## Solving problem

## SAMPLE PROBLEMS

4. For the difference amplifier circuit shown, determine the output voltage at terminal A.

(A) -18.13 V
(B) -6.07 V
(C) 6.07 V
(D) 15.45 V

## Solution:

By voltage division,
$v_{\text {in+ }}=25 V\left(\frac{3 \Omega}{5 \Omega+\Omega}\right)=9.375 \mathrm{~V}$
By the virtual short circuit between the input terminals, $\mathrm{v}_{\text {in }}=9.375 \mathrm{~V}$
Using Ohm's law, the current through the $15 \Omega$ resistor is
$I_{15}=\left(\frac{30 \mathrm{~V}-9.375 \mathrm{~V}}{15 \Omega}\right)=1.375 \mathrm{~V}$
The input impedance is infinite; therefore, $\mathrm{I}_{\mathrm{in}}=0$ and $\mathrm{I}_{15}=\mathrm{I}_{20}$.
Use Kirchoff's voltage law to find the output voltage at A.
$\mathrm{v}_{\mathrm{A}}=\mathrm{v}_{\text {in }}-20 \mathrm{I}_{20}=9.375 \mathrm{~V}-(20 \Omega)(1.375 \mathrm{~A})=-18.125 \mathrm{~V}$
Answer is A.

Problems 2 and 3 refer to the following figure.

2. What is the current, i ?
(A) -0.88 A
(B) -0.25 A
(C) 0 A
(D) 0.25 A

## Solution 2:

The input current in an op amp is so small that it is assumed to be zero.

Answer is C.
3. What is the output voltage, $\mathrm{v}_{\mathrm{o}}$ ?
(A) -7 V
(B) -6 V
(C) -1 V
(D) 6 V

## Solution 3:

This op amp circuit is a summing amplifier. Since $i=0$,
$i=\frac{v_{1}}{R_{1}}+\frac{v_{2}}{R_{2}}=\frac{3 \mathrm{~V}}{8 \Omega}+\frac{2 \mathrm{~V}}{4 \Omega}=0.875 \mathrm{~A}$
$\mathrm{V}_{\mathrm{o}}=-\mathrm{i}_{\mathrm{f}} \mathrm{R}_{\mathrm{f}}=-(0.875 \mathrm{~A})(8 \Omega)=-7 \mathrm{~V}$

Answer is A.
4. For the ideal op amp shown, what should be the value of resistor $\mathrm{R}_{\mathrm{f}}$ to obtain a gain of 5 ?

(A) $12.0 \mathrm{k} \Omega$
(B) $19.5 \mathrm{k} \Omega$
(C) $22.5 \mathrm{k} \Omega$
(D) $27.0 \mathrm{k} \Omega$

Solution:
Solution:
By voltage division, $v_{i n+}=v_{i(3 k \Omega)}^{(\underline{2 k \Omega})}=\begin{aligned} & 2 \\ & \overline{3}^{i}\end{aligned}$
By the virtual short circuit, $v=v_{i n-}=\frac{2}{3} v_{i}$
$i=\frac{v_{i n-}}{3 k \Omega}=\frac{\frac{2}{3} v_{i}}{3 k \Omega}$
Since the op amp draws no current, $\mathrm{i}_{\mathrm{f}}=\mathrm{i}$
$\frac{v_{o}-v_{i n-}}{R_{f}}=\frac{\frac{2}{3} v_{i}}{3 k \Omega}$

$\frac{\frac{13}{3}}{R_{f}}=\frac{\frac{2}{3}}{3 k \Omega}$
$\mathrm{R}_{\mathrm{f}}=19.5 \mathrm{k} \Omega$, Answer is B.
5. Evaluate the following amplifier circuit to determine the value of resistor $\mathrm{R}_{4}$ in order to obtain a voltage gain $\left(v_{0} / v_{i}\right)$ of -120 .

(A) $25 \Omega$
(B) $23 \mathrm{k} \Omega$
(C) $24 \mathrm{k} \Omega$
(D) $25 \mathrm{k} \Omega$

## Solution:


$\mathrm{v}_{\mathrm{in}+}$ is grounded, so $\mathrm{v}_{\mathrm{in} \text { - }}$ is also a virtual ground.
$\mathrm{v}_{\text {in }}=0$

Since $v_{\text {in }}=0, v_{i}=i_{1} R_{l}$ and $i_{l}=v_{i} / R_{1}$.

Since $v_{i n}=0, v_{x}=-i_{2} R_{2}$ and $i_{2}=-v_{x} / R_{2}$.

Similarly,
$v_{x}=-i_{3} R_{3}$
$v_{x}-v_{0}=-i_{4} R_{4}$

From Kirchhoff's current law,

$$
\begin{aligned}
& \mathrm{i}_{4}=\mathrm{i}_{2}+\mathrm{i}_{3} \\
& \frac{v_{x}-v_{o}}{R_{4}}=\frac{-v_{x}}{R_{2}}+\frac{-v_{x}}{R_{3}}
\end{aligned}
$$

Now, $\mathrm{v}_{\mathrm{o}}=-120 \mathrm{v}_{\mathrm{i}}$.

Also, $\mathrm{i}_{1}=\mathrm{i}_{2}$, so
$\frac{v_{i}}{R_{1}}=\frac{-v_{x}}{R_{2}}$
$v=-{\left(R_{2}\right)}_{v}$
${ }^{x} \quad\left(\overline{R_{1}}\right)^{i}$
$\left.\frac{-\left(R_{2}\right)_{v}-(-120 v)}{R_{4}}\right)_{i}^{\left(R_{2}\right)} v_{i}\left(\underline{R_{2}}\right)_{v_{i}}$
${ }_{120}\left(R_{1}\right)_{-1}$
$\frac{\left.\overline{R_{2}}\right|^{\prime}}{R_{4}}=\frac{1}{R_{2}}+\frac{1}{R_{3}}=\frac{R_{2}+R_{3}}{R_{2} R_{3}}$
$R_{4}=\frac{120\left(R_{1}\right)-\frac{\left(1 \times 10^{6} \Omega\right)-1}{R_{2}^{2}+R_{3}}}{R_{2} R_{3}}=\frac{\left.1205 \times 10^{5} \Omega\right)-1}{\frac{5 \times 10^{5} \Omega+100 \Omega}{\left(5 \times 10^{5} \Omega\right)(100 \Omega)}}=2.39 \times 10^{4} \Omega(24 \mathrm{k} \Omega)$

Answer is C.
13. For the circuit shown below:

(a) If $\mathrm{R}_{\mathrm{f}}=1 \mathrm{M} \Omega$ and $\mathrm{R}_{\mathrm{i}}=50 \Omega$, what is the voltage gain?

There are two ways to solve any problem involving an op amp. The first way is to use the formulas given in the Reference Handbook. Explicitly, $v_{o}=-\frac{R_{2}}{R_{l}} v_{a}+\left(1+\frac{R_{2}}{R_{l}}\right) v_{b}$ where $v_{a}$ is the input to the inverting terminal and $v_{b}$ is the input to the non-inverting terminal. In this case, there is no input to the non-inverting input and $v_{b}=0$. The formula reduces to the simple result $v_{o}=-\frac{R_{2}}{R_{2}} v_{a}$. Using the given circuit values, we get $v_{o}=-\frac{R_{l}{ }^{\prime}{ }^{a} v^{a}}{R_{I}{ }^{a}}=-\frac{R_{f}}{R_{i}} v_{i n}=-\frac{l M \Omega}{50 \Omega} v_{i n}=-20,000 v_{i n}$. The voltage gain is then $-20,000$. Note that any value over 100 is impractical for any real amplifier.

A more general way of solving any op amp circuit is to note that an ideal (and most real) op amps must satisify the virtual short assumption, i.e. that $\mathrm{V}_{+}=\mathrm{V}_{\text {. }}$. Using this assumption and KCL at an input node is adequate to solve most any op amp problem. In this case, KCl at the inverting input gives $+\frac{V_{\text {in }}}{R_{i}}-\frac{O-V_{\text {out }}}{R_{f}}=0$. Rearranging, $\frac{V_{\text {in }}}{R_{i}}+\frac{V_{\text {out }}}{R_{f}}=0$ and, solving for the voltage gain, $\frac{V_{\text {out }}}{V_{\text {in }}}=-\frac{R_{f}}{R_{i}}=-\frac{10^{6}}{50}=-20,000$ just as before
(b) If $\mathrm{V}_{\text {in }}=0.1$ volts, what is $\mathrm{V}_{\text {out }}$ ?

This is a continuation of (a). Using our voltage gain from (a) we get
$v_{o}=-20,000 v_{\text {in }}=-20,000(0.1$ volts $)=-2000$ volts
As mentioned previously, this is a ridiculous value for an output.
14. For the circuit shown below, $V_{1}=10 \sin (200 t)$ and $V_{2}=15 \sin 200 t$. $)$ What is $\mathrm{V}_{\text {out }}$ ? The op amp is ideal with infinite gain.


ANSWER:
Any problem with a capacitor (or inductor) in it and sinusoidal voltages immediately indicates that phasors are required. This means that $V_{1}$ and $V_{2}$ should be represented as phasors, and $\mathrm{C}_{\mathrm{f}}$ should be replaced by an impedance. This problem is not solved very well with the formulas in the Reference Handbook. This circuit is most easily solved using the virtual short assumption ( $\mathrm{V}_{+}=\mathrm{V}_{-}$ ), and using KCL at the inverting input. Note that the grounding of $\mathrm{V}_{+}$then requires that $\mathrm{V}_{-}=0$.
This is also called the virtual short assumption.
$+\frac{V_{2}-0}{R_{2}}+\frac{V_{l}-0}{R_{i}}-\frac{0-V_{\text {out }}}{I_{j \omega C}}=0$.
Rationalizing this expression gives $+\frac{V_{2}}{R_{2}}+\frac{V_{l}}{R_{i}}+j \omega C V_{\text {out }}=0$.
Solving for $\mathrm{V}_{\text {out }}$ gives $V_{\text {out }}=-\frac{V_{2}}{j \omega C R_{2}}-\frac{V_{1}}{j \omega C R_{i}}$
It is important to recognize that all sine functions should always be converted to cosines for proper phase in the phasor expressions, i.e. sin $200 \mathrm{t}=\cos 200 t-9 \emptyset^{\circ} \xi 1 \angle-90^{\circ}=-j$

Using the circuit parameters given,

$$
\begin{aligned}
V_{\text {out }} & =-\frac{-j 15}{j(200)\left(2 \infty 10^{-6}\right)\left(0.5 \infty 10^{6}\right)}-\frac{-j 10}{(200)\left(2 \infty 10^{-6}\right)\left(0.75 \infty 10^{6}\right)} \\
& =\frac{15}{200}+\frac{10}{300}=\frac{3}{40}+\frac{1}{30}
\end{aligned}
$$

The answer is then $V_{\text {ou }}(t)=\left(\frac{3}{40}+\frac{1}{3} \cos 200 t\right)$

## Diodes

1. A germanium diode is operated at $20^{\circ} \mathrm{C}$. A reverse bias of -1.5 volts results in a current of $70 \mu \mathrm{~A}$. Assuming that the temperature remains constant:
(a) What is the saturation current?
The general formula for the diode is $I=I_{s}\left(\frac{q V}{\eta k T}-1\right)$. Solving for $I_{s}$ gives $I_{s}=\frac{I}{e^{\frac{q V}{\eta k T}-1}}$. Using
the given parameters and recognizing that $\eta \wedge 1$ for a germanium diode, we get

As a point of information this current is essentially constant throughout the reverse bias region of the diode. See the curve below for a typical diode.

(b) What is the current that flows for a forward bias of +0.2 volts?

We use a commonly known result here to simplify the math, i.e. that, at room temperature,
$\frac{q}{k T} \uparrow{ }^{40}{ }_{\text {poolts }}$. For this problem, using the $\mathrm{I}_{\mathrm{S}}$ from (a),
$I=I_{s}\left(e_{\eta}{ }^{q V} \overline{k T}-1\right) \wedge I_{s}\left(e^{\frac{V\left(40 V_{\text {oults }}\right)}{\eta}} \quad 1\right)$
$=(70 \mu A)\left(e^{\left.\frac{\left.+0.2 \text { volts } 400_{\text {olts }}\right)}{(1)}-1\right)=(0 \mu \mu A)\left(n^{+8}-1\right) \wedge 0.209 \text { amperes } .}\right.$
(c) What is the current that flows for a forward bias of +0.2 volts at $40^{\circ} \mathrm{C}$ ?
2. A voltage $V=20+52 \sin 60 t($ volt)s is applied to the circuit shown below. The diode characteristics are static forward resistance $r_{f}=120 \Omega$ and dynamic resistance $r_{p}=100 \Omega$. What is the voltage across the inductance?


This is a trick question since the $\mathrm{V}_{2}{ }^{+}$voltage ( +20 volts) $>V_{I}^{+}( \pm 5 \sqrt{2}$ volts max.) the diode is always forward biased and always conducting. Therefore, the diode serves no useful purpose. The model for the diode then becomes an ideal diode (which can be ignored) and a $100 \Omega$ dynamic resistance.

The circuit becomes that of a voltage divider. We can use superposition to determine the voltage across the inductor. Since the inductor appears as a short to the DC source the contribution from that source is zero. For the AC source we have a reactive voltage divider.

$$
\begin{aligned}
V_{L} & =\frac{\left(60^{\left.\mathrm{rad}_{\text {sec }}\right)}(1)\right.}{\left(60^{\left.\mathrm{rad}_{\text {sec }}\right)}(1)+100 \Omega\right.}(-j 5 \sqrt{2} \text { volts }) \\
& =\frac{j 60}{j 60+100}(-j 5 \sqrt{2} \text { volts })=(0.26+j 0.44)(-j 5 \sqrt{2} \text { volts }) \\
& =(+3.12-j 1.87) \text { volts }=3.64 \angle-31^{\circ}
\end{aligned}
$$

The total voltage across the inductor is then the sum of the ac and dc voltages or $3.64 \angle-31^{\circ}$ volts.
3. At $25^{\circ} \mathrm{C}$ a germanium diode shows a saturation current of $100 \mu \mathrm{~A}$.
(a) What current would you expect at $100^{\circ} \mathrm{C}$ when the diode becomes "useless"?

This is simply formula evaluation. The saturation current doubles every $10^{\circ} \mathrm{C}$. Therefore,
$\begin{aligned} & \underline{I_{s 2}} \\ & I_{s 1}\end{aligned}=2 \frac{\Delta T}{10^{\circ} \mathrm{C}}=2 \quad \frac{100^{\circ} \mathrm{C}-25^{\circ} \mathrm{C}}{10^{\circ} \mathrm{C}}=2^{7.5}=181$. This gives $I=181 I=181100 \mu \mathrm{~A}=18.1 \mathrm{~mA}$
(b) What current would you expect at $0^{\circ} \mathrm{C}$ ?

Using the same approach we have
$I_{s 2}=2\left(\frac{\Delta T}{10^{\circ} \mathrm{C}}\right)_{s l}=2\left(\frac{0^{\circ} \mathrm{C}-25^{\circ} \mathrm{C}}{10^{\circ} \mathrm{C}}\right) I_{s l}=2^{-2.5} I_{s l}=0.177(100 \mu \mathrm{~A})=17.7 \mu \mathrm{~A}$
(c) At $25^{\circ} \mathrm{C}$, what current is predicted for a voltage of -0.5 volts?

This is purely formula evaluation.

$$
\begin{aligned}
& I=I_{s}\left(e_{\eta_{\eta} k T}^{q V}-1\right) \wedge I_{s}\left(e^{\frac{V\left(40 p_{\text {oolt }}\right)}{\eta}} \quad 1\right) \\
& =(100 \mu A)\left(e^{\frac{-0.5 \text { volst } 40)}{I}}-1\right) \uparrow-100 \mu \mathrm{~A}
\end{aligned}
$$

4. The forward resistance of the diodes is $\mathrm{R}_{\mathrm{f}}=5 \mathrm{k} \Omega$.
(a) What current is expected in the load resistor $\mathrm{R}_{\mathrm{L}}=5 \mathrm{k} \Omega$ if $V=100 \sin (\omega t)$ volts?

This is nothing but Ohm's Law evaluation. The diode is conducting in the positive half cycle and we use the forward resistance of the diode.
$I=\frac{100 \sin (\omega t)}{5000 \Omega+5000 \Omega}=0.01 \sin (\omega t)$ volts
(b) Plot current and voltage versus $\omega \mathrm{t}$ for $0<\omega \mathrm{t}<2 \pi$.

These are pretty simple to generate.


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