بانكى پرسيار(Question Bank) لە بابەتى الاستدلال الاحصائى (كورسى يەكەم) سالى خويندنى 2022-2023

Q1// Let $X \sim poi(\theta)$ and $Y = 4X$ by using transformation technique, find the p.d.f. of Y.

Q2// Let X have the binomial p.d.f. . $X \sim Bin\,(3\,,\,2/3)$, where $Y = X^2$, by using one-to-one transformation, find the p.d.f. of Y.

Q3// Let X_1 and X_2 be two stochastically independent r.v.'s that have Poisson distribution with means θ_1 , θ_2 respectively, the j.p.d.f. of X_1 and X_2 is;

$$f(x_1, x_2) = \begin{cases} \dfrac{\theta_1^{x_1}\,\theta_2^{x_2}\,e^{-\theta_1 - \theta_2}}{x_1!\,x_2!} & , \quad x_1 = 0,1,2,3,..... \quad , \quad x_2 = 0,1,2,3,..... \\ 0 & o.w \end{cases}$$

Where $Y_1 = X_1 + X_2$, $Y_2 = X_2$. **Find:** the j.p.d.f. of Y_1 and Y_2 , $f_1(y_1)$ and $f_2(y_2)$

Q4// Let X_1 and X_2 have a joint p.d.f as follows;

$$f(x_1, x_2) = \begin{cases} \left(\frac{2}{3}\right)^{x_1 + x_2}\left(\frac{1}{3}\right)^{2 - x_1 - x_2} & , \quad (x_1, x_2) = (0,0), (0,1), (1,0), (1,1) \\ 0 & o.w \end{cases}$$

Find the joint p.d.f of $Y_1 = X_1 - X_2$ and $Y_2 = X_1 + X_2$ and the marginal p.d.f of Y_1 and Y_2.

Q5// Let X have the p.d.f.;

$$f(x) = \begin{cases} 1 & , \quad 0 < x < 1 \\ 0 & o.w \end{cases}$$, where Y = -2lnX, **find** the p.d.f. of Y.

Q6// Let X is a uniform random variable on the interval (-2, 2), find the p.d.f. of Y;

1. Y = 4X + 3. **2.** Y = |X|.

Q7// Let $X \sim \Gamma(r/2, \theta)$, $Y = \dfrac{2X}{\theta}$, find; the p.d.f. of Y.

Q8// Let X have the p.d.f.;

$$f(x) = \begin{cases} 2xe^{-x^2} & , \quad 0 < x < \infty \\ 0 & o.w \end{cases}$$, where $Y = X^2$, **find** the p.d.f. of Y.

Q9// Let X_1 and X_2 be two stochastically independent r.v.'s, which have gamma distribution, with parameters (α, θ) and (β, θ) respectively, and $Y_1 = X_1 + X_2$, $Y_2 = \dfrac{X_1}{X_1 + X_2}$, find the j.p.d.f. of Y_1 and Y_2, $f(y_1, y_2)$, $f(y_1)$ and $f(y_2)$.

Q10// let $X_1, X_2, \ldots, X_n$ be a random sample of size (n) rssn taken from C.U(0,1). let $Y_1 < Y_2 < \ldots < Y_n$ be the order statistics of this sample. **Find** the p.d.f. of Y_1 and Y_n, the j.p.d.f. of Y_1 and Y_n

Q11// let $X_1, X_2, \ldots, X_n$ be a rssn taken from Exp($1/\theta$), let $Y_1 < Y_2 < \ldots < Y_n$ be the order statistics of this sample. **Find** $g(y_2)$, $g(y_{n-1})$ and when ($n = 4$) Find $g(y_1, y_3)$.

Q12// In a random sample of size (n) taken from exponential dist^n Exp(θ). Show that;

1. $T_1 = \overline{X}$ is unbiased estimator for the parameter (θ).

2. $T_2 = \frac{n}{n+1}\overline{X}^2$ is unbiased estimator for the parameter (θ^2).

Q13// In a random sample of size (n) from normal dist^n N(θ, σ^2). Show that;

1) $S^2 = \frac{1}{n-1}\sum(X_i - \overline{X})^2$ is unbiased estimator for the parameter (σ^2).

2) Is $T = \overline{X}^2$ unbiased estimator for θ^2.

Q14// In a random sample of size (n). Is $T = \overline{X}$ unbiased estimator for $\phi(\theta) = \theta$ of;

1. Ber(θ). **2.** Poisson(θ).

Q15// In a rss(n) from uniform dist^n C.U(0, θ).

1) Is Y_n unbiased in limit estimator for θ; (Note: Y_n estimator θ).

2) Is $\overline{X}$ unbiased in limit estimator for θ.

3) Is $\overline{X}$ unbiased in limit estimator for $\theta/2$.

Q16// In a random sample of size (n) from normal dist^n N(θ, σ^2). Is

$S^2 = \frac{1}{n}\sum(X_i - \overline{X})^2$ unbiased estimator for the parameter (σ^2).

Q17// Let $X_1, X_2, \ldots, X_n$ be a rss$n$ from Poisson dist^n, show that $\hat{\theta} = \overline{X}$ is consistent estimator for θ.

Q18// Let $X_1, X_2, \ldots, X_n$ be a rss$n$ from normal dist^n N(θ, σ^2), show that $S^2 = \frac{1}{n}\sum(X_i - \overline{X})^2$ is consistent estimator for σ^2.

Q19// Show that $\hat{\theta} = Y_n$ is consistent estimator for θ from C.U(0 , θ), (by theorem).

Q20// In a rss$n$, show that $\overline{X}$ is consistent estimator for the parameter θ, from;

1) N(θ, σ^2). **2)** Geo(θ).

Q21// Let $X_1, \ldots, X_n$ be a rss$n$ from exponential dist^n Exp($1/\theta$). Find the F.I. of X.

Q22// Let $X_1, X_2, \ldots, X_n$ be a rss$n$ from Bernoulli dist^n Ber(θ). Show that $\hat{\theta} = \sum X_i$ is sufficient estimator for the parameter θ.

Q23// Show that $\overline{X}$ is sufficient estimator for the mean of $N(\theta, \sigma^2)$.

Q24// In a rss$n$ from Poisson dist^n Poi(θ), is $\sum X_i$ sufficient estimator for θ?

Q25// Let $X_1, X_2, \ldots, X_n$ be a rss$n$ from a dist^n with p.d.f.:

$$f(x;\theta) = e^{2\theta - x} \quad , \quad x \geq 2\theta$$

Show that Y_1 is sufficient estimator for the parameter θ.

Q26// Let $X_1, X_2, \ldots, X_n$ be a rss$n$ from Poisson dist^n Poi(θ), show that $\hat{\theta} = \sum X_i$ is sufficient estimator for θ?

Q27// Let $X_1, X_2, \ldots, X_n$ be a rss$n$. Is $\overline{X}$ sufficient estimator for θ? of; **1)** Exp($1/\theta$). **2)** N(θ, σ^2).

Q28// Let $X_1, X_2, \ldots, X_n$ be a rss$n$ from Bernoulli dist^n Ber(θ). Show that $\hat{\theta} = \sum X_i$ is sufficient estimator for the parameter θ.

Q29// Let $X_1, X_2, \ldots, X_n$ be a rss$n$ from Poisson dist^n Poi(θ), show that $\hat{\theta} = \sum X_i$ is sufficient estimator for θ?

Q30// from Exp($1/\theta$). Is $\sum_{i=1}^{n} X_i$ sufficient estimator for θ? (by factorization theorem).

Q31// From Beta($\theta, \beta = 1$), $f(x;\theta) = \theta x^{\theta - 1} \quad , \quad 0 < x < 1$

Is $\prod_{i=1}^{n} X_i$ sufficient estimator for θ? (By factorization theorem).

Q32// Let $X_1, X_2, \ldots, X_n$ be a rss$n$. Is $\sum X_i^2$ sufficient estimator for θ? From N(0, θ).

Q33// Let $X_1, X_2, \ldots, X_n$ be a rss$n$ from Gamma dist^n $\Gamma(\alpha, 1/\theta)$, find the jointly sufficient estimators for the parameters (α, θ).

Q34// Let $X_1, X_2, \ldots, X_n$ be a rss$n$ from normal dist^n N(θ, σ^2), show that $\sum X_i$, $\sum X_i^2$ are the jointly sufficient estimators for the parameters (θ, σ^2) respectively.

Q35// Let $X_1, X_2, \ldots, X_n$ be a rss$n$ from C.U($\theta_1 - \theta_2$, $\theta_1 + \theta_2$), and $Y_1 < Y_2 < \ldots < Y_n$ be the order statistics, show that Y_1 and Y_n are the jointly sufficient estimators for the parameters (θ_1, θ_2) respectively.

Q36// Let $X_1, X_2, \ldots, X_n$ be a rss$n$ from Bernoulli dist^n Ber(θ), show that if the dist^n of X can be written in exponential form?

Q37// Let $X_1, X_2, \ldots, X_n$ be a rss$n$ from Poisson dist^n Poi(θ), show that if the dist^n of X can be written in exponential form?

Q38// Let $X_1, X_2, \ldots, X_n$ be a rss$n$ from exponential dist^n Exp(θ), show that if the exponential dist^n belongs to the exponential family?

Q39// Let $X_1, X_2, \ldots, X_n$ be a rss$n$ from normal dist^n N(0, θ), show that if the normal dist^n belongs to the exponential family?

Q40// In a rss$n$. Find minimal sufficient estimators for parameters of: **1)** Poisson(θ)**2)** Beta(α, β).

Q41// In a rss$n$. Find minimal sufficient estimators for θ from $\Gamma(2, \theta)$.

Q42// In a rss$n$. Find minimal sufficient estimators for θ, σ^2 from N(θ, σ^2).

Q43// Let X and Y be two random variables with j.p.d.f.;

$$f(x,y) = \frac{2}{\theta^2} e^{-(x+y)/\theta} \quad , \quad 0 < x < y < \infty$$

Show that;
1) $E(Y) = E(E(Y|X))$.
2) $Var(Y) \geq Var(E(Y|X))$.

Q44// In a rss3 from C.U(0, θ). Show that $[E(2Y_2) = E\{E(2Y_2 \mid Y_3)\}]$, and compare the variances of $(2Y_2)$ and $[E(2Y_2 \mid Y_3)]$.

Q45// Let X be a random variable from; **1)** Bernoulli dist^n. **2)** Poisson dist^n. **3)** Normal dist^n.

Show that the family of X is complete.

Q46// Let X be a r.v. with p.d.f.;

$$f(x;\theta) = \frac{1}{\theta} \quad , \quad 0 < x < \theta \quad , \quad \theta > 0$$

Show that $f(x; \theta)$ is complete?

Q47// Let X be a r.v. with p.d.f.;

$$f(x;\theta) = \frac{1}{2\theta} \quad , \quad -\theta < x < \theta \quad , \quad \theta > 0$$

Show that $f(x; \theta)$ is not complete? If it is then find the unique estimator for θ.

Q48// Let $X_1, X_2, \ldots, X_n$ be a rss$n$ from Poisson dist^n poi(θ). Show that $Y = \sum X_i$ is a complete sufficient estimator for θ. Find the unique continuous function of Y which is the best estimator for θ (M.V.U.E).

Q49// Let $X_1, X_2, \ldots, X_n$ be a rss$n$ from Bernoulli dist^n Ber(θ). Show that $Y = \sum X_i$ is a complete sufficient estimator for θ. Find the unique continuous function of Y which is the best estimator for θ (M.V.U.E).

Q50// Let $X_1, X_2, \ldots, X_n$ is a rss$n$ from Gamma dist^n $\Gamma(4, \theta)$, $0 < \theta < \infty$. **1)** Show that $Y = \sum X_i$ is a complete sufficient estimator for θ. **2)** Find the unique continuous function of Y which is the best estimator for θ (M.V.U.E).

Q51// Let $X_1, X_2, \ldots, X_n$ denote a random sample of size $n > 2$ from a dist^n with p.d.f. $f(x;\theta) = \theta e^{-\theta x} \quad 0 < x < \infty$, *and* $\theta > 0$. **1)** Show that $Y = \sum X_i$ is a complete sufficient estimator for θ. **2)** Prove that (n – 1)/Y is the unique continuous function of Y which is the best estimator for θ (M.V.U.E).

Q52// (Functions of Parameter): Let $X_1, X_2, \ldots, X_n$ denote a random sample from a dist^n which is Ber(1, θ), find the best estimator for the variance $n\theta(1 - \theta)$ of $Y = \sum X_i$ (M.V.U.E).

Q53// (Functions of Parameter): Let $X_1, X_2, \ldots, X_n$ denote a random sample from a dist^n which is N(0, θ). Then $Y = \sum X_i^2$ is a sufficient estimator for θ. Find the best estimator for θ^2 (M.V.U.E).

Q54// Let $X_1, X_2, \ldots, X_n$ be a rss$n$ from Poison dist^n Poi(θ), if $T = \overline{X}$ is an efficient estimator for $\phi(\theta) = \theta$.

Q55// Let $X_1, X_2, \ldots, X_n$ be a rss$n$ from exponential dist^n Exp(θ);

1) If $T = \overline{X}$ is an efficient estimator for $\phi(\theta) = \theta$.
2) Find RCLB for each of [$\phi(\theta) = \ln\theta, \phi(\theta) = 2\theta$].

Q56// In a rss$n$ from N(θ, σ^2). Show that;

1) If $T = \overline{X}$ is an efficient estimator for $\phi(\theta) = \theta$.
2) $S^2 = \dfrac{\sum(x_i - \overline{x})^2}{n}$ or $S^2 = \dfrac{\sum(x_i - \overline{x})^2}{n-1}$ is an efficient estimator for $\phi(\sigma^2) = \sigma^2$.

Q57// In a rss2 from Bernoulli dist^n Ber(θ), let $T_1 = X_1$ and $T_2 = \dfrac{\sum X_i}{n+1}$ be two estimators for parameter θ, show that which of them more efficient.

Q58// In a rss$n$ from normal dist^n N(θ, σ^2), let $S_1^2 = \dfrac{\sum(X_i - \overline{X})^2}{n}$ and $S_2^2 = \dfrac{\sum(X_i - \overline{X})^2}{n-1}$ be two estimators for parameter σ^2, show that which of them more efficient.

Q59// Given $f(x;\theta) = 1/\theta \quad , \quad 0 < x < \theta$, *with* $\theta > 0$, formally compute the reciprocal of;

$$n\, E\left(\left\{\frac{\partial \ln f(X;\theta)}{\partial \theta}\right\}^2\right)$$

Compare this with the variance of $(n + 1)\, Y_n / n$, where Y_n is the largest item of a random sample of size (n) from this distribution (nth order statistic)