1+2y')}{(x+2y)^2}$;
since $y'|_{(0,-1)} = -\frac{1}{2}$ we obtain $y''|_{(0,-1)} = \frac{(-2)\left(\frac{1}{2}\right) - (1)(0)}{4} = -\frac{1}{4}$

45. $y^2 + x^2 = y^4 - 2x$ at $(-2, 1)$ and $(-2, -1) \Rightarrow 2y\frac{dy}{dx} + 2x = 4y^3\frac{dy}{dx} - 2 \Rightarrow 2y\frac{dy}{dx} - 4y^3\frac{dy}{dx} = -2 - 2x$

$\Rightarrow \frac{dy}{dx}(2y - 4y^3) = -2 - 2x \Rightarrow \frac{dy}{dx} = \frac{x+1}{2y^3-y} \Rightarrow \left.\frac{dy}{dx}\right|_{(-2,1)} = -1$ and $\left.\frac{dy}{dx}\right|_{(-2,-1)} = 1$

46. $(x^2+y^2)^2 = (x-y)^2$ at $(1,0)$ and $(1,-1) \Rightarrow 2(x^2+y^2)\left(2x+2y\frac{dy}{dx}\right) = 2(x-y)\left(1-\frac{dy}{dx}\right)$
$\Rightarrow \frac{dy}{dx}[2y(x^2+y^2)+(x-y)] = -2x(x^2+y^2)+(x-y) \Rightarrow \frac{dy}{dx} = \frac{-2x(x^2+y^2)+(x-y)}{2y(x^2+y^2)+(x-y)} \Rightarrow \left.\frac{dy}{dx}\right|_{(1,0)} = -1$
and $\left.\frac{dy}{dx}\right|_{(1,-1)} = 1$

47. $x^2 + xy - y^2 = 1 \Rightarrow 2x + y + xy' - 2yy' = 0 \Rightarrow (x-2y)y' = -2x - y \Rightarrow y' = \frac{2x+y}{2y-x}$;
(a) the slope of the tangent line $m = y'|_{(2,3)} = \frac{7}{4} \Rightarrow$ the tangent line is $y - 3 = \frac{7}{4}(x-2) \Rightarrow y = \frac{7}{4}x - \frac{1}{2}$
(b) the normal line is $y - 3 = -\frac{4}{7}(x-2) \Rightarrow y = -\frac{4}{7}x + \frac{29}{7}$

48. $x^2 + y^2 = 25 \Rightarrow 2x + 2yy' = 0 \Rightarrow y' = -\frac{x}{y}$;
(a) the slope of the tangent line $m = y'|_{(3,-4)} = -\left.\frac{x}{y}\right|_{(3,-4)} = \frac{3}{4} \Rightarrow$ the tangent line is $y + 4 = \frac{3}{4}(x-3)$
$\Rightarrow y = \frac{3}{4}x - \frac{25}{4}$
(b) the normal line is $y + 4 = -\frac{4}{3}(x-3) \Rightarrow y = -\frac{4}{3}x$

49. $x^2y^2 = 9 \Rightarrow 2xy^2 + 2x^2yy' = 0 \Rightarrow x^2yy' = -xy^2 \Rightarrow y' = -\frac{y}{x}$;
(a) the slope of the tangent line $m = y'|_{(-1,3)} = -\left.\frac{y}{x}\right|_{(-1,3)} = 3 \Rightarrow$ the tangent line is $y - 3 = 3(x+1)$
$\Rightarrow y = 3x + 6$
(b) the normal line is $y - 3 = -\frac{1}{3}(x+1) \Rightarrow y = -\frac{1}{3}x + \frac{8}{3}$

50. $y^2 - 2x - 4y - 1 = 0 \Rightarrow 2yy' - 2 - 4y' = 0 \Rightarrow 2(y-2)y' = 2 \Rightarrow y' = \frac{1}{y-2}$;
(a) the slope of the tangent line $m = y'|_{(-2,1)} = -1 \Rightarrow$ the tangent line is $y - 1 = -1(x+2) \Rightarrow y = -x - 1$
(b) the normal line is $y - 1 = 1(x+2) \Rightarrow y = x + 3$

51. $6x^2 + 3xy + 2y^2 + 17y - 6 = 0 \Rightarrow 12x + 3y + 3xy' + 4yy' + 17y' = 0 \Rightarrow y'(3x+4y+17) = -12x - 3y$
$\Rightarrow y' = \frac{-12x-3y}{3x+4y+17}$;
(a) the slope of the tangent line $m = y'|_{(-1,0)} = \left.\frac{-12x-3y}{3x+4y+17}\right|_{(-1,0)} = \frac{6}{7} \Rightarrow$ the tangent line is $y - 0 = \frac{6}{7}(x+1)$
$\Rightarrow y = \frac{6}{7}x + \frac{6}{7}$
(b) the normal line is $y - 0 = -\frac{7}{6}(x+1) \Rightarrow y = -\frac{7}{6}x - \frac{7}{6}$

52. $x^2 - \sqrt{3}xy + 2y^2 = 5 \Rightarrow 2x - \sqrt{3}xy' - \sqrt{3}y + 4yy' = 0 \Rightarrow y'\left(4y - \sqrt{3}x\right) = \sqrt{3}y - 2x \Rightarrow y' = \frac{\sqrt{3}y-2x}{4y-\sqrt{3}x}$;
(a) the slope of the tangent line $m = y'|_{(\sqrt{3},2)} = \left.\frac{\sqrt{3}y-2x}{4y-\sqrt{3}x}\right|_{(\sqrt{3},2)} = 0 \Rightarrow$ the tangent line is $y = 2$
(b) the normal line is $x = \sqrt{3}$

53. $2xy + \pi \sin y = 2\pi \Rightarrow 2xy' + 2y + \pi(\cos y)y' = 0 \Rightarrow y'(2x + \pi\cos y) = -2y \Rightarrow y' = \frac{-2y}{2x+\pi\cos y}$;
(a) the slope of the tangent line $m = y'|_{(1,\frac{\pi}{2})} = \left.\frac{-2y}{2x+\pi\cos y}\right|_{(1,\frac{\pi}{2})} = -\frac{\pi}{2} \Rightarrow$ the tangent line is
$y - \frac{\pi}{2} = -\frac{\pi}{2}(x-1) \Rightarrow y = -\frac{\pi}{2}x + \pi$
(b) the normal line is $y - \frac{\pi}{2} = \frac{2}{\pi}(x-1) \Rightarrow y = \frac{2}{\pi}x - \frac{2}{\pi} + \frac{\pi}{2}$

54. $x \sin 2y = y \cos 2x \Rightarrow x(\cos 2y)2y' + \sin 2y = -2y \sin 2x + y' \cos 2x \Rightarrow y'(2x \cos 2y - \cos 2x)$
$= -\sin 2y - 2y \sin 2x \Rightarrow y' = \frac{\sin 2y + 2y \sin 2x}{\cos 2x - 2x \cos 2y}$;

(a) the slope of the tangent line $m = y'\big|_{\left(\frac{\pi}{4}, \frac{\pi}{2}\right)} = \frac{\sin 2y + 2y \sin 2x}{\cos 2x - 2x \cos 2y}\Big|_{\left(\frac{\pi}{4}, \frac{\pi}{2}\right)} = \frac{\pi}{\frac{\pi}{2}} = 2 \Rightarrow$ the tangent line is
$y - \frac{\pi}{2} = 2\left(x - \frac{\pi}{4}\right) \Rightarrow y = 2x$

(b) the normal line is $y - \frac{\pi}{2} = -\frac{1}{2}\left(x - \frac{\pi}{4}\right) \Rightarrow y = -\frac{1}{2}x + \frac{5\pi}{8}$

55. $y = 2\sin(\pi x - y) \Rightarrow y' = 2[\cos(\pi x - y)] \cdot (\pi - y') \Rightarrow y'[1 + 2\cos(\pi x - y)] = 2\pi \cos(\pi x - y)$
$\Rightarrow y' = \frac{2\pi \cos(\pi x - y)}{1 + 2\cos(\pi x - y)}$;

(a) the slope of the tangent line $m = y'\big|_{(1,0)} = \frac{2\pi \cos(\pi x - y)}{1 + 2\cos(\pi x - y)}\Big|_{(1,0)} = 2\pi \Rightarrow$ the tangent line is
$y - 0 = 2\pi(x - 1) \Rightarrow y = 2\pi x - 2\pi$

(b) the normal line is $y - 0 = -\frac{1}{2\pi}(x - 1) \Rightarrow y = -\frac{x}{2\pi} + \frac{1}{2\pi}$

56. $x^2 \cos^2 y - \sin y = 0 \Rightarrow x^2(2\cos y)(-\sin y)y' + 2x\cos^2 y - y'\cos y = 0 \Rightarrow y'\left[-2x^2 \cos y \sin y - \cos y\right]$
$= -2x\cos^2 y \Rightarrow y' = \frac{2x\cos^2 y}{2x^2 \cos y \sin y + \cos y}$;

(a) the slope of the tangent line $m = y'\big|_{(0,\pi)} = \frac{2x\cos^2 y}{2x^2 \cos y \sin y + \cos y}\Big|_{(0,\pi)} = 0 \Rightarrow$ the tangent line is $y = \pi$

(b) the normal line is $x = 0$

D

1. $A = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

2. $S = 4\pi r^2 \Rightarrow \frac{dS}{dt} = 8\pi r \frac{dr}{dt}$

3. (a) $V = \pi r^2 h \Rightarrow \frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$ (b) $V = \pi r^2 h \Rightarrow \frac{dV}{dt} = 2\pi r h \frac{dr}{dt}$
(c) $V = \pi r^2 h \Rightarrow \frac{dV}{dt} = \pi r^2 \frac{dh}{dt} + 2\pi r h \frac{dr}{dt}$

4. (a) $V = \frac{1}{3}\pi r^2 h \Rightarrow \frac{dV}{dt} = \frac{1}{3}\pi r^2 \frac{dh}{dt}$ (b) $V = \frac{1}{3}\pi r^2 h \Rightarrow \frac{dV}{dt} = \frac{2}{3}\pi r h \frac{dr}{dt}$
(c) $\frac{dV}{dt} = \frac{1}{3}\pi r^2 \frac{dh}{dt} + \frac{2}{3}\pi r h \frac{dr}{dt}$

5. (a) $\frac{dV}{dt} = 1$ volt/sec (b) $\frac{dI}{dt} = -\frac{1}{3}$ amp/sec
(c) $\frac{dV}{dt} = R\left(\frac{dI}{dt}\right) + I\left(\frac{dR}{dt}\right) \Rightarrow \frac{dR}{dt} = \frac{1}{I}\left(\frac{dV}{dt} - R\frac{dI}{dt}\right) \Rightarrow \frac{dR}{dt} = \frac{1}{I}\left(\frac{dV}{dt} - \frac{V}{I}\frac{dI}{dt}\right)$
(d) $\frac{dR}{dt} = \frac{1}{2}\left[1 - \frac{12}{2}\left(-\frac{1}{3}\right)\right] = \left(\frac{1}{2}\right)(3) = \frac{3}{2}$ ohms/sec, R is increasing

6. (a) $P = RI^2 \Rightarrow \frac{dP}{dt} = I^2 \frac{dR}{dt} + 2RI \frac{dI}{dt}$
(b) $P = RI^2 \Rightarrow 0 = \frac{dP}{dt} = I^2 \frac{dR}{dt} + 2RI \frac{dI}{dt} \Rightarrow \frac{dR}{dt} = -\frac{2RI}{I^2}\frac{dI}{dt} = -\frac{2\left(\frac{P}{I}\right)}{I^2}\frac{dI}{dt} = -\frac{2P}{I^3}\frac{dI}{dt}$

7. (a) $s = \sqrt{x^2 + y^2} = \left(x^2 + y^2\right)^{1/2} \Rightarrow \frac{ds}{dt} = \frac{x}{\sqrt{x^2 + y^2}}\frac{dx}{dt}$
(b) $s = \sqrt{x^2 + y^2} = \left(x^2 + y^2\right)^{1/2} \Rightarrow \frac{ds}{dt} = \frac{x}{\sqrt{x^2 + y^2}}\frac{dx}{dt} + \frac{y}{\sqrt{x^2 + y^2}}\frac{dy}{dt}$
(c) $s = \sqrt{x^2 + y^2} \Rightarrow s^2 = x^2 + y^2 \Rightarrow 2s\frac{ds}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt} \Rightarrow 2s \cdot 0 = 2x\frac{dx}{dt} + 2y\frac{dy}{dt} \Rightarrow \frac{dx}{dt} = -\frac{y}{x}\frac{dy}{dt}$

8. (a) $s = \sqrt{x^2 + y^2 + z^2} \Rightarrow s^2 = x^2 + y^2 + z^2 \Rightarrow 2s\frac{ds}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt} + 2z\frac{dz}{dt}$

$\Rightarrow \frac{ds}{dt} = \frac{x}{\sqrt{x^2+y^2+z^2}} \frac{dx}{dt} + \frac{y}{\sqrt{x^2+y^2+z^2}} \frac{dy}{dt} + \frac{z}{\sqrt{x^2+y^2+z^2}} \frac{dz}{dt}$

(b) From part (a) with $\frac{dx}{dt} = 0 \Rightarrow \frac{ds}{dt} = \frac{y}{\sqrt{x^2+y^2+z^2}} \frac{dy}{dt} + \frac{z}{\sqrt{x^2+y^2+z^2}} \frac{dz}{dt}$

(c) From part (a) with $\frac{ds}{dt} = 0 \Rightarrow 0 = 2x \frac{dx}{dt} + 2y \frac{dy}{dt} + 2z \frac{dz}{dt} \Rightarrow \frac{dx}{dt} + \frac{y}{x} \frac{dy}{dt} + \frac{z}{x} \frac{dz}{dt} = 0$

9. (a) $A = \frac{1}{2} ab \sin\theta \Rightarrow \frac{dA}{dt} = \frac{1}{2} ab \cos\theta \frac{d\theta}{dt}$ (b) $A = \frac{1}{2} ab \sin\theta \Rightarrow \frac{dA}{dt} = \frac{1}{2} ab \cos\theta \frac{d\theta}{dt} + \frac{1}{2} b \sin\theta \frac{da}{dt}$

(c) $A = \frac{1}{2} ab \sin\theta \Rightarrow \frac{dA}{dt} = \frac{1}{2} ab \cos\theta \frac{d\theta}{dt} + \frac{1}{2} b \sin\theta \frac{da}{dt} + \frac{1}{2} a \sin\theta \frac{db}{dt}$

10. Given $A = \pi r^2$, $\frac{dr}{dt} = 0.01$ cm/sec, and $r = 50$ cm. Since $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$, then $\frac{dA}{dt}\Big|_{r=50} = 2\pi(50)\left(\frac{1}{100}\right)$
$= \pi$ cm^2/min.

11. Given $\frac{d\ell}{dt} = -2$ cm/sec, $\frac{dw}{dt} = 2$ cm/sec, $\ell = 12$ cm and $w = 5$ cm.

(a) $A = \ell w \Rightarrow \frac{dA}{dt} = \ell \frac{dw}{dt} + w \frac{d\ell}{dt} \Rightarrow \frac{dA}{dt} = 12(2) + 5(-2) = 14$ cm^2/sec, increasing

(b) $P = 2\ell + 2w \Rightarrow \frac{dP}{dt} = 2 \frac{d\ell}{dt} + 2 \frac{dw}{dt} = 2(-2) + 2(2) = 0$ cm/sec, constant

(c) $D = \sqrt{w^2 + \ell^2} = (w^2 + \ell^2)^{1/2} \Rightarrow \frac{dD}{dt} = \frac{1}{2}(w^2+\ell^2)^{-1/2}\left(2w \frac{dw}{dt} + 2\ell \frac{d\ell}{dt}\right) \Rightarrow \frac{dD}{dt} = \frac{w \frac{dw}{dt} + \ell \frac{d\ell}{dt}}{\sqrt{w^2+\ell^2}}$
$= \frac{(5)(2)+(12)(-2)}{\sqrt{25+144}} = -\frac{14}{13}$ cm/sec, decreasing

12. (a) $V = xyz \Rightarrow \frac{dV}{dt} = yz \frac{dx}{dt} + xz \frac{dy}{dt} + xy \frac{dz}{dt} \Rightarrow \frac{dV}{dt}\Big|_{(4,3,2)} = (3)(2)(1) + (4)(2)(-2) + (4)(3)(1) = 2$ m^3/sec

(b) $S = 2xy + 2xz + 2yz \Rightarrow \frac{dS}{dt} = (2y+2z) \frac{dx}{dt} + (2x+2z) \frac{dy}{dt} + (2x+2y) \frac{dz}{dt}$
$\Rightarrow \frac{dS}{dt}\Big|_{(4,3,2)} = (10)(1) + (12)(-2) + (14)(1) = 0$ m^2/sec

(c) $\ell = \sqrt{x^2+y^2+z^2} = (x^2+y^2+z^2)^{1/2} \Rightarrow \frac{d\ell}{dt} = \frac{x}{\sqrt{x^2+y^2+z^2}} \frac{dx}{dt} + \frac{y}{\sqrt{x^2+y^2+z^2}} \frac{dy}{dt} + \frac{z}{\sqrt{x^2+y^2+z^2}} \frac{dz}{dt}$
$\Rightarrow \frac{d\ell}{dt}\Big|_{(4,3,2)} = \left(\frac{4}{\sqrt{29}}\right)(1) + \left(\frac{3}{\sqrt{29}}\right)(-2) + \left(\frac{2}{\sqrt{29}}\right)(1) = 0$ m/sec

13. Given: $\frac{dx}{dt} = 5$ ft/sec, the ladder is 13 ft long, and $x = 12$, $y = 5$ at the instant of time

(a) Since $x^2 + y^2 = 169 \Rightarrow \frac{dy}{dt} = -\frac{x}{y} \frac{dx}{dt} = -\left(\frac{12}{5}\right)(5) = -12$ ft/sec, the ladder is sliding down the wall

(b) The area of the triangle formed by the ladder and walls is $A = \frac{1}{2} xy \Rightarrow \frac{dA}{dt} = \left(\frac{1}{2}\right)\left(x \frac{dy}{dt} + y \frac{dx}{dt}\right)$. The area is changing at $\frac{1}{2}[12(-12) + 5(5)] = -\frac{119}{2} = -59.5$ ft^2/sec.

(c) $\cos\theta = \frac{x}{13} \Rightarrow -\sin\theta \frac{d\theta}{dt} = \frac{1}{13} \cdot \frac{dx}{dt} \Rightarrow \frac{d\theta}{dt} = -\frac{1}{13 \sin\theta} \cdot \frac{dx}{dt} = -\left(\frac{1}{5}\right)(5) = -1$ rad/sec

14. $s^2 = y^2 + x^2 \Rightarrow 2s \frac{ds}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt} \Rightarrow \frac{ds}{dt} = \frac{1}{s}\left(x \frac{dx}{dt} + y \frac{dy}{dt}\right) \Rightarrow \frac{ds}{dt} = \frac{1}{\sqrt{169}}[5(-442) + 12(-481)]$
$= -614$ knots

15. Let s represent the distance between the girl and the kite and x represents the horizontal distance between the girl and kite $\Rightarrow s^2 = (300)^2 + x^2 \Rightarrow \frac{ds}{dt} = \frac{x}{s} \frac{dx}{dt} = \frac{400(25)}{500} = 20$ ft/sec.

16. When the diameter is 3.8 in., the radius is 1.9 in. and $\frac{dr}{dt} = \frac{1}{3000}$ in/min. Also $V = 6\pi r^2 \Rightarrow \frac{dV}{dt} = 12\pi r \frac{dr}{dt}$
$\Rightarrow \frac{dV}{dt} = 12\pi(1.9)\left(\frac{1}{3000}\right) = 0.0076\pi$. The volume is changing at about 0.0239 in^3/min.

17. $V = \frac{1}{3}\pi r^2 h$, $h = \frac{3}{8}(2r) = \frac{3r}{4} \Rightarrow r = \frac{4h}{3} \Rightarrow V = \frac{1}{3}\pi\left(\frac{4h}{3}\right)^2 h = \frac{16\pi h^3}{27} \Rightarrow \frac{dV}{dt} = \frac{16\pi h^2}{9} \frac{dh}{dt}$

(a) $\frac{dh}{dt}\Big|_{h=4} = \left(\frac{9}{16\pi 4^2}\right)(10) = \frac{90}{256\pi} \approx 0.1119$ m/sec $= 11.19$ cm/sec

(b) $r = \frac{4h}{3} \Rightarrow \frac{dr}{dt} = \frac{4}{3} \frac{dh}{dt} = \frac{4}{3}\left(\frac{90}{256\pi}\right) = \frac{15}{32\pi} \approx 0.1492$ m/sec $= 14.92$ cm/sec

18. (a) $V = \frac{1}{3}\pi r^2 h$ and $r = \frac{15h}{2} \Rightarrow V = \frac{1}{3}\pi\left(\frac{15h}{2}\right)^2 h = \frac{75\pi h^3}{4} \Rightarrow \frac{dV}{dt} = \frac{225\pi h^2}{4}\frac{dh}{dt} \Rightarrow \left.\frac{dh}{dt}\right|_{h=5} = \frac{4(-50)}{225\pi(5)^2} = \frac{-8}{225\pi}$
≈ -0.0113 m/min $= -1.13$ cm/min

(b) $r = \frac{15h}{2} \Rightarrow \frac{dr}{dt} = \frac{15}{2}\frac{dh}{dt} \Rightarrow \left.\frac{dr}{dt}\right|_{h=5} = \left(\frac{15}{2}\right)\left(\frac{-8}{225\pi}\right) = \frac{-4}{15\pi} \approx -0.0849$ m/sec $= -8.49$ cm/sec

19. (a) $V = \frac{\pi}{3}y^2(3R - y) \Rightarrow \frac{dV}{dt} = \frac{\pi}{3}[2y(3R - y) + y^2(-1)]\frac{dy}{dt} \Rightarrow \frac{dy}{dt} = \left[\frac{\pi}{3}(6Ry - 3y^2)\right]^{-1}\frac{dV}{dt} \Rightarrow$ at $R = 13$ and $y = 8$ we have $\frac{dy}{dt} = \frac{1}{144\pi}(-6) = \frac{-1}{24\pi}$ m/min

(b) The hemisphere is on the circle $r^2 + (13 - y)^2 = 169 \Rightarrow r = \sqrt{26y - y^2}$ m

(c) $r = (26y - y^2)^{1/2} \Rightarrow \frac{dr}{dt} = \frac{1}{2}(26y - y^2)^{-1/2}(26 - 2y)\frac{dy}{dt} \Rightarrow \frac{dr}{dt} = \frac{13 - y}{\sqrt{26y - y^2}}\frac{dy}{dt} \Rightarrow \left.\frac{dr}{dt}\right|_{y=8} = \frac{13 - 8}{\sqrt{26 \cdot 8 - 64}}\left(\frac{-1}{24\pi}\right)$
$= \frac{-5}{288\pi}$ m/min

20. If $V = \frac{4}{3}\pi r^3$, $S = 4\pi r^2$, and $\frac{dV}{dt} = kS = 4k\pi r^2$, then $\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt} \Rightarrow 4k\pi r^2 = 4\pi r^2\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = k$, a constant. Therefore, the radius is increasing at a constant rate.

21. If $V = \frac{4}{3}\pi r^3$, $r = 5$, and $\frac{dV}{dt} = 100\pi$ ft^3/min, then $\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = 1$ ft/min. Then $S = 4\pi r^2 \Rightarrow \frac{dS}{dt} = 8\pi r\frac{dr}{dt} = 8\pi(5)(1) = 40\pi$ ft^2/min, the rate at which the surface area is increasing.

22. Let s represent the length of the rope and x the horizontal distance of the boat from the dock.

(a) We have $s^2 = x^2 + 36 \Rightarrow \frac{dx}{dt} = \frac{s}{x}\frac{ds}{dt} = \frac{s}{\sqrt{s^2 - 36}}\frac{ds}{dt}$. Therefore, the boat is approaching the dock at $\left.\frac{dx}{dt}\right|_{s=10} = \frac{10}{\sqrt{10^2 - 36}}(-2) = -2.5$ ft/sec.

(b) $\cos\theta = \frac{6}{r} \Rightarrow -\sin\theta\frac{d\theta}{dt} = -\frac{6}{r^2}\frac{dr}{dt} \Rightarrow \frac{d\theta}{dt} = \frac{6}{r^2\sin\theta}\frac{dr}{dt}$. Thus, $r = 10$, $x = 8$, and $\sin\theta = \frac{8}{10}$
$\Rightarrow \frac{d\theta}{dt} = \frac{6}{10^2\left(\frac{8}{10}\right)} \cdot (-2) = -\frac{3}{20}$ rad/sec

23. Let s represent the distance between the bicycle and balloon, h the height of the balloon and x the horizontal distance between the balloon and the bicycle. The relationship between the variables is $s^2 = h^2 + x^2$
$\Rightarrow \frac{ds}{dt} = \frac{1}{s}\left(h\frac{dh}{dt} + x\frac{dx}{dt}\right) \Rightarrow \frac{ds}{dt} = \frac{1}{85}[68(1) + 51(17)] = 11$ ft/sec.

24. (a) Let h be the height of the coffee in the pot. Since the radius of the pot is 3, the volume of the coffee is $V = 9\pi h \Rightarrow \frac{dV}{dt} = 9\pi\frac{dh}{dt} \Rightarrow$ the rate the coffee is rising is $\frac{dh}{dt} = \frac{1}{9\pi}\frac{dV}{dt} = \frac{10}{9\pi}$ in/min.

(b) Let h be the height of the coffee in the pot. From the figure, the radius of the filter $r = \frac{h}{2} \Rightarrow V = \frac{1}{3}\pi r^2 h$ $= \frac{\pi h^3}{12}$, the volume of the filter. The rate the coffee is falling is $\frac{dh}{dt} = \frac{4}{\pi h^2}\frac{dV}{dt} = \frac{4}{25\pi}(-10) = -\frac{8}{5\pi}$ in/min.

25. $y = QD^{-1} \Rightarrow \frac{dy}{dt} = D^{-1}\frac{dQ}{dt} - QD^{-2}\frac{dD}{dt} = \frac{1}{41}(0) - \frac{233}{(41)^2}(-2) = \frac{466}{1681}$ L/min $\Rightarrow$ increasing about 0.2772 L/min

26. (a) $\frac{dc}{dt} = (3x^2 - 12x + 15)\frac{dx}{dt} = (3(2)^2 - 12(2) + 15)(0.1) = 0.3$, $\frac{dr}{dt} = 9\frac{dx}{dt} = 9(0.1) = 0.9$, $\frac{dp}{dt} = 0.9 - 0.3 = 0.6$

(b) $\frac{dc}{dt} = (3x^2 - 12x - 45x^{-2})\frac{dx}{dt} = (3(1.5)^2 - 12(1.5) - 45(1.5)^{-2})(0.05) = -1.5625$, $\frac{dr}{dt} = 70\frac{dx}{dt} = 70(0.05) = 3.5$, $\frac{dp}{dt} = 3.5 - (-1.5625) = 5.0625$

27. Let $P(x, y)$ represent a point on the curve $y = x^2$ and θ the angle of inclination of a line containing P and the origin. Consequently, $\tan\theta = \frac{y}{x} \Rightarrow \tan\theta = \frac{x^2}{x} = x \Rightarrow \sec^2\theta\frac{d\theta}{dt} = \frac{dx}{dt} \Rightarrow \frac{d\theta}{dt} = \cos^2\theta\frac{dx}{dt}$. Since $\frac{dx}{dt} = 10$ m/sec and $\left.\cos^2\theta\right|_{x=3} = \frac{x^2}{y^2 + x^2} = \frac{3^2}{9^2 + 3^2} = \frac{1}{10}$, we have $\left.\frac{d\theta}{dt}\right|_{x=3} = 1$ rad/sec.

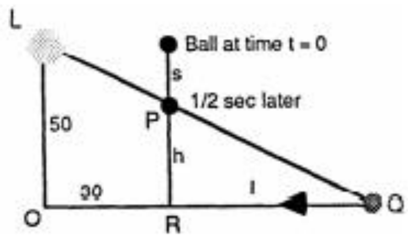
28. $y = (-x)^{1/2}$ and $\tan\theta = \frac{y}{x} \Rightarrow \tan\theta = \frac{(-x)^{1/2}}{x} \Rightarrow \sec^2\theta\frac{d\theta}{dt} = \frac{\left(\frac{1}{2}\right)(-x)^{-1/2}(-1)x - (-x)^{1/2}(1)}{x^2}\frac{dx}{dt}$

$\Rightarrow \frac{d\theta}{dt} = \left(\frac{\frac{-x}{2\sqrt{-x}} - \sqrt{-x}}{x^2} \right) (\cos^2\theta) \left(\frac{dx}{dt}\right)$. Now, $\tan\theta = \frac{2}{-4} = -\frac{1}{2} \Rightarrow \cos\theta = -\frac{2}{\sqrt{5}} \Rightarrow \cos^2\theta = \frac{4}{5}$. Then $\frac{d\theta}{dt} = \left(\frac{\frac{4}{2} - 2}{16}\right)\left(\frac{4}{5}\right)(-8) = \frac{2}{5}$ rad/sec.

29. The distance from the origin is $s = \sqrt{x^2 + y^2}$ and we wish to find $\left.\frac{ds}{dt}\right|_{(5,12)}$
$= \frac{1}{2}\left(x^2 + y^2\right)^{-1/2} \left.\left(2x\,\frac{dx}{dt} + 2y\,\frac{dy}{dt}\right)\right|_{(5,12)} = \frac{(5)(-1) + (12)(-5)}{\sqrt{25 + 144}} = -5$ m/sec

30. When s represents the length of the shadow and x the distance of the man from the streetlight, then $s = \frac{3}{5}x$.
   (a) If I represents the distance of the tip of the shadow from the streetlight, then $I = s + x \Rightarrow \frac{dI}{dt} = \frac{ds}{dt} + \frac{dx}{dt}$ (which is velocity not speed) $\Rightarrow \left|\frac{dI}{dt}\right| = \left|\frac{3}{5}\,\frac{dx}{dt} + \frac{dx}{dt}\right| = \left|\frac{8}{5}\right| \left|\frac{dx}{dt}\right| = \frac{8}{5}\,|-5| = 8$ ft/sec, the speed the tip of the shadow is moving along the ground.
   (b) $\frac{ds}{dt} = \frac{3}{5}\,\frac{dx}{dt} = \frac{3}{5}(-5) = -3$ ft/sec, so the length of the shadow is <u>decreasing</u> at a rate of 3 ft/sec.

31. Let $s = 16t^2$ represent the distance the ball has fallen, h the distance between the ball and the ground, and I the distance between the shadow and the point directly beneath the ball. Accordingly, $s + h = 50$ and since the triangle LOQ and triangle PRQ are similar we have $I = \frac{30h}{50 - h} \Rightarrow h = 50 - 16t^2$ and $I = \frac{30\left(50 - 16t^2\right)}{50 - \left(50 - 16t^2\right)}$
$= \frac{1500}{16t^2} - 30 \Rightarrow \frac{dI}{dt} = -\frac{1500}{8t^3} \Rightarrow \left.\frac{dI}{dt}\right|_{t=\frac{1}{2}} = -1500$ ft/sec.



32. Let s = distance of car from foot of perpendicular in the textbook diagram $\Rightarrow \tan\theta = \frac{s}{132} \Rightarrow \sec^2\theta\,\frac{d\theta}{dt} = \frac{1}{132}\,\frac{ds}{dt}$
$\Rightarrow \frac{d\theta}{dt} = \frac{\cos^2\theta}{132}\,\frac{ds}{dt}$; $\frac{ds}{dt} = -264$ and $\theta = 0 \Rightarrow \frac{d\theta}{dt} = -2$ rad/sec. A half second later the car has traveled 132 ft right of the perpendicular $\Rightarrow |\theta| = \frac{\pi}{4}$, $\cos^2\theta = \frac{1}{2}$, and $\frac{ds}{dt} = 264$ (since s increases) $\Rightarrow \frac{d\theta}{dt} = \frac{\left(\frac{1}{2}\right)}{132}(264) = 1$ rad/sec

E

1. l'Hôpital: $\lim_{x \to 2} \frac{x-2}{x^2-4} = \frac{1}{2x}\Big|_{x=2} = \frac{1}{4}$ or $\lim_{x \to 2} \frac{x-2}{x^2-4} = \lim_{x \to 2} \frac{x-2}{(x-2)(x+2)} = \lim_{x \to 2} \frac{1}{x+2} = \frac{1}{4}$

2. l'Hôpital: $\lim_{x \to 0} \frac{\sin 5x}{x} = \frac{5\cos 5x}{1}\Big|_{x=0} = 5$ or $\lim_{x \to 0} \frac{\sin 5x}{x} = 5 \lim_{5x \to 0} \frac{\sin 5x}{5x} = 5 \cdot 1 = 5$

3. l'Hôpital: $\lim_{x \to \infty} \frac{5x^2-3x}{7x^2+1} = \lim_{x \to \infty} \frac{10x-3}{14x} = \lim_{x \to \infty} \frac{10}{14} = \frac{5}{7}$ or $\lim_{x \to \infty} \frac{5x^2-3x}{7x^2+1} = \lim_{x \to \infty} \frac{5-\frac{3}{x}}{7+\frac{1}{x^2}} = \frac{5}{7}$

4. l'Hôpital: $\lim_{x \to 1} \frac{x^3-1}{4x^3-x-3} = \lim_{x \to 1} \frac{3x^2}{12x^2-1} = \frac{3}{11}$ or $\lim_{x \to 1} \frac{x^3-1}{4x^3-x-3} = \lim_{x \to 1} \frac{(x-1)(x^2+x+1)}{(x-1)(4x^2+4x+3)}$
$= \lim_{x \to 1} \frac{(x^2+x+1)}{(4x^2+4x+3)} = \frac{3}{11}$

5. l'Hôpital: $\lim_{x \to 0} \frac{1-\cos x}{x^2} = \lim_{x \to 0} \frac{\sin x}{2x} = \lim_{x \to 0} \frac{\cos x}{2} = \frac{1}{2}$ or $\lim_{x \to 0} \frac{1-\cos x}{x^2} = \lim_{x \to 0} \left[\frac{(1-\cos x)}{x^2}\left(\frac{1+\cos x}{1+\cos x}\right)\right]$
$= \lim_{x \to 0} \frac{\sin^2 x}{x^2(1+\cos x)} = \lim_{x \to 0} \left[\left(\frac{\sin x}{x}\right)\left(\frac{\sin x}{x}\right)\left(\frac{1}{1+\cos x}\right)\right] = \frac{1}{2}$

6. l'Hôpital: $\lim_{x \to \infty} \frac{2x^2+3x}{x^3+x+1} = \lim_{x \to \infty} \frac{4x+3}{3x^2+1} = \lim_{x \to \infty} \frac{4}{6x} = 0$ or $\lim_{x \to \infty} \frac{2x^2+3x}{x^3+x+1} = \lim_{x \to \infty} \frac{\frac{2}{x}+\frac{3}{x^2}}{1+\frac{1}{x^2}+\frac{1}{x^3}} = \frac{0}{1} = 0$

7. $\lim_{t \to 0} \frac{\sin t^2}{t} = \lim_{t \to 0} \frac{2t\cos t^2}{1} = 0$

8. $\lim_{x \to \pi/2} \frac{2x-\pi}{\cos x} = \lim_{\theta \to \pi/2} \frac{2}{-\sin x} = \frac{2}{-1} = -2$

9. $\lim_{\theta \to \pi} \frac{\sin \theta}{\pi-\theta} = \lim_{\theta \to \pi} \frac{\cos \theta}{-1} = \frac{-1}{-1} = 1$

10. $\lim_{x \to \pi/2} \frac{1-\sin x}{1+\cos 2x} = \lim_{x \to \pi/2} \frac{-\cos x}{-2\sin 2x} = \lim_{x \to \pi/2} \frac{\sin x}{-4\cos 2x} = \frac{1}{-4(-1)} = \frac{1}{4}$

11. $\lim_{x \to \pi/4} \frac{\sin x - \cos x}{x - \frac{\pi}{4}} = \lim_{x \to \pi/4} \frac{\cos x + \sin x}{1} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}$

12. $\lim_{x \to \pi/3} \frac{\cos x - \frac{1}{2}}{x - \frac{\pi}{3}} = \lim_{x \to \pi/3} \frac{-\sin x}{1} = -\frac{\sqrt{3}}{2}$

13. $\lim_{x \to \pi/2} -\left(x - \frac{\pi}{2}\right) \tan x = \lim_{x \to \pi/2} \frac{-\left(x-\frac{\pi}{2}\right)\sin x}{\cos x} = \lim_{x \to \pi/2} \frac{\left(\frac{\pi}{2}-x\right)\cos x + \sin x(-1)}{-\sin x} = \frac{-1}{-1} = 1$

14. $\lim_{x \to 0} \frac{2x}{x+7\sqrt{x}} = \lim_{x \to 0} \frac{2}{1+\frac{7}{2\sqrt{x}}} = \lim_{x \to 0} \frac{4\sqrt{x}}{2\sqrt{x}+7} = \frac{4 \cdot 0}{2 \cdot 0+7} = 0$

15. $\lim_{x \to 1} \frac{2x^2-(3x+1)\sqrt{x}+2}{x-1} = \lim_{x \to 1} \frac{2x^2-3x^{3/2}-x^{1/2}+2}{x-1} = \lim_{x \to 1} \frac{4x-\frac{9}{2}x^{1/2}-\frac{1}{2\sqrt{x}}}{1} = -1$

16. $\lim_{x \to 2} \frac{\sqrt{x^2+5}-3}{x^2-4} = \lim_{x \to 2} \frac{\frac{1}{2}(x^2+5)^{-1/2}(2x)}{2x} = \lim_{x \to 2} \frac{1}{2\sqrt{x^2+5}} = \frac{1}{6}$

17. $\lim_{x \to 0} \frac{\sqrt{a(a+x)}-a}{x} = \lim_{x \to 0} \frac{a}{2\sqrt{a^2+ax}} = \frac{a}{2\sqrt{a^2}} = \frac{1}{2}$, where $a > 0$.

18. $\lim\limits_{t \to 0} \frac{10(\sin t - t)}{t^3} = \lim\limits_{t \to 0} \frac{10(\cos t - 1)}{3t^2} = \lim\limits_{t \to 0} \frac{10(-\sin t)}{6t} = \lim\limits_{t \to 0} \frac{-10 \cos t}{6} = \frac{-10 \cdot 1}{6} = -\frac{5}{3}$

19. $\lim\limits_{x \to 0} \frac{x(\cos x - 1)}{\sin x - x} = \lim\limits_{x \to 0} \frac{-x\sin x + \cos x - 1}{\cos x - 1} = \lim\limits_{x \to 0} \frac{-x\cos x - 2\sin x}{-\sin x} = \lim\limits_{x \to 0} \frac{x\cos x + 2\sin x}{\sin x} = \lim\limits_{x \to 0} \frac{-x\sin x + 3\cos x}{\cos x} = \frac{3}{1} = 3$

20. $\lim\limits_{h \to 0} \frac{\sin(a+h) - \sin a}{h} = \lim\limits_{h \to 0} \frac{\cos(a+h) - \cos a}{1} = 0$

21. $\lim\limits_{r \to 1} \frac{a(r^n - 1)}{r - 1} = \lim\limits_{r \to 1} \frac{a(n r^{n-1})}{1} = an \lim\limits_{r \to 1} r^{n-1} = an$, where n is a positive integer.

22. $\lim\limits_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{\sqrt{x}}\right) = \lim\limits_{x \to 0^+} \left(\frac{1 - \sqrt{x}}{x}\right) = \left(\begin{array}{c}\text{l'Hopital's rule} \\ \text{does not apply}\end{array}\right) = \lim\limits_{x \to 0^+} \left(1 - \sqrt{x}\right) \cdot \frac{1}{x} = \infty$

23. $\lim\limits_{x \to \infty} \left(x - \sqrt{x^2 + x}\right) = \lim\limits_{x \to \infty} \left(x - \sqrt{x^2 + x}\right)\left(\frac{x + \sqrt{x^2 + x}}{x + \sqrt{x^2 + x}}\right) = \lim\limits_{x \to \infty} \frac{x^2 - (x^2 + x)}{x + \sqrt{x^2 + x}} = \lim\limits_{x \to \infty} \frac{-\frac{x}{x}}{\frac{x}{x} + \frac{\sqrt{x^2 + x}}{\sqrt{x^2}}}$

$= \lim\limits_{x \to \infty} \frac{-1}{1 + \sqrt{1 + \frac{1}{x}}} = -\frac{1}{2} \left(\begin{array}{c}\text{l'Hopital's rule} \\ \text{is unnecessary}\end{array}\right)$

24. $\lim\limits_{x \to \infty} x \tan\left(\frac{1}{x}\right) = \lim\limits_{x \to \infty} \frac{\tan\left(\frac{1}{x}\right)}{\frac{1}{x}} = \lim\limits_{x \to \infty} \frac{-\frac{1}{x^2} \sec^2\left(\frac{1}{x}\right)}{-\frac{1}{x^2}} = \lim\limits_{x \to \infty} \sec^2\left(\frac{1}{x}\right) = \sec^2 0 = 1$

25. $\lim\limits_{x \to \pm\infty} \frac{3x - 5}{2x^2 - x + 2} = \lim\limits_{x \to \pm\infty} \frac{3}{4x - 1} = 0$

26. $\lim\limits_{x \to 0} \frac{\sin 7x}{\tan 11x} = \lim\limits_{x \to 0} \frac{7\cos(7x)}{11\sec^2(11x)} = \frac{7 \cdot 1}{11 \cdot 1} = \frac{7}{11}$

27. $\lim\limits_{x \to \infty} \frac{\sqrt{9x + 1}}{\sqrt{x + 1}} = \sqrt{\lim\limits_{x \to \infty} \frac{9x + 1}{x + 1}} = \sqrt{\lim\limits_{x \to \infty} \frac{9}{1}} = \sqrt{9} = 3$

28. $\lim\limits_{x \to 0^+} \frac{\sqrt{x}}{\sqrt{\sin x}} = \sqrt{\frac{1}{\lim\limits_{x \to 0^+} \frac{\sin x}{x}}} = \sqrt{\frac{1}{1}} = 1$

29. $\lim\limits_{x \to \pi/2^-} \frac{\sec x}{\tan x} = \lim\limits_{x \to \pi/2^-} \left(\frac{1}{\cos x}\right)\left(\frac{\cos x}{\sin x}\right) = \lim\limits_{x \to \pi/2^-} \frac{1}{\sin x} = 1$

30. $\lim\limits_{x \to 0^+} \frac{\cot x}{\csc x} = \lim\limits_{x \to 0^+} \frac{\left(\frac{\cos x}{\sin x}\right)}{\left(\frac{1}{\sin x}\right)} = \lim\limits_{x \to 0^+} \cos x = 1$

31. Part (b) is correct because part (a) is neither in the $\frac{0}{0}$ nor $\frac{\infty}{\infty}$ form and so l'Hôpital's rule may not be used.

32. Answers may vary.

(a) $f(x) = 3x + 1; g(x) = x$

$\lim_{x \to \infty} \frac{f(x)}{g(x)} = \lim_{x \to \infty} \frac{3x+1}{x} = \lim_{x \to \infty} \frac{3}{1} = 3$

(b) $f(x) = x + 1; g(x) = x^2$

$\lim_{x \to \infty} \frac{f(x)}{g(x)} = \lim_{x \to \infty} \frac{x+1}{x^2} = \lim_{x \to \infty} \frac{1}{2x} = 0$

(c) $f(x) = x^2; g(x) = x + 1$

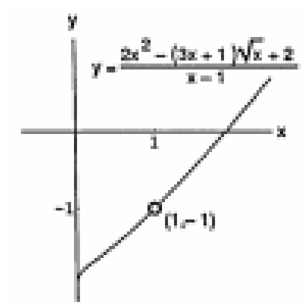
$\lim_{x \to \infty} \frac{f(x)}{g(x)} = \lim_{x \to \infty} \frac{x^2}{x+1} = \lim_{x \to \infty} \frac{2x}{1} = \infty$

33. If f(x) is to be continuous at $x = 0$, then $\lim_{x \to 0} f(x) = f(0) \Rightarrow c = f(0) = \lim_{x \to 0} \frac{9x - 3\sin 3x}{5x^3} = \lim_{x \to 0} \frac{9 - 9\cos 3x}{15x^2}$
$= \lim_{x \to 0} \frac{27 \sin 3x}{30x} = \lim_{x \to 0} \frac{81 \cos 3x}{30} = \frac{27}{10}$.

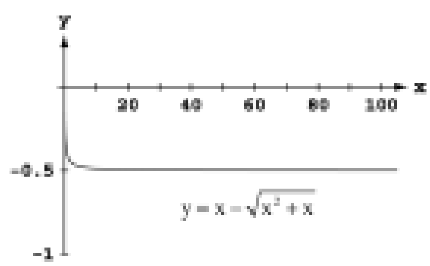
34. (a) For $x \neq 0$, $f'(x) = \frac{d}{dx}(x + 2) = 1$ and $g'(x) = \frac{d}{dx}(x + 1) = 1$. Therefore, $\lim_{x \to 0} \frac{f'(x)}{g'(x)} = \frac{1}{1} = 1$, while $\lim_{x \to 0} \frac{f(x)}{g(x)}$
$= \frac{x+2}{x+1} = \frac{0+2}{0+1} = 2$.

(b) This does not contradict l'Hôpital's rule because neither f nor g is differentiable at $x = 0$ (as evidenced by the fact that neither is continuous at $x = 0$), so l'Hôpital's rule does not apply.

35. The graph indicates a limit near -1. The limit leads to the indeterminate form $\frac{0}{0}$: $\lim_{x \to 1} \frac{2x^2 - (3x+1)\sqrt{x} + 2}{x-1}$
$= \lim_{x \to 1} \frac{2x^2 - 3x^{3/2} - x^{1/2} + 2}{x-1} = \lim_{x \to 1} \frac{4x - \frac{9}{2}x^{1/2} - \frac{1}{2}x^{-1/2}}{1}$
$= \frac{4 - \frac{9}{2} - \frac{1}{2}}{1} = \frac{4-5}{1} = -1$



36. (a)



(b) The limit leads to the indeterminate form $\infty - \infty$:

$\lim_{x \to \infty} \left(x - \sqrt{x^2 + x}\right) = \lim_{x \to \infty} \left(x - \sqrt{x^2 + x}\right)\left(\frac{x + \sqrt{x^2+x}}{x + \sqrt{x^2+x}}\right) = \lim_{x \to \infty} \left(\frac{x^2 - (x^2 + x)}{x + \sqrt{x^2 + x}}\right) = \lim_{x \to \infty} \frac{-x}{x + \sqrt{x^2 + x}}$
$= \lim_{x \to \infty} \frac{-1}{1 + \sqrt{1 + \frac{1}{x}}} = \frac{-1}{1 + \sqrt{1 + 0}} = -\frac{1}{2}$

F

5. $y = \ln 3x \Rightarrow y' = \left(\frac{1}{3x}\right)(3) = \frac{1}{x}$

6. $y = \ln kx \Rightarrow y' = \left(\frac{1}{kx}\right)(k) = x$

7. $y = \ln(t^2) \Rightarrow \frac{dy}{dt} = \left(\frac{1}{t^2}\right)(2t) = \frac{2}{t}$

8. $y = \ln\left(t^{3/2}\right) \Rightarrow \frac{dy}{dt} = \left(\frac{1}{t^{3/2}}\right)\left(\frac{3}{2}t^{1/2}\right) = \frac{3}{2t}$

9. $y = \ln \frac{3}{x} = \ln 3x^{-1} \Rightarrow \frac{dy}{dx} = \left(\frac{1}{3x^{-1}}\right)\left(-3x^{-2}\right) = -\frac{1}{x}$

10. $y = \ln \frac{10}{x} = \ln 10x^{-1} \Rightarrow \frac{dy}{dx} = \left(\frac{1}{10x^{-1}}\right)\left(-10x^{-2}\right) = -\frac{1}{x}$

11. $y = \ln(\theta + 1) \Rightarrow \frac{dy}{d\theta} = \left(\frac{1}{\theta+1}\right)(1) = \frac{1}{\theta+1}$

12. $y = \ln(2\theta + 2) \Rightarrow \frac{dy}{d\theta} = \left(\frac{1}{2\theta+2}\right)(2) = \frac{1}{\theta+1}$

13. $y = \ln x^3 \Rightarrow \frac{dy}{dx} = \left(\frac{1}{x^3}\right)(3x^2) = \frac{3}{x}$

14. $y = (\ln x)^3 \Rightarrow \frac{dy}{dx} = 3(\ln x)^2 \cdot \frac{d}{dx}(\ln x) = \frac{3(\ln x)^2}{x}$

15. $y = t(\ln t)^2 \Rightarrow \frac{dy}{dt} = (\ln t)^2 + 2t(\ln t) \cdot \frac{d}{dt}(\ln t) = (\ln t)^2 + \frac{2t \ln t}{t} = (\ln t)^2 + 2\ln t$

16. $y = t\sqrt{\ln t} = t(\ln t)^{1/2} \Rightarrow \frac{dy}{dt} = (\ln t)^{1/2} + \frac{1}{2}t(\ln t)^{-1/2} \cdot \frac{d}{dt}(\ln t) = (\ln t)^{1/2} + \frac{t(\ln t)^{-1/2}}{2t}$
$= (\ln t)^{1/2} + \frac{1}{2(\ln t)^{1/2}}$

17. $y = \frac{x^4}{4}\ln x - \frac{x^4}{16} \Rightarrow \frac{dy}{dx} = x^3 \ln x + \frac{x^4}{4} \cdot \frac{1}{x} - \frac{4x^3}{16} = x^3 \ln x$

18. $y = \frac{x^3}{3}\ln x - \frac{x^3}{9} \Rightarrow \frac{dy}{dx} = x^2 \ln x + \frac{x^3}{3} \cdot \frac{1}{x} - \frac{3x^2}{9} = x^2 \ln x$

19. $y = \frac{\ln t}{t} \Rightarrow \frac{dy}{dt} = \frac{t\left(\frac{1}{t}\right) - (\ln t)(1)}{t^2} = \frac{1 - \ln t}{t^2}$

20. $y = \frac{1 + \ln t}{t} \Rightarrow \frac{dy}{dt} = \frac{t\left(\frac{1}{t}\right) - (1 + \ln t)(1)}{t^2} = \frac{1 - 1 - \ln t}{t^2} = -\frac{\ln t}{t^2}$

21. $y = \frac{\ln x}{1 + \ln x} \Rightarrow y' = \frac{(1 + \ln x)\left(\frac{1}{x}\right) - (\ln x)\left(\frac{1}{x}\right)}{(1 + \ln x)^2} = \frac{\frac{1}{x} + \frac{\ln x}{x} - \frac{\ln x}{x}}{(1 + \ln x)^2} = \frac{1}{x(1 + \ln x)^2}$

22. $y = \frac{x \ln x}{1 + \ln x} \Rightarrow y' = \frac{(1 + \ln x)\left(\ln x + x \cdot \frac{1}{x}\right) - (x \ln x)\left(\frac{1}{x}\right)}{(1 + \ln x)^2} = \frac{(1 + \ln x)^2 - \ln x}{(1 + \ln x)^2} = 1 - \frac{\ln x}{(1 + \ln x)^2}$

23. $y = \ln(\ln x) \Rightarrow y' = \left(\frac{1}{\ln x}\right)\left(\frac{1}{x}\right) = \frac{1}{x \ln x}$

24. $y = \ln(\ln(\ln x)) \Rightarrow y' = \frac{1}{\ln(\ln x)} \cdot \frac{d}{dx}(\ln(\ln x)) = \frac{1}{\ln(\ln x)} \cdot \frac{1}{\ln x} \cdot \frac{d}{dx}(\ln x) = \frac{1}{x(\ln x)\ln(\ln x)}$

25. $y = \theta[\sin(\ln \theta) + \cos(\ln \theta)] \Rightarrow \frac{dy}{d\theta} = [\sin(\ln \theta) + \cos(\ln \theta)] + \theta\left[\cos(\ln \theta) \cdot \frac{1}{\theta} - \sin(\ln \theta) \cdot \frac{1}{\theta}\right]$
$= \sin(\ln \theta) + \cos(\ln \theta) + \cos(\ln \theta) - \sin(\ln \theta) = 2\cos(\ln \theta)$

26. $y = \ln(\sec \theta + \tan \theta) \Rightarrow \frac{dy}{d\theta} = \frac{\sec \theta \tan \theta + \sec^2 \theta}{\sec \theta + \tan \theta} = \frac{\sec \theta(\tan \theta + \sec \theta)}{\tan \theta + \sec \theta} = \sec \theta$

27. $y = \ln \frac{1}{x\sqrt{x+1}} = -\ln x - \frac{1}{2}\ln(x+1) \Rightarrow y' = -\frac{1}{x} - \frac{1}{2}\left(\frac{1}{x+1}\right) = -\frac{2(x+1)+x}{2x(x+1)} = -\frac{3x+2}{2x(x+1)}$

28. $y = \frac{1}{2}\ln \frac{1+x}{1-x} = \frac{1}{2}[\ln(1+x) - \ln(1-x)] \Rightarrow y' = \frac{1}{2}\left[\frac{1}{1+x} - \left(\frac{1}{1-x}\right)(-1)\right] = \frac{1}{2}\left[\frac{1-x+1+x}{(1+x)(1-x)}\right] = \frac{1}{1-x^2}$

29. $y = \frac{1 + \ln t}{1 - \ln t} \Rightarrow \frac{dy}{dt} = \frac{(1 - \ln t)\left(\frac{1}{t}\right) - (1 + \ln t)\left(\frac{-1}{t}\right)}{(1 - \ln t)^2} = \frac{\frac{1}{t} - \frac{\ln t}{t} + \frac{1}{t} + \frac{\ln t}{t}}{(1 - \ln t)^2} = \frac{2}{t(1 - \ln t)^2}$

30. $y = \sqrt{\ln\sqrt{t}} = \left(\ln t^{1/2}\right)^{1/2} \Rightarrow \frac{dy}{dt} = \frac{1}{2}\left(\ln t^{1/2}\right)^{-1/2} \cdot \frac{d}{dt}\left(\ln t^{1/2}\right) = \frac{1}{2}\left(\ln t^{1/2}\right)^{-1/2} \cdot \frac{1}{t^{1/2}} \cdot \frac{d}{dt}\left(t^{1/2}\right)$
$= \frac{1}{2}\left(\ln t^{1/2}\right)^{-1/2} \cdot \frac{1}{t^{1/2}} \cdot \frac{1}{2}t^{-1/2} = \frac{1}{4t\sqrt{\ln\sqrt{t}}}$

31. $y = \ln(\sec(\ln\theta)) \Rightarrow \frac{dy}{d\theta} = \frac{1}{\sec(\ln\theta)} \cdot \frac{d}{d\theta}(\sec(\ln\theta)) = \frac{\sec(\ln\theta)\tan(\ln\theta)}{\sec(\ln\theta)} \cdot \frac{d}{d\theta}(\ln\theta) = \frac{\tan(\ln\theta)}{\theta}$

32. $y = \ln\frac{\sqrt{\sin\theta\cos\theta}}{1+2\ln\theta} = \frac{1}{2}(\ln\sin\theta + \ln\cos\theta) - \ln(1+2\ln\theta) \Rightarrow \frac{dy}{d\theta} = \frac{1}{2}\left(\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}\right) - \frac{\frac{2}{\theta}}{1+2\ln\theta}$
$= \frac{1}{2}\left[\cot\theta - \tan\theta - \frac{4}{\theta(1+2\ln\theta)}\right]$

33. $y = \ln\left(\frac{(x^2+1)^5}{\sqrt{1-x}}\right) = 5\ln(x^2+1) - \frac{1}{2}\ln(1-x) \Rightarrow y' = \frac{5\cdot 2x}{x^2+1} - \frac{1}{2}\left(\frac{1}{1-x}\right)(-1) = \frac{10x}{x^2+1} + \frac{1}{2(1-x)}$

34. $y = \ln\sqrt{\frac{(x+1)^5}{(x+2)^{20}}} = \frac{1}{2}[5\ln(x+1) - 20\ln(x+2)] \Rightarrow y' = \frac{1}{2}\left(\frac{5}{x+1} - \frac{20}{x+2}\right) = \frac{5}{2}\left[\frac{(x+2)-4(x+1)}{(x+1)(x+2)}\right]$
$= -\frac{5}{2}\left[\frac{3x+2}{(x+1)(x+2)}\right]$

35. $y = \int_{x^2/2}^{x^2} \ln\sqrt{t}\,dt \Rightarrow \frac{dy}{dx} = \left(\ln\sqrt{x^2}\right) \cdot \frac{d}{dx}(x^2) - \left(\ln\sqrt{\frac{x^2}{2}}\right) \cdot \frac{d}{dx}\left(\frac{x^2}{2}\right) = 2x\ln|x| - x\ln\frac{|x|}{\sqrt{2}}$

36. $y = \int_{\sqrt{x}}^{\sqrt[3]{x}} \ln t\,dt \Rightarrow \frac{dy}{dx} = \left(\ln\sqrt[3]{x}\right) \cdot \frac{d}{dx}\left(\sqrt[3]{x}\right) - \left(\ln\sqrt{x}\right) \cdot \frac{d}{dx}\left(\sqrt{x}\right) = \left(\ln\sqrt[3]{x}\right)\left(\frac{1}{3}x^{-2/3}\right) - \left(\ln\sqrt{x}\right)\left(\frac{1}{2}x^{-1/2}\right)$
$= \frac{\ln\sqrt[3]{x}}{3\sqrt[3]{x^2}} - \frac{\ln\sqrt{x}}{2\sqrt{x}}$

37. $\int_{-3}^{-2} \frac{1}{x}\,dx = [\ln|x|]_{-3}^{-2} = \ln 2 - \ln 3 = \ln\frac{2}{3}$

38. $\int_{-1}^{0} \frac{3}{3x-2}\,dx = [\ln|3x-2|]_{-1}^{0} = \ln 2 - \ln 5 = \ln\frac{2}{5}$

39. $\int \frac{2y}{y^2-25}\,dy = \ln|y^2-25| + C$

40. $\int \frac{8r}{4r^2-5}\,dr = \ln|4r^2-5| + C$

41. $\int_0^{\pi} \frac{\sin t}{2-\cos t}\,dt = [\ln|2-\cos t|]_0^{\pi} = \ln 3 - \ln 1 = \ln 3$; or let $u = 2-\cos t \Rightarrow du = \sin t\,dt$ with $t = 0$
$\Rightarrow u = 1$ and $t = \pi \Rightarrow u = 3 \Rightarrow \int_0^{\pi} \frac{\sin t}{2-\cos t}\,dt = \int_1^3 \frac{1}{u}\,du = [\ln|u|]_1^3 = \ln 3 - \ln 1 = \ln 3$

42. $\int_0^{\pi/3} \frac{4\sin\theta}{1-4\cos\theta}\,d\theta = [\ln|1-4\cos\theta|]_0^{\pi/3} = \ln|1-2| = -\ln 3 = \ln\frac{1}{3}$; or let $u = 1-4\cos\theta \Rightarrow du = 4\sin\theta\,d\theta$
with $\theta = 0 \Rightarrow u = -3$ and $\theta = \frac{\pi}{3} \Rightarrow u = -1 \Rightarrow \int_0^{\pi/3} \frac{4\sin\theta}{1-4\cos\theta}\,d\theta = \int_{-3}^{-1} \frac{1}{u}\,du = [\ln|u|]_{-3}^{-1} = -\ln 3 = \ln\frac{1}{3}$

43. Let $u = \ln x \Rightarrow du = \frac{1}{x}\,dx$; $x = 1 \Rightarrow u = 0$ and $x = 2 \Rightarrow u = \ln 2$;
$\int_1^2 \frac{2\ln x}{x}\,dx = \int_0^{\ln 2} 2u\,du = [u^2]_0^{\ln 2} = (\ln 2)^2$

44. Let $u = \ln x \Rightarrow du = \frac{1}{x}\,dx$; $x = 2 \Rightarrow u = \ln 2$ and $x = 4 \Rightarrow u = \ln 4$;
$\int_2^4 \frac{dx}{x\ln x} = \int_{\ln 2}^{\ln 4} \frac{1}{u}\,du = [\ln u]_{\ln 2}^{\ln 4} = \ln(\ln 4) - \ln(\ln 2) = \ln\left(\frac{\ln 4}{\ln 2}\right) = \ln\left(\frac{\ln 2^2}{\ln 2}\right) = \ln\left(\frac{2\ln 2}{\ln 2}\right) = \ln 2$

45. Let $u = \ln x \Rightarrow du = \frac{1}{x}\,dx$; $x = 2 \Rightarrow u = \ln 2$ and $x = 4 \Rightarrow u = \ln 4$;
$\int_2^4 \frac{dx}{x(\ln x)^2} = \int_{\ln 2}^{\ln 4} u^{-2}\,du = \left[-\frac{1}{u}\right]_{\ln 2}^{\ln 4} = -\frac{1}{\ln 4} + \frac{1}{\ln 2} = -\frac{1}{\ln 2^2} + \frac{1}{\ln 2} = -\frac{1}{2\ln 2} + \frac{1}{\ln 2} = \frac{1}{2\ln 2} = \frac{1}{\ln 4}$

46. Let $u = \ln x \Rightarrow du = \frac{1}{x}\,dx$; $x = 2 \Rightarrow u = \ln 2$ and $x = 16 \Rightarrow u = \ln 16$;
$\int_2^{16} \frac{dx}{2x\sqrt{\ln x}} = \frac{1}{2}\int_{\ln 2}^{\ln 16} u^{-1/2}\,du = \left[u^{1/2}\right]_{\ln 2}^{\ln 16} = \sqrt{\ln 16} - \sqrt{\ln 2} = \sqrt{4\ln 2} - \sqrt{\ln 2} = 2\sqrt{\ln 2} - \sqrt{\ln 2} = \sqrt{\mathrm{l}}$

47. Let $u = 6 + 3\tan t \Rightarrow du = 3\sec^2 t\,dt$;
$\int \frac{3\sec^2 t}{6+3\tan t}\,dt = \int \frac{du}{u} = \ln|u| + C = \ln|6 + 3\tan t| + C$

48. Let $u = 2 + \sec y \Rightarrow du = \sec y \tan y\,dy$;
$\int \frac{\sec y \tan y}{2+\sec y}\,dy = \int \frac{du}{u} = \ln|u| + C = \ln|2 + \sec y| + C$

49. Let $u = \cos\frac{x}{2} \Rightarrow du = -\frac{1}{2}\sin\frac{x}{2}\,dx \Rightarrow -2\,du = \sin\frac{x}{2}\,dx$; $x = 0 \Rightarrow u = 1$ and $x = \frac{\pi}{2} \Rightarrow u = \frac{1}{\sqrt{2}}$;
$\int_0^{\pi/2} \tan\frac{x}{2}\,dx = \int_0^{\pi/2} \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\,dx = -2\int_1^{1/\sqrt{2}} \frac{du}{u} = \left[-2\ln|u|\right]_1^{1/\sqrt{2}} = -2\ln\frac{1}{\sqrt{2}} = 2\ln\sqrt{2} = \ln 2$

50. Let $u = \sin t \Rightarrow du = \cos t\,dt$; $t = \frac{\pi}{4} \Rightarrow u = \frac{1}{\sqrt{2}}$ and $t = \frac{\pi}{2} \Rightarrow u = 1$;
$\int_{\pi/4}^{\pi/2} \cot t\,dt = \int_{\pi/4}^{\pi/2} \frac{\cos t}{\sin t}\,dt = \int_{1/\sqrt{2}}^{1} \frac{du}{u} = \left[\ln|u|\right]_{1/\sqrt{2}}^{1} = -\ln\frac{1}{\sqrt{2}} = \ln\sqrt{2}$

51. Let $u = \sin\frac{\theta}{3} \Rightarrow du = \frac{1}{3}\cos\frac{\theta}{3}\,d\theta \Rightarrow 6\,du = 2\cos\frac{\theta}{3}\,d\theta$; $\theta = \frac{\pi}{2} \Rightarrow u = \frac{1}{2}$ and $\theta = \pi \Rightarrow u = \frac{\sqrt{3}}{2}$;
$\int_{\pi/2}^{\pi} 2\cot\frac{\theta}{3}\,d\theta = \int_{\pi/2}^{\pi} \frac{2\cos\frac{\theta}{3}}{\sin\frac{\theta}{3}}\,d\theta = 6\int_{1/2}^{\sqrt{3}/2} \frac{du}{u} = 6\left[\ln|u|\right]_{1/2}^{\sqrt{3}/2} = 6\left(\ln\frac{\sqrt{3}}{2} - \ln\frac{1}{2}\right) = 6\ln\sqrt{3} = \ln 27$

52. Let $u = \cos 3x \Rightarrow du = -3\sin 3x\,dx \Rightarrow -2\,du = 6\sin 3x\,dx$; $x = 0 \Rightarrow u = 1$ and $x = \frac{\pi}{12} \Rightarrow u = \frac{1}{\sqrt{2}}$;
$\int_0^{\pi/12} 6\tan 3x\,dx = \int_0^{\pi/12} \frac{6\sin 3x}{\cos 3x}\,dx = -2\int_1^{1/\sqrt{2}} \frac{du}{u} = -2\left[\ln|u|\right]_1^{1/\sqrt{2}} = -2\ln\frac{1}{\sqrt{2}} - \ln 1 = 2\ln\sqrt{2} = \ln 2$

53. $\int \frac{dx}{2\sqrt{x}+2x} = \int \frac{dx}{2\sqrt{x}\,(1+\sqrt{x})}$; let $u = 1 + \sqrt{x} \Rightarrow du = \frac{1}{2\sqrt{x}}\,dx$; $\int \frac{dx}{2\sqrt{x}\,(1+\sqrt{x})} = \int \frac{du}{u} = \ln|u| + C$
$= \ln\left|1 + \sqrt{x}\right| + C = \ln\left(1 + \sqrt{x}\right) + C$

54. Let $u = \sec x + \tan x \Rightarrow du = (\sec x \tan x + \sec^2 x)\,dx = (\sec x)(\tan x + \sec x)\,dx \Rightarrow \sec x\,dx = \frac{du}{u}$;
$\int \frac{\sec x\,dx}{\sqrt{\ln(\sec x + \tan x)}} = \int \frac{du}{u\sqrt{\ln u}} = \int (\ln u)^{-1/2} \cdot \frac{1}{u}\,du = 2(\ln u)^{1/2} + C = 2\sqrt{\ln(\sec x + \tan x)} + C$

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5. $\ln y = 2t + 4 \Rightarrow e^{\ln y} = e^{2t+4} \Rightarrow y = e^{2t+4}$

6. $\ln y = -t + 5 \Rightarrow e^{\ln y} = e^{-t+5} \Rightarrow y = e^{-t+5}$

7. $\ln(y - 40) = 5t \Rightarrow e^{\ln(y-40)} = e^{5t} \Rightarrow y - 40 = e^{5t} \Rightarrow y = e^{5t} + 40$

8. $\ln(1 - 2y) = t \Rightarrow e^{\ln(1-2y)} = e^t \Rightarrow 1 - 2y = e^t \Rightarrow -2y = e^t - 1 \Rightarrow y = -\left(\frac{e^t - 1}{2}\right)$

9. $\ln(y - 1) - \ln 2 = x + \ln x \Rightarrow \ln(y - 1) - \ln 2 - \ln x = x \Rightarrow \ln\left(\frac{y-1}{2x}\right) = x \Rightarrow e^{\ln\left(\frac{y-1}{2x}\right)} = e^x \Rightarrow \frac{y-1}{2x} = e^x$
$\Rightarrow y - 1 = 2xe^x \Rightarrow y = 2xe^x + 1$

10. $\ln(y^2 - 1) - \ln(y + 1) = \ln(\sin x) \Rightarrow \ln\left(\frac{y^2-1}{y+1}\right) = \ln(\sin x) \Rightarrow \ln(y - 1) = \ln(\sin x) \Rightarrow e^{\ln(y-1)} = e^{\ln(\sin x)}$
$\Rightarrow y - 1 = \sin x \Rightarrow y = \sin x + 1$

11. (a) $e^{2k} = 4 \Rightarrow \ln e^{2k} = \ln 4 \Rightarrow 2k \ln e = \ln 2^2 \Rightarrow 2k = 2\ln 2 \Rightarrow k = \ln 2$
(b) $100e^{10k} = 200 \Rightarrow e^{10k} = 2 \Rightarrow \ln e^{10k} = \ln 2 \Rightarrow 10k \ln e = \ln 2 \Rightarrow 10k = \ln 2 \Rightarrow k = \frac{\ln 2}{10}$
(c) $e^{k/1000} = a \Rightarrow \ln e^{k/1000} = \ln a \Rightarrow \frac{k}{1000} \ln e = \ln a \Rightarrow \frac{k}{1000} = \ln a \Rightarrow k = 1000 \ln a$

12. (a) $e^{5k} = \frac{1}{4} \Rightarrow \ln e^{5k} = \ln 4^{-1} \Rightarrow 5k \ln e = -\ln 4 \Rightarrow 5k = -\ln 4 \Rightarrow k = -\frac{\ln 4}{5}$
(b) $80e^k = 1 \Rightarrow e^k = 80^{-1} \Rightarrow \ln e^k = \ln 80^{-1} \Rightarrow k \ln e = -\ln 80 \Rightarrow k = -\ln 80$
(c) $e^{(\ln 0.8)k} = 0.8 \Rightarrow \left(e^{\ln 0.8}\right)^k = 0.8 \Rightarrow (0.8)^k = 0.8 \Rightarrow k = 1$

13. (a) $e^{-0.3t} = 27 \Rightarrow \ln e^{-0.3t} = \ln 3^3 \Rightarrow (-0.3t)\ln e = 3\ln 3 \Rightarrow -0.3t = 3\ln 3 \Rightarrow t = -10\ln 3$
(b) $e^{kt} = \frac{1}{2} \Rightarrow \ln e^{kt} = \ln 2^{-1} = kt \ln e = -\ln 2 \Rightarrow t = -\frac{\ln 2}{k}$
(c) $e^{(\ln 0.2)t} = 0.4 \Rightarrow \left(e^{\ln 0.2}\right)^t = 0.4 \Rightarrow 0.2^t = 0.4 \Rightarrow \ln 0.2^t = \ln 0.4 \Rightarrow t\ln 0.2 = \ln 0.4 \Rightarrow t = \frac{\ln 0.4}{\ln 0.2}$

14. (a) $e^{-0.01t} = 1000 \Rightarrow \ln e^{-0.01t} = \ln 1000 \Rightarrow (-0.01t)\ln e = \ln 1000 \Rightarrow -0.01t = \ln 1000 \Rightarrow t = -100\ln 1000$
(b) $e^{kt} = \frac{1}{10} \Rightarrow \ln e^{kt} = \ln 10^{-1} = kt \ln e = -\ln 10 \Rightarrow kt = -\ln 10 \Rightarrow t = -\frac{\ln 10}{k}$
(c) $e^{(\ln 2)t} = \frac{1}{2} \Rightarrow \left(e^{\ln 2}\right)^t = 2^{-1} \Rightarrow 2^t = 2^{-1} \Rightarrow t = -1$

15. $e^{\sqrt{t}} = x^2 \Rightarrow \ln e^{\sqrt{t}} = \ln x^2 \Rightarrow \sqrt{t} = 2\ln x \Rightarrow t = 4(\ln x)^2$

16. $e^{x^2} e^{2x+1} = e^t \Rightarrow e^{x^2+2x+1} = e^t \Rightarrow \ln e^{x^2+2x+1} = \ln e^t \Rightarrow t = x^2 + 2x + 1$

17. $y = e^{-5x} \Rightarrow y' = e^{-5x} \frac{d}{dx}(-5x) \Rightarrow y' = -5e^{-5x}$

18. $y = e^{2x/3} \Rightarrow y' = e^{2x/3} \frac{d}{dx}\left(\frac{2x}{3}\right) \Rightarrow y' = \frac{2}{3}e^{2x/3}$

19. $y = e^{5-7x} \Rightarrow y' = e^{5-7x} \frac{d}{dx}(5 - 7x) \Rightarrow y' = -7e^{5-7x}$

20. $y = e^{\left(4\sqrt{x}+x^2\right)} \Rightarrow y' = e^{\left(4\sqrt{x}+x^2\right)} \frac{d}{dx}\left(4\sqrt{x} + x^2\right) \Rightarrow y' = \left(\frac{2}{\sqrt{x}} + 2x\right)e^{\left(4\sqrt{x}+x^2\right)}$

21. $y = xe^x - e^x \Rightarrow y' = (e^x + xe^x) - e^x = xe^x$

22. $y = (1 + 2x)e^{-2x} \Rightarrow y' = 2e^{-2x} + (1 + 2x)e^{-2x} \frac{d}{dx}(-2x) \Rightarrow y' = 2e^{-2x} - 2(1 + 2x)e^{-2x} = -4xe^{-2x}$

23. $y = (x^2 - 2x + 2)e^x \Rightarrow y' = (2x - 2)e^x + (x^2 - 2x + 2)e^x = x^2e^x$

24. $y = (9x^2 - 6x + 2)e^{3x} \Rightarrow y' = (18x - 6)e^{3x} + (9x^2 - 6x + 2)e^{3x} \frac{d}{dx}(3x) \Rightarrow y' = (18x - 6)e^{3x} + 3(9x^2 - 6x + 2)e^{3x}$
$= 27x^2e^{3x}$

25. $y = e^{\theta}(\sin\theta + \cos\theta) \Rightarrow y' = e^{\theta}(\sin\theta + \cos\theta) + e^{\theta}(\cos\theta - \sin\theta) = 2e^{\theta}\cos\theta$

26. $y = \ln(3\theta e^{-\theta}) = \ln 3 + \ln\theta + \ln e^{-\theta} = \ln 3 + \ln\theta - \theta \Rightarrow \frac{dy}{d\theta} = \frac{1}{\theta} - 1$

27. $y = \cos\left(e^{-\theta^2}\right) \Rightarrow \frac{dy}{d\theta} = -\sin\left(e^{-\theta^2}\right)\frac{d}{d\theta}\left(e^{-\theta^2}\right) = \left(-\sin\left(e^{-\theta^2}\right)\right)\left(e^{-\theta^2}\right)\frac{d}{d\theta}(-\theta^2) = 2\theta e^{-\theta^2}\sin\left(e^{-\theta^2}\right)$

28. $y = \theta^3e^{-2\theta}\cos 5\theta \Rightarrow \frac{dy}{d\theta} = (3\theta^2)\left(e^{-2\theta}\cos 5\theta\right) + (\theta^3\cos 5\theta)e^{-2\theta}\frac{d}{d\theta}(-2\theta) - 5(\sin 5\theta)\left(\theta^3e^{-2\theta}\right)$
$= \theta^2e^{-2\theta}(3\cos 5\theta - 2\theta\cos 5\theta - 5\theta\sin 5\theta)$

29. $y = \ln(3te^{-t}) = \ln 3 + \ln t + \ln e^{-t} = \ln 3 + \ln t - t \Rightarrow \frac{dy}{dt} = \frac{1}{t} - 1 = \frac{1-t}{t}$

30. $y = \ln(2e^{-t}\sin t) = \ln 2 + \ln e^{-t} + \ln\sin t = \ln 2 - t + \ln\sin t \Rightarrow \frac{dy}{dt} = -1 + \left(\frac{1}{\sin t}\right)\frac{d}{dt}(\sin t) = -1 + \frac{\cos t}{\sin t}$
$= \frac{\cos t - \sin t}{\sin t}$

31. $y = \ln\frac{e^{\theta}}{1+e^{\theta}} = \ln e^{\theta} - \ln\left(1 + e^{\theta}\right) = \theta - \ln\left(1+e^{\theta}\right) \Rightarrow \frac{dy}{d\theta} = 1 - \left(\frac{1}{1+e^{\theta}}\right)\frac{d}{d\theta}\left(1 + e^{\theta}\right) = 1 - \frac{e^{\theta}}{1+e^{\theta}} = \frac{1}{1+e^{\theta}}$

32. $y = \ln\frac{\sqrt{\theta}}{1+\sqrt{\theta}} = \ln\sqrt{\theta} - \ln\left(1 + \sqrt{\theta}\right) \Rightarrow \frac{dy}{d\theta} = \left(\frac{1}{\sqrt{\theta}}\right)\frac{d}{d\theta}\left(\sqrt{\theta}\right) - \left(\frac{1}{1+\sqrt{\theta}}\right)\frac{d}{d\theta}\left(1 + \sqrt{\theta}\right)$
$= \left(\frac{1}{\sqrt{\theta}}\right)\left(\frac{1}{2\sqrt{\theta}}\right) - \left(\frac{1}{1+\sqrt{\theta}}\right)\left(\frac{1}{2\sqrt{\theta}}\right) = \frac{\left(1+\sqrt{\theta}\right) - \sqrt{\theta}}{2\theta\left(1+\sqrt{\theta}\right)} = \frac{1}{2\theta\left(1+\sqrt{\theta}\right)} = \frac{1}{2\theta\left(1+\theta^{1/2}\right)}$

33. $y = e^{(\cos t + \ln t)} = e^{\cos t}e^{\ln t} = te^{\cos t} \Rightarrow \frac{dy}{dt} = e^{\cos t} + te^{\cos t}\frac{d}{dt}(\cos t) = (1 - t\sin t)e^{\cos t}$

34. $y = e^{\sin t}(\ln t^2 + 1) \Rightarrow \frac{dy}{dt} = e^{\sin t}(\cos t)(\ln t^2 + 1) + \frac{2}{t}e^{\sin t} = e^{\sin t}\left[(\ln t^2 + 1)(\cos t) + \frac{2}{t}\right]$

35. $\int_0^{\ln x}\sin e^t\,dt \Rightarrow y' = \left(\sin e^{\ln x}\right)\cdot\frac{d}{dx}(\ln x) = \frac{\sin x}{x}$

36. $y = \int_{e^{4\sqrt{x}}}^{e^{2x}}\ln t\,dt \Rightarrow y' = (\ln e^{2x})\cdot\frac{d}{dx}(e^{2x}) - \left(\ln e^{4\sqrt{x}}\right)\cdot\frac{d}{dx}\left(e^{4\sqrt{x}}\right) = (2x)(2e^{2x}) - \left(4\sqrt{x}\right)\left(e^{4\sqrt{x}}\right)\cdot\frac{d}{dx}\left(4\sqrt{x}\right)$
$= 4xe^{2x} - 4\sqrt{x}\,e^{4\sqrt{x}}\left(\frac{2}{\sqrt{x}}\right) = 4xe^{2x} - 8e^{4\sqrt{x}}$

37. $\ln y = e^y\sin x \Rightarrow \left(\frac{1}{y}\right)y' = (y'e^y)(\sin x) + e^y\cos x \Rightarrow y'\left(\frac{1}{y} - e^y\sin x\right) = e^y\cos x$
$\Rightarrow y'\left(\frac{1 - ye^y\sin x}{y}\right) = e^y\cos x \Rightarrow y' = \frac{ye^y\cos x}{1 - ye^y\sin x}$

38. $\ln xy = e^{x+y} \Rightarrow \ln x + \ln y = e^{x+y} \Rightarrow \frac{1}{x} + \left(\frac{1}{y}\right)y' = (1 + y')e^{x+y} \Rightarrow y'\left(\frac{1}{y} - e^{x+y}\right) = e^{x+y} - \frac{1}{x}$
$\Rightarrow y'\left(\frac{1 - ye^{x+y}}{y}\right) = \frac{xe^{x+y} - 1}{x} \Rightarrow y' = \frac{y\left(xe^{x+y} - 1\right)}{x\left(1 - ye^{x+y}\right)}$

39. $e^{2x} = \sin(x + 3y) \Rightarrow 2e^{2x} = (1 + 3y')\cos(x + 3y) \Rightarrow 1 + 3y' = \frac{2e^{2x}}{\cos(x+3y)} \Rightarrow 3y' = \frac{2e^{2x}}{\cos(x+3y)} - 1$
$\Rightarrow y' = \frac{2e^{2x} - \cos(x+3y)}{3\cos(x+3y)}$

40. $\tan y = e^x + \ln x \Rightarrow (\sec^2 y)\, y' = e^x + \frac{1}{x} \Rightarrow y' = \frac{(xe^x+1)\cos^2 y}{x}$

41. $\int (e^{3x} + 5e^{-x})\, dx = \frac{e^{3x}}{3} - 5e^{-x} + C$

42. $\int (2e^x - 3e^{-2x})\, dx = 2e^x + \frac{3}{2}e^{-2x} + C$

43. $\int_{\ln 2}^{\ln 3} e^x\, dx = [e^x]_{\ln 2}^{\ln 3} = e^{\ln 3} - e^{\ln 2} = 3 - 2 = 1$

44. $\int_{-\ln 2}^{0} e^{-x}\, dx = [-e^{-x}]_{-\ln 2}^{0} = -e^0 + e^{\ln 2} = -1 + 2 = 1$

45. $\int 8e^{(x+1)}\, dx = 8e^{(x+1)} + C$

46. $\int 2e^{(2x-1)}\, dx = e^{(2x-1)} + C$

47. $\int_{\ln 4}^{\ln 9} e^{x/2}\, dx = \left[2e^{x/2}\right]_{\ln 4}^{\ln 9} = 2\left[e^{(\ln 9)/2} - e^{(\ln 4)/2}\right] = 2\left(e^{\ln 3} - e^{\ln 2}\right) = 2(3-2) = 2$

48. $\int_0^{\ln 16} e^{x/4}\, dx = \left[4e^{x/4}\right]_0^{\ln 16} = 4\left(e^{(\ln 16)/4} - e^0\right) = 4\left(e^{\ln 2} - 1\right) = 4(2-1) = 4$

49. Let $u = r^{1/2} \Rightarrow du = \frac{1}{2}r^{-1/2}\, dr \Rightarrow 2\, du = r^{-1/2}\, dr$;
$\int \frac{e^{\sqrt{r}}}{\sqrt{r}}\, dr = \int e^{r^{1/2}} \cdot r^{-1/2}\, dr = 2\int e^u\, du = 2e^u + C = 2e^{r^{1/2}} + C = 2e^{\sqrt{r}} + C$

50. Let $u = -r^{1/2} \Rightarrow du = -\frac{1}{2}r^{-1/2}\, dr \Rightarrow -2\, du = r^{-1/2}\, dr$;
$\int \frac{e^{-\sqrt{r}}}{\sqrt{r}}\, dr = \int e^{-r^{1/2}} \cdot r^{-1/2}\, dr = -2\int e^u\, du = -2e^{-r^{1/2}} + C = -2e^{-\sqrt{r}} + C$

51. Let $u = -t^2 \Rightarrow du = -2t\, dt \Rightarrow -du = 2t\, dt$;
$\int 2te^{-t^2}\, dt = -\int e^u\, du = -e^u + C = -e^{-t^2} + C$

52. Let $u = t^4 \Rightarrow du = 4t^3\, dt \Rightarrow \frac{1}{4}du = t^3\, dt$;
$\int t^3 e^{t^4}\, dt = \frac{1}{4}\int e^u\, du = \frac{1}{4}e^{t^4} + C$

53. Let $u = \frac{1}{x} \Rightarrow du = -\frac{1}{x^2}dx \Rightarrow -du = \frac{1}{x^2}dx$;
$\int \frac{e^{1/x}}{x^2}dx = \int -e^u\, du = -e^u + C = -e^{1/x} + C$

54. Let $u = -x^{-2} \Rightarrow du = 2x^{-3}\, dx \Rightarrow \frac{1}{2}du = x^{-3}\, dx$;
$\int \frac{e^{-1/x^2}}{x^3}dx = \int e^{-x^{-2}} \cdot x^{-3}\, dx = \frac{1}{2}\int e^u\, du = \frac{1}{2}e^u + C = \frac{1}{2}e^{-x^{-2}} + C = \frac{1}{2}e^{-1/x^2} + C$

55. Let $u = \tan\theta \Rightarrow du = \sec^2\theta\, d\theta$; $\theta = 0 \Rightarrow u = 0$, $\theta = \frac{\pi}{4} \Rightarrow u = 1$;
$\int_0^{\pi/4}\left(1 + e^{\tan\theta}\right)\sec^2\theta\, d\theta = \int_0^{\pi/4}\sec^2\theta\, d\theta + \int_0^1 e^u\, du = [\tan\theta]_0^{\pi/4} + [e^u]_0^1 = \left[\tan\left(\frac{\pi}{4}\right) - \tan(0)\right] + (e^1 - e^0)$
$= (1-0) + (e-1) = e$

56. Let $u = \cot\theta \Rightarrow du = -\csc^2\theta\, d\theta$; $\theta = \frac{\pi}{4} \Rightarrow u = 1$, $\theta = \frac{\pi}{2} \Rightarrow u = 0$;
$\int_{\pi/4}^{\pi/2}\left(1 + e^{\cot\theta}\right)\csc^2\theta\, d\theta = \int_{\pi/4}^{\pi/2}\csc^2\theta\, d\theta - \int_1^0 e^u\, du = [-\cot\theta]_{\pi/4}^{\pi/2} - [e^u]_1^0 = \left[-\cot\left(\frac{\pi}{2}\right) + \cot\left(\frac{\pi}{4}\right)\right] - (e^0 - e^1)$
$= (0+1) - (1-e) = e$

57. Let $u = \sec\pi t \Rightarrow du = \pi\sec\pi t\tan\pi t\, dt \Rightarrow \frac{du}{\pi} = \sec\pi t\tan\pi t\, dt$;
$\int e^{\sec(\pi t)}\sec(\pi t)\tan(\pi t)\, dt = \frac{1}{\pi}\int e^u\, du = \frac{e^u}{\pi} + C = \frac{e^{\sec\pi t}}{\pi} + C$

58. Let $u = \csc(\pi + t) \Rightarrow du = -\csc(\pi+t)\cot(\pi+t)\, dt$;
$\int e^{\csc(\pi+t)}\csc(\pi+t)\cot(\pi+t)\, dt = -\int e^u\, du = -e^u + C = -e^{\csc(\pi+t)} + C$

59. Let $u = e^v \Rightarrow du = e^v\, dv \Rightarrow 2\, du = 2e^v\, dv$; $v = \ln\frac{\pi}{6} \Rightarrow u = \frac{\pi}{6}$, $v = \ln\frac{\pi}{2} \Rightarrow u = \frac{\pi}{2}$;
$\int_{\ln(\pi/6)}^{\ln(\pi/2)} 2e^v\cos e^v\, dv = 2\int_{\pi/6}^{\pi/2}\cos u\, du = [2\sin u]_{\pi/6}^{\pi/2} = 2\left[\sin\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{6}\right)\right] = 2\left(1 - \frac{1}{2}\right) = 1$

60. Let $u = e^{x^2} \Rightarrow du = 2xe^{x^2}\, dx$; $x = 0 \Rightarrow u = 1$, $x = \sqrt{\ln\pi} \Rightarrow u = e^{\ln\pi} = \pi$;
$\int_0^{\sqrt{\ln\pi}} 2xe^{x^2}\cos\left(e^{x^2}\right)dx = \int_1^{\pi}\cos u\, du = [\sin u]_1^{\pi} = \sin(\pi) - \sin(1) = -\sin(1) \approx -0.84147$

61. Let $u = 1 + e^r \Rightarrow du = e^r\, dr$;
$\int \frac{e^r}{1+e^r}dr = \int \frac{1}{u}du = \ln|u| + C = \ln(1+e^r) + C$

62. $\int \frac{1}{1+e^x}dx = \int \frac{e^{-x}}{e^{-x}+1}dx$;
let $u = e^{-x} + 1 \Rightarrow du = -e^{-x}\, dx \Rightarrow -du = e^{-x}\, dx$;
$\int \frac{e^{-x}}{e^{-x}+1}dx = -\int \frac{1}{u}du = -\ln|u| + C = -\ln(e^{-x}+1) + C$

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49. $\int_0^1 2^{-\theta}\, d\theta = \int_0^1 \left(\frac12\right)^\theta d\theta = \left[\frac{\left(\frac12\right)^\theta}{\ln\left(\frac12\right)}\right]_0^1 = \frac{\frac12}{\ln\left(\frac12\right)} - \frac{1}{\ln\left(\frac12\right)} = -\frac{\frac12}{\ln\left(\frac12\right)} = \frac{-1}{2(\ln 1 - \ln 2)} = \frac{1}{2\ln 2}$

50. $\int_{-2}^0 5^{-\theta}\, d\theta = \int_{-2}^0 \left(\frac15\right)^\theta d\theta = \left[\frac{\left(\frac15\right)^\theta}{\ln\left(\frac15\right)}\right]_{-2}^0 = \frac{1}{\ln\left(\frac15\right)} - \frac{\left(\frac15\right)^{-2}}{\ln\left(\frac15\right)} = \frac{1}{\ln\left(\frac15\right)}(1-25) = \frac{-24}{\ln 1 - \ln 5} = \frac{24}{\ln 5}$

51. Let $u = x^2 \Rightarrow du = 2x\,dx \Rightarrow \frac12 du = x\,dx$; $x = 1 \Rightarrow u = 1$, $x = \sqrt2 \Rightarrow u = 2$;
$\int_1^{\sqrt2} x2^{(x^2)}\, dx = \int_1^2 \left(\frac12\right) 2^u\, du = \frac12\left[\frac{2^u}{\ln 2}\right]_1^2 = \left(\frac{1}{2\ln 2}\right)(2^2 - 2^1) = \frac{1}{\ln 2}$

52. Let $u = x^{1/2} \Rightarrow du = \frac12 x^{-1/2}\, dx \Rightarrow 2\, du = \frac{dx}{\sqrt x}$; $x = 1 \Rightarrow u = 1$, $x = 4 \Rightarrow u = 2$;
$\int_1^4 \frac{2^{\sqrt x}}{\sqrt x}\, dx = \int_1^4 2^{x^{1/2}} \cdot x^{-1/2}\, dx = 2\int_1^2 2^u\, du = \left[\frac{2^{(u+1)}}{\ln 2}\right]_1^2 = \left(\frac{1}{\ln 2}\right)(2^3 - 2^2) = \frac{4}{\ln 2}$

53. Let $u = \cos t \Rightarrow du = -\sin t\, dt \Rightarrow -du = \sin t\, dt$; $t = 0 \Rightarrow u = 1$, $t = \frac{\pi}{2} \Rightarrow u = 0$;
$\int_0^{\pi/2} 7^{\cos t}\sin t\, dt = -\int_1^0 7^u\, du = \left[-\frac{7^u}{\ln 7}\right]_1^0 = \left(\frac{-1}{\ln 7}\right)(7^0 - 7) = \frac{6}{\ln 7}$

54. Let $u = \tan t \Rightarrow du = \sec^2 t\, dt$; $t = 0 \Rightarrow u = 0$, $t = \frac{\pi}{4} \Rightarrow u = 1$;
$\int_0^{\pi/4} \left(\frac13\right)^{\tan t}\sec^2 t\, dt = \int_0^1 \left(\frac13\right)^u du = \left[\frac{\left(\frac13\right)^u}{\ln\left(\frac13\right)}\right]_0^1 = \left(-\frac{1}{\ln 3}\right)\left[\left(\frac13\right)^1 - \left(\frac13\right)^0\right] = \frac{2}{3\ln 3}$

55. Let $u = x^{2x} \Rightarrow \ln u = 2x\ln x \Rightarrow \frac1u \frac{du}{dx} = 2\ln x + (2x)\left(\frac1x\right) \Rightarrow \frac{du}{dx} = 2u(\ln x + 1) \Rightarrow \frac12 du = x^{2x}(1 + \ln x)$ d
$x = 2 \Rightarrow u = 2^4 = 16$, $x = 4 \Rightarrow u = 4^8 = 65{,}536$;
$\int_2^4 x^{2x}(1 + \ln x)\, dx = \frac12 \int_{16}^{65{,}536} du = \frac12 [u]_{16}^{65{,}536} = \frac12(65{,}536 - 16) = \frac{65{,}520}{2} = 32{,}760$

56. Let $u = \ln x \Rightarrow du = \frac1x dx$; $x = 1 \Rightarrow u = 0$, $x = 2 \Rightarrow u = \ln 2$;
$\int_1^2 \frac{2^{\ln x}}{x}\, dx = \int_0^{\ln 2} 2^u\, du = \left[\frac{2^u}{\ln 2}\right]_0^{\ln 2} = \left(\frac{1}{\ln 2}\right)(2^{\ln 2} - 2^0) = \frac{2^{\ln 2} - 1}{\ln 2}$

57. $\int 3x^{\sqrt3}\, dx = \frac{3x^{(\sqrt3+1)}}{\sqrt3+1} + C$

58. $\int x^{(\sqrt2-1)}\, dx = \frac{x^{\sqrt2}}{\sqrt2} + C$

59. $\int_0^3 \left(\sqrt2 + 1\right) x^{\sqrt2}\, dx = \left[x^{(\sqrt2+1)}\right]_0^3 = 3^{(\sqrt2+1)}$

60. $\int_1^e x^{(\ln 2)-1}\, dx = \left[\frac{x^{\ln 2}}{\ln 2}\right]_1^e = \frac{e^{\ln 2} - 1^{\ln 2}}{\ln 2} = \frac{2-1}{\ln 2} = \frac{1}{\ln 2}$

61. $\int \frac{\log_{10} x}{x}\, dx = \int \left(\frac{\ln x}{\ln 10}\right)\left(\frac1x\right) dx$; $\left[u = \ln x \Rightarrow du = \frac1x dx\right]$
$\to \int \left(\frac{\ln x}{\ln 10}\right)\left(\frac1x\right) dx = \frac{1}{\ln 10}\int u\, du = \left(\frac{1}{\ln 10}\right)\left(\frac12 u^2\right) + C = \frac{(\ln x)^2}{2\ln 10} + C$

62. $\int_1^4 \frac{\log_2 x}{x}\, dx = \int_1^4 \left(\frac{\ln x}{\ln 2}\right)\left(\frac1x\right) dx$; $\left[u = \ln x \Rightarrow du = \frac1x dx;\ x = 1 \Rightarrow u = 0,\ x = 4 \Rightarrow u = \ln 4\right]$
$\to \int_1^4 \left(\frac{\ln x}{\ln 2}\right)\left(\frac1x\right) dx = \int_0^{\ln 4} \left(\frac{1}{\ln 2}\right) u\, du = \left(\frac{1}{\ln 2}\right)\left[\frac12 u^2\right]_0^{\ln 4} = \left(\frac{1}{\ln 2}\right)\left[\frac12(\ln 4)^2\right] = \frac{(\ln 4)^2}{2\ln 2} = \frac{(\ln 4)^2}{\ln 4} = \ln 4$

63. $\int_1^4 \frac{\ln 2 \log_2 x}{x}\, dx = \int_1^4 \left(\frac{\ln 2}{x}\right)\left(\frac{\ln x}{\ln 2}\right) dx = \int_1^4 \frac{\ln x}{x}\, dx = \left[\frac12(\ln x)^2\right]_1^4 = \frac12\left[(\ln 4)^2 - (\ln 1)^2\right] = \frac12(\ln 4)^2$
$= \frac12(2\ln 2)^2 = 2(\ln 2)^2$

64. $\int_1^e \frac{2\ln 10(\log_{10} x)}{x}\, dx = \int_1^e \frac{(\ln 10)(2\ln x)}{(\ln 10)}\left(\frac1x\right) dx = \left[(\ln x)^2\right]_1^e = (\ln e)^2 - (\ln 1)^2 = 1$

64. $\int_1^e \frac{2 \ln 10 (\log_{10} x)}{x}\, dx = \int_1^e \frac{(\ln 10)(2 \ln x)}{(\ln 10)} \left(\frac{1}{x}\right) dx = \left[(\ln x)^2\right]_1^e = (\ln e)^2 - (\ln 1)^2 = 1$

65. $\int_0^2 \frac{\log_2 (x+2)}{x+2}\, dx = \frac{1}{\ln 2} \int_0^2 [\ln (x+2)] \left(\frac{1}{x+2}\right) dx = \left(\frac{1}{\ln 2}\right) \left[\frac{(\ln (x+2))^2}{2}\right]_0^2 = \left(\frac{1}{\ln 2}\right) \left[\frac{(\ln 4)^2}{2} - \frac{(\ln 2)^2}{2}\right]$
$= \left(\frac{1}{\ln 2}\right) \left[\frac{4(\ln 2)^2}{2} - \frac{(\ln 2)^2}{2}\right] = \frac{3}{2} \ln 2$

66. $\int_{1/10}^{10} \frac{\log_{10} (10x)}{x}\, dx = \frac{10}{\ln 10} \int_{1/10}^{10} [\ln (10x)] \left(\frac{1}{10x}\right) dx = \left(\frac{10}{\ln 10}\right) \left[\frac{(\ln (10x))^2}{20}\right]_{1/10}^{10} = \left(\frac{10}{\ln 10}\right) \left[\frac{(\ln 100)^2}{20} - \frac{(\ln 1)^2}{2}\right]$
$= \left(\frac{10}{\ln 10}\right) \left[\frac{4(\ln 10)^2}{20}\right] = 2 \ln 10$

67. $\int_0^9 \frac{2 \log_{10} (x+1)}{x+1}\, dx = \frac{2}{\ln 10} \int_0^9 \ln (x+1) \left(\frac{1}{x+1}\right) dx = \left(\frac{2}{\ln 10}\right) \left[\frac{(\ln (x+1))^2}{2}\right]_0^9 = \left(\frac{2}{\ln 10}\right) \left[\frac{(\ln 10)^2}{2} - \frac{(\ln 1)^2}{2}\right]$
$= \ln 10$

68. $\int_2^3 \frac{2 \log_2 (x-1)}{x-1}\, dx = \frac{2}{\ln 2} \int_2^3 \ln (x-1) \left(\frac{1}{x-1}\right) dx = \left(\frac{2}{\ln 2}\right) \left[\frac{(\ln (x-1))^2}{2}\right]_2^3 = \left(\frac{2}{\ln 2}\right) \left[\frac{(\ln 2)^2}{2} - \frac{(\ln 1)^2}{2}\right] = \ln 2$

69. $\int \frac{dx}{x \log_{10} x} = \int \left(\frac{\ln 10}{\ln x}\right) \left(\frac{1}{x}\right) dx = (\ln 10) \int \left(\frac{1}{\ln x}\right) \left(\frac{1}{x}\right) dx;\ \left[u = \ln x \ \Rightarrow\ du = \frac{1}{x}\, dx\right]$
$\rightarrow (\ln 10) \int \left(\frac{1}{\ln x}\right) \left(\frac{1}{x}\right) dx = (\ln 10) \int \frac{1}{u}\, du = (\ln 10) \ln |u| + C = (\ln 10) \ln |\ln x| + C$

70. $\int \frac{dx}{x (\log_8 x)^2} = \int \frac{dx}{x \left(\frac{\ln x}{\ln 8}\right)^2} = (\ln 8)^2 \int \frac{(\ln x)^{-2}}{x}\, dx = (\ln 8)^2 \frac{(\ln x)^{-1}}{-1} + C = -\frac{(\ln 8)^2}{\ln x} + C$

71. $\int_1^{\ln x} \frac{1}{t}\, dt = \left[\ln |t|\right]_1^{\ln x} = \ln |\ln x| - \ln 1 = \ln (\ln x),\ x > 1$

72. $\int_1^{e^x} \frac{1}{t}\, dt = \left[\ln |t|\right]_1^{e^x} = \ln e^x - \ln 1 = x \ln e = x$

73. $\int_1^{1/x} \frac{1}{t}\, dt = \left[\ln |t|\right]_1^{1/x} = \ln \left|\frac{1}{x}\right| - \ln 1 = (\ln 1 - \ln |x|) - \ln 1 = -\ln x,\ x > 0$

74. $\frac{1}{\ln a} \int_1^x \frac{1}{t}\, dt = \left[\frac{1}{\ln a} \ln |t|\right]_1^x = \frac{\ln x}{\ln a} - \frac{\ln 1}{\ln a} = \log_a x,\ x > 0$

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13. $\alpha = \sin^{-1}\left(\frac{5}{13}\right) \Rightarrow \cos\alpha = \frac{12}{13}$, $\tan\alpha = \frac{5}{12}$, $\sec\alpha = \frac{13}{12}$, $\csc\alpha = \frac{13}{5}$, and $\cot\alpha = \frac{12}{5}$

14. $\alpha = \tan^{-1}\left(\frac{4}{3}\right) \Rightarrow \sin\alpha = \frac{4}{5}$, $\cos\alpha = \frac{3}{5}$, $\sec\alpha = \frac{5}{3}$, $\csc\alpha = \frac{5}{4}$, and $\cot\alpha = \frac{3}{4}$

15. $\alpha = \sec^{-1}\left(-\sqrt{5}\right) \Rightarrow \sin\alpha = \frac{2}{\sqrt{5}}$, $\cos\alpha = -\frac{1}{\sqrt{5}}$, $\tan\alpha = -2$, $\csc\alpha = \frac{\sqrt{5}}{2}$, and $\cot\alpha = -\frac{1}{2}$

16. $\alpha = \sec^{-1}\left(-\frac{\sqrt{13}}{2}\right) \Rightarrow \sin\alpha = \frac{3}{\sqrt{13}}$, $\cos\alpha = -\frac{2}{\sqrt{13}}$, $\tan\alpha = -\frac{3}{2}$, $\csc\alpha = \frac{\sqrt{13}}{3}$, and $\cot\alpha = -\frac{2}{3}$

17. $\sin\left(\cos^{-1}\frac{\sqrt{2}}{2}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

18. $\sec\left(\cos^{-1}\frac{1}{2}\right) = \sec\left(\frac{\pi}{3}\right) = 2$

19. $\tan\left(\sin^{-1}\left(-\frac{1}{2}\right)\right) = \tan\left(-\frac{\pi}{6}\right) = -\frac{1}{\sqrt{3}}$

20. $\cot\left(\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right) = \cot\left(-\frac{\pi}{3}\right) = -\frac{1}{\sqrt{3}}$

21. $\csc\left(\sec^{-1}2\right) + \cos\left(\tan^{-1}\left(-\sqrt{3}\right)\right) = \csc\left(\cos^{-1}\left(\frac{1}{2}\right)\right) + \cos\left(-\frac{\pi}{3}\right) = \csc\left(\frac{\pi}{3}\right) + \cos\left(-\frac{\pi}{3}\right) = \frac{2}{\sqrt{3}} + \frac{1}{2} = \frac{4+\sqrt{3}}{2\sqrt{3}}$

22. $\tan\left(\sec^{-1}1\right) + \sin\left(\csc^{-1}(-2)\right) = \tan\left(\cos^{-1}\frac{1}{1}\right) + \sin\left(\sin^{-1}\left(-\frac{1}{2}\right)\right) = \tan(0) + \sin\left(-\frac{\pi}{6}\right) = 0 + \left(-\frac{1}{2}\right) = -\frac{1}{2}$

23. $\sin\left(\sin^{-1}\left(-\frac{1}{2}\right) + \cos^{-1}\left(-\frac{1}{2}\right)\right) = \sin\left(-\frac{\pi}{6} + \frac{2\pi}{3}\right) = \sin\left(\frac{\pi}{2}\right) = 1$

24. $\cot\left(\sin^{-1}\left(-\frac{1}{2}\right) - \sec^{-1}2\right) = \cot\left(-\frac{\pi}{6} - \cos^{-1}\left(\frac{1}{2}\right)\right) = \cot\left(-\frac{\pi}{6} - \frac{\pi}{3}\right) = \cot\left(-\frac{\pi}{2}\right) = 0$

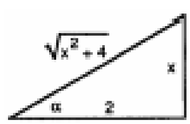
25. $\sec\left(\tan^{-1}1 + \csc^{-1}1\right) = \sec\left(\frac{\pi}{4} + \sin^{-1}\frac{1}{1}\right) = \sec\left(\frac{\pi}{4} + \frac{\pi}{2}\right) = \sec\left(\frac{3\pi}{4}\right) = -\sqrt{2}$

26. $\sec\left(\cot^{-1}\sqrt{3} + \csc^{-1}(-1)\right) = \sec\left(\frac{\pi}{6} + \sin^{-1}\left(\frac{1}{-1}\right)\right) = \sec\left(\frac{\pi}{2} - \frac{\pi}{3} - \frac{\pi}{2}\right) = \sec\left(-\frac{\pi}{3}\right) = 2$

27. $\sec^{-1}\left(\sec\left(-\frac{\pi}{6}\right)\right) = \sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

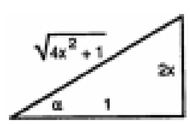
28. $\cot^{-1}\left(\cot\left(-\frac{\pi}{4}\right)\right) = \cot^{-1}(-1) = \frac{3\pi}{4}$

29. $\alpha = \tan^{-1}\frac{x}{2}$ indicates the diagram



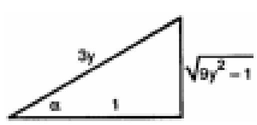
$\Rightarrow \sec\left(\tan^{-1}\frac{x}{2}\right) = \sec\alpha = \frac{\sqrt{x^2+4}}{2}$

30. $\alpha = \tan^{-1}2x$ indicates the diagram



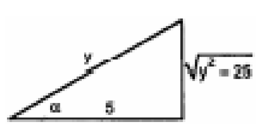
$\Rightarrow \sec\left(\tan^{-1}2x\right) = \sec\alpha = \sqrt{4x^2+1}$

31. $\alpha = \sec^{-1}3y$ indicates the diagram



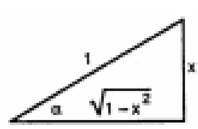
$\Rightarrow \tan\left(\sec^{-1}3y\right) = \tan\alpha = \sqrt{9y^2-1}$

32. $\alpha = \sec^{-1}\frac{y}{5}$ indicates the diagram

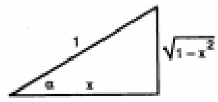


$\Rightarrow \tan\left(\sec^{-1}\frac{y}{5}\right) = \tan\alpha = \frac{\sqrt{y^2-25}}{5}$

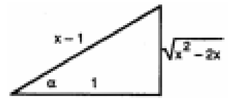
33. $\alpha = \sin^{-1}x$ indicates the diagram



$\Rightarrow \cos\left(\sin^{-1}x\right) = \cos\alpha = \sqrt{1-x^2}$

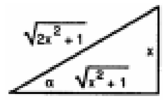
34. $\alpha = \cos^{-1} x$ indicates the diagram 

$\Rightarrow \tan\left(\cos^{-1} x\right) = \tan\alpha = \frac{\sqrt{1-x^2}}{x}$

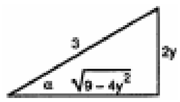
35. $\alpha = \tan^{-1}\sqrt{x^2-2x}$ indicates the diagram 

$\Rightarrow \sin\left(\tan^{-1}\sqrt{x^2-2x}\right)$

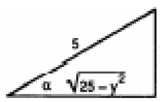
$= \sin\alpha = \frac{\sqrt{x^2-2x}}{x-1}$

36. $\alpha = \tan^{-1}\frac{x}{\sqrt{x^2+1}}$ indicates the diagram 

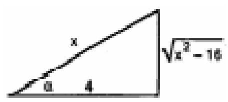
$\Rightarrow \sin\left(\tan^{-1}\frac{x}{\sqrt{x^2+1}}\right) = \sin\alpha = \frac{x}{\sqrt{2x^2+1}}$

37. $\alpha = \sin^{-1}\frac{2y}{3}$ indicates the diagram 

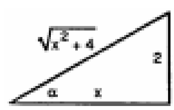
$\Rightarrow \cos\left(\sin^{-1}\frac{2y}{3}\right) = \cos\alpha = \frac{\sqrt{9-4y^2}}{3}$

38. $\alpha = \sin^{-1}\frac{y}{5}$ indicates the diagram 

$\Rightarrow \cos\left(\sin^{-1}\frac{y}{5}\right) = \cos\alpha = \frac{\sqrt{25-y^2}}{5}$

39. $\alpha = \sec^{-1}\frac{x}{4}$ indicates the diagram 

$\Rightarrow \sin\left(\sec^{-1}\frac{x}{4}\right) = \sin\alpha = \frac{\sqrt{x^2-16}}{x}$

40. $\alpha = \sec^{-1}\frac{\sqrt{x^2+4}}{x}$ indicates the diagram 

$\Rightarrow \sin\left(\sec^{-1}\frac{\sqrt{x^2+4}}{x}\right) = \sin\alpha = \frac{2}{\sqrt{x^2+4}}$

41. $\lim_{x\to 1^-} \sin^{-1} x = \frac{\pi}{2}$

42. $\lim_{x\to -1^+} \cos^{-1} x = \pi$

43. $\lim_{x\to\infty} \tan^{-1} x = \frac{\pi}{2}$

44. $\lim_{x\to -\infty} \tan^{-1} x = -\frac{\pi}{2}$

45. $\lim_{x\to\infty} \sec^{-1} x = \frac{\pi}{2}$

46. $\lim_{x\to -\infty} \sec^{-1} x = \lim_{x\to -\infty} \cos^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2}$

47. $\lim_{x\to\infty} \csc^{-1} x = \lim_{x\to\infty} \sin^{-1}\left(\frac{1}{x}\right) = 0$

48. $\lim_{x\to -\infty} \csc^{-1} x = \lim_{x\to -\infty} \sin^{-1}\left(\frac{1}{x}\right) = 0$

49. $y = \cos^{-1}\left(x^2\right) \Rightarrow \frac{dy}{dx} = -\frac{2x}{\sqrt{1-\left(x^2\right)^2}} = \frac{-2x}{\sqrt{1-x^4}}$

50. $y = \cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1} x \Rightarrow \frac{dy}{dx} = \frac{1}{|x|\sqrt{x^2-1}}$

51. $y = \sin^{-1}\sqrt{2}\,t \Rightarrow \frac{dy}{dt} = \frac{\sqrt{2}}{\sqrt{1-\left(\sqrt{2}\,t\right)^2}} = \frac{\sqrt{2}}{\sqrt{1-2t^2}}$

52. $y = \sin^{-1}(1-t) \Rightarrow \frac{dy}{dt} = \frac{-1}{\sqrt{1-(1-t)^2}} = \frac{-1}{\sqrt{2t-t^2}}$

53. $y = \sec^{-1}(2s+1) \Rightarrow \frac{dy}{ds} = \frac{2}{|2s+1|\sqrt{(2s+1)^2-1}} = \frac{2}{|2s+1|\sqrt{4s^2+4s}} = \frac{1}{|2s+1|\sqrt{s^2+s}}$

54. $y = \sec^{-1} 5s \Rightarrow \frac{dy}{ds} = \frac{5}{|5s|\sqrt{(5s)^2-1}} = \frac{1}{|s|\sqrt{25s^2-1}}$

55. $y = \csc^{-1}\left(x^2+1\right) \Rightarrow \frac{dy}{dx} = -\frac{2x}{|x^2+1|\sqrt{\left(x^2+1\right)^2-1}} = \frac{-2x}{\left(x^2+1\right)\sqrt{x^4+2x^2}}$

56. $y = \csc^{-1}\left(\frac{x}{2}\right) \Rightarrow \frac{dy}{dx} = -\frac{\left(\frac{1}{2}\right)}{\left|\frac{x}{2}\right|\sqrt{\left(\frac{x}{2}\right)^2 - 1}} = \frac{-1}{|x|\sqrt{\frac{x^2-4}{4}}} = \frac{-2}{|x|\sqrt{x^2-4}}$

57. $y = \sec^{-1}\left(\frac{1}{t}\right) = \cos^{-1} t \Rightarrow \frac{dy}{dt} = \frac{-1}{\sqrt{1-t^2}}$

58. $y = \sin^{-1}\left(\frac{3}{t^2}\right) = \csc^{-1}\left(\frac{t^2}{3}\right) \Rightarrow \frac{dy}{dt} = -\frac{\left(\frac{2t}{3}\right)}{\left|\frac{t^2}{3}\right|\sqrt{\left(\frac{t^2}{3}\right)^2 - 1}} = \frac{-2t}{t^2\sqrt{\frac{t^4-9}{9}}} = \frac{-6}{t\sqrt{t^4-9}}$

59. $y = \cot^{-1}\sqrt{t} = \cot^{-1} t^{1/2} \Rightarrow \frac{dy}{dt} = -\frac{\left(\frac{1}{2}\right)t^{-1/2}}{1+\left(t^{1/2}\right)^2} = \frac{-1}{2\sqrt{t}(1+t)}$

60. $y = \cot^{-1}\sqrt{t-1} = \cot^{-1}(t-1)^{1/2} \Rightarrow \frac{dy}{dt} = -\frac{\left(\frac{1}{2}\right)(t-1)^{-1/2}}{1+\left[(t-1)^{1/2}\right]^2} = \frac{-1}{2\sqrt{t-1}(1+t-1)} = \frac{-1}{2t\sqrt{t-1}}$

61. $y = \ln\left(\tan^{-1} x\right) \Rightarrow \frac{dy}{dx} = \frac{\left(\frac{1}{1+x^2}\right)}{\tan^{-1} x} = \frac{1}{\left(\tan^{-1} x\right)\left(1+x^2\right)}$

62. $y = \tan^{-1}(\ln x) \Rightarrow \frac{dy}{dx} = \frac{\left(\frac{1}{x}\right)}{1+(\ln x)^2} = \frac{1}{x\left[1+(\ln x)^2\right]}$

63. $y = \csc^{-1}\left(e^t\right) \Rightarrow \frac{dy}{dt} = -\frac{e^t}{\left|e^t\right|\sqrt{\left(e^t\right)^2 - 1}} = \frac{-1}{\sqrt{e^{2t}-1}}$

64. $y = \cos^{-1}\left(e^{-t}\right) \Rightarrow \frac{dy}{dt} = -\frac{-e^{-t}}{\sqrt{1-\left(e^{-t}\right)^2}} = \frac{e^{-t}}{\sqrt{1-e^{-2t}}}$

65. $y = s\sqrt{1-s^2} + \cos^{-1} s = s\left(1-s^2\right)^{1/2} + \cos^{-1} s \Rightarrow \frac{dy}{ds} = \left(1-s^2\right)^{1/2} + s\left(\frac{1}{2}\right)\left(1-s^2\right)^{-1/2}(-2s) - \frac{1}{\sqrt{1-s^2}}$
$= \sqrt{1-s^2} - \frac{s^2}{\sqrt{1-s^2}} - \frac{1}{\sqrt{1-s^2}} = \sqrt{1-s^2} - \frac{s^2+1}{\sqrt{1-s^2}} = \frac{1-s^2-s^2-1}{\sqrt{1-s^2}} = \frac{-2s^2}{\sqrt{1-s^2}}$

66. $y = \sqrt{s^2-1} - \sec^{-1} s = \left(s^2-1\right)^{1/2} - \sec^{-1} s \Rightarrow \frac{dy}{ds} = \left(\frac{1}{2}\right)\left(s^2-1\right)^{-1/2}(2s) - \frac{1}{|s|\sqrt{s^2-1}} = \frac{s}{\sqrt{s^2-1}} - \frac{1}{|s|\sqrt{s^2-1}}$
$= \frac{s|s|-1}{|s|\sqrt{s^2-1}}$

67. $y = \tan^{-1}\sqrt{x^2-1} + \csc^{-1} x = \tan^{-1}\left(x^2-1\right)^{1/2} + \csc^{-1} x \Rightarrow \frac{dy}{dx} = \frac{\left(\frac{1}{2}\right)\left(x^2-1\right)^{-1/2}(2x)}{1+\left[\left(x^2-1\right)^{1/2}\right]^2} - \frac{1}{|x|\sqrt{x^2-1}}$
$= \frac{1}{x\sqrt{x^2-1}} - \frac{1}{|x|\sqrt{x^2-1}} = 0$, for $x > 1$

68. $y = \cot^{-1}\left(\frac{1}{x}\right) - \tan^{-1} x = \frac{\pi}{2} - \tan^{-1}\left(x^{-1}\right) - \tan^{-1} x \Rightarrow \frac{dy}{dx} = 0 - \frac{-x^{-2}}{1+\left(x^{-1}\right)^2} - \frac{1}{1+x^2} = \frac{1}{x^2+1} - \frac{1}{1+x^2} = 0$

69. $y = x\sin^{-1} x + \sqrt{1-x^2} = x\sin^{-1} x + \left(1-x^2\right)^{1/2} \Rightarrow \frac{dy}{dx} = \sin^{-1} x + x\left(\frac{1}{\sqrt{1-x^2}}\right) + \left(\frac{1}{2}\right)\left(1-x^2\right)^{-1/2}(-2x)$
$= \sin^{-1} x + \frac{x}{\sqrt{1-x^2}} - \frac{x}{\sqrt{1-x^2}} = \sin^{-1} x$

70. $y = \ln\left(x^2+4\right) - x\tan^{-1}\left(\frac{x}{2}\right) \Rightarrow \frac{dy}{dx} = \frac{2x}{x^2+4} - \tan^{-1}\left(\frac{x}{2}\right) - x\left[\frac{\left(\frac{1}{2}\right)}{1+\left(\frac{x}{2}\right)^2}\right] = \frac{2x}{x^2+4} - \tan^{-1}\left(\frac{x}{2}\right) - \frac{2x}{4+x^2}$
$= -\tan^{-1}\left(\frac{x}{2}\right)$

71. $\int \frac{1}{\sqrt{9-x^2}}\,dx = \sin^{-1}\left(\frac{x}{3}\right) + C$

72. $\int \frac{1}{\sqrt{1-4x^2}}\,dx = \frac{1}{2}\int \frac{2}{\sqrt{1-(2x)^2}}\,dx = \frac{1}{2}\int \frac{du}{\sqrt{1-u^2}}$, where $u = 2x$ and $du = 2\,dx$
$= \frac{1}{2}\sin^{-1} u + C = \frac{1}{2}\sin^{-1}(2x) + C$

73. $\int \frac{1}{17+x^2}\,dx = \int \frac{1}{\left(\sqrt{17}\right)^2+x^2}\,dx = \frac{1}{\sqrt{17}}\tan^{-1}\frac{x}{\sqrt{17}} + C$

74. $\int \frac{1}{9+3x^2}\,dx = \frac{1}{3}\int \frac{1}{\left(\sqrt{3}\right)^2+x^2}\,dx = \frac{1}{3\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C = \frac{\sqrt{3}}{9}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$

75. $\int \frac{dx}{x\sqrt{25x^2-2}} = \int \frac{du}{u\sqrt{u^2-2}}$, where $u = 5x$ and $du = 5\,dx$
$= \frac{1}{\sqrt{2}}\sec^{-1}\left|\frac{u}{\sqrt{2}}\right| + C = \frac{1}{\sqrt{2}}\sec^{-1}\left|\frac{5x}{\sqrt{2}}\right| + C$

76. $\int \frac{dx}{x\sqrt{5x^2-4}} = \int \frac{du}{u\sqrt{u^2-4}}$, where $u = \sqrt{5}x$ and $du = \sqrt{5}\,dx$
$= \frac{1}{2}\sec^{-1}\left|\frac{u}{2}\right| + C = \frac{1}{2}\sec^{-1}\left|\frac{\sqrt{5}x}{2}\right| + C$

77. $\int_0^1 \frac{4\,ds}{\sqrt{4-s^2}} = \left[4\sin^{-1}\frac{s}{2}\right]_0^1 = 4\left(\sin^{-1}\frac{1}{2} - \sin^{-1} 0\right) = 4\left(\frac{\pi}{6} - 0\right) = \frac{2\pi}{3}$

78. $\int_0^{3\sqrt{2}/4} \frac{ds}{\sqrt{9-4s^2}} = \frac{1}{2}\int_0^{3\sqrt{2}/2} \frac{du}{\sqrt{9-u^2}}$, where $u = 2s$ and $du = 2\,ds$; $s = 0 \Rightarrow u = 0$, $s = \frac{3\sqrt{2}}{4} \Rightarrow u = \frac{3\sqrt{2}}{2}$
$= \left[\frac{1}{2}\sin^{-1}\frac{u}{3}\right]_0^{3\sqrt{2}/2} = \frac{1}{2}\left(\sin^{-1}\frac{\sqrt{2}}{2} - \sin^{-1} 0\right) = \frac{1}{2}\left(\frac{\pi}{4} - 0\right) = \frac{\pi}{8}$

79. $\int_0^2 \frac{dt}{8+2t^2} = \frac{1}{\sqrt{2}}\int_0^{2\sqrt{2}} \frac{du}{8+u^2}$, where $u = \sqrt{2}t$ and $du = \sqrt{2}\,dt$; $t = 0 \Rightarrow u = 0$, $t = 2 \Rightarrow u = 2\sqrt{2}$
$= \left[\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{8}}\tan^{-1}\frac{u}{\sqrt{8}}\right]_0^{2\sqrt{2}} = \frac{1}{4}\left(\tan^{-1}\frac{2\sqrt{2}}{\sqrt{8}} - \tan^{-1} 0\right) = \frac{1}{4}\left(\tan^{-1} 1 - \tan^{-1} 0\right) = \frac{1}{4}\left(\frac{\pi}{4} - 0\right) = \frac{\pi}{16}$

80. $\int_{-2}^2 \frac{dt}{4+3t^2} = \frac{1}{\sqrt{3}}\int_{-2\sqrt{3}}^{2\sqrt{3}} \frac{du}{4+u^2}$, where $u = \sqrt{3}t$ and $du = \sqrt{3}\,dt$; $t = -2 \Rightarrow u = -2\sqrt{3}$, $t = 2 \Rightarrow u = 2\sqrt{3}$
$= \left[\frac{1}{\sqrt{3}} \cdot \frac{1}{2}\tan^{-1}\frac{u}{2}\right]_{-2\sqrt{3}}^{2\sqrt{3}} = \frac{1}{2\sqrt{3}}\left[\tan^{-1}\sqrt{3} - \tan^{-1}\left(-\sqrt{3}\right)\right] = \frac{1}{2\sqrt{3}}\left[\frac{\pi}{3} - \left(-\frac{\pi}{3}\right)\right] = \frac{\pi}{3\sqrt{3}}$

81. $\int_{-1}^{-\sqrt{2}/2} \frac{dy}{y\sqrt{4y^2-1}} = \int_{-2}^{-\sqrt{2}} \frac{du}{u\sqrt{u^2-1}}$, where $u = 2y$ and $du = 2\,dy$; $y = -1 \Rightarrow u = -2$, $y = -\frac{\sqrt{2}}{2} \Rightarrow u = -\sqrt{2}$
$= \left[\sec^{-1}|u|\right]_{-2}^{-\sqrt{2}} = \sec^{-1}\left|-\sqrt{2}\right| - \sec^{-1}|-2| = \frac{\pi}{4} - \frac{\pi}{3} = -\frac{\pi}{12}$

82. $\int_{-2/3}^{-\sqrt{2}/3} \frac{dy}{y\sqrt{9y^2-1}} = \int_{-2}^{-\sqrt{2}} \frac{du}{u\sqrt{u^2-1}}$, where $u = 3y$ and $du = 3\,dy$; $y = -\frac{2}{3} \Rightarrow u = -2$, $y = -\frac{\sqrt{2}}{3} \Rightarrow u = -\sqrt{2}$
$= \left[\sec^{-1}|u|\right]_{-2}^{-\sqrt{2}} = \sec^{-1}\left|-\sqrt{2}\right| - \sec^{-1}|-2| = \frac{\pi}{4} - \frac{\pi}{3} = -\frac{\pi}{12}$

83. $\int \frac{3\,dr}{\sqrt{1-4(r-1)^2}} = \frac{3}{2} \int \frac{du}{\sqrt{1-u^2}}$, where $u = 2(r-1)$ and $du = 2\,dr$
$= \frac{3}{2} \sin^{-1} u + C = \frac{3}{2} \sin^{-1} 2(r-1) + C$

84. $\int \frac{6\,dr}{\sqrt{4-(r+1)^2}} = 6 \int \frac{du}{\sqrt{4-u^2}}$, where $u = r+1$ and $du = dr$
$= 6 \sin^{-1} \frac{u}{2} + C = 6 \sin^{-1} \left(\frac{r+1}{2}\right) + C$

85. $\int \frac{dx}{2+(x-1)^2} = \int \frac{du}{2+u^2}$, where $u = x-1$ and $du = dx$
$= \frac{1}{\sqrt{2}} \tan^{-1} \frac{u}{\sqrt{2}} + C = \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x-1}{\sqrt{2}}\right) + C$

86. $\int \frac{dx}{1+(3x+1)^2} = \frac{1}{3} \int \frac{du}{1+u^2}$, where $u = 3x+1$ and $du = 3\,dx$
$= \frac{1}{3} \tan^{-1} u + C = \frac{1}{3} \tan^{-1} (3x+1) + C$

87. $\int \frac{dx}{(2x-1)\sqrt{(2x-1)^2-4}} = \frac{1}{2} \int \frac{du}{u\sqrt{u^2-4}}$, where $u = 2x-1$ and $du = 2\,dx$
$= \frac{1}{2} \cdot \frac{1}{2} \sec^{-1} \left|\frac{u}{2}\right| + C = \frac{1}{4} \sec^{-1} \left|\frac{2x-1}{2}\right| + C$

88. $\int \frac{dx}{(x+3)\sqrt{(x+3)^2-25}} = \int \frac{du}{u\sqrt{u^2-25}}$, where $u = x+3$ and $du = dx$
$= \frac{1}{5} \sec^{-1} \left|\frac{u}{5}\right| + C = \frac{1}{5} \sec^{-1} \left|\frac{x+3}{5}\right| + C$

89. $\int_{-\pi/2}^{\pi/2} \frac{2\cos\theta\,d\theta}{1+(\sin\theta)^2} = 2 \int_{-1}^{1} \frac{du}{1+u^2}$, where $u = \sin\theta$ and $du = \cos\theta\,d\theta$; $\theta = -\frac{\pi}{2} \Rightarrow u = -1$, $\theta = \frac{\pi}{2} \Rightarrow u = 1$
$= \left[2\tan^{-1} u\right]_{-1}^{1} = 2\left(\tan^{-1} 1 - \tan^{-1}(-1)\right) = 2\left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right] = \pi$

90. $\int_{\pi/6}^{\pi/4} \frac{\csc^2 x\,dx}{1+(\cot x)^2} = -\int_{\sqrt{3}}^{1} \frac{du}{1+u^2}$, where $u = \cot x$ and $du = -\csc^2 x\,dx$; $x = \frac{\pi}{6} \Rightarrow u = \sqrt{3}$, $x = \frac{\pi}{4} \Rightarrow u = 1$
$= \left[-\tan^{-1} u\right]_{\sqrt{3}}^{1} = -\tan^{-1} 1 + \tan^{-1} \sqrt{3} = -\frac{\pi}{4} + \frac{\pi}{3} = \frac{\pi}{12}$

91. $\int_{0}^{\ln\sqrt{3}} \frac{e^x\,dx}{1+e^{2x}} = \int_{1}^{\sqrt{3}} \frac{du}{1+u^2}$, where $u = e^x$ and $du = e^x\,dx$; $x = 0 \Rightarrow u = 1$, $x = \ln\sqrt{3} \Rightarrow u = \sqrt{3}$
$= \left[\tan^{-1} u\right]_{1}^{\sqrt{3}} = \tan^{-1} \sqrt{3} - \tan^{-1} 1 = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$

92. $\int_{1}^{e^{\pi/4}} \frac{4\,dt}{t(1+\ln^2 t)} = 4 \int_{0}^{\pi/4} \frac{du}{1+u^2}$, where $u = \ln t$ and $du = \frac{1}{t}\,dt$; $t = 1 \Rightarrow u = 0$, $t = e^{\pi/4} \Rightarrow u = \frac{\pi}{4}$
$= \left[4\tan^{-1} u\right]_{0}^{\pi/4} = 4\left(\tan^{-1} \frac{\pi}{4} - \tan^{-1} 0\right) = 4\tan^{-1} \frac{\pi}{4}$

93. $\int \frac{y\,dy}{\sqrt{1-y^4}} = \frac{1}{2} \int \frac{du}{\sqrt{1-u^2}}$, where $u = y^2$ and $du = 2y\,dy$
$= \frac{1}{2} \sin^{-1} u + C = \frac{1}{2} \sin^{-1} y^2 + C$

94. $\int \frac{\sec^2 y\,dy}{\sqrt{1-\tan^2 y}} = \int \frac{du}{\sqrt{1-u^2}}$, where $u = \tan y$ and $du = \sec^2 y\,dy$
$= \sin^{-1} u + C = \sin^{-1} (\tan y) + C$

J.

1. (a) x^2 (b) $\frac{x^3}{3}$ (c) $\frac{x^3}{3} - x^2 + x$

2. (a) $3x^2$ (b) $\frac{x^8}{8}$ (c) $\frac{x^8}{8} - 3x^2 + 8x$

3. (a) x^{-3} (b) $-\frac{x^{-2}}{3}$ (c) $-\frac{x^{-2}}{3} + x^2 + 3x$

4. (a) $-x^{-2}$ (b) $-\frac{x^{-4}}{4} + \frac{x^3}{3}$ (c) $\frac{x^{-2}}{2} + \frac{x^2}{2} - x$

5. (a) $\frac{-1}{x}$ (b) $\frac{-5}{x}$ (c) $2x + \frac{5}{x}$

6. (a) $\frac{1}{x^2}$ (b) $\frac{-1}{4x^2}$ (c) $\frac{x^4}{4} + \frac{1}{2x^2}$

7. (a) $\sqrt{x^3}$ (b) $\sqrt{x}$ (c) $\frac{2}{3}\sqrt{x^3} + 2\sqrt{x}$

8. (a) $x^{4/3}$ (b) $\frac{1}{2}x^{2/3}$ (c) $\frac{3}{4}x^{4/3} + \frac{3}{2}x^{2/3}$

9. (a) $x^{2/3}$ (b) $x^{1/3}$ (c) $x^{-1/3}$

10. (a) $x^{1/2}$ (b) $x^{-1/2}$ (c) $x^{-3/2}$

11. (a) $\cos(\pi x)$ (b) $-3\cos x$ (c) $\frac{-\cos(\pi x)}{\pi} + \cos(3x)$

12. (a) $\sin(\pi x)$ (b) $\sin\left(\frac{\pi x}{2}\right)$ (c) $\left(\frac{2}{\pi}\right)\sin\left(\frac{\pi x}{2}\right) + \pi \sin x$

13. (a) $\tan x$ (b) $2\tan\left(\frac{x}{3}\right)$ (c) $-\frac{2}{3}\tan\left(\frac{3x}{2}\right)$

14. (a) $-\cot x$ (b) $\cot\left(\frac{3x}{2}\right)$ (c) $x + 4\cot(2x)$

15. (a) $-\csc x$ (b) $\frac{1}{5}\csc(5x)$ (c) $2\csc\left(\frac{\pi x}{2}\right)$

16. (a) $\sec x$ (b) $\frac{4}{3}\sec(3x)$ (c) $\frac{2}{\pi}\sec\left(\frac{\pi x}{2}\right)$

17. $\int (x+1)\,dx = \frac{x^2}{2} + x + C$

18. $\int (5-6x)\,dx = 5x - 3x^2 + C$

19. $\int \left(3t^2 + \frac{t}{2}\right) dt = t^3 + \frac{t^2}{4} + C$

20. $\int \left(\frac{t^2}{2} + 4t^3\right) dt = \frac{t^3}{6} + t^4 + C$

21. $\int (2x^3 - 5x + 7)\, dx = \frac{1}{2}x^4 - \frac{5}{2}x^2 + 7x + C$

22. $\int (1 - x^2 - 3x^5)\, dx = x - \frac{1}{3}x^3 - \frac{1}{2}x^6 + C$

23. $\int \left(\frac{1}{x^2} - x^2 - \frac{1}{3}\right) dx = \int \left(x^{-2} - x^2 - \frac{1}{3}\right) dx = \frac{x^{-1}}{-1} - \frac{x^3}{3} - \frac{1}{3}x + C = -\frac{1}{x} - \frac{x^3}{3} - \frac{x}{3} + C$

24. $\int \left(\frac{1}{5} - \frac{2}{x^3} + 2x\right) dx = \int \left(\frac{1}{5} - 2x^{-3} + 2x\right) dx = \frac{1}{5}x - \left(\frac{2x^{-2}}{-2}\right) + \frac{2x^2}{2} + C = \frac{x}{5} + \frac{1}{x^2} + x^2 + C$

25. $\int x^{-1/3}\, dx = \frac{x^{2/3}}{\frac{2}{3}} + C = \frac{3}{2}x^{2/3} + C$

26. $\int x^{-5/4}\, dx = \frac{x^{-1/4}}{-\frac{1}{4}} + C = \frac{-4}{\sqrt[4]{x}} + C$

27. $\int \left(\sqrt{x} + \sqrt[3]{x}\right) dx = \int \left(x^{1/2} + x^{1/3}\right) dx = \frac{x^{3/2}}{\frac{3}{2}} + \frac{x^{4/3}}{\frac{4}{3}} + C = \frac{2}{3}x^{3/2} + \frac{3}{4}x^{4/3} + C$

28. $\int \left(\frac{\sqrt{x}}{2} + \frac{2}{\sqrt{x}}\right) dx = \int \left(\frac{1}{2}x^{1/2} + 2x^{-1/2}\right) dx = \frac{1}{2}\left(\frac{x^{3/2}}{\frac{3}{2}}\right) + 2\left(\frac{x^{1/2}}{\frac{1}{2}}\right) + C = \frac{1}{3}x^{3/2} + 4x^{1/2} + C$

29. $\int \left(8y - \frac{2}{y^{1/4}}\right) dy = \int \left(8y - 2y^{-1/4}\right) dy = \frac{8y^2}{2} - 2\left(\frac{y^{3/4}}{\frac{3}{4}}\right) + C = 4y^2 - \frac{8}{3}y^{3/4} + C$

30. $\int \left(\frac{1}{7} - \frac{1}{y^{5/4}}\right) dy = \int \left(\frac{1}{7} - y^{-5/4}\right) dy = \frac{1}{7}y - \left(\frac{y^{-1/4}}{-\frac{1}{4}}\right) + C = \frac{y}{7} + \frac{4}{y^{1/4}} + C$

31. $\int 2x\left(1 - x^{-3}\right) dx = \int \left(2x - 2x^{-2}\right) dx = \frac{2x^2}{2} - 2\left(\frac{x^{-1}}{-1}\right) + C = x^2 + \frac{2}{x} + C$

32. $\int x^{-3}(x + 1)\, dx = \int \left(x^{-2} + x^{-3}\right) dx = \frac{x^{-1}}{-1} + \left(\frac{x^{-2}}{-2}\right) + C = -\frac{1}{x} - \frac{1}{2x^2} + C$

33. $\int \frac{t\sqrt{t} + \sqrt{t}}{t^2}\, dt = \int \left(\frac{t^{3/2}}{t^2} + \frac{t^{1/2}}{t^2}\right) dt = \int \left(t^{-1/2} + t^{-3/2}\right) dt = \frac{t^{1/2}}{\frac{1}{2}} + \left(\frac{t^{-1/2}}{-\frac{1}{2}}\right) + C = 2\sqrt{t} - \frac{2}{\sqrt{t}} + C$

34. $\int \frac{4 + \sqrt{t}}{t^3}\, dt = \int \left(\frac{4}{t^3} + \frac{t^{1/2}}{t^3}\right) dt = \int \left(4t^{-3} + t^{-5/2}\right) dt = 4\left(\frac{t^{-2}}{-2}\right) + \left(\frac{t^{-3/2}}{-\frac{3}{2}}\right) + C = -\frac{2}{t^2} - \frac{2}{3t^{3/2}} + C$

35. $\int -2\cos t\, dt = -2\sin t + C$

36. $\int -5\sin t\, dt = 5\cos t + C$

37. $\int 7\sin\frac{\theta}{3}\, d\theta = -21\cos\frac{\theta}{3} + C$

38. $\int 3\cos 5\theta\, d\theta = \frac{3}{5}\sin 5\theta + C$

39. $\int -3\csc^2 x\, dx = 3\cot x + C$

40. $\int -\frac{\sec^2 x}{3}\, dx = -\frac{\tan x}{3} + C$

41. $\int \frac{\csc\theta\cot\theta}{2}\, d\theta = -\frac{1}{2}\csc\theta + C$

42. $\int \frac{2}{5}\sec\theta\tan\theta\, d\theta = \frac{2}{5}\sec\theta + C$

43. $\int (4\sec x\tan x - 2\sec^2 x)\, dx = 4\sec x - 2\tan x + C$

44. $\int \frac{1}{2}(\csc^2 x - \csc x\cot x)\, dx = -\frac{1}{2}\cot x + \frac{1}{2}\csc x + C$

45. $\int (\sin 2x - \csc^2 x)\, dx = -\frac{1}{2}\cos 2x + \cot x + C$

46. $\int (2\cos 2x - 3\sin 3x)\, dx = \sin 2x + \cos 3x + C$

47. $\int \frac{1 + \cos 4t}{2}\, dt = \int \left(\frac{1}{2} + \frac{1}{2}\cos 4t\right) dt = \frac{1}{2}t + \frac{1}{2}\left(\frac{\sin 4t}{4}\right) + C = \frac{t}{2} + \frac{\sin 4t}{8} + C$

48. $\int \frac{1 - \cos 6t}{2}\, dt = \int \left(\frac{1}{2} - \frac{1}{2}\cos 6t\right) dt = \frac{1}{2}t - \frac{1}{2}\left(\frac{\sin 6t}{6}\right) + C = \frac{t}{2} - \frac{\sin 6t}{12} + C$

49. $\int (1+\tan^2\theta)\, d\theta = \int \sec^2\theta\, d\theta = \tan\theta + C$

50. $\int (2+\tan^2\theta)\, d\theta = \int (1+1+\tan^2\theta)\, d\theta = \int (1+\sec^2\theta)\, d\theta = \theta + \tan\theta + C$

51. $\int \cot^2 x\, dx = \int (\csc^2 x - 1)\, dx = -\cot x - x + C$

52. $\int (1-\cot^2 x)\, dx = \int (1-(\csc^2 x - 1))\, dx = \int (2-\csc^2 x)\, dx = 2x + \cot x + C$

53. $\int \cos\theta(\tan\theta + \sec\theta)\, d\theta = \int (\sin\theta + 1)\, d\theta = -\cos\theta + \theta + C$

54. $\int \frac{\csc\theta}{\csc\theta - \sin\theta}\, d\theta = \int \left(\frac{\csc\theta}{\csc\theta - \sin\theta}\right)\left(\frac{\sin\theta}{\sin\theta}\right) d\theta = \int \frac{1}{1-\sin^2\theta}\, d\theta = \int \frac{1}{\cos^2\theta}\, d\theta = \int \sec^2\theta\, d\theta = \tan\theta + C$

K.

1. $\int_{-2}^{0} (2x+5)\, dx = \left[x^2+5x\right]_{-2}^{0} = (0^2+5(0)) - ((-2)^2+5(-2)) = 6$

2. $\int_{-3}^{4} \left(5 - \frac{x}{2}\right) dx = \left[5x - \frac{x^2}{4}\right]_{-3}^{4} = \left(5(4) - \frac{4^2}{4}\right) - \left(5(-3) - \frac{(-3)^2}{4}\right) = \frac{133}{4}$

3. $\int_{0}^{4} \left(3x - \frac{x^3}{4}\right) dx = \left[\frac{3x^2}{2} - \frac{x^4}{16}\right]_{0}^{4} = \left(\frac{3(4)^2}{2} - \frac{4^4}{16}\right) - \left(\frac{3(0)^2}{2} - \frac{(0)^4}{16}\right) = 8$

4. $\int_{-2}^{2} (x^3 - 2x + 3)\, dx = \left[\frac{x^4}{4} - x^2 + 3x\right]_{-2}^{2} = \left(\frac{2^4}{4} - 2^2 + 3(2)\right) - \left(\frac{(-2)^4}{4} - (-2)^2 + 3(-2)\right) = 12$

5. $\int_{0}^{1} (x^2 + \sqrt{x})\, dx = \left[\frac{x^3}{3} + \frac{2}{3}x^{3/2}\right]_{0}^{1} = \left(\frac{1}{3} + \frac{2}{3}\right) - 0 = 1$

6. $\int_{0}^{5} x^{3/2}\, dx = \left[\frac{2}{5}x^{5/2}\right]_{0}^{5} = \frac{2}{5}(5)^{5/2} - 0 = 2(5)^{3/2} = 10\sqrt{5}$

7. $\int_{1}^{32} x^{-6/5}\, dx = \left[-5x^{-1/5}\right]_{1}^{32} = \left(-\frac{5}{2}\right) - (-5) = \frac{5}{2}$

8. $\int_{-2}^{-1} \frac{2}{x^2}\, dx = \int_{-2}^{-1} 2x^{-2}\, dx = \left[-2x^{-1}\right]_{-2}^{-1} = \left(\frac{-2}{-1}\right) - \left(\frac{-2}{-2}\right) = 1$

9. $\int_0^{\pi} \sin x \, dx = [-\cos x]_0^{\pi} = (-\cos \pi) - (-\cos 0) = -(-1) - (-1) = 2$

10. $\int_0^{\pi} (1 + \cos x) \, dx = [x + \sin x]_0^{\pi} = (\pi + \sin \pi) - (0 + \sin 0) = \pi$

11. $\int_0^{\pi/3} 2 \sec^2 x \, dx = [2 \tan x]_0^{\pi/3} = \left(2 \tan\left(\frac{\pi}{3}\right)\right) - (2 \tan 0) = 2\sqrt{3} - 0 = 2\sqrt{3}$

12. $\int_{\pi/6}^{5\pi/6} \csc^2 x \, dx = [-\cot x]_{\pi/6}^{5\pi/6} = \left(-\cot\left(\frac{5\pi}{6}\right)\right) - \left(-\cot\left(\frac{\pi}{6}\right)\right) = -\left(-\sqrt{3}\right) - \left(-\sqrt{3}\right) = 2\sqrt{3}$

13. $\int_{\pi/4}^{3\pi/4} \csc \theta \cot \theta \, d\theta = [-\csc \theta]_{\pi/4}^{3\pi/4} = \left(-\csc\left(\frac{3\pi}{4}\right)\right) - \left(-\csc\left(\frac{\pi}{4}\right)\right) = -\sqrt{2} - \left(-\sqrt{2}\right) = 0$

14. $\int_0^{\pi/3} 4 \sec u \tan u \, du = [4 \sec u]_0^{\pi/3} = 4 \sec\left(\frac{\pi}{3}\right) - 4 \sec 0 = 4(2) - 4(1) = 4$

15. $\int_{\pi/2}^{0} \frac{1 + \cos 2t}{2} \, dt = \int_{\pi/2}^{0} \left(\frac{1}{2} + \frac{1}{2} \cos 2t\right) dt = \left[\frac{1}{2} t + \frac{1}{4} \sin 2t\right]_{\pi/2}^{0} = \left(\frac{1}{2}(0) + \frac{1}{4} \sin 2(0)\right) - \left(\frac{1}{2}\left(\frac{\pi}{2}\right) + \frac{1}{4} \sin 2\left(\frac{\pi}{2}\right)\right)$
$= -\frac{\pi}{4}$

16. $\int_{-\pi/3}^{\pi/3} \frac{1 - \cos 2t}{2} \, dt = \int_{-\pi/3}^{\pi/3} \left(\frac{1}{2} - \frac{1}{2} \cos 2t\right) dt = \left[\frac{1}{2} t - \frac{1}{4} \sin 2t\right]_{-\pi/3}^{\pi/3}$
$= \left(\frac{1}{2}\left(\frac{\pi}{3}\right) - \frac{1}{4} \sin 2\left(\frac{\pi}{3}\right)\right) - \left(\frac{1}{2}\left(-\frac{\pi}{3}\right) - \frac{1}{4} \sin 2\left(-\frac{\pi}{3}\right)\right) = \frac{\pi}{6} - \frac{1}{4} \sin \frac{2\pi}{3} + \frac{\pi}{6} + \frac{1}{4} \sin\left(\frac{-2\pi}{3}\right) = \frac{\pi}{3} - \frac{\sqrt{3}}{4}$

17. $\int_{-\pi/2}^{\pi/2} (8y^2 + \sin y) \, dy = \left[\frac{8y^3}{3} - \cos y\right]_{-\pi/2}^{\pi/2} = \left(\frac{8\left(\frac{\pi}{2}\right)^3}{3} - \cos \frac{\pi}{2}\right) - \left(\frac{8\left(-\frac{\pi}{2}\right)^3}{3} - \cos\left(-\frac{\pi}{2}\right)\right) = \frac{2\pi^3}{3}$

18. $\int_{-\pi/3}^{-\pi/4} \left(4 \sec^2 t + \frac{\pi}{t^2}\right) dt = \int_{-\pi/3}^{-\pi/4} \left(4 \sec^2 t + \pi t^{-2}\right) dt = \left[4 \tan t - \frac{\pi}{t}\right]_{-\pi/3}^{-\pi/4}$
$= \left(4 \tan\left(-\frac{\pi}{4}\right) - \frac{\pi}{\left(-\frac{\pi}{4}\right)}\right) - \left(4 \tan\left(\frac{\pi}{3}\right) - \frac{\pi}{\left(-\frac{\pi}{3}\right)}\right) = (4(-1) + 4) - \left(4\left(-\sqrt{3}\right) + 3\right) = 4\sqrt{3} - 3$

19. $\int_1^{-1} (r + 1)^2 \, dr = \int_1^{-1} (r^2 + 2r + 1) \, dr = \left[\frac{r^3}{3} + r^2 + r\right]_1^{-1} = \left(\frac{(-1)^3}{3} + (-1)^2 + (-1)\right) - \left(\frac{1^3}{3} + 1^2 + 1\right) = -\frac{8}{3}$

20. $\int_{-\sqrt{3}}^{\sqrt{3}} (t + 1)(t^2 + 4) \, dt = \int_{-\sqrt{3}}^{\sqrt{3}} (t^3 + t^2 + 4t + 4) \, dt = \left[\frac{t^4}{4} + \frac{t^3}{3} + 2t^2 + 4t\right]_{-\sqrt{3}}^{\sqrt{3}}$
$= \left(\frac{\left(\sqrt{3}\right)^4}{4} + \frac{\left(\sqrt{3}\right)^3}{3} + 2\left(\sqrt{3}\right)^2 + 4\sqrt{3}\right) - \left(\frac{\left(-\sqrt{3}\right)^4}{4} + \frac{\left(-\sqrt{3}\right)^3}{3} + 2\left(-\sqrt{3}\right)^2 + 4\left(-\sqrt{3}\right)\right) = 10\sqrt{3}$

21. $\int_{\sqrt{2}}^{1} \left(\frac{u^7}{2} - \frac{1}{u^5}\right) du = \int_{\sqrt{2}}^{1} \left(\frac{u^7}{2} - u^{-5}\right) du = \left[\frac{u^8}{16} + \frac{1}{4u^4}\right]_{\sqrt{2}}^{1} = \left(\frac{1^8}{16} + \frac{1}{4(1)^4}\right) - \left(\frac{\left(\sqrt{2}\right)^8}{16} + \frac{1}{4\left(\sqrt{2}\right)^4}\right) = -\frac{3}{4}$

22. $\int_{1/2}^{1} \left(\frac{1}{v^3} - \frac{1}{v^4}\right) dv = \int_{1/2}^{1} (v^{-3} - v^{-4}) \, dv = \left[\frac{-1}{2v^2} + \frac{1}{3v^3}\right]_{1/2}^{1} = \left(\frac{-1}{2(1)^2} + \frac{1}{3(1)^3}\right) - \left(\frac{-1}{2\left(\frac{1}{2}\right)^2} + \frac{1}{3\left(\frac{1}{2}\right)^3}\right) = -\frac{5}{6}$

23. $\int_1^{\sqrt{2}} \frac{s^2 + \sqrt{s}}{s^2} \, ds = \int_1^{\sqrt{2}} \left(1 + s^{-3/2}\right) ds = \left[s - \frac{2}{\sqrt{s}}\right]_1^{\sqrt{2}} = \left(\sqrt{2} - \frac{2}{\sqrt{\sqrt{2}}}\right) - \left(1 - \frac{2}{\sqrt{1}}\right) = \sqrt{2} - 2^{3/4} + 1$
$= \sqrt{2} - \sqrt[4]{8} + 1$

24. $$\int_9^4 \frac{1-\sqrt{u}}{\sqrt{u}}\,du = \int_9^4 \left(u^{-1/2} - 1\right) du = \left[2\sqrt{u} - u\right]_9^4 = \left(2\sqrt{4} - 4\right) - \left(2\sqrt{9} - 9\right) = 3$$

25. $$\int_{-4}^{4} |x|\,dx = \int_{-4}^{0} |x|\,dx + \int_0^4 |x|\,dx = -\int_{-4}^{0} x\,dx + \int_0^4 x\,dx = \left[-\frac{x^2}{2}\right]_{-4}^{0} + \left[\frac{x^2}{2}\right]_0^4 = \left(-\frac{0^2}{2} + \frac{(-4)^2}{2}\right) + \left(\frac{4^2}{2} - \frac{0^2}{2}\right)$$
$$= 16$$

26. $$\int_0^{\pi} \frac{1}{2}\left(\cos x + |\cos x|\right) dx = \int_0^{\pi/2} \frac{1}{2}(\cos x + \cos x)\,dx + \int_{\pi/2}^{\pi} \frac{1}{2}(\cos x - \cos x)\,dx = \int_0^{\pi/2} \cos x\,dx = [\sin x]_0^{\pi/2}$$
$$- \sin \frac{\pi}{2} - \sin 0 - 1$$